MASTER YOUR MATHS

15 Complete Sample Papers for CBSE Class 12 (2026 Board)

Based on the Latest CBSE Syllabus

MR. AMIT MITTAL

PGT Mathematics, St. Francis School Baraut

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Preface

CBSE's continuous reforms in the assessment system highlight a strong shift towards Competency-Based Education (CBE), where understanding, reasoning, and real-life application stand above memorization. Mathematics, as a discipline, remains central to nurturing essential 21st-century skills such as logical reasoning, analytical thinking, quantitative literacy, and decision-making.

This book, "Master Your Maths – 15 Complete Sample Papers for CBSE Class 12 (2026 Board)", has been crafted in alignment with the Latest CBSE Syllabus and Assessment Framework 2026, ensuring that students receive rich, meaningful, and exam-oriented practice.

The sample papers in this book incorporate the full spectrum of Bloom's Taxonomy:

- *Remembering & understanding – strengthening foundations*
- *Applying – using concepts in real-life situations*
- *Analysing & evaluating – developing higher-order thinking*
- *Creating – encouraging students to think beyond routine methods*

To support CBSE's vision of competency-driven assessment, each paper includes:

- ✓ *Case-based questions*
- ✓ *Data-based and source-based items*
- ✓ *Real-life mathematical applications*
- ✓ *Higher Order Thinking Skills (HOTS)*
- ✓ *Integrated, interdisciplinary questions*
- ✓ *Variety in question typology as per CBSE 2026 Blueprint*

With carefully graded difficulty, each paper allows students to progress from confidence-building exercises to advanced problem-solving tasks. Practising these papers in timed conditions will help learners develop speed, accuracy, and exam temperament.

I encourage students to use this book as a tool for reflection: identify errors, analyse patterns, and strengthen weaker areas. Learning mathematics is a journey—made meaningful through consistent practice and mindful revision.

I sincerely hope this book supports every learner in not only achieving excellent marks but also in developing a deep and lasting understanding of Mathematics.

Mr. Amit Mittal
PGT Mathematics
St. Francis School, Baraut

TIME MANAGEMENT FOR THE CBSE 2026 BOARD EXAM

Success in Mathematics depends not only on understanding concepts but also on solving questions within the time limit. Many students know the method but struggle to complete the paper on time. Effective time management can significantly improve marks and reduce exam stress.

This page gives a simple, practical strategy that students can follow while attempting each of the 10 sample papers in this book—and during the actual CBSE Board Examination.

Section A – Objective Type (1 Mark Each)

Section A consists of 20 objective-type questions, each carrying one mark. Most questions in this section are direct and can be solved within 10–20 seconds, though a few may require slightly more time. A good strategy is to aim for **one minute per question** and comfortably complete this entire section within **30 minutes**, ensuring accuracy while keeping enough time for later sections.

Section B – Short Answer (2 Marks Each)

Section B contains five two-mark questions that involve short calculations, reasoning, or simple steps. Since these questions are slightly lengthier than Section A, you should aim to spend **two minutes per question**, and keep **five minutes extra** as a safety margin. This allows you to complete Section B smoothly in about **15 minutes**, without rushing through important details.

Section C – Application-Based (3 Marks Each)

Section C has six questions of three marks each. These require explanation, proper steps, and sometimes a construction-based question may appear here. It is advisable to spend **four minutes per question** and keep **six minutes extra** for flexibility. With this plan, the whole section can be completed in **30 minutes**, ensuring clear and well-presented solutions.

Section D – Long Answer (4 Marks Each)

Section D comprises four long-answer questions, each worth four marks. These questions involve multiple steps, conceptual understanding, and careful presentation. You should aim to spend **ten minutes per question**, along with **ten extra minutes** for review. This brings the total time for Section D to about **50 minutes**, allowing you to complete the longest part of the paper without pressure.

Section E – Case-Based Questions

Section E consists of three case-based questions that require careful reading and application of concepts. You should plan **10 minutes for each case study**, making it **30 minutes** for the full section, and keep an additional **5 minutes extra** for safety.

Extra 20 Minutes – Final Checking

After completing all five sections of the paper, you will still have around **20 minutes** left. This time should be used wisely for rechecking your solutions, verifying calculations, correcting small mistakes, and ensuring that all diagrams, graphs, and constructions are neat and accurate.

Proper utilisation of these final 20 minutes helps you submit a clean, error-free and confident answer sheet.

Blueprint for CBSE 2026 Pre-Board Exams

(NOT OFFICIAL)

CLASS - XII **Maximum Marks : 80**

SUBJECT – MATHEMATICS (041) **Time Allowed : 3 Hours**

S. NO.	CHAPTER	UNIT (MARKS)	SEC A (MCQ & AR) (1 MARK)	SEC B (VSA) (02 MARKS)	SEC C (SA) (03 MARKS)	SEC D (LA) (05 MARKS)	SEC E Case Study-Based (04 MARKS)	TOTAL MARKS
1	REL & FUN	I(08)	01	00	00	00	01	**05**
2	ITF		01	01	00	00	00	**03**
3	MATRICES	II(10)	03	00	00	00	00	**03**
4	DETERMINANTS		02	00	00	01	00	**07**
5	CONT.& DIFF.	III(35)	02	02(1 Int. choice)	00	00	00	**06**
6	APP OF DERI.		00	00	00	01(1 Int. choice)	01	**09**
7	INTEGRALS		01	00	03(1 Int. choice)	00	00	**10**
8	APP OF INTE.		00	00	00	01	00	**05**
9	DIFF. EQN.		02	00	01(1 Int. choice)	00	00	**05**
10	VECTORS	IV(14)	03	01(1 Int. choice)	00	00	00	**05**
11	THREE-D		02	01	00	01(1 Int. choice)	00	**09**
12	LPP	V(05)	02	00	01	00	00	**05**
13	PROBABILITY	VI(08)	01	00	01(1 Int. choice)	00	01	**08**
NUMBER OF QUESTIONS (TOTAL MARKS)			**18+02 (20)**	**05 (10)**	**06 (18)**	**04 (20)**	**03 (12)**	**38 (80)**

ST Francis School Baraut

ESTD. 2016

Series : SFS/02/01

SET – 1

ROLL No.

Q.P Code 10/02/01

Candidates must write the Q.P Code on the title page of the Answer book.

- Please check that this question paper contains 7 printed pages.
- Please check that this question paper contains 38 questions.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please write down the serial number of the question in the answer-book before attempting it.
- 15 minute time has been allotted to read this question paper.
- The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे | *अधिकतम अंक : 80*

Time allowed : 3 hours | *Maximum Marks : 80*

General Instructions:

Read the following instructions very carefully and strictly follow them:

(*i*) *This Question paper contains 38 questions. **All** questions are **compulsory**.*

(*ii*) *This Question paper is divided into **five** Sections – **A**, **B**, **C**, **D** and **E**.*

(*iii*) *In **Section A**, Questions no. **1** to **18** are **multiple choice questions** (MCQs) and Questions no. **19** and **20** are **Assertion-Reason based** questions of **1 mark each**.*

(*iv*) *In **Section B**, Questions no. **21** to **25** are **Very Short Answer (VSA)-type** questions, carrying **2 marks each**.*

(*v*) *In **Section C**, Questions no. **26** to **31** are **Short Answer (SA)-type** questions, carrying **3 marks each**.*

(*vi*) *In **Section D**, Questions no. **32** to **35** are **Long Answer (LA)-type** questions, carrying **5 marks each**.*

(*vii*) *In **Section E**, Questions no. **36** to **38** are **Case study-based questions**, carrying **4 marks each**.*

(*viii*) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(*ix*) *Use of calculators is **not** allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. Which of these relations on set A where $A = \{1, 2, 3, 4\}$ are equivalence relation?

[NCERT Part-I, Page 2]

(*a*) $R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 4)\}$

(*b*) $R_2 = \{(1, 4), (2, 2), (3, 3), (4, 1), (4, 2), (4, 4)\}$

(*c*) $R_3 = \{(1, 1), (1, 2), (1, 3)\}$

(*d*) $R_4 = \{(1, 1), (1, 2), (1, 4), (2, 2), (2, 4), (3, 3), (4, 1), (4, 4)\}$

2. The value of $\cos^{-1}\left(\frac{1}{2}\right) + 3\sin^{-1}\left(\frac{1}{2}\right)$ is equal to **[Conceptual Application]**

(*a*) $\frac{\pi}{4}$ (*b*) $\frac{\pi}{6}$ (*c*) $\frac{2\pi}{3}$ (*d*) $\frac{5\pi}{6}$

3. The principal value of $\text{cosec}^{-1}(2)$ is **[NCERT Part-I, Page 22]**

(*a*) $\frac{\pi}{3}$ (*b*) $\frac{\pi}{6}$ (*c*) $\frac{3\pi}{2}$ (*d*) $\frac{\pi}{5}$

4. The order of the matrix $\begin{bmatrix} a & 1 & x \\ a^2-b & \sqrt{3} & 2 \\ \frac{-2}{5} & 5 & 0 \end{bmatrix}$ is [NCERT Part-I, Page 36]

(a) 3×3 (b) 2×2 (c) 3×2 (d) 2×3

5. Maximum value of $\Delta = \begin{vmatrix} 1 & 1 & 1+\cos\theta \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1 \end{vmatrix}$, θ is a real number is [Integrated Question]

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $-\frac{3}{4}$

6. The function f, defined by $f(x) = \begin{cases} kx^2, & \text{if } x \geq 1 \\ 4, & \text{if } x < 1 \end{cases}$ is continuous at $x = 1$, then $k =$ [NCERT Part-I, Page 105]

(a) 4 (b) –4 (c) 2 (d) –3

7. Function f, defined by $f(x) = \begin{cases} \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ is [Conceptual Application]

(a) continuous at $x = 0$ (b) not continuous at $x = 0$
(c) differentiable at $x = 0$ (d) None of these

8. If $x = at^2, y = 2at$, then $\frac{d^2y}{dx^2}$ is [NCERT Part-I, Page 134-135]

(a) $\frac{1}{t}$ (b) $-\frac{1}{t^2}$ (c) at^2 (d) $\frac{-1}{2at^3}$

9. $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin x}$ equals to [NCERT Part-II, Page 241, 268]

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{3}{2}$

10. The value of integral $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16\sin 2x} dx$ is [Integrated Question]

(a) log 2 (b) $\frac{1}{20}\log 2$ (c) $\frac{1}{20}\log 3$ (d) log 5

11. The value of integral $\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x . \log\left(\frac{1+x}{1-x}\right) dx$ is [NCERT Part-II, Page 273-274]

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) None of these

12. The area of the region included between $y^2 = x$ and the line $y = x$ is [Conceptual Application]

(a) $\frac{25}{6}$ sq units (b) $\frac{1}{6}$ sq units (c) $\frac{27}{8}$ sq units (d) $\frac{29}{6}$ sq units

13. Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is [Conceptual Application]

(a) π sq units (b) $\frac{\pi}{2}$ sq units (c) $\frac{\pi}{3}$ sq units (d) $\frac{\pi}{4}$ sq units

14. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx$ is equal to [NCERT Part-II, Page 235-236]

(a) $\cos\sqrt{x} + C$ (b) $2\cos\sqrt{x} + C$ (c) $-2\cos\sqrt{x} + C$ (d) $\sqrt{x}\cos\sqrt{x} + C$

15. The degree of the differential equation $\frac{d^3y}{dx^3} + 3\left(\frac{d^2y}{dx^2}\right)^2 - \frac{dy}{dx} + y = 0$ [NCERT Part-II, Page 302]

(a) 0 (b) 1 (c) 2 (d) 3

16. The solution of the equation $(2y-1)dx - (2x+3)dy = 0$ is [NCERT Part-II, Page 306-307]

(a) $\frac{2x-1}{2y+3} = C$ (b) $\frac{2y+1}{2x-3} = C$ (c) $\frac{2x+3}{2y-1} = C$ (d) $\frac{2x-1}{2y-1} = C$

17. If two events are independent, then [Conceptual Application]

(a) they must be mutually exclusive. (b) the sum of their probabilities must be equal to 1.

(c) (a) and (b) both are correct. (d) None of the above is correct.

18. Let A and B be two events such that $P(A) = \frac{3}{8}, P(B) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$. Then $P(A/B).P(A'/B)$ is equal to [NCERT Part-II, Page 408-409]

(a) $\frac{2}{5}$ (b) $\frac{3}{8}$ (c) $\frac{3}{20}$ (d) $\frac{6}{25}$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (a), (b), (c) and (d) as given below.)

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** The minor of 5 in the determinant $\begin{vmatrix} 2 & 5 & 8 \\ 0 & 3 & 7 \\ -1 & -2 & 6 \end{vmatrix}$ is -7. [NCERT Part-I, Page 84]

Reason (R): The determinant that is left by cancelling the row and column intersecting at a particular element is called the minor.

20. **Assertion (A):** If $\vec{a} = \hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = -\hat{i} + 3\hat{j} + 5\hat{k}$ represent the two adjacent sides of a parallelogram, the area of parallelogram is $3\sqrt{14}$ square units. [Integrated Question]

Reason (R): $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = |\vec{a}|^2 - |\vec{b}|^2$

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Which is greater $\tan 1$ or $\tan^{-1} 1$? [Conceptual Application]

22. Find the non-zero values of x satisfying the matrix equation

$$x\begin{bmatrix} 2x & 12 \\ 3 & x \end{bmatrix} + 2\begin{bmatrix} 8 & 5x \\ 4 & 4x \end{bmatrix} = 2\begin{bmatrix} x^2+8 & 44 \\ 10 & 6x \end{bmatrix}$$ [NCERT Part-I, Page 41, 44]

OR

If $A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ then find $A' - B'$. [NCERT Part-I, Page 46, 61]

23. Find the general solution of the differential equation $(x + y)dy + (x - y)\,dx = 0$.
[NCERT Part-II, Page 313-314]

24. If for three non-zero vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$, $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, then show that $\vec{b} = \vec{c}$.
[Integrated Question]

OR

If θ is the angle between two vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$, find $\sin\theta$. [NCERT Part-II, Page 363-364]

25. A coin is tossed thrice and all eight outcomes are assumed equally likely. Let the event E be "the first throw results in head" and event F be "the last throw results in tail". Find whether events E and F are independent.
[NCERT Part-II, Page 418]

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. Let T be set of all triangles in a plane with R a relation in T given by $R = \{(T_1, T_2) : T_1 \cong T_2\}$. Show that R is an equivalence relation.
[NCERT Part-I, Page 2]

27. Find the local maxima and local minima, if any of the function f, given by
$f(x) = \sin x + \cos x,\ 0 < x < \frac{\pi}{2}$.
[NCERT Part-I, Page 166]

28. Determine the constant a and b, such that the function $f(x) = \begin{cases} ax^2 + 2b & , \text{ if } x > 2 \\ 2 & , \text{ if } x = 2 \\ 2ax - 2b & , \text{ if } x < 2 \end{cases}$ is continuous.
[NCERT Part-I, Page 105]

29. For the differential equation, find a particular solution satisfying the given condition
$x(x^2 - 1)\frac{dy}{dx} = 1;\ y = 0$ when $x = 2$.
[NCERT Part-II, Page 306-307]

OR

If $y = \sin(\sin x)$ prove that $\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y\cos^2 x = 0$
[NCERT Part-II, Page 121, 137]

30. Find $\int \frac{2\cos x}{(1 - \sin x)(1 + \sin^2 x)}\,dx$
[Integrated Question]

OR

Find $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{16 + 9\sin 2x}\,dx$
[Integrated Question]

31. Find $\int \frac{2}{(1 - x)(1 + x^2)}dx$
[NCERT Part-II, Page 252-253]

OR

Evaluate $\int \frac{dx}{1 + 8\sin^2 x}$
[Integrated Question]

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. In the determinant $\begin{vmatrix} 1 & 3 & -4 \\ 1 & 0 & 6 \\ 2 & 1 & 4 \end{vmatrix}$, find the cofactor of each element and hence evaluate
$a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$.
[NCERT Part-I, Page 84]

OR

Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations:
$x + 3z = -9,\ -x + 2y - 2z = -4,\ 2x - 3y + 4z = 3$
[NCERT Part-I, Page 94-95]

33. Find the shortest distance between the two lines: **[NCERT Part-II, Page 386-387]**

$\vec{r} = (2+\lambda)\hat{i} + (12-\lambda)\hat{j} + (\lambda+1)\hat{k}$

$\vec{r} = (3\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$

OR

The scalar product of the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ with the unit vector along the sum of vectors $\vec{b} = 2\hat{i} + 3\hat{j} - 15\hat{k}$ and $\vec{c} = \lambda\hat{i} + 3\hat{j} + 13\hat{k}$ is equal to one. Find the value of λ and hence find unit vector along $\vec{b} + \vec{c}$. **[Conceptual Application]**

34. Solve the LPP graphically **[NCERT Part-II, Page 397-398]**

Maximise $Z = 15x + 2y$, subject to constraints:

$x - 2y \le 2,\ 3x + 2y \le 12,\ -3x + 2y \le 3,\ x \ge 0,\ y \ge 0$

35. Find the particular solution of the differential equation $2x^2\frac{dy}{dx} - 2xy + y^2 = 0;\ y(e) = e$.

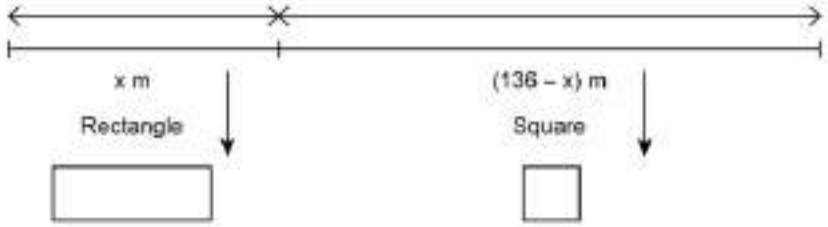
[NCERT Part-II, Page 313-314]

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. A wire of length 136 m is to be cut into two pieces. One of the pieces is to be made into square and another into a rectangle as shown. **[Conceptual Application]**



The length of rectangle is twice its breadth.

