KENDRIYA VIDYALAYA SANGATHAN
HYDERABAD REGION

MATHS MODEL PAPERS MARKING SCHEME

CLASS - 12

INDEX

KENDRIYA VIDYALAYA SANGATHAN: HYDERABAD REGION

MODEL QUESTION PAPER - 1 (2025 - 26)

SUBJECT: MATHEMATICS (041)

MARKING SCHEME

………

SECTION: A (Solution of MCQs of 1 Mark each)

1	2	3	4	5	6	7	8	9	10
B	A	C	A	D	D	A	B	D	C
11	12	13	14	15	16	17	18	19	20
A	D	D	C	A	A	C	C	D	C

SECTION-B

Q.No.	**HINTS/SOLUTION**	Marks
21.	Let $p = \|a\|b + \|b\|a \;\; and \;\; q = \|a\|b - \|b\|a$ Then, $p.q = (\|a\|b + \|b\|a).(\|a\|b - \|b\|a)$ For proving $p.q = 0$ Therefore, $\|a\|b + \|b\|a$ and $\|a\|b - \|b\|a$ are perpendicular to each other for any non zero vector a and b.	½ 1 ½
22.	Given $A = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$ For $A^2 == \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix}$ Simplifying, $A^2-5A+4I+X=0$ For $X = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{pmatrix}$	1 ½ ½
23.	Let $y = (sinx)^y$ Taking log on both sides, we get logy = ylog(sinx) on differentiating both sides, we get $$\frac{dy}{dx} = \frac{y^2 cotx}{(1 - ylogsinx)}$$ (OR) $y = \log tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$ Differentiating with respect to x $\frac{dy}{dx} = \frac{1}{cosx}$ $\frac{dy}{dx} - secx = 0$	½ 1 ½ $1\frac{1}{2}$ $\frac{1}{2}$

<table>
<tr><td>24.</td><td>$Let\ I = \int_0^{2\pi} |sinx|\, dx$
$$I = \int_0^{\pi} |sinx|\, dx + \int_{\pi}^{2\pi} |sinx|\, dx$$
$I = \int_0^{\pi} sinx\, dx - \int_{\pi}^{2\pi} sinx\, dx = [-cosx]_0^{\pi} - [-cosx]_{\pi}^{2\pi} = 4$
(OR)
A=$\int_0^3 y$ dx = $\int_0^3 \sqrt{x}$ dx
A=$2\sqrt{3}$ sq. units</td><td>1
1
1
1</td></tr>
<tr><td>25.</td><td>Given line is $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2} = \mu$ then,
General point on the line is R$(3\mu - 2, 2\mu - 1, 2\mu + 3)$
Distance from R to P(1, 3, 3) is5 units
$\therefore \mu = 0, 2.$
∴ R(-2, -1, 3) or R(4 , 3 , 7)</td><td>1
1</td></tr>
</table>

SECTION-C

<table>
<tr><td>26.</td><td>Given, $3y = ax^2 + 1 \ldots (i)$
And, $At\ x = 1, \frac{dy}{dt} = 2\frac{dx}{dt} \ldots (ii)$
Now differentiating equation (i) w.r.t. t, we get
$$3\frac{dy}{dt} = 3ax^2\frac{dx}{dt}$$
Put the value of $\frac{dy}{dt}$ from equation(ii), we get
a = 2
Hence the value of a=2.
(OR)
For correct $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$
For $\frac{dy}{dx} = cot\frac{\theta}{2}$
For $\frac{d^2y}{dx^2} = -\frac{1}{4a}cosec^4\frac{\theta}{2}$</td><td>1
1
1
1
1
1</td></tr>
<tr><td>27.</td><td>Given, $\int_0^{\frac{\pi}{4}} \frac{sinx+cosx}{9+16sin2x}dx$
Since, $sin2x = 1 - (sinx - cosx)^2$
Let, $sinx - cosx = t\ then, dx = \frac{dt}{sinx+cosx}$
When, x=0 =>t=-1 and when x=$\frac{\pi}{4}$ $=> t = 0$
Simplifying $I = \int_{-1}^{0} \frac{1}{9+16(1-t^2)}dt = \frac{1}{40}log9$</td><td>1
2</td></tr>
<tr><td>28.</td><td>Given, $|\overrightarrow{a}| = 3, |\overrightarrow{b}| = 4\ and |\overrightarrow{c}| = 2.$ and $\vec{a} + \vec{b} + \vec{c} = \overrightarrow{0}$
So, $\overrightarrow{(a + b + c)}\overrightarrow{(a + b + c)} = 0$
Simplifying, $\mu = \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -\frac{29}{2},$</td><td>1
2</td></tr>
<tr><td>29.</td><td>Given differential equation $(xy - x^2)dy = y^2dx.$</td><td></td></tr>
</table>

