Pre Board MT -02

INFINITY THINK BEYOND.....

AN EDUCATIONAL INSTITUTE

12 th

Syllabus: Mock Test 02 : Ch – Continuity and Differentiability ,Application of Derivatives

Time: 90 min **Maximum marks :40**

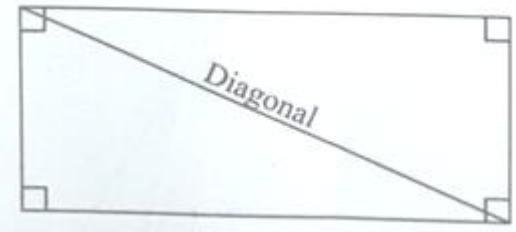
INSTRUCTIONS TO THE STUDENTS

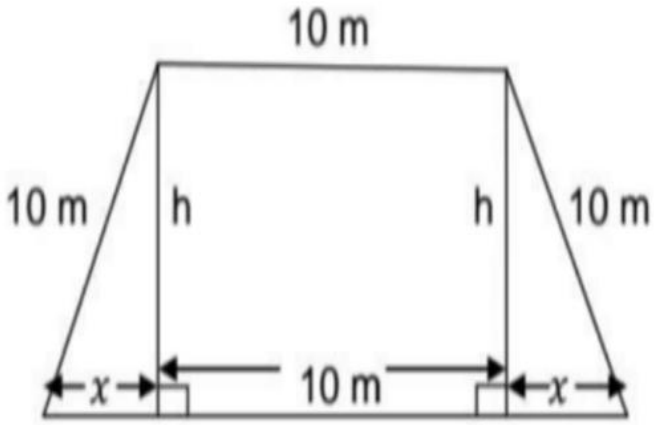
1 . Read each question carefully .
2. Mark of each question is mention in front of question .
3. Attempt one question in internal choice based question .
4. Use of calculators is not allowed.
5. No negative marking .

SECTION A

(Questions 1 – 10 carry 1 marks)

1	For what value of k , the function given below is continuous at x=0 ? $F(x) = \begin{cases} \frac{\sqrt{4+x}-2}{x} & , x \neq 0 \\ k & , x = 0 \end{cases}$ (a)0 (b)1/4 (c) 1 (d) 4	**1**
2	The interval in which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing is : (a)(-1, ∞) (b) (-2,-1) (c) (- ∞, -2) (d)[-1,1]	**1**
3	$F(x) = x^2 - 6x + 5$ is strictly increasing in interval: (a)(-∞ , 3) (b) (3, ∞) (c) (- ∞, ∞) (d) (-3, ∞)	**1**
4	The maximum value of $f(x) = 5 + \lvert \sin x - 2 \rvert$ is : (a)6 (b)7 (c) 8 (d) 9	**1**
5	The value of k , for which $f(x) = \begin{cases} \frac{\sqrt{3}\cos x + \sin x}{x + \frac{\pi}{3}} & x \neq \frac{-\pi}{3} \\ k & , x = \frac{-\pi}{3} \end{cases}$, $x = \frac{-\pi}{3}$ is continuous at $x = \frac{-\pi}{3}$ is : (a)1 (b) -1 (c) 2 (d) -2	**1**
6	If $Y = \tan^{-1}\left(\frac{1-\cos 2x}{\sin 2x}\right)$, then $\frac{dy}{dx}$ is $\left[x \in \left(0, \frac{\pi}{4}\right)\right]$ (a) 1 (b) -1 (c) 2 (d) $\frac{1}{2}$	**1**
7	The function f(x) = [x], where [x]denotes the greatest integer less than or equal to x is continuous at (a)x= 1 (b) x= 1.5 (c) x= -2 (d) x= 4	**1**
8	If the sum of two numbers is 3 , then the maximum value of the product of the first and the square of second is (a)1 (b) 4 (c) 3 (d) 0	**1**
9	The value of x for which the polynomial $2x^3 - 9x^2 + 12x + 4$ is a decreasing function of x is ? (a)-1,< x <1 (b) 0 < x < 2 (c) x > 3 (d) 1<x< 2	**1**
10	Two statements are given, one labeled Assertion (A) and the other labelled Reason(R) Select the correct answer from the options (A), (B), (C) and (D) as given below . (a)Both A and R are true and R is the correct explanation for A. (b) Both A and R are true and R is not the correct explanation for A. (c) A is true but R is false. (d) A is false but R is true	**1**

	Assertion (A): If the side of a square is incresing at the rate of 0.2 cm /sec, then rate of increase of is perimeter is 0.08 cm / sec. **Reason (R) :** perimeter of a square = 4 (side)	
	SECTION B **(Questions 11 – 13 carry 2 marks)**	
11	If $x^y = y^x$, then find $\frac{dy}{dx}$.	**2**
12	A spherical ice – ball melts uniformly . When its radius is 10cm , determine the rate of changes of its volume with respect to the radius .	**2**
13	(a)If $\tan^{-1}[x^3 + y^3] = a^{2025}$, then find $\frac{dy}{dx}$ **OR** (b)If y = 5cosx – 3sinx , prove that $\frac{d^2y}{dx^2}$ +y =0	**2**
	SECTION C **(Questions 14 – 15 carry 3 marks)**	
14	Find the maximum value of $\frac{\sin x \cos x}{\sin x + \cos x}$ in the interval $[0, \frac{\pi}{2}]$	**3**
15	If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$	**3**
	SECTION D **(Questions 16 – 17 carry 5 marks)**	
16	If $(x-a)^2 + (y-b)^2 = c^2$, for some c> 0 , prove that $\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$ is the constant independent of a and b.	**5**
17	Find the interval in which the given function is increasing and decreasing F(x) = $\frac{4\sin x - 2x - x\cos x}{2+\cos x}$ **OR** Show that the semi – vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.	**5**
	SECTION E **(Questions 18 – 19 carry 4 marks)**	
18	When a rectangular sheet of iron heated, its length decrease at the rate of 6cm/s and width increase at the rate of 5cm/s . Let x and y are length and width of its rectangular sheet respectively at any time 't' .  Based on the above information answer the following questions: (i)What are the values of $\frac{dx}{dt}$ and $\frac{dy}{dt}$? (ii)What is the rate of changes of perimeter of the sheet? (iii)(a)Find the rate of changes of the length of diagonal of the sheet when x= 3 and y = 4 cm . **OR** (b) Find the rate of changes of the ratio of length and width of the sheet when x= 3cm and y= 4.	**4**
19	The front gate of a building is in the shape of a trapezium as shown below. Its three sides other than base are 10 m each. The height of the gate is h metres. On the basis of this information and figure given below answer the following questions:	**4**



(i)Let the area of the gate be A. Write the area of the gate as a function of x.

(ii)Find the critical points of the function

(iii)(a)Use first derivates test to find the maximum area of the gate of the building in m^2.

Or

(b)Use second derivative test to find the maximum area of the gate of the building in m^2.