Answer the questions based on above information.

(*i*) Find the area of rectangle.

(*ii*) Find the area of square.

(*iii*) What length of wire is used to make rectangle, so that combined area of square and rectangle is minimum?

OR

(*iii*) Find the length of the side of square so that combined area of square and rectangle is minimum.

Case Study - 2

37. Two students Vivek and Mohan appears in an examination. Assume that the two events "Vivek passes" and "Mohan passes" are independent. The probability of Vivek passing an examination is $\frac{3}{5}$ and Mohan passing the examination is $\frac{4}{5}$. **[Conceptual Application]**

Answer the questions based on above information.

(*i*) What is the probability that both Vivek and Mohan pass the examination?

(*ii*) What is the probability that only Vivek passes the examination?

(*iii*) What is the probability that only Mohan passes the examination?

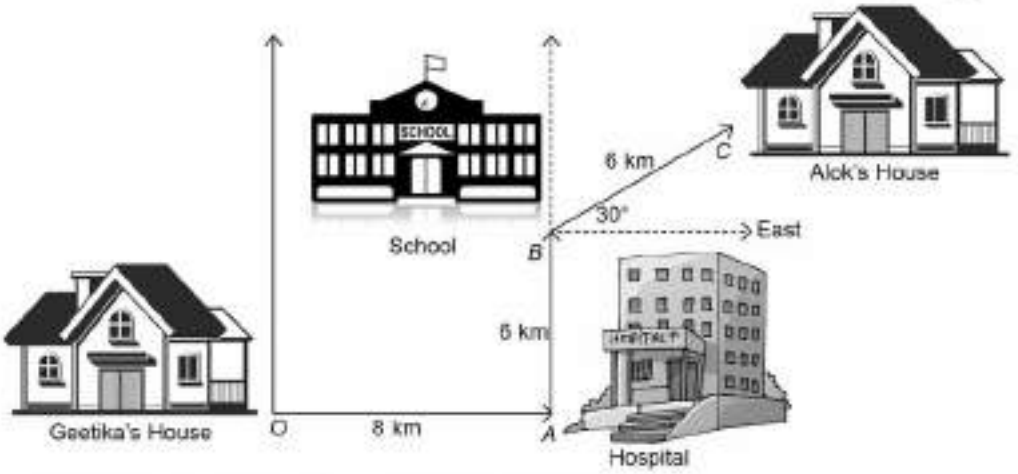
OR

(*iii*) Find the probability that only one of them pass the examination.

Case Study - 3

38. Geetika's house is situated at Shalimar Bagh at point O. For going to Alok's house, she first travels 8 km by bus in the East to reach point A, where a hospital is situated. From the hospital, Geetika takes an auto-rickshaw and goes 6 km in the North to reach point B, where school is situated. From school, she travels by bus to reach Alok's house which is at 30° East, 6 km from point B.

[Conceptual Application]



Based on the above information, answer the following questions.

(*i*) What is the vector distance between Geetika's house and school?

(*ii*) How much distance Geetika travels to reach school?

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ESTD. 2016

Series : SFS/02/02

SET – 2

ROLL No.

Q.P Code 12/02/02

Candidates must write the Q.P Code on the title page of the Answer book.

- Please check that this question paper contains 7 printed pages.
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गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे | *अधिकतम अंक : 80*

Time allowed : 3 hours | *Maximum Marks : 80*

General Instructions:

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(ii) *This Question paper is divided into **five** Sections – **A**, **B**, **C**, **D** and **E**.*

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(v) *In **Section C**, Questions no. **26** to **31** are **Short Answer (SA)-type** questions, carrying **3 marks each**.*

(vi) *In **Section D**, Questions no. **32** to **35** are **Long Answer (LA)-type** questions, carrying **5 marks each**.*

(vii) *In **Section E**, Questions no. **36** to **38** are **Case study-based questions**, carrying **4 marks each**.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. Let R be a relation on the set N given by $R = \{(a, b) : a = b - 3, b > 5\}$. Then, **[NCERT Part-I, Page 2]**

(a) $(1, 2) \in R$ (b) $(5, 9) \in R$ (c) $(7, 10) \in R$ (d) $(10, 7) \in R$

2. Let 'f' : $R - \{2\} \to R - \{1\}$ be a function defined by $f(x) = \dfrac{x-1}{x-2}$, then 'f' is **[NCERT Part-I, Page 7]**

(a) into function (b) many one function

(c) bijective function (d) many one, into function.

3. If $AB = C$ then orders of matrices A, B, C is equal to **[NCERT Part-I, Page 51-52]**

(a) $A_{2\times 3}, B_{3\times 2}, C_{2\times 3}$ (b) $A_{3\times 2}, B_{2\times 3}, C_{3\times 3}$

(c) $A_{3\times 3}, B_{2\times 3}, C_{3\times 3}$ (d) $A_{3\times 2}, B_{2\times 3}, C_{3\times 2}$

4. If $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ -4 & 3 \end{bmatrix}, C = \begin{bmatrix} 2 & 12 \\ -7 & 9 \end{bmatrix}$ then $5A - 3B + C$ is equal to **[NCERT Part-I, Page 46]**

(a) $\begin{bmatrix} -1 & -1 \\ -20 & 20 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 1 \\ 20 & 20 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ 20 & 20 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -1 \\ 20 & 20 \end{bmatrix}$

5. The value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$ is **[NCERT Part-I, Page 21]**

(a) $\frac{13\pi}{6}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{5}$

6. Solution of the differential equation $x\sqrt{1-y^2}\,dx + y\sqrt{1-x^2}\,dy = 0$ is **[NCERT Part-II, Page 306-307]**

(a) $\sqrt{1+x^2}+\sqrt{1-y^2}=C$ (b) $\sqrt{1-x^2}+\sqrt{1-y^2}=C$

(c) $\sqrt{1-x^2}-\sqrt{1-y^2}=C$ (d) $\left(\sqrt{1-x^2}\right)\left(\sqrt{1-y^2}\right)=C$

7. Integrating factor for the solution of differential equation $(x-y^3)dy + y\,dx = 0$ is **[NCERT Part-II, Page 323]**

(a) $\frac{1}{y}$ (b) $\log y$ (c) y (d) y^2

8. The area enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is equal to **[Conceptual Application]**

(a) $\pi^2 ab$ sq units (b) πab sq units (c) $\pi a^2 b$ sq units (d) πab^2 sq units

9. The area enclosed by the circle $x^2 + y^2 = 2$ is equal to **[Conceptual Application]**

(a) 4π sq units (b) $2\sqrt{2}\,\pi$ sq units (c) $4\pi^2$ sq units (d) 2π sq units

10. $\int_0^{\frac{\pi}{4}} \sqrt{1-\sin 2x}\,dx =$ **[NCERT Part-II, Page 241]**

(a) $\sqrt{2}-1$ (b) $\sqrt{2}+1$ (c) $\sqrt{2}$ (d) $\sqrt{3}$

11. $\int_0^2 (x^2+3)dx =$ **[NCERT Part-II, Page 268]**

(a) $\frac{25}{3}$ (b) $\frac{26}{3}$ (c) 8 (d) $\frac{23}{3}$

12. The function $f(x) = \cos x - 2px$ is decreasing for **[NCERT Part-I, Page 153]**

(a) $p < \frac{1}{2}$ (b) $p > \frac{1}{2}$ (c) $p < 2$ (d) $p > 2$

13. For the function $y = x^3 + 21$, the value of x, when y increases 75 times as fast as x, is **[NCERT Part-I, Page 147-148]**

(a) ± 3 (b) $\pm 5\sqrt{3}$ (c) ± 5 (d) None of these

14. The function f, defined by $f(x) = \frac{x+1}{1+\sqrt{1+x}}$ is continuous at $x = 0$, when $f(0) =$ **[NCERT Part-I, Page 105]**

(a) 1 (b) -2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

15. The function $f(x) = \begin{cases} \frac{|x-4|}{2(x-4)}, & \text{if } x \neq 4 \\ 0, & \text{if } x = 4 \end{cases}$ is **[NCERT Part-I, Page 105]**

(a) continuous at all $x \in R$ (b) discontinuous at $x = 4$

(c) continuous only when $x = 4$ (d) None of these

16. The general point on the line $\vec{r} = (2\hat{i}+\hat{j}-4\hat{k})+\lambda(3\hat{i}+2\hat{j}-\hat{k})$ is **[NCERT Part-II, Page 382]**

(a) $(2, 1, -4)$ (b) $(3, 2, -1)$ (c) $(-1, 1, 3)$ (d) $(2+3\lambda, 1+2\lambda, -4-\lambda)$

17. A and B are two independent events, $P(A) = 0.2$ and $P(B) = 0.8$, then $P(A \cap \overline{B}) =$ **[Conceptual Application]**

(a) 0.03 (b) 0.04 (c) 0.4 (d) 0.3

18. A die is tossed twice. The probability of getting 1, 2, 3, 4 on first toss and 4, 5, 6 on the second toss is [Conceptual Application]

(a) $\frac{2}{5}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{9}$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (a), (b), (c) and (d) as given below.)

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** Degree of the differential equation $\frac{dy}{dx} + \cos\left(\frac{dy}{dx}\right) = 0$ is not defined. [NCERT Part-II, Page 302]

Reason (R): The degree of those differential equations are not defined which can not be written as polynomial of derivatives.

20. **Assertion (A):** If k is a scalar and A is a 3×3 square matrix, then $|kA|$ is equal to $k^3 |A|$. [NCERT Part-II, Page 80]

Reason (R): If every element of a third order determinant of value Δ is multiplied by n, then the value of new determinant is $n^3\Delta$.

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Solve the differential equation $\frac{dy}{dx} = y^3 \operatorname{cosec} x$, given that $y\left(\frac{\pi}{4}\right) = 2$. [NCERT Part-II, Page 306-307]

22. A and B throw a pair of dice alternatively. A wins if he throws 6 before B and B wins if he throws 7 before A throws 6. If A begins, show that the odds in favour of A are 30 : 31. [Conceptual Application]

OR

E and F are events of an experiment such that $P(E) = 0.8$, $P(F) = 0.7$, $P(E \cap F) = 0.6$. Find $P\left(\frac{\overline{E}}{\overline{F}}\right)$. [NCERT Part-II, Page 408]

23. Find the area of the parallelogram whose one side and a diagonal are represented by coinitial vector $3\hat{i} - \hat{k}$ and $2\hat{i} + \hat{j} - 4\hat{k}$. [Conceptual Application]

24. If $\cos^{-1} x + \cos^{-1} y = 2\pi$, then find the value of $x^{100} + y^{200} + \frac{1}{x^{100}\, y^{200}}$. [Conceptual Application]

25. Given matrix $A = \begin{bmatrix} 4 & 3 \\ -1 & 2 \end{bmatrix}$, show that $A + A'$ is symmetric matrix and $A - A'$ is a skew symmetric matrix. [NCERT Part-I, Page 63-64]

OR

If A is a square matrix such that $A^2 = A$, show that $(I + A)^3 = 7A + I$. [Conceptual Application]

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. If $x = a \sin 2t(1 + \cos 2t)$ and $y = b \cos 2t(1 - \cos 2t)$, show that $\left(\frac{dy}{dx}\right)_{\text{at } t=\frac{\pi}{4}} = \frac{b}{a}$.

[NCERT Part-I, Page 134-135]

27. Show that $y = \log(1 + x) - \frac{2x}{2 + x}$, $x > -1$ is an increasing function of x throughout its domain.

[NCERT Part-I, Page 153]

28. Evaluate $\int_2^3 \frac{x-1}{(x+1)^3} e^x dx$. **[NCERT Part-II, Page 262-263]**

OR

Solve the differential equation: $\frac{dy}{dx} = \sin(x + y) + \cos(x + y)$. **[Integrated Question]**

29. Let $f : W \to W$ be defined as $f(n) = n - 1$, if n is odd and $f(n) = n + 1$, if n is even. Show that f is injective as well as surjective. Here, W represents the set of whole numbers. **[NCERT Part-I, Page 7]**

30. Using integration, find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$. **[Conceptual Application]**

OR

Using the method of integration, find the area of the ΔABC, coordinates of whose vertices are $A(2, 0)$, $B(4, 5)$ and $C(6, 3)$. **[Conceptual Application]**

31. If $y = \frac{\log x}{x}$, show that $\frac{d^2y}{dx^2} = \frac{2 \log x - 3}{x^3}$. **[NCERT Part-I, Page 137]**

OR

Examine the differentiability of the function $f(x) = \begin{cases} x[x], & \text{if } 0 \le x < 2 \\ (x-1)x, & \text{if } 2 \le x < 3 \end{cases}$ at $x = 2$.

[NCERT Part-I, Page 105]

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. Show that lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ intersect. Also find their point of intersection. **[Conceptual Application]**

OR

Find the value of λ, so that the lines $\frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{5z-10}{11}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other. **[NCERT Part-II, Page 383-384]**

33. If $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$, find A^{-1}. How we can use A^{-1} to solve the system of equations: $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2$; $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5$; $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = -4$? **[NCERT Part-I, Page 94-95]**

OR

Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and state how we can use it to solve the system of equations $x - y + z = 4; x - 2y - 2z = 9; 2x + y + 3z = 1$. **[NCERT Part-I, Page 94-95]**

34. Find the particular solution of the differential equation $\tan x \cdot \frac{dy}{dx} = 2x \tan x + x^2 - y; (\tan x \neq 0)$ given that $y = 0$ when $x = \frac{\pi}{2}$. **[NCERT Part-II, Page 322-323]**

35. Solve the following linear programming problem graphically.

Minimise $Z = 3x + y + 39500$ **[NCERT Part-II, Page 397-398]**

subject to the constraints,

$x \geq 0, y \geq 0, \ x + y \leq 7000, x \leq 4500, y \leq 3000, x + y \geq 3500$

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. A person has to reach a company for an interview. He has four options to reach the company i.e. by metro, by bus, by scooter or by other means of transport. The probabilities of using these means are $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$ respectively. The probabilities that he will be late if he comes by metro, bus or scooter are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$ respectively, but if he comes by other means he will not be late. Using the above information answer the following questions. **[Conceptual Application]**

(*i*) What is the conditional probability of reaching late by other means of transport?

(*ii*) What is the probability that he travelled by bus and was late?

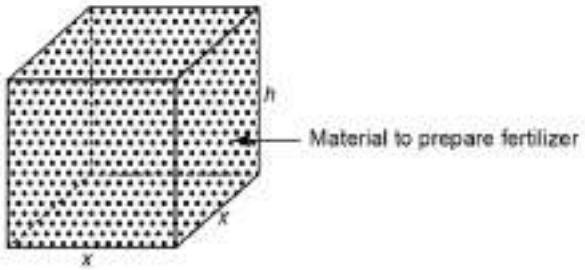
(*iii*) What is the probability of reaching late?

OR

(*iii*) What is the probability that person comes by Metro, given that when he arrives the company, he is late?

Case Study - 2

37. A village panchayat wants to dug out a square base tank for preparing fertilizers and wants capacity to be 250 cu metres. On calculations, it was found that cost of the land is ₹ 50 per square meter and cost of digging increases with depth and for the whole tank is ₹ 400 $(\text{depth})^2$. Tank is shown as below. **[Conceptual Application]**



(*i*) If the side of the square base is x m and the height of the tank is h m, then establish relation between x and h.

(*ii*) Find the cost C for digging the tank in terms of x and h.

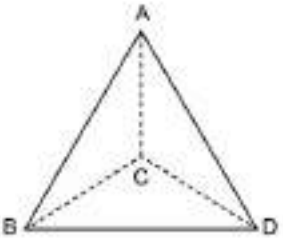
(*iii*) Find the cost C in terms of h only .

OR

(*iii*) Find the value of h for which cost C is minimum.

Case Study - 3

38. A building is to be constructed in the form of a triangular pyramid, $ABCD$ as shown in the figure. Let its angular points are $A(0, 1, 2)$, $B(3, 0, 1)$, $C(4, 3, 6)$ and $D(2, 3, 2)$ and G be the point of intersection of the medians of ΔBCD. **[Conceptual Application]**



(*i*) Find the coordinates of point G.

(*ii*) Find the length of vector $\overrightarrow{AG}$.

ST Francis School Baraut

ESTD. 2016

Series : SFS/02/03 **SET – 3**

ROLL No. **Q.P Code 12/02/03**

Candidates must write the Q.P Code on the title page of the Answer book.