	$\frac{dy}{dx} = \frac{y^2}{(xy-x^2)}$ ……..(i) homogeneous. Put y=vx => $\frac{dy}{dx} = v + x\frac{dv}{dx}$ Simplifying and getting solution as V=logvcx $$\frac{y}{x} = logcy => y = ce^{\frac{y}{x}}$$ (OR) Given, $(x^2+1)\frac{dy}{dx} + 2xy = \sqrt{x^2+4}$. $\frac{dy}{dx} + \frac{2xy}{x^2+1} = \frac{\sqrt{x^2+4}}{(x^2+1)}$, Equation is the linear form. . I.F= $e^{\int \frac{2x}{x^2+1}dx} = x^2+1$ Solution is $$(x^2+1)y = \frac{x}{2}\sqrt{x^2+4} + 2\, log\, log\, \left|x+\sqrt{x^2+4}\right| + C$$	1 1 1 1 2
30.	Minimize Z = 5x+10y subject to the constraints; $x+2y \le 120;\ x+y \ge 6;\ x-2y \ge 0;\ x,y \ge 0$ For appropriate diagram, Possible points for maximum Z are A(40,20), B(60,0), $C(120,0)$ and D(60,30) Therefore the Minimum value of Z is 300. at B(60, 0)	 1 1 1
31.	No.of coins with head on both sides=(n-1) No.of fair coins=(n+2) Let event E_1:Picking a coin with head on both sides ; P(E_1)=$\frac{n-1}{2n+1}$ E_2:Picking a fair coin ; P(E_2)= $\frac{n+2}{2n+1}$ A:Getting a head on tossing the coin P(A/E_1)=1 ; P(A/E_2)=1/2 P(A)=P(E_1)P(A/E_1)+P(E_2)P(A/E_2)= $\frac{3n}{2(2n+1)} = \frac{31}{42}$ $or\ n = 31.$ (OR) Let the events be: A: Mehul is selected B: Rashi is selected Then according to the question, A and B are independent events and $P(A)$=0.4, $P(A\cap\overline{B})$ +$P(B\cap\overline{A})$ =0.5 Let $P(B)$=x Then $P(A\cap\overline{B})$ +$P(B\cap\overline{A})$ =0.5 ⇒$P(A)P(\overline{B})$+$P(B)P(\overline{A})$=0.5 ⇒0.4(1−x) +x(1−0.4) =0.5 ⇒0.4−0.4x+0.6x=0.5 ⇒0.2x=0.5−0.4=0.1 ⇒x=$\frac{0.1}{0.2}$=$\frac{1}{2}$ = 0.5	 1 1 1 1

	So, probability of selection of Rashi = 0.5 Probability of selection of at least one of them = 1−$P(\bar{A}\cap\bar{B})$ =1−$P(\bar{A})P(\bar{B})$ =1−0.6×0.5 =1−0.3=0.7	1 1

SECTION-D

32.	Let equation of line through (1, 1, 1) be $\frac{x-1}{a}=\frac{y-1}{b}=\frac{z-1}{c}$……………….(i) Line(i) perpendicular to the lines $\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4}$ and $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ Therefore a+2b+4c=0 and 2a+3b+4c=0 ∴ dr's are -4, 4, -1 and 4, -4, 1 ∴ Cartesian equation is $\frac{x-1}{4}=\frac{y-1}{-4}=\frac{z-1}{1}$ and vector equation is r=(i+j+k)+$\mu(4i-4j+k)$ and $\theta = cos^{-1}(\frac{24}{\sqrt{609}})$.	½ 1 1 1 ½ ½ ½
33.	Let$\frac{1}{x}=p;\ \frac{1}{y}=q; and\ \frac{1}{z}=r,$ then the given equations become 2p+3q+10r=4; 4p-6q+5r=1; 6p+9q-20r=2 |A|=1200 Thus A is non singular matrix. Therefore, its inverse exist. adjA= $\begin{pmatrix} 75 & 150 & 75 \\ 110 & -110 & 30 \\ 72 & 0 & -24 \end{pmatrix}$ $A^{-1}=\frac{1}{1200}\begin{pmatrix} 75 & 150 & 75 \\ 110 & -110 & 30 \\ 72 & 0 & -24 \end{pmatrix}$ Simplifying, we get $x=2; y=3; and\ z=5$ (OR) AB = I So, A^{-1}=B and B^{-1}=A Given system of equations is 3x−6y−z=3, 2x−5y−z+2=0, −2x+4y+z=5 In matrix form it can be written as: $AX=C$, where $X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}[xyz]$ and $C=\begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix}$ Here |A|=−3−0+2=−1≠0 So, the system is consistent and has unique solution given by the expression $X=A^{-1}C=BC$ X=$\begin{bmatrix} 2 \\ -3 \\ 21 \end{bmatrix}$; Thus x=2,y=−3,z=21	1 2 1 1 1 ½ ½ ½ ½ ½ 1 ½
34.	Let $f(x)=x+sin2x,\ \ f'(x)=1+2cos2x$	1