- Please check that this question paper contains 7 printed pages.
- Please check that this question paper contains 38 questions.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please write down the serial number of the question in the answer-book before attempting it.
- 15 minute time has been allotted to read this question paper.
- The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे *अधिकतम अंक : 80*

Time allowed : 3 hours *Maximum Marks : 80*

General Instructions:

Read the following instructions very carefully and strictly follow them:

(*i*) *This Question paper contains 38 questions. All questions are* ***compulsory.***

(*ii*) *This Question paper is divided into* ***five*** *Sections – A, B, C, D and E.*

(*iii*) *In* ***Section A****, Questions no.* ***1*** *to* ***18*** *are* ***multiple choice questions*** *(MCQs) and Questions no.* ***19*** *and* ***20*** *are* ***Assertion-Reason based*** *questions of* ***1 mark each.***

(*iv*) *In* ***Section B****, Questions no.* ***21*** *to* ***25*** *are* ***Very Short Answer (VSA)-type*** *questions, carrying* ***2 marks each.***

(*v*) *In* ***Section C****, Questions no.* ***26*** *to* ***31*** *are* ***Short Answer (SA)-type*** *questions, carrying* ***3 marks each.***

(*vi*) *In* ***Section D****, Questions no.* ***32*** *to* ***35*** *are* ***Long Answer (LA)-type*** *questions, carrying* ***5 marks each.***

(*vii*) *In* ***Section E****, Questions no.* ***36*** *to* ***38*** *are* ***Case study-based questions****, carrying* ***4 marks each.***

(*viii*) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(*ix*) *Use of calculators is* ***not*** *allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. If $\sin^{-1}x = y$, then **[NCERT Part-I, Page 19]**

(*a*) $0 \le y < \pi$ (*b*) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

(*c*) $0 < y < \pi$ (*d*) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

2. Let R be a relation on the set L of lines defined by $l_1 R\, l_2$ if l_1 is perpendicular to l_2, then relation R is **[NCERT Part-I, Page 2]**

(*a*) reflexive and symmetric (*b*) symmetric and transitive

(*c*) equivalence relation (*d*) symmetric

3. Let R be a relation on the set N of natural numbers defined by nRm, if n divides m, then R is **[NCERT Part-I, Page 2]**

(*a*) reflexive and symmetric (*b*) transitive and symmetric

(*c*) equivalence (*d*) reflexive, transitive but not symmetric

4. $y = e^{-x} + ax + b$ is a solution of differential equation **[NCERT Part-II, Page 304-305]**

(*a*) $e^{-x}y'' = 1$ (*b*) $e^{x}y'' = 1$

(*c*) $e^{x}(y')^2 = 1$ (*d*) $e^{-x}(y')^2 = 1$

5. The area bounded by the curve $y = |x|$, the x-axis and between $x = -2$ to $x = 0$ is **[Conceptual Application]**

(a) 4 sq units (b) $\frac{3}{2}$ sq units

(c) 1 sq unit (d) 2 sq units

6. The degree of the differential equation $\left(1+\frac{dy}{dx}\right)^3 = \left(\frac{dy}{dx}\right)^2$ is **[NCERT Part-II, Page 302]**

(a) 1 (b) 2 (c) 3 (d) 4

7. The value of λ for which $\int \frac{4x^3 + \lambda 4^x}{4^x + x^4} dx = \log|4^x + x^4| + C$ is **[NCERT Part-II, Page 226-227]**

(a) 1 (b) $\log_e 4$ (c) $\log_4 e$ (d) 4

8. The point(s) on the curve $y = x^2$, at which y-coordinate is changing six times as fast as x-coordinate is/are **[NCERT Part-I, Page 147-148]**

(a) (2, 4) (b) (3, 9) (c) (3, 9), (9, 3) (d) (6, 2)

9. A function f is said to be continuous for $x \in R$, if **[Conceptual Application]**

(a) it is continuous at $x = 0$ (b) differentiable at $x = 0$

(c) continuous at two points (d) differentiable for $x \in R$

10. The rate of change of area of a circle with respect to its radius is **[NCERT Part-I, Page 147-148]**

(a) 2π (b) πr (c) $2\pi r$ (d) π

11. A function which is continuous at $x = 1$, but not differentiable at $x = 1$ is **[Conceptual Application]**

(a) $|x|$ (b) $[x]$ (c) $sgn(x)$ (d) $|x - 1|$

12. Direction ratios of the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$ are **[NCERT Part-II, Page 382]**

(a) 2, 6, 3 (b) –2, 6, 3

(c) 2, – 6, 3 (d) None of these

13. Projection of a line segment joining the points (2, 0, 5) and (0, 3, 1) on the line whose direction ratios are 2, 3, 6 is **[Conceptual Application]**

(a) $-\frac{19}{7}$ (b) $\frac{19}{49}$ (c) $\frac{19}{7}$ (d) 19

14. If $\vec{a}$, $\vec{b}$, $\vec{c}$ be the position vectors of vertices A, B, C of a parallelogram $ABCD$, then the position vector of D is **[NCERT Part-II, Page 339, 344]**

(a) $\vec{a} + \vec{c} - \vec{b}$ (b) $\vec{a} - \vec{c} + \vec{b}$

(c) $\vec{a} - \vec{c} - \vec{b}$ (d) $\vec{c} - \vec{a} + \vec{b}$

15. Area of parallelogram, whose diagonals are along vectors $\hat{i} + 2\hat{k}$ and $2\hat{j} - 3\hat{k}$ is **[Conceptual Application]**

(a) $\sqrt{29}$ sq units (b) $(-4\hat{i} + 3\hat{j} + 2\hat{k})$ sq units

(c) $\frac{1}{2}\sqrt{29}$ sq units (d) None of these

16. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ then **[Conceptual Application]**

(a) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$ (b) $\vec{a} + \vec{b} = \vec{b} + \vec{c} = \vec{c} + \vec{a}$

(c) $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = 2(\vec{a} \times \vec{b})$ (d) None of these

17. If A and B are independent events then which of the following is not true **[NCERT Part-II, Page 417]**

(a) $P(A \cap B) = 0$ (b) $P(\overline{A} \cap B) = P(\overline{A})\,P(B)$

(c) $P(A \cap \overline{B}) = P(A)P(\overline{B})$ (d) $P(A \cup B) = 1 - P(\overline{A} \cap \overline{B})$

18. Bag A contains 3 red and 5 black balls and bag B contains 2 red and 4 black balls. A ball is drawn from one of the bags. The probability that ball drawn is red is **[NCERT Part-II, Page 424]**

(a) $\frac{17}{24}$ (b) $\frac{17}{48}$ (c) $\frac{3}{8}$ (d) $\frac{1}{3}$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (a), (b), (c) and (d) as given below.)

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** The area bounded by the curve $|x| + |y| = 1$ is 2 sq units. **[Conceptual Application]**

Reason (R): Curve is symmetrical to both axes and area bounded in each quadrant is 1 unit.

20. **Assertion (A):** The function $y = [x(x-2)]^2$ is increasing in $(0, 1) \cup (2, \infty)$. **[Conceptual Application]**

Reason (R): $\frac{dy}{dx} = 0$, when $x = 0, 1, 2$

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Solve the differential equation $xy\,dy = (y + 5)\,dx$, given that $y(5) = 0$. **[NCERT Part-II, Page 306-307]**

22. If A and B are events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$, find $P(\overline{A} \cap \overline{B})$. **[Conceptual Application]**

OR

Two dice are thrown simultaneously. If A be the event "getting 6 on the first die" and B be the event "getting 2 on the second die". Are the events independent? **[NCERT Part-II, Page 418]**

23. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors of equal magnitude, then show that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to the vectors $\vec{a}, \vec{b}$ and $\vec{c}$. **[Conceptual Application]**

24. Find the value of $\sin^{-1}\left|\cos\left(\frac{33\pi}{5}\right)\right|$. **[Conceptual Application]**

25. If $A = \begin{bmatrix} 0 & 3 \\ 2 & -5 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 4a \\ -8 & 5b \end{bmatrix}$, then find the values of k, a and b. **[NCERT Part-I, Page 41]**

OR

If $A = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$, find a matrix C such that $5A + 3B + 2C$ is a null matrix.

[NCERT Part-I, Page 43-44]

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. If $x = a(\theta - \sin\theta), y = a(1 + \cos\theta)$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$. **[Integrated Question]**

27. Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ is always increasing on R.

[NCERT Part-I, Page 153]

28. Evaluate $\int \sin^{-1}\sqrt{\frac{x}{2+x}}\,dx$. **[Integrated Question]**

OR

Show that the differential equation, **[NCERT Part-II, Page 313-314]**

$xe^{y/x} - y + x\frac{dy}{dx} = 0$ is homogeneous and find the particular solution, given that $y = 0$ when $x = e$.

29. Let the relation S in the set of all real numbers R, be defined as $aSb \Leftrightarrow 1 + ab > 0$ for all $a, b \in R$. Is relation S an equivalence relation? **[NCERT Part-I, Page 2]**

30. Sketch the graph $y = |x - 1|$. Evaluate $\int_{-2}^{1}|x-1|dx$. What does the value of this integral represent on the graph? **[NCERT Part-II, Page 267]**

OR

Using method of integration find the area of the triangle ABC, coordinates of whose vertices are $A(1, -2), B(3, 5)$ and $C(5, 2)$. **[Conceptual Application]**

31. If $\tan^{-1}\left(\frac{y}{x}\right) = \log\sqrt{x^2+y^2}$, prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$. **[NCERT Part-I, Page 121]**

OR

If function f is differentiable at $x = a$, find $\lim_{x\to a}\frac{xf(a) - af(x)}{x-a}$. **[Integrated Question]**

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. If $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}, \vec{b} = 4\hat{i} - 7\hat{j} + \hat{k}$, find a vector $\vec{c}$ such that $\vec{a}\times\vec{c} = \vec{b}$ and $\vec{a}\cdot\vec{c} = 6$. **[Conceptual Application]**

OR

Find the vector and Cartesian equations of a line passing through $(1, 2, -4)$ and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$. **[Conceptual Application]**

33. For the matrices A and B, verify that $(AB)' = B'A'$, where $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$, $B = [-1 \quad 2 \quad 1]$.

[NCERT Part-I, Page 61]

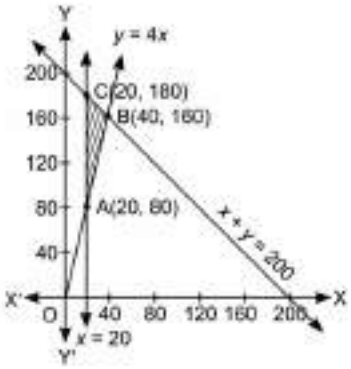
OR

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then show that $A^2 - 5A + 7I = O$. Hence, find A^{-1}. [Conceptual Application]

34. Solve the differential equation: [NCERT Part-II, Page 322-323]

$x\frac{dy}{dx} + y = x\cos x + \sin x$, given that $y = 1$ when $x = \frac{\pi}{2}$.

35. The feasible region of the system of linear constraints is shown below: [Conceptual Application]



Answer each of the following:

(*i*) If $Z = ax + by$ and $Z_{(20,\ 180)}$ is equal to $Z_{(40,\ 160)}$, what is relation between a and b?

(*ii*) If $Z = 400x + 300y$ be objective function, find value of maximum Z and the point of maximum.

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. Three persons A, B and C apply for the job of a manager in a private company the chances of their selection is given by the relation $4A = 2B = C$. The probability that if selected A, B and C can bring changes to improve profitability of the company are 0.8, 0.5 and 0.3 respectively.

[Conceptual Application]

(*i*) What is the probability that C is selected as a manager?

(*ii*) What is conditional probability that if change has taken place it is due to B?

(*iii*) What is the conditional probability that change does not take place due to selection of A?

OR

(*iii*) Find the probability that change does not take place.

Case Study - 2

37. An advertisement firm is supplied with decorative wire pieces of 34 m each and are asked to cut the wire into two pieces. From one piece a circular sign board is to be made and from other a square one and the idea is to keep the sum of the areas enclosed by a circle and square to be minimum for writing slogans. **[Conceptual Application]**

(*i*) If wire is cut at x m from one end and made into a circle of radius r, then find r.

(*ii*) Find the area enclosed by the circular ring.

(*iii*) Find the area enclosed by square frame.

OR

(*iii*) Find the value of x when combined area is minimum.

Case Study - 3

38. Three shopkeepers A, B and C go to a store to buy stationary. A purchase 12 dozen notebooks, 5 dozen pens and 6 dozen pencils. B purchase 10 dozen notebooks, 6 dozen pens and 7 dozen pencils. C purchase 11 dozen notebooks, 13 dozen pens and 8 dozen pencils. A notebook costs ₹ 40, a pen costs ₹ 12 and a pencil costs ₹ 3. **[Conceptual Application]**

(*i*) Represent the number of items purchased by shopkeepers A, B and C in matrix form.

(*ii*) If X represents the matrix formed in above part (*i*) and Y represents the matrix formed by the cost of each item, then find XY.

ST Francis School Baraut

Series : SFS/02/04

SET – 4

ROLL No.

Q.P Code 12/02/04

Candidates must write the Q.P Code on the title page of the Answer book.

- Please check that this question paper contains 7 printed pages.
- Please check that this question paper contains 38 questions.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please write down the serial number of the question in the answer-book before attempting it.
- 15 minute time has been allotted to read this question paper.
- The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे | *अधिकतम अंक : 80*

Time allowed : 3 hours | *Maximum Marks : 80*

General Instructions:

Read the following instructions very carefully and strictly follow them:

(*i*) *This Question paper contains* ***38*** *questions.* ***All*** *questions are* ***compulsory.***

(*ii*) *This Question paper is divided into* ***five*** *Sections –* ***A, B, C, D*** *and* ***E.***

(*iii*) *In* ***Section A****, Questions no.* ***1*** *to* ***18*** *are* ***multiple choice questions*** *(MCQs) and Questions no.* ***19*** *and* ***20*** *are* ***Assertion-Reason based*** *questions of* ***1 mark each.***

(*iv*) *In* ***Section B****, Questions no.* ***21*** *to* ***25*** *are* ***Very Short Answer (VSA)-type*** *questions, carrying* ***2 marks each.***

(*v*) *In* ***Section C****, Questions no.* ***26*** *to* ***31*** *are* ***Short Answer (SA)-type*** *questions, carrying* ***3 marks each.***

(*vi*) *In* ***Section D****, Questions no.* ***32*** *to* ***35*** *are* ***Long Answer (LA)-type*** *questions, carrying* ***5 marks each.***

(*vii*) *In* ***Section E****, Questions no.* ***36*** *to* ***38*** *are* ***Case study-based questions****, carrying* ***4 marks each.***

(*viii*) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(*ix*) *Use of calculators is* ***not*** *allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. If A is a square matrix, then $A + A'$ is a **[NCERT Part-I, Page 61]**

(*a*) non-singular matrix (*b*) symmetric matrix

(*c*) skew-symmetric matrix (*d*) unit matrix

2. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then $A^2 + 2A$ equals **[Conceptual Application]**

(*a*) $4A$ (*b*) $3A$

(*c*) $2A$ (*d*) A

3. If $A = \begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -1 \\ -1 & 3 \end{bmatrix}$, then $(A + B)^{-1}$ equals **[Conceptual Application]**

(*a*) $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ (*b*) $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

(*c*) $\begin{bmatrix} 1 & -2 \\ -1 & -1 \end{bmatrix}$ (*d*) None of these

4. If the function, $f(x) = \frac{6\cos x}{\pi - 2x}$ is continuous at $x = \frac{\pi}{2}$, then the value of $f\left(\frac{\pi}{2}\right)$ is **[NCERT Part-I, Page 105]**

(a) 1 (b) 2
(c) 3 (d) –3

5. If the directions cosines of a line are <k, k, k>, then **[Conceptual Application]**

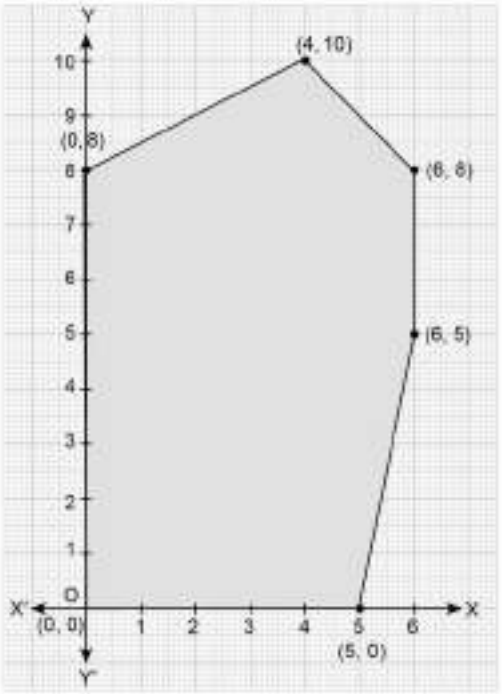
(a) $k > 0$ (b) $0 < k < 1$
(c) $k = 1$ (d) $k = \frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$

6. Integrating factor for the solution of differential equation $x\frac{dy}{dx} - y = x^4 - 3x$ is **[NCERT Part-II, Page 323]**

(a) x (b) $\log x$ (c) $\frac{1}{x}$ (d) $-x$

7. The feasible region for an LPP is shown below: **[NCERT Part-II, Page 398]**

Let $Z = 3x - 4y$ be the objective function. Minimum of Z occurs at



(a) (0, 0) (b) (0, 8) (c) (5, 0) (d) (4, 10)

8. The angle between the vectors $(\vec{i} + \vec{j})$ and $(\vec{i} - \vec{j})$ is **[NCERT Part-II, Page 356]**

(a) $\frac{\pi}{2}$ (b) $\frac{2\pi}{3}$
(c) $-\frac{\pi}{3}$ (d) $\frac{5\pi}{6}$

9. The value of $\int_0^{\frac{\pi}{2}} \frac{\sin^{2023} x}{\cos^{2023} x + \sin^{2023} x} dx$ is **[NCERT Part-II, Page 273-274]**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$
(c) 0 (d) 1

10. The solution set of equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$ is **[NCERT Part-I, Page 78-79]**

(a) $\{0, 1\}$ (b) $\{1, 2\}$

(c) $\{1, 5\}$ (d) $\{2, -1\}$

11. The optimal value of the objective function is attained at the points **[Conceptual Application]**

(a) given by intersection of inequation with y-axis only.

(b) given by intersection of inequation with x-axis only.

(c) given by corner points of the feasible region.

(d) None of these

12. The projection of vector $\vec{a} = (2\hat{i} - \hat{j} + \hat{k})$ on vector $\vec{b} = (\hat{i} + 2\hat{j} + 2\hat{k})$ is **[NCERT Part-II, Page 357-358]**

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) 2 (d) $\sqrt{6}$

13. If each element of third order determinant of value Δ is multiplied by 4, then value of the new determinant is: **[NCERT Part-I, Page 80]**

(a) Δ (b) $21\ \Delta$

(c) $64\ \Delta$ (d) $128\ \Delta$

14. From the set $\{1, 2, 3, 4, 5\}$, two numbers a and b are chosen one by one without replacement. The probability that a / b is an integer is **[Conceptual Application]**

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) $\frac{3}{5}$

15. Find the product of the order and degree of differential equation **[NCERT Part-II, Page 301-302]**

$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 - y^2 = 0.$

(a) 4 (b) 1

(c) 3 (d) 5

16. The two vectors $(\hat{j} + \hat{k})$ and $(3\hat{i} - \hat{j} + 4\hat{k})$ represent the two sides AB and AC respectively of a triangle ABC. The length of the median through A is **[Conceptual Application]**

(a) $\frac{\sqrt{34}}{2}$ units (b) $2\sqrt{3}$ units

(c) $3\sqrt{2}$ units (d) None of these

17. If $y = \sin x^0$, then $\frac{dy}{dx}$ is equal to **[NCERT Part-I, Page 119]**

(a) $\cos x^0$ (b) $\sec x^0 \tan x^0$

(c) $\frac{\pi}{180} \cos x^0$ (d) None of these

18. If a, b, c are the angles that a line makes with the positive direction of x, y, z axis respectively, then the direction cosines of the line are **[NCERT Part-II, Page 377-378]**

(a) $\sin a, \sin b, \sin c$ (b) $\cos a, \cos b, \cos c$

(c) $\tan a, \tan b, \tan c$ (d) $\cos^2 a, \cos^2 b, \cos^2 c$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (*a*), (*b*), (*c*) and (*d*) as given below.)

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true but R is not the correct explanation of A.

(*c*) A is true but R is false.

(*d*) A is false but R is true.

19. Let R be a relation defined by $R = \{(x, y) : |x - y| = 0\}$, on set $A = \{1, 3, 5\}$, then **[NCERT Part-I, Page 2]**

Assertion (*A*): R is symmetric.

Reason (*R*): A relation on set A is said to be symmetric if for $(a, b) \in R$, we have $(b, a) \in R$.

20. If $u = f(\tan x)$, $v = g(\sec x)$ and $f'(1) = 2$, $g'(\sqrt{2}) = 4$, then **[NCERT Part-I, Page 121]**

Assertion (*A*): $\left(\frac{du}{dv}\right)_{x=\frac{\pi}{4}} = \frac{1}{\sqrt{2}}$

Reason(*R*): If $u = f(x)$, $v = g(x)$, then the derivative of u with respect to v is $\frac{du}{dv} = \frac{\left(\frac{du}{dx}\right)}{\left(\frac{dv}{dx}\right)} = \frac{f'(x)}{g'(x)}$

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Write the principal value of $\tan^{-1}\left[2\sin\left(\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$. **[NCERT Part-I, Page 27]**

OR

If $\cos^{-1}p + \cos^{-1}q + \cos^{-1}r = 3\pi$, then find the value of $pq + qr + rp$. **[Conceptual Application]**

22. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimetres of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm. **[NCERT Part-I, Page 147-148]**

23. Find the point(s) on the curve $x^2 = 2y$ which is(are) nearest to the point $(0, 5)$. **[NCERT Part-I, Page 153]**

OR

Find the intervals in which the function f given by $f(x) = 2x^2 - 3x$ is **[NCERT Part-I, Page 153]**

(*a*) strictly increasing (*b*) strictly decreasing.

24. Evaluate : $\int_{-1}^{5} |x - 3|\, dx$ **[Conceptual Application]**

25. Show that $f(x) = \tan^{-1}(\sin x + \cos x)$ is increasing on $\left(0, \frac{\pi}{4}\right)$. **[NCERT Part-I, Page 153]**

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. Evaluate: $\int \frac{1}{x(x^4 - 1)}\, dx$ **[NCERT Part-II, Page 252-253]**

27. A coin is biased so that the head is three times as likely to occur as tail. If the coin is tossed twice, find the probability that (*i*) head appears exactly once (*ii*) At least one head appear. **[Conceptual Application]**.

28. Evaluate: $\int e^{2x}\cos x\,dx$ **[NCERT Part-II, Page 262-263]**

OR

Evaluate: $\int_0^{\pi} \frac{x\tan x}{\sec x+\tan x}\,dx$ **[NCERT Part-II, Page 274]**

29. If $y=\log(1+\sin x)$, then prove that **[NCERT Part-I, Page 137]**

$$\frac{d^2y}{dx^2}=-\frac{1}{1+\sin x}.$$

OR

Find the general solution of differential equation: $\log\left(\frac{dy}{dx}\right)=3x+4y$ **[NCERT Part-II, Page 306-307]**

30. Solve the following LPP graphically. **[NCERT Part-II, Page 397-398]**

Maximize $Z=8x+9y$ subject to the constraints given below :

$2x+3y\le 6;\quad 3x-2y\le 6;\quad y\le 1;\quad x,y\ge 0$

OR

Solve the following LPP graphically. **[NCERT Part-II, Page 397-398]**

Minimize $Z=3x+5y$ subject to constraints given below:

$x+3y\ge 3;\ x+y\le 2;\ x,y\ge 0$

31. If $x=\sin t$ and $y=\sin pt$, then prove that $(1-x^2)\frac{d^2y}{dx^2}=x\frac{dy}{dx}-p^2y$. **[Conceptual Application]**

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. AOB is a positive quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ where $OA=a$, $OB=b$. Find the area between the arc AB and chord AB of the ellipse. **[Conceptual Application]**

33. Show that the relation R on the set Z of all integers, given by $R=\{(a,b):5 \text{ divides } (a-b)\}$ is an equivalence relation. **[NCERT Part-I, Page 2]**

OR

Prove that the function $f:N\to N$, defined by $f(x)=x^2+x+1$ is one-one but not onto. **[NCERT Part-I, Page 7]**

34. If $A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}$, find A^{-1} and hence solve the system of equations **[NCERT Part-I, Page 94-95]**

$x+2y+z=4,\ -x+y+z=0$ and $x-3y+z=2$.

35. The points $A(1, 2, 3)$, $B(-1, -2, -3)$ and $C(2, 3, 2)$ are three vertices of a parallelogram $ABCD$. Find the cartesian equation of CD. **[NCERT Examplar, Page 221]**

OR

Find the shortest distance between the lines: **[NCERT Part-II, Page 386-387]**

$\vec{r}=(8+3\lambda)\hat{i}-\hat{j}(9+16\lambda)+(10+7\lambda)\hat{k}$ and $\vec{r}=15\hat{i}+29\hat{j}+5\hat{k}+\mu(3\hat{i}+8\hat{j}-5\hat{k})$.

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36.

An academy in a town has 500 students on its list and collects fixed charges of ₹ 300 per student per month. The academy proposes to increases the monthly fee and it is believed that for every increase of ₹ 1, one student will discontinue the service. If academy increases ₹ x per month per student, then

(*i*) express the collection (revenue) in terms of 'x'. **[Conceptual Application]**

(*ii*) find the critical point of revenue function obtained in part (*i*).

(*iii*) use first derivative test to find the value of 'x' that maximize revenue function.

OR

(*iii*) use second derivative test to find the value of 'x' that maximize revenue function.

Case Study - 2

37. In answering a question on a multiple choice test for class XII, a student either knows the answer or guesses. Let 3/4 be the probability that he knows the answer and 1/4 be the probability that he guesses. Assume that a student who guesses at the answer will be correct with probability 1/4. **[Conceptual Application]**

Let E_1, E_2, E be the events that the student knows the answer, guesses the answer and answers correctly respectively.

Based on the above information, answer the following:

(*i*) What is the value of $P(E/E_1)$ and $P(E/E_2)$?

(*ii*) What is the probability that the student answered it correctly?

(*iii*) What is the probability that the student knows the answer given that he answered it correctly?

OR

(*iii*) What is the probability that the student guesses the answer given that he answered it correctly?

ST Francis School Baraut

ESTD. 2016

Series : SFS/02/05 **SET – 5**

ROLL No. **Q.P Code 12/02/05**

Candidates must write the Q.P Code on the title page of the Answer book.

- Please check that this question paper contains 7 printed pages.
- Please check that this question paper contains 38 questions.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
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गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे *अधिकतम अंक : 80*

Time allowed : 3 hours *Maximum Marks : 80*

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) *This Question paper contains* ***38*** *questions.* ***All*** *questions are* ***compulsory.***

(ii) *This Question paper is divided into* ***five*** *Sections –* ***A, B, C, D*** *and* ***E.***

(iii) *In* ***Section A,*** *Questions no.* ***1*** *to* ***18*** *are* ***multiple choice questions*** *(MCQs) and Questions no.* ***19*** *and* ***20*** *are* ***Assertion-Reason based*** *questions of* ***1 mark each.***

(iv) *In* ***Section B,*** *Questions no.* ***21*** *to* ***25*** *are* ***Very Short Answer (VSA)-type*** *questions, carrying* ***2 marks each.***

(v) *In* ***Section C,*** *Questions no.* ***26*** *to* ***31*** *are* ***Short Answer (SA)-type*** *questions, carrying* ***3 marks each.***

(vi) *In* ***Section D,*** *Questions no.* ***32*** *to* ***35*** *are* ***Long Answer (LA)-type*** *questions, carrying* ***5 marks each.***

(vii) *In* ***Section E,*** *Questions no.* ***36*** *to* ***38*** *are* ***Case study-based questions,*** *carrying* ***4 marks each.***

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(ix) *Use of calculators is* ***not*** *allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. If points $(-3, 0), (3, 0), (0, k)$ are collinear, then value of k is **[NCERT Part-I, Page 82]**

(a) -6 (b) 0 (c) 6 (d) None of these

2. If A and B are invertible matrices of the same order then which of the following is correct? **[Conceptual Application]**

(a) $|A| = 0, |B| \neq 0$ (b) $|A| \neq 0, |B| = 0$ (c) $|A| \neq 0, |B| \neq 0$ (d) $|A + B| = |A| + |B|$

3. If $A = [a_{ij}]$ is a matrix of order 2×2, such that $a_{ij} = \begin{cases} i - j, & \text{for } i = j \\ i^2 \cdot j, & \text{for } i \neq j \end{cases}$, then A^2 is equal to **[Conceptual Application]**

(a) $\begin{bmatrix} 0 & 2 \\ 4 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 4 \\ 8 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 8 \\ 8 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$

4. If A is a square matrix, then $A - A'$ is **[NCERT Part-I, Page 63-64]**

(a) a diagonal matrix (b) a scalar matrix

(c) a symmetric matrix (d) a skew symmetric matrix

5. If $\vec{a}$, $\vec{b}$ and $|\vec{a} \times \vec{b}|$ are unit vectors, then angle between $\vec{a}$ and $\vec{b}$ is **[Conceptual Application]**

(a) $90°$ (b) $30°$ (c) $45°$ (d) $60°$

6. If position vectors of points P and Q are $2\hat{i} - \hat{j} + 4\hat{k}$ and $5\hat{j} + 7\hat{k}$ respectively, then $\overrightarrow{QP}$ is **[NCERT Part-II, Page 339]**

(a) $-2\hat{i} + 6\hat{j} + 3\hat{k}$ (b) $2\hat{i} + 4\hat{j} + 11\hat{k}$ (c) $-2\hat{i} - 4\hat{j} - 11\hat{k}$ (d) $2\hat{i} - 6\hat{j} - 3\hat{k}$

7. Given A is a square matrix of order 3 and $|A| = 5$ then $|A \cdot Adj A|$ is **[NCERT Part-I, Page 89-90]**

(a) 5 (b) 125 (c) 25 (d) 625

8. Charvi is trying to find the general solution of the following differential equations. **[Conceptual Application]**

(i) $x^2\frac{dy}{dx} = x + y$ (ii) $(x^2 + y^2)\frac{dy}{dx} = 2xy$ (iii) $\frac{dy}{dx} = e^{x+y} + x^2e^y$ (iv) $(x - y^3)dy + ydx = 0$

For which of the following we need to find integrating factor of its solution?

(a) only (i) (b) (i) and (ii) (c) only (iii) (d) (i) and (iv)

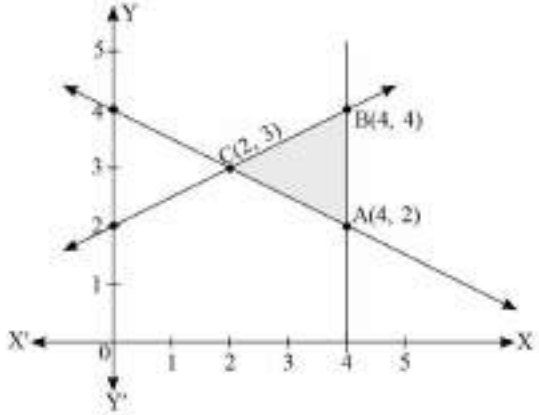
9. $\int_{-0.5}^{1} e^{|x|}dx$ is **[Conceptual Application]**

(a) $e - e^{0.5}$ (b) $e + e^{0.5} - 2$ (c) $e^{0.5} - 2$ (d) $2e - 2$

10. If $f(x) = \begin{cases} \frac{\lambda|x-2|}{x-2}, & \text{for } x < 2 \\ 5 + x, & \text{for } x \geq 2 \end{cases}$ is continuous at $x = 2$, then value of λ is **[NCERT Part-I, Page 105]**

(a) 7 (b) –7 (c) 1 (d) –1

11. The feasible region to the linear constraints of a linear programming problem is given as **[Conceptual Application]**



Which of the following is constraint related to AC?

(a) $2x + y - 10 \geq 0$ (b) $x + 2y \leq 0$ (c) $x + 2y - 8 \geq 0$ (d) $2x + y - 11 \leq 0$

12. If $\vec{a} = 3\hat{j} - 4\hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$, then vector whose magnitude is that of $|\vec{b}|$ and along $\vec{a}$ is **[Conceptual Application]**

(a) $21\hat{j} - 28\hat{k}$ (b) $\frac{3}{5}\hat{j} - \frac{4}{5}\hat{k}$ (c) $\frac{21}{5}\hat{j} - \frac{28}{5}\hat{k}$ (d) $14\hat{i} - 21\hat{j} + 42\hat{k}$

13. The corner points of the bounded feasible region determined by a system of linear constraints are $A(0, 8)$, $B(3, 2)$ and $C(4, 6)$. If the objective function is $Z = mx + ny$, $m, n > 0$ such that $Z_A = Z_C$ then $m : n$ is **[NCERT Part-II, Page 398]**

(a) 1 : 2 (b) 2 : 1 (c) 1 : –2 (d) 2 : – 1

14. The function $f(x) = |x + 1|$, $x \in R$ is differentiable for **[NCERT Part-I, Page 118-119]**

(a) $x = 1$ only (b) $x = -1$ only (c) $x \in R - \{1\}$ (d) $x \in R - \{-1\}$

15. If a line makes angles 45° and 60° with x and y axis respectively, then the obtuse angle which it makes with z-axis is **[NCERT Part-II, Page 377-378]**

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{5\pi}{6}$

16. If lines $\vec{r} = 2\hat{j} + \lambda(\hat{i} - 3\hat{j} + \hat{k})$ and $\vec{r} = \hat{i} - \hat{j} + 4\hat{k} + \mu(3\hat{i} - p\hat{j} + 3\hat{k})$ are parallel, then value of p is **[Conceptual Application]**

(a) –3 (b) 9 (c) –9 (d) 3

17. If A and B throw a coin in turn the one to get head first wins the game, then probability of B winning the game is [Conceptual Application]

(a) $\frac{1}{4}$ (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) $\frac{1}{8}$

18. General solution of the differential equation $\frac{dy}{dx} = x + y$ is [NCERT Part-II, Page 322-323]

(a) $e^x \cdot y = x + C$ (b) $\log|1 + x + y| = x + C$ (c) $x + y + 1 = C$ (d) $e^x \cdot y = x^2 + C$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (a), (b), (c) and (d) as given below.)

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** Let R be the relation defined as $R = \{(1, 1), (2, 2), (3, 3), (1, 2)\}$ on set $A = \{1, 2, 3\}$, then relation R is an equivalence relation. [NCERT Part-I, Page 2]

Reason (R): A relation is said to be equivalence, if it is reflexive, symmetric and transitive.

20. Let $f(x) = e^{-x}, x \in R$ be a given function, then [NCERT Part-I, Page 153]

Assertion (A): function $f(x) = e^{-x}$ increases for $x > 0$.

Reason (R): A function 'f' is said to be increasing in R, if $f'(x) > 0, x \in R$

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Show that local maximum or local minimum value does not exist for $f(x) = \log_e x, x > 0$. [NCERT Part-I, Page 166]

OR

The cost of manufacturing x units is given by $C(x) = 2x^3 - x^2 + 4x + 5$, find the marginal cost when 10 units are produced. [NCERT Part-I, Page 147-148]

22. Find the critical points, if any for the function $f(x) = \sin 2x - x, -\frac{\pi}{2} < x < \frac{\pi}{2}$. [NCERT Part-I, Page 164]

23. Find the principal value of $\sin^{-1}\left(\cos\frac{11\pi}{3}\right)$. [Conceptual Application]

OR

Find the principal value of $\tan^{-1}(-1) + \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) - \sin^{-1}\left(\frac{-1}{2}\right)$. [NCERT Part-I, Page 25]

24. Solve the integral: $\int x(5 - x)^{50} dx$. [NCERT Part-II, Page 235-236]

25. Find the interval(s) for which the function $f(x) = x^3 - 3x^2 + 3x - 7, x \in R$ is increasing. [NCERT Part-I, Page 153]

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. If $\cos y = x \cos(a + y)$ with $\cos a \neq \pm 1$, then prove that $\frac{dy}{dx} = \frac{\cos^2(a + y)}{\sin a}$. [NCERT Part-I, Page 122-123]

27. A and B throw a die alternatively one of them gets a number greater than 4 and wins the game, if A starts the game, find the probability of B winning the game. [Conceptual Application]

28. If $x = \sin t$ and $y = \sin pt$, prove that

$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0.$ **[NCERT Part-I, Page 134-135]**

OR

Show that the differential equation $2xy\frac{dy}{dx} = x^2 + 3y^2$ is homogeneous and solve it.

[NCERT Part-II, Page 313-314]

29. Evaluate: $\int \sin x \cdot \sin 2x \cdot \sin 3x \, dx$. **[NCERT Part-II, Page 241]**

OR

Evaluate: $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$. **[Integrated Question]**

30. Solve the following linear programing problem graphically: **[NCERT Part-II, Page 397-398]**

Minimise $Z = x + 2y$,

subject to the constraints

$2x + y \geq 3, x + 2y \geq 6, x \geq 0, y \geq 0.$

OR

Solve the following linear programming problem graphically: **[NCERT Part-II, Page 397-398]**

Maximise $Z = 50x + 60y$,

subject to the constraints

$x \geq 0, y \geq 0, 5x + 8y \leq 200, 10x + 8y \leq 240.$

31. Evaluate: $\int \frac{(x^2+1)(x^2+3)}{(x^2+4)(x^2+5)} dx$. **[NCERT Part-II, Page 253]**

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, then find $(AB)^{-1}$. **[Integrated Question]**

33. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. Also, find their point of intersection. **[Conceptual Application]**

OR

Reduce the equations of the following lines in Cartesian form **[Conceptual Application]**

$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$.