	For maxima or minima put $f'(x) = 0$, we get $$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \epsilon [0,2\pi].$$ Then, we evaluate the value of f at critical points $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ and at the end points of the interval$[0,2\pi]$. We get the maximum value 2π at $x = 2\pi$ and minimum value is 0 at x=0.	2 1 1
35.	Area bounded by the ellipse = 4×(Area of shaded region in the first quadrant only) = 4× $\int_{x=a}^{y=b} \|y\|\, dx$ = 4× $\int_0^2 \|y\|\, dx => 4\int_0^2 \frac{3}{2}\sqrt{4-x^2}\ dx$ $$4\int_0^2 \frac{3}{2}\sqrt{2^2-x^2}\ dx$$ $$=> 6[\frac{x}{2}\sqrt{4-x^2} + \frac{2^2}{2} sin^{-1}\frac{x}{2}]_0^2$$ Required area =$6\pi\ sq. unit$.	1 1 2 1

SECTION-E

Case Study-1

36.	(i)(A) For f(x) to be defined x - 2 ≠ 0 . i.e., x ≠ 2 ∴ Domain of f = R – {2} (ii).B Let y=f(x), then $y = \frac{x-1}{x-2} => x = \frac{2y-1}{y-1}$ Since, $x \in R - \{2\}, therefore\ y \neq 1$ Hence, range of $f = R - \{1\}$ (iii). (D) We have ,$g(x) = 2f(x) - 1 => 2\left(\frac{x-1}{x-2}\right) - 1 => \frac{x}{x-2}$ OR (iii). (A)	1 1 2

Case Study-2

37.	(i)(C) (ii)(C) Area(A)=length*breadth A=xy $$A = x\left(\frac{p-2x}{2}\right) ==> \frac{px-2x^2}{2}$$ (iii)(D) Put $\frac{dA}{dx} = 0$, we get $x = \frac{p}{4}$. And $\frac{d^2A}{dx^2} < 0$, Therefore, area is maximum at $x = \frac{p}{4}$	1 1

	(OR) (i)(C) We have $$y=\frac{p-2x}{2}=\frac{p}{2}-\frac{p}{4}=\frac{p}{4}$$	2

Case Study-3

38	(i) P (Shell fired from exactly one of them hits the plane)= P [(Shell from A hits the plane and shell from B does not hit the plane) or (shell from A does not hit the plane and shell from B hits the plane)]=0.3 × 0.8 + 0.7 × 0.2 = 0.24 + 0.14 = 0.38. (ii) P (Shell fired from B hit the plane/Exactly one of them hit the plane) $=\frac{P(shell\ fired\ from\ B\ hit\ the\ plane \cap Exactly\ one\ of\ them\ hit\ the\ plane)}{P(Exactly\ one\ of\ them\ hit\ the\ plane)}$ $=\frac{P(Shell\ from\ only\ B\ hit\ the\ plane)}{P(Exactly\ one\ of\ them\ hit\ the\ plane)}$ $=\frac{0.14}{0.38}=\frac{7}{19}$ (Since,$P(\underline{A}\cap B)=P(\underline{A})\times P(B)=(1-0.3)\times 0.2=0.14$)	2 2

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KENDRIYA VIDYALAYA SANGATHAN: HYDERABAD REGION

MODEL QUESTION PAPER - 2 (2025 - 26)

SUBJECT: MATHEMATICS (041)

MARKING SCHEME

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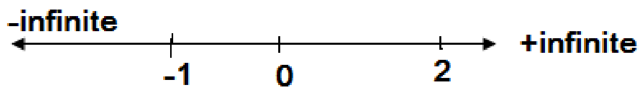
SECTION-A		
1	D	1
2	A	1
3	C	1
4	A	1
5	B	1
6	D	1
7	A	1
8	C	1
9	D	1
10	B	1
11	C	1
12	A	1
13	D	1
14	C	1
15	D	1
16	B	1
17	A	1
18	D	1
19	A	1