Also, find the end points of a line segment which is perpendicular to both the lines if end points lie on the given lines.

34. Find the area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$. **[Conceptual Application]**

35. Prove that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by

$R = \{(a, b) : |a - b| \text{ is even}\}$ is an equivalence relation. Show that all the elements of $\{1, 3, 5\}$ are related to each other and all the elements of $\{2, 4\}$ are related to each other but no element of $\{1, 3, 5\}$ is related to any element of $\{2, 4\}$. **[NCERT Part-I, Page 2]**

OR

Show that the signum function $f: R \to R$, given by $f(x) = \text{sgn}(x)$ is neither one-one nor onto.

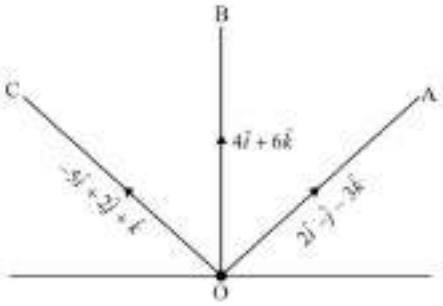
[NCERT Part-I, Page 7]

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36.



Three players A, B and C start from the same point in three different directions given by the vector $2\hat{i}-\hat{j}-3\hat{k}$, $4\hat{i}+6\hat{k}$ and $-5\hat{i}+2\hat{j}+\hat{k}$ respectively. **[Conceptual Application]**

(*i*) What is the magnitude of distance covered by player A?

(*ii*) Write the magnitudes of distance covered by players in descending order.

(*iii*) Find the magnitude of the resultant of vectors representing the directions of the players A, B and C.

OR

(*iii*) Write the direction cosines of the resultant of vectors representing the directions of A, B and C.

Case Study - 2

37. In a factory which manufactures bolts, machines A, B and C, manufacture 25%, 35% and 40% of the bolts respectively. Of their output 5, 4 and 2 per cent are respectively defective bolts. A bolt is drawn at random from the total output and is found to be defective. **[Conceptual Application]**

(*i*) Find the probability that machine A manufactures the defective bolts.

(*ii*) Find the probability of processing defective bolt.

(*iii*) A bolt was selected for quality and is found to be defective. What is the probability that it was manufactured by machine C?

OR

(*iii*) A bolt was selected for quality and is found to be defective. What is the probability that it was not manufactured by machine A?

Case Study - 3

38. Sand is pouring from a pipe at the rate of 12 cm^3/s. The falling sand forms a cone on the ground in such a way that height of the cone is always one-sixth of the radius of the base. Suppose V, h and r represent volume of sand cone, height of the cone and radius of base of sand cone at any given time t. **[Conceptual Application]**

(*i*) Write the expression for volume V of cone in terms of height h of the cone.

(*ii*) What is the rate of change of height of the cone, when height of sand cone is 3 cm?

ST Francis School Baraut

Series : SFS/02/06

SET – 6

ROLL No.

Q.P Code 12/02/06

Candidates must write the Q.P Code on the title page of the Answer book.

- Please check that this question paper contains 7 printed pages.
- Please check that this question paper contains 38 questions.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please write down the serial number of the question in the answer-book before attempting it.
- 15 minute time has been allotted to read this question paper.
- The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे | *अधिकतम अंक : 80*

Time allowed : 3 hours | *Maximum Marks : 80*

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) *This Question paper contains 38 questions. All questions are* ***compulsory.***

(ii) *This Question paper is divided into* ***five*** *Sections – A, B, C, D and E.*

(iii) *In* ***Section A****, Questions no. 1 to* ***18*** *are* ***multiple choice questions*** *(MCQs) and Questions no.* ***19*** *and* ***20*** *are* ***Assertion-Reason based*** *questions of* ***1 mark each.***

(iv) *In* ***Section B****, Questions no.* ***21*** *to* ***25*** *are* ***Very Short Answer*** *(VSA)-type questions, carrying* ***2 marks each.***

(v) *In* ***Section C****, Questions no.* ***26*** *to* ***31*** *are* ***Short Answer*** *(SA)-type questions, carrying* ***3 marks each.***

(vi) *In* ***Section D****, Questions no.* ***32*** *to* ***35*** *are* ***Long Answer*** *(LA)-type questions, carrying* ***5 marks each.***

(vii) *In* ***Section E****, Questions no.* ***36*** *to* ***38*** *are* ***Case study-based questions****, carrying* ***4 marks each.***

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(ix) *Use of calculators is* ***not*** *allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. If $D(0, 6)$, $E(5, 4)$ and $F(3, 10)$ are the mid points of sides AB, BC and CA respectively of a ΔABC, then area of ΔABC is **[NCERT Part-I, Page 82]**

(a) 13 sq units (b) 26 sq units (c) 39 sq units (d) 52 sq units

2. If A is an invertible matrix of order 3 and $|A| = 3$, then if $|3A| = k|A|$, the value of k is **[NCERT Part-I, Page 80]**

(a) 243 (b) 27 (c) 81 (d) 9

3. If matrix $A = \begin{bmatrix} 0 & 2 \\ -1 & 0 \end{bmatrix}$, then the value of $A^2 + 3I_2$ is **[Conceptual Application]**

(a) $\begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (d) I_2

4. If $A = \begin{bmatrix} 3 & -1 & 2 \\ 0 & 4 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, then value of $|A^{-1}|$ is **[NCERT Part-I, Page 90]**

(a) $\frac{1}{12}$ (b) 12 (c) $-\frac{1}{12}$ (d) -12

5. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular unit vectors, then the value of $|2\vec{a} + \vec{b} + \vec{c}|$ is **[Conceptual Application]**

(a) $-\sqrt{3}$ (b) $-\sqrt{6}$ (c) $\sqrt{6}$ (d) $\sqrt{3}$

6. The position vector of a point which divides the join of points with position vectors $\vec{a} - 2\vec{b}$ and $2\vec{a} + \vec{b}$ externally in the ratio 3 : 5 is **[NCERT Part-II, Page 353]**

(a) $\frac{-\vec{a} - 13\vec{b}}{2}$ (b) $\frac{\vec{a} + 13\vec{b}}{2}$ (c) $\frac{11\vec{a} - 7\vec{b}}{8}$ (d) $\frac{13\vec{a} - \vec{b}}{8}$

7. If A and B are square matrices of order 3 such that $|A| = 4$ and $AB = 2I$, then value of $|B|$ is **[NCERT Part-II, Page 90]**

(a) $\frac{1}{2}$ (b) 2 (c) 4 (d) 1

8. The degree of differential equation $\left(1 + \frac{dy}{dx}\right)^2 + x + \sin\left(\frac{d^2y}{dx^2}\right) = 0$ is **[NCERT Part-II, Page 302]**

(a) 1 (b) 2 (c) 0 (d) not defined

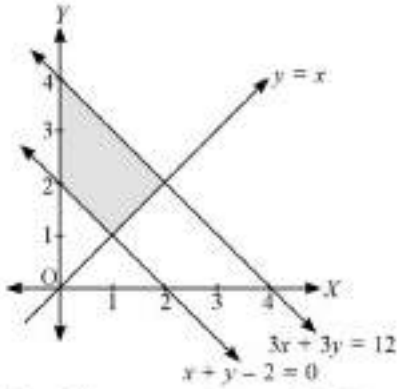
9. Value of $\int_0^{\pi} |\cos x| dx$ is **[NCERT Part-II, Page 273-274]**

(a) 2 (b) 0 (c) 3 (d) 4

10. If $f(x) = \begin{cases} \frac{e^{x+\lambda} - e^{\lambda}}{x}, & x \neq 0 \\ e^4, & x = 0 \end{cases}$ is continuous at $x = 0$, then value of λ is **[NCERT Part-I, Page 105]**

(a) e^{λ} (b) e^4 (c) 4 (d) e^{λ}

11. The feasible region corresponding to the linear constraints of linear programming problem is given in figure. Which of the following is not a constraint to the given linear programming problem? **[Conceptual Application]**



(a) $x + y - 2 \leq 0$ (b) $x \geq 0$ (c) $y \geq x$ (d) $3x + 3y \leq 12$

12. If $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = 225$ and $|\vec{a}| = 5$, then value of $|\vec{b}|$ is **[NCERT Part-II, Page 356, 363]**

(a) 25 (b) 5 (c) 9 (d) 3

13. The corner points of the bounded feasible region determined by system of linear constraints are (1, 3), (2, 5), (6, 3) and (0, 4). Let objective function be $Z = px + qy$, $p, q > 0$. If Z has same value at (2, 5) and (6, 3), then if $p = kq$, then value of k is **[NCERT Part-II, Page 398]**

(a) 2 (b) $\frac{1}{2}$ (c) -2 (d) $-\frac{1}{2}$

14. Which of the following function is continuous as well as differentiable for $x \in R$? **[Conceptual Application]**

(a) Absolute value function or Modulus function (b) Polynomial function
(c) Signum function (d) Greatest integer function

15. Direction cosines of the line $\frac{2x - 1}{3} = \frac{2 - y}{5}$, $z = 4$ are **[NCERT Part-II, Page 379-380]**

(a) $\frac{3}{\sqrt{109}}, \frac{-10}{\sqrt{109}}, 0$ (b) $\frac{3}{\sqrt{109}}, \frac{10}{\sqrt{109}}, 0$

(c) $\frac{3}{\sqrt{109}}, \frac{-10}{\sqrt{109}}, \frac{1}{\sqrt{109}}$ (d) $\frac{3}{\sqrt{34}}, \frac{5}{\sqrt{34}}, 0$

16. If a line makes angles α, β, γ with x, y and z-axes respectively, then value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is [NCERT Part-II, Page 377-378]

(a) -2 (b) 1 (c) 0 (d) 2

17. If A and B are two independent events such that $P(A) = 0.4$, $P(B) = 0.3$, then $P(A \cup B)$ is [NCERT Part-II, Page 418]

(a) 0.27 (b) 0.58 (c) 0.60 (d) 0.72

18. General solution of the differential equation $\frac{dy}{dx} = 2x \cdot e^{x^2 - y}$ is [NCERT Part-II, Page 306-307]

(a) $e^{x^2 - y} = C$ (b) $e^y + e^{x^2} = C$ (c) $e^{-y} + e^{x^2} = C$ (d) $e^y - e^{x^2} = C$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (a), (b), (c) and (d) as given below.)

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** The function 'f' defined by $f : R - \{0\} \to R$ as $f(x) = \frac{1}{x}$ is a bijective function. [NCERT Part-I, Page 7]

Reason (R): A function is said to be bijective if it is injective as well as surjective.

20. If the total revenue received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$ in ₹, then

Assertion (A): the marginal revenue when 5 units are sold is ₹ 66. [Integrated Question]

Reason (R): If for a function 'f', $f''(x_1) < 0$, where 'x_1' is a solution of $f'(x) = 0$, then function 'f' attains local maximum at 'x_1'.

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Differentiate $\tan^{-1}\left(\frac{1 + \cos x}{\sin x}\right)$ with respect to x. [NCERT Part-I, Page 124]

OR

If $x = at^2$, $y = 2bt$, then find (i) $\frac{dy}{dx}$ (ii) $\frac{d^2y}{dx^2}$ [NCERT Part-I, Page 134-135]

22. Find the critical points for the function $f(x) = x^3 - 3x^2 + 6x - 100$, if any. [NCERT Part-I, Page 164]

23. Evaluate $\sin\left[\cos^{-1}\left(\cos\frac{7\pi}{4}\right)\right]$. [Conceptual Application]

OR

Find the domain of $\cos^{-1}(3x + 4)$. [NCERT Part-I, Page 21]

24. Evaluate: $\int \cos x(\operatorname{cosec} x + 1)dx$. [NCERT Part-II, Page 241]

25. Find the interval for which the function $f(x) = \sin 3x$ is increasing in $\left(0, \frac{\pi}{2}\right)$. [NCERT Part-I, Page 153]

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. If $e^{x+y} = e^x + e^y$, then prove that $\frac{dy}{dx} = -e^{y-x}$. [NCERT Part-I, Page 122-123]

27. A speaks truth in 80% cases and B speaks truth in 90% cases. In what percentage of cases are they likely to agree with each other in stating the same fact? **[Conceptual Application]**

28. Find $\frac{dy}{dx}$, if $y = (\sin x)^{\tan x} + (\cos x)^{\sec x}$. **[NCERT Part-I, Page 130]**

OR

Find the particular solution of the differential equation
$(1 - y^2)(1 + \log x)dx + 2xydy = 0$, given that $y = 0$ when $x = 1$. **[NCERT Part-II, Page 306-307]**

29. Evaluate: $\int \frac{\sin\theta}{(5 - \sin^2\theta)(2 - \cos^2\theta)} d\theta$. **[NCERT Part-II, Page 253]**

OR

Evaluate: $\int_0^{3/2} |x \sin \pi x| dx$. **[NCERT Part-II, Page 273-274]**

30. Solve the following linear programming problem graphically: **[NCERT Part-II, Page 397-398]**
Maximize $Z = 1000x + 600y$
Subject to the constraints
$x + y \le 200, x \ge 20, y - 4x \ge 0, x, y \ge 0$.

OR

Solve the following linear programming problem graphically: **[NCERT Part-II, Page 397-398]**
Minimize $Z = 3x + 4y + 370$
Subject to the constraints
$x, y \ge 0, x + y \le 60, x + y \ge 10, x \le 40, y \le 50$.

31. Evaluate: $\int \sqrt{5 - 3x - x^2}\, dx$. **[NCERT Part-II, Page 264-265]**

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1}. Hence, solve the system of equations $2x + 3y + z = 0$, $3x - 2y - z = 5$, $5x - 4y - 2z = 9$. **[NCERT Part-I, Page 94-95]**

33. Find the vector and Cartesian equations of a line passing through the point $(1, 2, -4)$ and perpendicular to the lines **[Conceptual Application]**

$$\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$$

and

$$\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k}).$$

OR

Find the image of the point with position vector $(2\hat{i} - \hat{j} + 5\hat{k})$ in the line $\vec{r} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$. **[Conceptual Application]**

34. Using integration, find the area of the triangle ABC, the coordinates of whose vertices are $A(4, 1)$, $B(6, 6)$ and $C(8, 4)$. **[Conceptual Application]**

35. Let $f: N \to N$, N is the set of natural numbers, be defined as $f(x) = x + 1$, if x is odd and $f(x) = x - 1$, if x is even. Show that 'f' is a bijective function. **[NCERT Part-I, Page 7]**

OR

Let set $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and R be a relation in $A \times A$ defined by $(a, b)\, R\, (c, d)$ if $a + d = b + c$ for $(a, b), (c, d) \in A \times A$. Show that R is an equivalence relation. Also obtain equivalence class $[(3, 7)]$.
[NCERT Part-I, Page 2, 4]

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts *(i)*, *(ii)*, *(iii)* of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. Two boys are flying kites on the terrace of their respective houses. The straight strings of kites are in the form of skew lines and their equations are given by **[Conceptual Application]**

$$\vec{r} = (3\hat{i} + 5\hat{j} + 7\hat{k}) + \lambda(\hat{i} - 2\hat{j} + \hat{k})$$

and $$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}.$$

(*i*) Find a vector perpendicular to both the strings.

(*ii*) State the condition so that strings intersect each other.

(*iii*) Find the shortest distance between the strings.

OR

(*iii*) Along which direction vectors are the two strings?

Case Study - 2

37. There are two shops in a village named Rama general store and Green general store. Rama general store, stores 30 tin of pure mustard oil and 40 tin of adulterated mustard oil. While Green general store, stores 50 tin of pure mustard oil and 60 tin of adulterated mustard oil. Rani wants to purchase one tin of mustard oil from any of the shops. **[Conceptual Application]**

(*i*) Find the probability of getting an adulterated mustard oil tin from Green general store.

(*ii*) Find the probability of purchasing a pure tin of mustard oil.

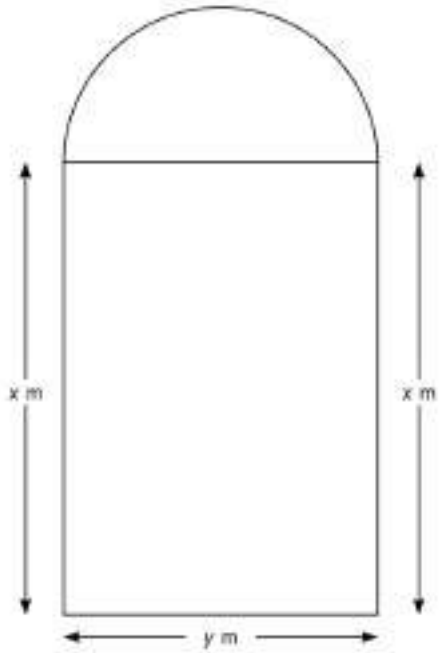
(*iii*) If the tin of mustard oil purchased is pure, what is the probability that it was purchased from Rama general store?

OR

(*iii*) What is the probability of purchasing a pure mustard oil tin from a Green general store or adulterated mustard oil tin from Rama general store?

Case Study - 3

38.



A person wants to purchase a 3 bedroom flat in Noida. When he went to have a look at the flat, he noticed a window in the form of a rectangle surmounted by semicircular shape. The perimeter of window is found to be 10 m. If x m is length of rectangular portion of window and y m is its breadth and semicircular portion is on breadth, then **[Conceptual Application]**

(*i*) find the area of the window in terms of y.

(*ii*) find the value of y for which area of the window is maximum.

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Series : SFS/02/07 **SET – 7**

ROLL No. **Q.P Code 12/02/07**

Candidates must write the Q.P Code on the title page of the Answer book.

- Please check that this question paper contains 7 printed pages.
- Please check that this question paper contains 38 questions.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please write down the serial number of the question in the answer-book before attempting it.
- 15 minute time has been allotted to read this question paper.
- The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे *अधिकतम अंक : 80*

Time allowed : 3 hours *Maximum Marks : 80*

General Instructions:

Read the following instructions very carefully and strictly follow them:

(*i*) *This Question paper contains 38 questions. All questions are* ***compulsory.***

(*ii*) *This Question paper is divided into* ***five*** *Sections – **A**, **B**, **C**, **D** and **E**.*

(*iii*) *In* ***Section A****, Questions no. **1** to **18** are* ***multiple choice questions*** *(MCQs) and Questions no. **19** and **20** are* ***Assertion-Reason based*** *questions of* ***1 mark each.***

(*iv*) *In* ***Section B****, Questions no. **21** to **25** are* ***Very Short Answer (VSA)-type*** *questions, carrying* ***2 marks each.***

(*v*) *In* ***Section C****, Questions no. **26** to **31** are* ***Short Answer (SA)-type*** *questions, carrying **3** **marks each.***

(*vi*) *In* ***Section D****, Questions no. **32** to **35** are* ***Long Answer (LA)-type*** *questions, carrying **5** **marks each.***

(*vii*) *In* ***Section E****, Questions no. **36** to **38** are* ***Case study-based questions****, carrying **4** **marks each.***

(*viii*) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(*ix*) *Use of calculators is* ***not*** *allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. If A_{ij} represents cofactor of an element a_{ij} of a determinant corresponding to a matrix $A = [a_{ij}]$, then for matrix $A = \begin{bmatrix} 1 & 4 & -2 \\ 0 & 1 & 3 \\ 1 & 2 & -2 \end{bmatrix}$, A_{32} is **[NCERT Part-I, Page 84]**

   (*a*) 3 (*b*) −3 (*c*) 5 (*d*) −5

2. The matrix $\begin{bmatrix} 3 & -1 \\ 2 & k \end{bmatrix}$ is invertible for **[NCERT Part-I, Page 90]**

   (*a*) $k = 0$ (*b*) $k \neq 3$ (*c*) $k = \frac{2}{3}$ (*d*) $k \neq \frac{-2}{3}$

3. If $A = [a_{ij}]$ is a matrix of order 2×3 such that $a_{ij} = 2j - i$, then a_{32} is **[Conceptual Application]**

   (*a*) 4 (*b*) does not exist (*c*) 1 (*d*) −1

4. The values of x for which $\begin{vmatrix} 6 & -2 \\ 2 & 4 \end{vmatrix} = x^2 - 12x$ are **[NCERT Part-I, Page 77]**

   (*a*) −2, 14 (*b*) 2, −14 (*c*) −2, −14 (*d*) 2, 14

5. The value(s) of p for which vector $p(\hat{i} - \hat{j} + \hat{k})$ represents a unit vector is **[NCERT Part-II, Page 341]**

   (*a*) $\frac{1}{\sqrt{3}}$ (*b*) $\pm\sqrt{3}$ (*c*) ± 1 (*d*) $\pm\frac{1}{\sqrt{3}}$

6. If $|\vec{a}| = 4$ and $-3 \le \lambda \le 1$ then the range of $|\lambda\vec{a}|$ is **[Conceptual Application]**

(a) $[0, 8]$ (b) $[-12, 8]$ (c) $[0, 12]$ (d) $[8, 12]$

7. If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$ then value of x is **[NCERT Part-I, Page 90]**

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) 2 (d) -2

8. Integrating factor for the solution of differential equation $\sin^2 x \frac{dy}{dx} + y = \cot x$ is **[NCERT Part-II, Page 322-323]**

(a) $e^{-\cot x}$ (b) $\cot x$ (c) $-\cot x$ (d) $e^{\cot x}$

9. $\int_{-1}^{1} |1 - x| dx$ is equal to **[NCERT Part-II, Page 273-274]**

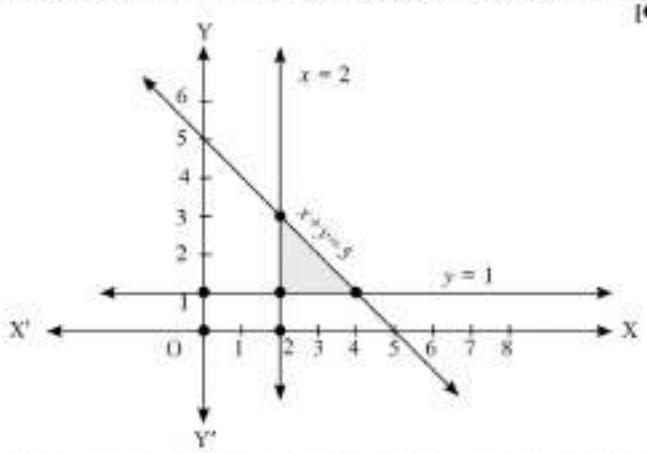
(a) 1 (b) 2 (c) 3 (d) -3

10. A function 'f' is said to be continuous for $x \in R$ if **[Conceptual Application]**

(a) 'f' is continuous at $x = 0$ (b) 'f' is differentiable at $x = 0$

(c) 'f' is continuous for $R - \{0\}$ (d) 'f' is differentiable for $x \in R$

11. The feasible region corresponding to the linear constraints of a linear programming problem is given as: **[Conceptual Application]**



Which of the following is not constraint to the given linear programming problem?

(a) $x \ge 2$ (b) $x + y \ge 5$ (c) $y \ge 1$ (d) $y \ge 0$

12. Cosine of an angle between the vectors $\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \hat{k}$ is **[NCERT Part-II, Page 355-356]**

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $-\frac{1}{3}$ (d) $\cos^{-1}\left(-\frac{1}{3}\right)$

13. The corner points of the bounded feasible region determined by the linear constraints are $A(1, 3)$, $B(0, 4)$, $C(2, 5)$ and $D(6, 2)$. Let $Z = px + qy$, the condition on p and q such that minimum Z occurs on any point of segment AD is **[NCERT Part-II, Page 398]**

(a) $p + 5q = 0$ (b) $p = 5q$ (c) $5p = q$ (d) $5p + q = 0$

14. The function $f(x) = |x| + |x - 1|$ is **[Integrated Question]**

(a) continuous at $x = 0, 1$ but not differentiable. (b) not continuous at $x = 0, 1$ but differentiable.

(c) not continuous and differentiable at $x = 0, 1$. (d) continuous as well as differentiable at $x = 0, 1$.

15. Direction cosines of a vector equally inclined to the coordinate axes are **[NCERT Part-II, Page 349]**

(a) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ (b) 1, 1, 1 (c) 0, 0, 0 (d) None of these

16. The equations of y-axis are **[NCERT Part-II, Page 382]**

(a) $\frac{x}{0} = \frac{y}{1} = \frac{z}{1}$ (b) $\frac{x-1}{1} = \frac{y}{0} = \frac{z}{1}$ (c) $\frac{x-1}{0} = \frac{y}{1} = \frac{z+5}{0}$ (d) $\frac{x}{0} = \frac{y-1}{1} = \frac{z}{0}$

17. Three persons fire a target and their probabilities of hitting a target are 0.2, 0.3 and 0.5 respectively. The probability that target is hit is **[Conceptual Application]**

(a) 0.993 (b) 0.94 (c) 0.72 (d) 0.90

18. If p and q represent degree and order of a differential equation $\left(\frac{dy}{dx}\right)^3 + 3y\left(\frac{d^2y}{dx^2}\right) = 0$, then value of $2p - 3q$ is **[NCERT Part-II, Page 301-302]**

(a) 1 (b) -4 (c) 8 (d) 3

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (a), (b), (c) and (d) as given below.)

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** A reflexive relation in a given set A is an identity relation also. **[Integrated Question]**
Reason (R): The number of one-one functions from set $A = \{1, 2\}$ to set $A = \{1, 2\}$ are 2.

20. If 'f' be a function defined by $f(x) = x^2 - x + 1$ for $x \in R$, then
Assertion (A): the function 'f' is neither increasing nor decreasing on $[-1, 1]$. **[Integrated Question]**
Reason (R): Marginal cost related to cost function $C(x)$ is $C'(x)$, where x represents number of units produced.

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Is the function $f(x) = \log(\cos x)$ strictly decreasing on $\left(0, \frac{\pi}{2}\right)$ or not? **[NCERT Part-I, Page 153]**

OR

Find whether the function $y = x^3 + x^2 + x + 1$ has local maxima or local minima. **[NCERT Part-I, Page 166]**

22. If at $x = 1$, the function $f(x) = x^4 - 62x^2 + ax + 9$ attains its maximum value on the interval $[0, 2]$, then find a. **[NCERT Part-I, Page 164]**

23. Represent $\sec\left[\tan^{-1}\left(\frac{y}{3}\right)\right]$ in terms of y. **[Conceptual Application]**

OR

Write the branch of $\tan^{-1} x$ other than principal value branch in which it becomes bijection.

24. If $\int \frac{1}{\sqrt{4 - 9x^2}}dx = \frac{1}{3}\sin^{-1}(ax) + C$, then find the value of a. **[NCERT Part-II, Page 244]**

25. Without using derivatives, find the maximum and minimum value for the function $f(x) = |3\sin x - 2|$. **[Conceptual Application]**

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. If $y = e^{a\sin^{-1}x}$, $-1 \le x \le 1$, then show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$. **[NCERT Part-I, Page 137]**

27. A card from a pack of 52 cards is lost. From the remaining cards, 2 cards are drawn without replacement and found to be both aces. Find the probability that lost card is an ace.

28. If $x = a(\cos\theta + \log\tan\frac{\theta}{2})$ and $y = a\sin\theta$, find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$. [NCERT Part-II, Page 134-135]

OR

Solve the differential equation:

$(x^2 - yx^2)dy + (y^2 + x^2y^2)dx = 0$, given that $y = 1$ when $x = 1$. [NCERT Part-II, Page 306-307]

29. Evaluate: $\int\left[\frac{1}{\log x} - \frac{1}{(\log x)^2}\right]dx$. [NCERT Part-II, Page 259-260]

OR

Evaluate: $\int_0^{\frac{\pi}{2}}(2\log\sin x - \log\sin 2x)dx$. [NCERT Part-II, Page 273-274]

30. Solve the following linear programming problem: [NCERT Part-II, Page 397-398]

Maximize $Z = x + 2y$

Subject to the constraints

$x + 2y \geq 100,\ 2x - y \leq 0,\ 2x + y \leq 200,\ x \geq 0,\ y \geq 0$.

OR

Solve the following linear programming problem: [NCERT Part-II, Page 397-398]

Minimize $Z = 6x + 3y$

Subject to the constraints

$4x + y \geq 80,\ x + 5y \geq 115,\ 3x + 2y \leq 150,\ x \geq 0,\ y \geq 0$.

31. Evaluate: $\int\sin(\log x)dx$. [NCERT Part-II, Page 259-260]

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. Using matrices, solve the following system of equations

$x + y - z = 3,\ 2x + 3y + z = 10,\ 3x - y - 7z = 1$. [NCERT Part-I, Page 94-95]

33. Find the value of λ, so that the lines L_1: $\frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2}$ and L_2: $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles. Also, find equations of a line passing through point $(3, -1, 7)$ and parallel to L_2. [Conceptual Application]

OR

Find the vector and cartesian equations of a line passing through the point $(1, 2, -4)$ and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-20}{8} = \frac{z-5}{-5}$. [Conceptual Application]

34. Using integration, find the area of the region bounded by the following lines:

$5x - 2y - 10 = 0,\ x + y - 9 = 0,\ 2x - 5y - 4 = 0$. [Conceptual Application]

35. Show that the relation S on the set R of real numbers defined as $S = \{(a, b) : a \leq b\}$ is reflexive and transitive but not symmetric. [NCERT Part-I, Page 2]

OR

Show that the function $f: N \to N$ defined by $f(x) = x^2 + x + 1$ is one-one but not onto. [NCERT Part-I, Page 7]

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. Two friends live in the houses A and B whose locations are represented by coordinates (3, 1, 2) and (–1, 4, 0) respectively. **[Conceptual Application]**

Using the above information, answer the following questions:

(*i*) If the points $A(3, 1, 2)$ and $B(-1, 4, 0)$ are joined by vector $\overrightarrow{AB}$, then find $\overrightarrow{AB}$.

(*ii*) Find the unit vector in the direction of $\overrightarrow{AB}$.

(*iii*) Write a vector of magnitude 5 units along the vector $\overrightarrow{AB}$.

OR

(*iii*) Write the direction cosines of $\overrightarrow{AB}$.

Case Study - 2

37. A coach is training three players for badminton competition. He observes that player A can win 4 out of 5 matches. Player B can win 3 out of 4 matches and player C can win 2 out of 3 matches played.

(*i*) What is the probability that all the three will win the matches in a competition? **[Conceptual Application]**

(*ii*) What is the probability that A loses but B and C win the game?

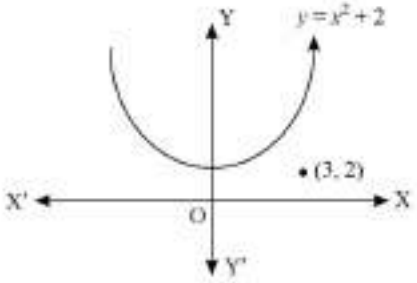
(*iii*) What is the probability that at least one out of A, B and C will win the game?

OR

(*iii*) What is the probability that none of them will win the game?

Case Study - 3

38. The jet of an enemy is flying along the curve $y = x^2 + 2$ and a soldier is placed at the point (3, 2). If at a given time, jet is at point (x, y), then **[Conceptual Application]**



(*i*) what is the distance D between the jet and the soldier in terms of x?

(*ii*) for what value of x jet is nearest to the soldier?

ST Francis School
Baraut

Series : SFS/02/08

SET – 8

ROLL No.

Q.P Code 12/02/08

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गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे | *अधिकतम अंक : 80*

Time allowed : 3 hours | *Maximum Marks : 80*

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) *This Question paper contains* **38** *questions.* ***All*** *questions are* ***compulsory.***

(ii) *This Question paper is divided into* ***five*** *Sections –* ***A****,* ***B****,* ***C****,* ***D*** *and* ***E****.*

(iii) *In* ***Section A****, Questions no.* ***1*** *to* ***18*** *are* ***multiple choice questions*** *(MCQs) and Questions no.* ***19*** *and* ***20*** *are* ***Assertion-Reason based*** *questions of* ***1 mark each.***

(iv) *In* ***Section B****, Questions no.* ***21*** *to* ***25*** *are* ***Very Short Answer (VSA)-type*** *questions, carrying* ***2 marks each.***

(v) *In* ***Section C****, Questions no.* ***26*** *to* ***31*** *are* ***Short Answer (SA)-type*** *questions, carrying* ***3 marks each.***

(vi) *In* ***Section D****, Questions no.* ***32*** *to* ***35*** *are* ***Long Answer (LA)-type*** *questions, carrying* ***5 marks each.***

(vii) *In* ***Section E****, Questions no.* ***36*** *to* ***38*** *are* ***Case study-based questions,*** *carrying* ***4 marks each.***

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(ix) *Use of calculators is* ***not*** *allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. The domain of $\sin^{-1}\sqrt{x-1}$ is **[NCERT Part-I, Page 19]**

(a) $[-1, 1]$ (b) $[0, 1]$ (c) $[-1, -2]$ (d) $[1, 2]$

2. Let $f: R \to R$ defined by $f(x) = 3x$. Choose the correct answer. **[NCERT Part-I, Page 7]**

(a) f is one-one and onto (b) f is many-one and onto

(c) f is one-one but not onto (d) f is neither one-one nor onto

3. $\int_0^1 \log(1+x)dx =$ **[NCERT Part-II, Page 259-260, 268]**

(a) $\log 4$ (b) $\log\left(\frac{4}{e}\right)$ (c) $\log e^4$ (d) $\frac{1}{\log 4}$

4. $\int \frac{\cot x}{\sqrt[4]{\sin x}}dx =$ **[NCERT Part-II, Page 235-236]**

(a) $\frac{-3}{\sqrt[4]{\sin x}} + C$ (b) $\frac{-2}{\sin^3 x} + C$ (c) $\frac{3}{\sin^{\frac{1}{3}} x} + C$ (d) $\frac{2}{\cos x} + C$

5. The area bounded by the curve $y = |x|$, $y = 0$ and $x = 3$ in first quadrant is **[Conceptual Application]**

(a) $\frac{9}{2}$ sq units (b) 9 sq units (c) $\frac{25}{2}$ sq units (d) 25 sq units

6. The area bounded by $y^2 = x$ and $x = 3$ is given by **[Conceptual Application]**

(a) $\int_0^3 (3 - y^2)dy$ (b) $\int_0^3 \sqrt{y}\, dy$ (c) $2\int_0^{\sqrt{3}} (3 - y^2)dy$ (d) $2\int_0^3 (y^2 + 3)dy$

7. If $y = \sin 5x$, then $\frac{d^2y}{dx^2}$ is **[NCERT Part-I, Page 137]**

(a) $-\sin 5x$ (b) $-25 \sin 5x$ (c) $-\cos 5x$ (d) $-25 \cos 5x$

8. $\frac{d}{dx}(x^x)$ is equal to **[NCERT Part-I, Page 130]**

(a) x^{x-1} (b) $x \log x$ (c) $x^x(1 + \log x)$ (d) xx^{x-1}

9. Diameter of a sphere is $\frac{3}{2}(2x + 5)$, the rate of change of its surface area with respect to x is **[NCERT Part-I, Page 147]**

(a) $18\pi (2x + 5)$ (b) $\frac{3}{2}$ (c) $9\pi (2x + 5)$ (d) $\frac{3}{4}$

10. The edge of a cube is increasing at the rate of 0.3 cm/s, the rate of change of its surface area when edge is 3 cm is **[NCERT Part-I, Page 147]**

(a) 10.8 cm (b) 10.8 cm^2 (c) 10.8 cm^2/s (d) 10.8 cm/s

11. The degree of the differential equation **[NCERT Part-II, Page 302]**

$\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$ is

(a) 1 (b) 2 (c) 3 (d) Not defined

12. The direction cosines of line of support of vector $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$ are **[NCERT Part-II, Page 349]**

(a) <1, –2, 2> (b) $<\frac{1}{3}, \frac{-2}{3}, \frac{2}{3}>$ (c) $<\frac{1}{\sqrt{3}}, \frac{-2}{\sqrt{3}}, \frac{2}{\sqrt{3}}>$ (d) <–1, 2, 2>

13. The position vector of a point through which the line $\vec{r} = 2\hat{i} - \hat{j} + 4\hat{k} + \lambda(\hat{i} - \hat{j} - \hat{k})$ passes is **[NCERT Part-II, Page 382]**

(a) <2, –1, 4> (b) $\hat{i} - \hat{j} - \hat{k}$ (c) $2\hat{i} - \hat{j} + 4\hat{k}$ (d) (3, –2, 3)

14. If $\vec{a} \times \vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $|\vec{a}| = 2$, $|\vec{b}| = 1$, then the angle between $\vec{a}$ and $\vec{b}$ is **[NCERT Part-II, Page 363-364]**

(a) 30° (b) 60° (c) 90° (d) 120°

15. The line joining the points (0, 5, 4) and (1, 3, 6) has direction ratios proportional to **[NCERT Part-II, Page 379-380]**

(a) <–1, 2, –2> (b) <1, 2, 2> (c) (1, –2, 0) (d) (1, 3, 0)

16. If $\vec{a}$ and $\vec{b}$ are non-zero vectors, such that $\vec{a} \cdot \vec{b} = 0$, then **[NCERT Part-II, Page 356]**

(a) $\vec{a}$ is parallel to $\vec{b}$ (b) $\vec{a}$ and $\vec{b}$ are collinear

(c) $\vec{a}$ is perpendicular to $\vec{b}$ (d) None of these

17. If A and B are events such that $P(A|B) = P(B|A)$, then **[NCERT Part-II, Page 408]**

(a) $A \subset B$ but $A \neq B$ (b) $A = B$ (c) $A \cap B = \phi$ (d) $P(A) = P(B)$

18. If $P(A) = \frac{1}{2}$, $P(B) = 0$, then $P(A|B)$ is **[NCERT Part-II, Page 408]**

(a) 0 (b) $\frac{1}{2}$ (c) not defined (d) 1

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (a), (b), (c) and (d) as given below.)

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** For a bounded feasible region maximum or minimum of objective function lies at the corner points of the feasible region. **[NCERT Part-II, Page 398-399]**

Reason (R): Maximum or minimum lies at the corner points of feasible region.

20. **Assertion (A):** The domain of the function **[NCERT Part-I, Page 19, 21, 24]**

$\sin^{-1}x + \cos^{-1}x + \tan^{-1}x$ is $[-1, 1]$

Reason (R): $\sin^{-1}x$, $\cos^{-1}x$ are defined for $|x| \le 1$ and $\tan^{-1}x$ is defined for all x.

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. For the differential equation, find a particular solution satisfying the given condition

$(1 + \sin^2 x)\,dy + (1 + y^2)\cos x\,dx = 0$, given that, when $x = \frac{\pi}{2}$, $y = 0$. **[NCERT Part-II, Page 306-307]**

22. A and B throw a pair of dice turn by turn. The first to throw 9 is awarded a prize. If A starts the game, show that the probability of A getting the prize is $\frac{9}{17}$. **[Conceptual Application]**

OR

Assume that each born child is equally likely to be a boy or a girl. If a family has two children what is the conditional probability that both are girls, given that **[NCERT Part-II, Page 408]**

(i) the youngest is a girl.

(ii) at least one is a girl.

23. Find the length and the foot of the perpendicular drawn from the point $(2, -1, 5)$ on the line

$\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$. **[Conceptual Application]**

24. Find the value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{-1}{2}\right) + \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$. **[NCERT Part-I, Page 27]**

25. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 7$. Show that $f(A) = 0$, hence find A^7.

[NCERT Part-I, Page 47, 51]

OR

Find the matrix X so that $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$. **[NCERT Part-I, Page 51, 41]**

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. If $x = \frac{1}{y}$, then show that $\frac{dx}{dy} = -\sqrt{\frac{1+x^4}{1+y^4}}$. **[NCERT Part-I, Page 122-123]**

27. A point on the hypotenuse of a right-angled triangle is at distance a and b from the sides. Show that the minimum length of the hypotenuse is $(a^{2/3} + b^{2/3})^{3/2}$ **[NCERT Part-I, Page 166]**

28. Evaluate $\int_0^1 |3x - 2|\,dx$. **[Conceptual Application]**

OR

Find the general solution of the differential equation, $x \cos y\, dy = (xe^x \log x + e^x)dx$. **[NCERT Part-II, Page 306-307]**

29. Let S be the set of points in a plane and R be a relation in S defined as $R = \{(A, B): d(A, B) < 2\}$, where $d(A, B)$ represents the distance between the points A and B. Is R an equivalence relation? **[Conceptual Application]**

30. Using integration find the area of the region: $\{(x, y): 9x^2 + y^2 \le 36 \text{ and } 3x + y \ge 6\}$. **[Conceptual Application]**

OR

Make a rough sketch of the region given below and find its area using integration $\{(x, y): 0 \le y \le x^2 + 3; 0 \le y \le 2x + 3, 0 \le x \le 3\}$. **[Conceptual Application]**

31. Prove that the function $y = Sgn\,[x - 1]$ is not differentiable at $x = 1$. **[NCERT Part-I, Page 119]**

OR

Find $\frac{dy}{dx}$, if $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$. **[NCERT Part-I, Page 119]**

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. Find the distance of the point $A(1, 8, 4)$ and its image in the line joining the points $B(0, -1, 3)$ and $C(2, -3, -1)$. **[Conceptual Application]**

OR

If the vertices A, B, C of a ΔABC are $(1, 2, 3)$, $(-1, 0, 0)$ and $(0, 1, 2)$ respectively, what is the magnitude of $\angle ABC$? **[NCERT Part-II, Page 356]**

33. If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$. **[NCERT Part-I, Page 90]**

OR

If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1} and hence prove that $A^2 - 4A - 5I = O$. **[Conceptual Application]**

34. Show that the differential equation
$2ye^{\frac{x}{y}}\,dx + (y - 2xe^{\frac{x}{y}})\,dy = 0$ is homogeneous. Find the particular solution of this differential equation, given that $x = 0$ when $y = 1$. **[NCERT Part-II, Page 312-314]**

35. Solve the following linear programming problem graphically. **[NCERT Part-II, Page 397-398]**
Maximise $Z = x + y$
subject to constraints $x \geq 3, y \geq 4$ and $6x + 9y \leq 72$

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. In a group activity class, there are 10 students whose ages are 16, 17, 15, 14, 19, 17, 16, 19, 16 and 15 years. One student is selected at random such that each has equal chance of being chosen and age of the student is recorded.

On the basis of the above information, answer the following questions: **[Conceptual Application]**

(*i*) Find the probability that the age of the selected student is a composite number.

(*ii*) Let X be the age of the selected student. What can be the value of X?

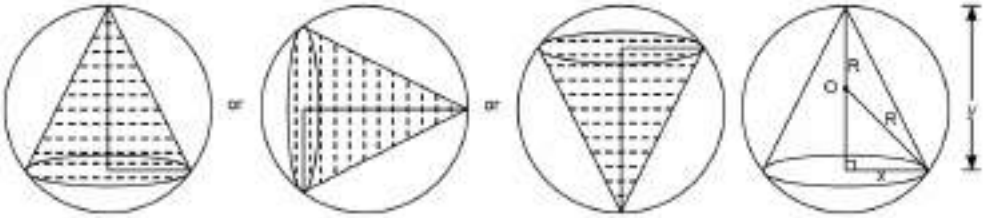
(*iii*) A student was selected and his age was found to be less than 16, what in the probability that his age is 13?

OR

(*iii*) A student was selected at random and his age was found to be greater than 15 years. Find the probability that his age is a prime number.

Case Study - 2

37. A company dealing in gift items decided to make three-dimensional gift items and a core team was given the responsibility for the same. After discussions core team suggested to insert a right circular cone in a sphere touching the surface of a sphere and at the same time gets illuminated when taking rounds. They presented the different dimensions also keeping the cost factor into account. The gift item looks like as shown **[Conceptual Application]**



(*i*) If a cone of base radius x and height y is inscribed in sphere of radius R then establish relation between x, y and R.

(*ii*) What is the volume of a cone in terms of y alone?

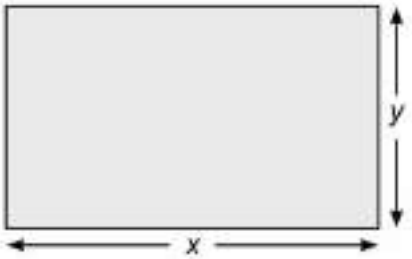
(*iii*) What should be the length of altitude of a cone so that the volume of inscribed cone is maximum?

OR

(*iii*) Find the maximum volume of a cone inscribed in a sphere.

Case Study - 3

38. Manjit wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2. **[NCERT Part-I, Page 34-35, 94]**



(*i*) Express the given situation in the form of linear equation in 2 variables.

(*ii*) Express above system of equations in matrix form and hence find dimensions of plot.

ST Francis School
Baraut

Series : SFS/02/09

SET – 9

ROLL No.

Q.P Code 12/02/09

Candidates must write the Q.P Code on the title page of the Answer book.

- Please check that this question paper contains 7 printed pages.
- Please check that this question paper contains 38 questions.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please write down the serial number of the question in the answer-book before attempting it.
- 15 minute time has been allotted to read this question paper.
- The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे | *अधिकतम अंक : 80*

Time allowed : 3 hours | *Maximum Marks : 80*

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) *This Question paper contains* ***38*** *questions. All questions are* ***compulsory.***

(ii) *This Question paper is divided into* ***five*** *Sections –* ***A, B, C, D*** *and* ***E.***

(iii) *In* ***Section A,*** *Questions no.* ***1*** *to* ***18*** *are* ***multiple choice questions*** *(MCQs) and Questions no.* ***19*** *and* ***20*** *are* ***Assertion-Reason based*** *questions of* ***1 mark each.***

(iv) *In* ***Section B,*** *Questions no.* ***21*** *to* ***25*** *are* ***Very Short Answer (VSA)-type*** *questions, carrying* ***2 marks each.***

(v) *In* ***Section C,*** *Questions no.* ***26*** *to* ***31*** *are* ***Short Answer (SA)-type*** *questions, carrying* ***3 marks each.***

(vi) *In* ***Section D,*** *Questions no.* ***32*** *to* ***35*** *are* ***Long Answer (LA)-type*** *questions, carrying* ***5 marks each.***

(vii) *In* ***Section E,*** *Questions no.* ***36*** *to* ***38*** *are* ***Case study-based questions,*** *carrying* ***4 marks each.***

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(ix) *Use of calculators is* ***not*** *allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. The minor of the element of second row and third column (a_{23}) in the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & 7 \end{vmatrix}$ is **[NCERT Part-I, Page 84]**

(a) –22 (b) 13 (c) –13 (d) 22

2. The area of the triangle with vertices (–1, 2), (4, 0) and (3, 9) is **[NCERT Part-I, Page 82]**

(a) $\frac{43}{2}$ sq units (b) $-\frac{43}{4}$ sq units (c) 21 sq units (d) 42 sq units

3. If $\begin{bmatrix} y+2x & 5 \\ -x & 3 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ -2 & 3 \end{bmatrix}$, then the value of y is **[NCERT Part-I, Page 41]**

(a) 11 (b) 3 (c) –3 (d) 1

4. The principal value of $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ is **[NCERT Part-I, Page 19]**

(a) $\frac{\pi}{3}$ (b) $-\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $-\frac{\pi}{2}$

5. Let $A = \{1, 2, 3, ..n\}$ and $B = \{a, b\}$. Then the number of surjections from A into B is [Conceptual Application]

(a) ${}^nP_2$ (b) $2^n - 2$ (c) $2^n - 1$ (d) None of these

6. The area bounded by the curve $2x^2 + y^2 = 2$ is [Conceptual Application]

(a) π sq units (b) $\sqrt{2}\pi$ sq units (c) $\frac{\pi}{2}$ sq units (d) 2π sq units

7. The area bounded by the lines $|x| - 2 + |y| = 0$ is [Conceptual Application]

(a) 1 sq unit (b) 2 sq units (c) 4 sq units (d) 8 sq units

8. The value of $\int_8^{13} \frac{\sqrt{21-x}}{\sqrt{x}+\sqrt{21-x}}\,dx$ is [NCERT Part-II, Page 274]

(a) $\frac{21}{2}$ (b) 0 (c) $\frac{5}{2}$ (d) None of these

9. The value of $\int_0^{\frac{\pi}{2}} \frac{1}{1+\tan^3 x}\,dx$ is [NCERT Part-II, Page 274]

(a) 0 (b) 1 (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

10. The absolute maximum value of $y = x^3 - 3x + 2$ in $0 \le x \le 2$ is [NCERT Part-I, Page 172]

(a) 4 (b) 6 (c) 2 (d) 0

11. It is given that at $x = 1$, the function $f(x) = x^4 - 62x^2 + ax + 9$ attains its maximum value, on the interval $[0, 2]$. The value of a is [NCERT Part-I, Page 166]

(a) 20 (b) –120 (c) 120 (d) 52

12. A function 'f' is said to be continuous at $x = a$, if [NCERT Part-I, Page 105]

(a) $\lim_{x \to a} f(x)$ exists (b) $\lim_{x \to a^-} f(x)$ does not exist

(c) $f(a)$ exists (d) None of these

13. If $y = a\sin^3 t, x = a\cos^3 t$, then $\frac{dy}{dx}$ at $t = \frac{3\pi}{4}$ is [NCERT Part-I, Page 134-135]

(a) –1 (b) $-\frac{1}{\sqrt{3}}$ (c) $-\sqrt{3}$ (d) 1

14. The general solution of the differential equation $\frac{dy}{dx} = e^{\frac{x^2}{2}} + xy$ is [NCERT Part-II, Page 322-323]

(a) $y = Ce^{\frac{-x^2}{2}}$ (b) $y = Ce^{\frac{x^2}{2}}$ (c) $y = (x + C)e^{\frac{x^2}{2}}$ (d) $y = (C - x)e^{\frac{x^2}{2}}$

15. Degree of the differential equation $\frac{d^2y}{dx^2} + \sin\left(\frac{dy}{dx}\right) = 0$ is [NCERT Part-II, Page 302]

(a) 2 (b) 1 (c) 0 (d) not defined

16. Integrating factor of the differential equation $\frac{dy}{dx} = x + y$ is [NCERT Part-II, Page 324]

(a) –1 (b) 1 (c) e^{-x} (d) e^{-y}

17. Given $P(\overline{A}) = 0.4$, $P(B) = 0.2$ and $P(A/B) = 0.5$, then $P(A \cup \overline{B})$ is [NCERT Part-II, Page 408]

(a) 0.35 (b) 0.9 (c) 0.65 (d) 0.55

18. Mother, father and son line up at random for a family picture. If A is the event 'Son on one end' and B is the event 'Father in the middle', then $P(A/B)$ is [NCERT Part-II, Page 408]

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) $\frac{1}{3}$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (a), (b), (c) and (d) as given below.)

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** The area of parallelogram with diagonals $\vec{a}$ and $\vec{b}$ is $\frac{1}{2}|\vec{a} \times \vec{b}|$ **[Conceptual Application]**

Reason (R): If $\vec{a}$ and $\vec{b}$ represent the adjacent sides of a triangle, then the area of triangle can be obtained by evaluating $|\vec{a} \times \vec{b}|$

20. **Assertion (A):** $\sin^{-1}(0.76)$ is defined. **[NCERT Part-I, Page 19]**

Reason (R): $\sin^{-1}(0.76)$ is defined because $\sin^{-1}x$ is defined for all real numbers.

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{x+1}{2-y}$, $(y \neq 2)$. **[NCERT Part-II, Page 306-307]**

22. A and B throw a coin in turn till one of them gets a 'tail' and wins the game. If A starts the game find the probability of B winning the game. **[Conceptual Application]**

OR

There is a group of 50 people who are patriotic out of which 20 believe in non-violence. Two persons are selected at random out of them. What is the probability that least one of the selected persons is non-violent. **[Conceptual Application]**

23. For any two vectors $\vec{a}$ and $\vec{b}$, prove that $(\vec{a} \times \vec{b})^2 = \vec{a}^2 \vec{b}^2 - (\vec{a} \cdot \vec{b})^2$. **[NCERT Part-II, Page 356, 363]**

24. Find the value of $\tan^2(\sec^{-1}3) - \cot^2(\operatorname{cosec}^{-1}4)$. **[Conceptual Application]**

25. For a square matrix A, show that $A + A'$ is a symmetric matrix and $A - A'$ is a skew symmetric matrix. **[NCERT Part-I, Page 63]**

OR

Solve the following matrix equation for x, $\begin{bmatrix} x & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = O$. **[NCERT Part-I, Page 94-95]**

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. If $(\sin x)^y = x + y$, find $\frac{dy}{dx}$. **[NCERT Part-I, Page 122-123]**

27. Find the intervals in which the function f given by $f(x) = \sin^4 x + \cos^4 x$ in $\left[0, \frac{\pi}{2}\right]$ is increasing or decreasing. **[NCERT Part-I, Page 152-153]**

28. Evaluate $\int_0^a \frac{1}{x+\sqrt{a^2-x^2}}\,dx$ [NCERT Part-II, Page 235-236, 274]

OR

Find the general solution of the differential equation, $2x^2\frac{dy}{dx} - 2xy + y^2 = 0$.

[NCERT Part-II, Page 313-314]

29. Show that the relation R on the set $A = \{x \in Z : 0 \le x \le 12\}$, given by $R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$ is an equivalence relation. Find the equivalence classes related to 1 and 3 respectively.

[NCERT Part-I, Page 2, 4]

30. Find the area of the region in the first quadrant enclosed by the x-axis, the line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$. [Conceptual Application]

OR

Using the method of integration, find the area of the triangle ABC, coordinates of whose vertices are $A(1, 2)$, $B(2, 0)$ and $C(4, 3)$. [Conceptual Application]

31. Differentiate $x^{x\cos x} + \frac{x^2+1}{x^2-1}$ w.r.t. x. [NCERT Part-I, Page 130]

OR

Show that the function $f(x) = |x - 3|, x \in R$ is continuous but not differentiable at $x = 3$.

[NCERT Part-I, Page 118-119]

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. If $\vec{\alpha} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{\beta} = 2\hat{i} + \hat{j} - 4\hat{k}$, then express $\vec{\beta}$ in the form $\vec{\beta} = \vec{\beta_1} + \vec{\beta_2}$, where $\vec{\beta_1}$ is parallel to $\vec{\alpha}$ and $\vec{\beta_2}$ is perpendicular to $\vec{\alpha}$. [Conceptual Application]

OR

Find the vector and Cartesian equations of the line passing through the point $A(1, 2, -4)$ and perpendicular to the lines $\frac{x-4}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-1}{1} = \frac{y+2}{-3} = \frac{z-3}{5}$. [Conceptual Application]

33. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, find adj A and verify that $A(\text{adj } A) = (\text{adj } A)A = |A|\, I_3$.

[NCERT Part-I, Page 88]

OR

Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and state how it can be used to solve the system of equations $x - y + z = 4; x - 2y - 2z = 9; 2x + y + 3z = 1$. [NCERT Part-I, Page 94-95]

34. Find the general solution of the differential equation: $(1 + y^2) + (x - e^{\tan^{-1}y})\frac{dy}{dx} = 0$

[NCERT Part-II, Page 322-323]

35. Solve the following linear programming problem (LPP) graphically. [NCERT Part-II, Page 397-398]

Maximise: $Z = 0.08x + 0.10y$

subject to the constraints, $x \ge 2000$, $y \ge 4000$,

$x + y \le 12000$.

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. Based on a situation and complaint, higher authorities decided to talk to concerned persons as the situation relates to confidential matter of the organisation. **[Conceptual Application]**

From previous experience, organisation know, that person A speaks truth in 85% of the cases and person B tells a lie in 30% of the cases.

(*i*) What is the probability of B speaking the truth?

(*ii*) What is the probability of A speaking the truth?

(*iii*) Organisation decided to have them face to face and ask questions from both, what is the probability that they contradict each other?

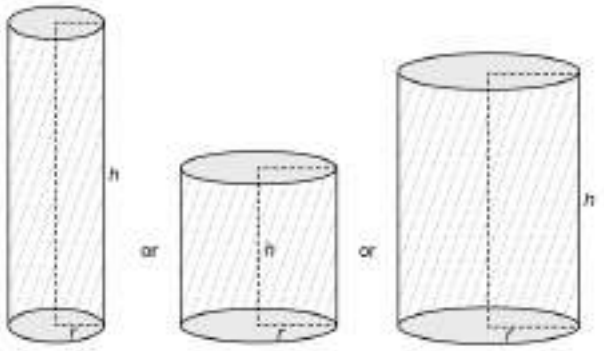
OR

(*iii*) In what per cent of cases both are speaking truth?

Case Study - 2

37. A company is launching a new product and decided to pack the product in the form of a closed right circular cylinder of volume 432π cubic units and having minimum surface area as shown. They tried different options and tried to get the solution by answering the questions given below:

[NCERT Part-I, Page 160]



(*i*) If r is radius of base of cylinder and h is the height of cylinder then establish the relation between r and h.

(*ii*) Find total surface area in terms of r only.

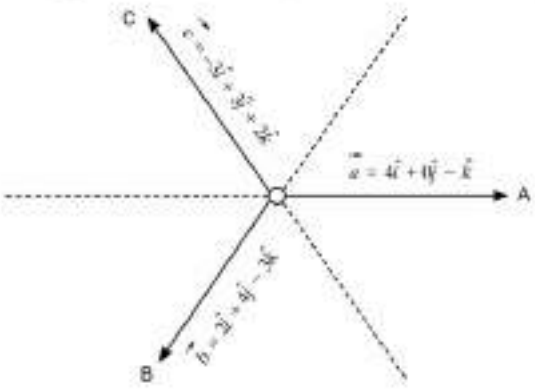
(*iii*) Find the radius r for minimum surface area.

OR

(*iii*) Find the minimum surface area.

Case Study - 3

38. Teams A, B, C went for playing a tug of war game. Teams A, B, C have attached a rope to a metal ring and is trying to pull the ring into their own area (team areas shown below). **[Conceptual Application]**



Team A pulls along vector $\vec{a} = 4\hat{i} - \hat{k}$

Team B pulls along vector $\vec{b} = 2\hat{i} + 4\hat{j} - 3\hat{k}$

Team C pulls along vector $\vec{c} = -3\hat{i} + 3\hat{j} + 2\hat{k}$

(*i*) What are direction cosines of line along which team A pulls?

(*ii*) What is the magnitude of the resultant force exerted by teams?

ST Francis School
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Series : SFS/02/10 **SET – 10**

ROLL No. **Q.P Code 12/02/10**

Candidates must write the Q.P Code on the title page of the Answer book.

- Please check that this question paper contains 7 printed pages.
- Please check that this question paper contains 38 questions.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please write down the serial number of the question in the answer-book before attempting it.
- 15 minute time has been allotted to read this question paper.
- The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे *अधिकतम अंक : 80*

Time allowed : 3 hours *Maximum Marks : 80*

General Instructions:

Read the following instructions very carefully and strictly follow them:

(*i*) *This Question paper contains 38 questions.* ***All*** *questions are* ***compulsory.***

(*ii*) *This Question paper is divided into* ***five*** *Sections – **A**, **B**, **C**, **D** and **E**.*

(*iii*) *In* ***Section A****, Questions no.* ***1*** *to* ***18*** *are* ***multiple choice questions*** *(MCQs) and Questions no.* ***19*** *and* ***20*** *are* ***Assertion-Reason based*** *questions of* ***1 mark each.***

(*iv*) *In* ***Section B****, Questions no.* ***21*** *to* ***25*** *are* ***Very Short Answer*** *(VSA)-type questions, carrying* ***2 marks each.***

(*v*) *In* ***Section C****, Questions no.* ***26*** *to* ***31*** *are* ***Short Answer*** *(SA)-type questions, carrying* ***3 marks each.***

(*vi*) *In* ***Section D****, Questions no.* ***32*** *to* ***35*** *are* ***Long Answer*** *(LA)-type questions, carrying* ***5 marks each.***

(*vii*) *In* ***Section E****, Questions no.* ***36*** *to* ***38*** *are* ***Case study-based questions****, carrying* ***4 marks each.***

(*viii*) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(*ix*) *Use of calculators is* ***not*** *allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. If $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$ and $x \in N$, then the value of x is **[NCERT Part-I, Page 77]**

(*a*) 1 (*b*) 5 (*c*) 2 (*d*) 4

2. If $A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$, then A^2 is **[NCERT Part-I, Page 50-51]**

(*a*) $\begin{bmatrix} 0 & 4 \\ 4 & 0 \end{bmatrix}$ (*b*) $\begin{bmatrix} 4 & 0 \\ 4 & 0 \end{bmatrix}$ (*c*) $\begin{bmatrix} 0 & 4 \\ 0 & 4 \end{bmatrix}$ (*d*) $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

3. If matrices A and B are inverse of each other then **[NCERT Part-I, Page 69]**

(*a*) $AB = BA$ (*b*) $AB = BA = I$ (*c*) $AB = BA = O$ (*d*) $AB = O, BA = I$

4. If $y = A \cos 2x + B \sin 2x$, then $\frac{d^2y}{dx^2}$ equals **[NCERT Part-I, Page 137]**

(*a*) y (*b*) $-y$ (*c*) $-4y$ (*d*) $4y$

5. A function $f(x) = \frac{x}{x-5}$, is not a continuous function for x equal to **[NCERT Part-I, Page 105]**

(*a*) 5 (*b*) –5 (*c*) $R - \{5\}$ (*d*) 0

6. If $y = 2^{\sqrt{x}}$, then $\frac{dy}{dx}$ is [NCERT Part-I, Page 130]

(a) $\frac{2^{\sqrt{x}}}{2\sqrt{x}}$ (b) $\sqrt{x}\ 2^{\sqrt{x}-1}$ (c) $\frac{2^{\sqrt{x}}}{2\sqrt{x}} \log_e 2$ (d) $\sqrt{x} \cdot 2^{\sqrt{x}-1} \log x^2$

7. If $f(x) = \frac{1}{4x^2 + 2x + 1}$, then its maximum value is [Conceptual Application]

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) 1 (d) $\frac{5}{4}$

8. The area bounded by the line $y = 4x$, the y-axis and the line $y = 2$ is [Conceptual Application]

(a) 2 sq units (b) 4 sq units (c) $\frac{1}{4}$ sq units (d) $\frac{1}{2}$ sq units

9. Area of the region bounded by the curve $y = \cos x$ between $x = 0$ and $x = 2\pi$ is [Conceptual Application]

(a) 4 sq units (b) 3 sq units (c) 2 sq units (d) 1 sq unit

10. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$ equals to [NCERT Part-II, Page 235-236]

(a) $2e^{\sqrt{x}} + C$ (b) $\frac{1}{\sqrt{x}} + C$ (c) $2\sqrt{x} + C$ (d) $e^{\sqrt{x}} + C$

11. $\int \frac{\sin^6 x}{\cos^8 x} dx$ is equal to [NCERT Part-II, Page 241]

(a) $\frac{\sin^7 x}{\cos^7 x} + C$ (b) $\frac{1}{7} \tan^7 x + C$ (c) $\tan^6 x + C$ (d) $\sec^8 x + C$

12. A line makes equal angles with coordinate axes, direction cosines of line are [NCERT Part-II, Page 377-378]

(a) 1, 1, 1 (b) $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$

(c) $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

13. If the direction cosines of a given line are $\frac{1}{k}, \frac{1}{k}, \frac{1}{k}$ then, the value of k is [NCERT Part-II, Page 377-378]

(a) $\frac{1}{\sqrt{2}}$ (b) $\pm\frac{1}{\sqrt{3}}$ (c) 1 (d) $\pm\sqrt{3}$

14. Given vector $\vec{a}$, then $-2\vec{a}$ is a vector whose [NCERT Part-II, Page 346]

(a) magnitude is twice that of $\vec{a}$ and direction is same as that of $\vec{a}$

(b) magnitude is twice that of $\vec{a}$ and direction is opposite to that of $\vec{a}$

(c) magnitude is same as that of $\vec{a}$ and direction is opposite to that of $\vec{a}$

(d) None of these

15. Position vectors of points A and B are $\vec{a} + \vec{b}$ and $2\vec{a} - \vec{b}$. Then $\overrightarrow{AB}$ equal to [NCERT Part-II, Page 339]

(a) $3\vec{a}$ (b) $-\vec{a} + 2\vec{b}$ (c) $\vec{a} - 2\vec{b}$ (d) None of these

16. Vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = \frac{2}{3}$ and $(\vec{a} \times \vec{b})$ is a unit vector. The angle between $\vec{a}$ and $\vec{b}$ is [NCERT Part-II, Page 364]

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

17. A pair of dice is thrown and it is known that the second die always exhibits an odd number. Then the probability that the sum obtained on two dice is 7, is [NCERT Part-II, Page 406-408]

(a) $\frac{1}{6}$ (b) $\frac{5}{6}$ (c) $\frac{1}{2}$ (d) none of these

18. A man is known to speak truth in 3 out of 4 times. He throws a dice and reports that it is a six. Then the probability that it is actually a six is **[NCERT Part-II, Page 406-408]**

(*a*) $\frac{4}{9}$ (*b*) $\frac{2}{7}$ (*c*) $\frac{1}{6}$ (*d*) $\frac{3}{8}$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (*a*), (*b*), (*c*) and (*d*) as given below.)

(*a*) Both *A* and *R* are true and *R* is the correct explanation of *A*.

(*b*) Both *A* and *R* are true but *R* is not the correct explanation of *A*.

(*c*) *A* is true but *R* is false.

(*d*) *A* is false but *R* is true.

19. **Assertion (*A*):** The smaller area enclosed by the ellipse $9x^2 + y^2 = 36$ and the line $3x + y = 6$ in first quadrant is $(3\pi - 6)$ square units. **[Conceptual Application]**

Reason (*R*): Area bounded by the curves $y = f(x), y = g(x)$ between $x = a$ and $x = b$ is $\int_a^b \{f(x) - g(x)\}dx$ $f(x) \geq g(x)$ for $x \in [a, b]$

20. **Assertion (*A*):** Both $\sin x$ and $\cos x$ are decreasing function in $\left(\frac{\pi}{2}, \pi\right)$. **[Conceptual Application]**

Reason (*R*): If a differentiable function decreases in an interval (a, b), then its derivative also decreases in (a, b).

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Find the particular solution of the differential equation $\frac{dy}{dx} = 1 + x + y + xy$, given that $y = 0$ when $x = 1$. **[NCERT Part-II, Page 306-307]**

22. Let *E* and *F* be the events with $P(E) = \frac{3}{5}, P(F) = \frac{3}{10}$ and $P(E \cup F) = \frac{7}{10}$. Are *E* and *F* independent events? **[NCERT Part-II, Page 418]**

OR

Three balls are drawn one by one with replacement from a bag containing 5 white and 4 red balls. Find the probability of drawing 2 red and 1 white balls.

23. If the sum of two unit vectors is a unit vector, then find the magnitude of their difference.

[Conceptual Application]

24. Find the principal value of $\sec^{-1}(-2)$. **[NCERT Part-I, Page 22-23]**

25. Find *A* and *B*, if $A + B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $2A + 3B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$. **[Conceptual Application]**

OR

If $f(x) = x^2 - 4x + 1$, find $f(A)$, when $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$. **[Conceptual Application]**

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. If $e^y = y^x$, show that $\frac{dy}{dx} = \frac{(\log y)^2}{\log y - 1}$. **[NCERT Part-I, Page 130]**

27. Find the intervals in which the function $f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51$ is

(*i*) strictly increasing (*ii*) strictly decreasing **[NCERT Part-I, Page 152-153]**

28. Find $\int \frac{e^x}{(e^x - 1)^2 (e^x + 2)} dx$. **[Integrated Question]**

OR

Solve the differential equation, $\frac{dy}{dx} + 2y = xe^{4x}$ **[NCERT Part-II, Page 322-323]**

29. Show that the function $f : R \to R$ defined by $f(x) = \frac{x}{x^2 + 1}$, for all $x \in R$ is not one one. **[NCERT Part-I, Page 7]**

30. Using integration, find the area of the region bounded by the triangle whose vertices are (–1, 2), (1, 5) and (3, 4). **[Conceptual Application]**

OR

Find the area bounded by the curves $y = |x - 1|$ and $y = 1$, using integration. **[Conceptual Application]**

31. Discuss the differentiability of the function $f(x) = x|x|$ at $x = 0$. **[NCERT Part-I, Page 118-119]**

OR

If $x = a\ (\cos\theta + \log\tan\frac{\theta}{2})$ and $y = a\sin\theta$, find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$. **[NCERT Part-I, Page 134-135]**

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. Show that the line through the points (1, – 1, 2) and (3, 4, – 2) is perpendicular to the line through the points(0, 3, 2) and (3, 5, 6). **[Conceptual Application]**

OR

Find the shortest distance between the lines **[NCERT Part-II, Page 387-388]**

$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$.

33. Express the following matrix as the sum of a symmetric and a skew symmetric matrix and verify your result: **[NCERT Part-I, Page 64]**

$$\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

OR

If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1} and hence prove that $A^2 - 4A - 5I = O$. **[Conceptual Application]**

34. Find the general solution of the differential equation $\frac{dy}{dx} - y = \sin x$. [NCERT Part-II, Page 322-323]

35. Solve the following *LPP* graphically [NCERT Part-II, Page 397-398]

Minimise $Z = 5x + 10y$

subject to constraints

$x + 2y \leq 120$

$x + y \geq 60$

$x - 2y \geq 0$ and $x, y \geq 0$

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. During the interview a candidate is asked to pick three cards out of 8 black and 2 white cards which are kept upside down. Maximum marks are allotted to candidate who picks both the white cards and least marks are allotted when none of the white cards is picked. **[Conceptual Application]**

(*i*) What is the probability that he pickup 3 white cards?

(*ii*) What is the probability that he will pick exactly 1 white card ?

(*iii*) What is the probability of getting maximum marks?

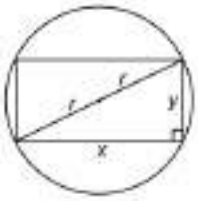
OR

(*iii*) Find the probability that he gets the least marks in an interaction.

Case Study - 2

37. Organisations innovate different methods to keep their employees in good mental health. For that they give them freedom to work from any place. A company for their employees developed an area with a rectangular space enclosed by circle for refreshing and working as shown.

[Conceptual Application]



(*i*) What does rectangle enclosed by a circle means?

(*ii*) If a rectangle of sides x and y is inscribed in a circle of radius r, then establish relation between x, y and r.

(*iii*) Find the area of a rectangle in terms of x only.

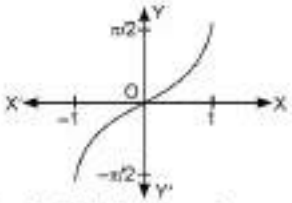
OR

(*iii*) Find the maximum area where employees can sit and work or enjoy.

Case Study - 3

38. Learning and evaluation go side by side and it helps the student to judge himself/herself where he/she stands regarding understanding of the subject. One day in Mathematics class, teacher asked the students to answer the questions with respect to the graph drawn on the board.

[NCERT Part-I, Page 20]



(*i*) The figure represents graph of which inverse trigonometric function?

(*ii*) What is the value of the given function, where $x = \frac{-1}{2}$?