20	D	1
	SECTION-B	
21	We know that, $(\cos^{-1} cos\, x\,) = x, if\ x \in [0, \pi]$. Now, $\left(\cos^{-1} cos\, \frac{13\pi}{6}\right) = \left[\cos^{-1} cos\, \left(2\pi + \frac{\pi}{6}\right)\right], \left(\because\ \frac{13\pi}{6} \notin [0, \pi]\right)$ $= \left[\cos^{-1} cos\, \frac{\pi}{6}\right], (\because\ \cos^{-1} cos\,(2\pi + x) = cos\ cos\, x\ ) = \frac{\pi}{6}, \left(\because\ \frac{\pi}{6} \in [0, \pi]\right)$ **OR** $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$ $= \frac{\pi}{3} - (\pi - \sec^{-1} 2\,), [\because\ (-x) = \pi - x\ ]$ $= \frac{\pi}{3} - \left(\pi - \frac{\pi}{3}\right)$ $= \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$	1 1 1 1
22	x = a $\tan^3 \theta$ Differentiating w.r.t. θ $\frac{dx}{d\theta}$ = a ×3 $\tan^2 \theta \sec^2 \theta$ Again, y = a $\sec^3 \theta$ Differentiating w.r.t. θ $\frac{dy}{d\theta}$ = a ×3 $\sec^2 \theta \sec \theta \tan \theta$ = 3 a $\tan \theta \sec^3 \theta$ $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ =3 a $\tan \theta \sec^3 \theta$/ a ×3 $\tan^2 \theta \sec^2 \theta$ $= \frac{sec\ \theta}{tan\ \theta}$ = cosec θ	0.5 0.5 1
23	$\int \frac{sec^2 x}{\sqrt{tan^2 x + 4}}\, dx$ $Let, tan\ x = z$ $\therefore sec^2 x\ dx = dz$ $\int \frac{sec^2 x}{\sqrt{tan^2 x + 4}}\, dx = \int \frac{dz}{\sqrt{z^2 + 2^2}}$	0.5

	$= log\ \left\|z + \sqrt{z^2 + 2^2}\right\| + c$ $= log\ \left\|tan\, x\ + \sqrt{tan^2 x + 4}\right\| + c$ **OR** $\int \sqrt{1 - sin\ 2x}\, dx, \qquad \frac{\pi}{4} < x < \frac{\pi}{2}$ $= \int \sqrt{sin^2 x + cos^2 x - 2. sin\ x\ . cos\ x}\ dx$ $= \int \sqrt{(sin\ x\ - cos\ x\ )^2} dx$ $= \int (sin\ x\ - cos\ x\ ) dx\ \left(in\ \frac{\pi}{4} < x < \frac{\pi}{2}, sin\ x\ \ is\ greater\ than\ cos\ x\ \right)$ $= -cos\ x\ - sin\ x\ + c$	0.5 1 1 1
24	$\vec{a} \times \vec{b} = \|\vec{a}\|\|\vec{b}\|\ sin\ \theta\ \ \hat{n}\ , where\ \hat{n}\ is\ a\ unit\ vector.$ $\Rightarrow \left\|\vec{a} \times \vec{b}\right\| = \left\|\|\vec{a}\|\left\|\vec{b}\right\| sin\ \theta\ \hat{n}\right\|$ $\Rightarrow \sqrt{(-3)^2 + 1^2 + 2^2} = \|\ \vec{a}\|\left\|\ \vec{b}\right\|\|sin\ \theta\ \ \|\|\hat{n}\ \|$ $\Rightarrow \sqrt{9 + 1 + 4} = 2.7. \|sin\ \theta\ \ \|. 1$ $\Rightarrow \sqrt{14} = 14\ \|sin\ \theta\ \ \|$ $\Rightarrow \frac{\sqrt{14}}{14} = \ \|sin\ \theta\ \ \|$ $\Rightarrow \frac{1}{\sqrt{14}} = \ \|sin\ \theta\ \ \|$ $\Rightarrow \|sin\ \theta\ \ \| = \frac{1}{\sqrt{14}}$ $\Rightarrow sin\ \theta = \pm \frac{1}{\sqrt{14}}$ $\Rightarrow \theta = \left(\pm \frac{1}{\sqrt{14}}\right)$	1 1
25	$S = \{1, 2, 3, 4, 5, 6\}$ $A = \{\ 2, 4, 6\}$ $B = \{1, 2, 3\}$	

	$A \cap B = \{2\}$ $P(A) = \frac{3}{6} = \frac{1}{2}$ $P(B) = \frac{3}{6} = \frac{1}{2}$ $P(A \cap B) = \frac{1}{6}$ $P(A).P(B) = \frac{1}{2}.\frac{1}{2} = \frac{1}{4}$ $\therefore P(A \cap B) \neq P(A).P(B)$ $Hence, A\ and\ B\ are\ not\ independent$	1.5 0.5
SECTION-C		
26	Here, $(\cos y)^x = (\sin x)^y$ Taking log both sides we get $\log (\cos y)^x = \log (\sin x)^y$ Or, $x \log \cos y = y \log \sin x$ Or, $y \log \sin x = x \log \cos y$ Differentiating both sides w.r.t x $y \times \frac{\cos x}{\sin x} + \log \sin x \frac{dy}{dx} = x \times \frac{-\sin y}{\cos y} \frac{dy}{dx} + \log \cos y$ Or, $y \cot x + \log \sin x \frac{dy}{dx} = -x \tan y \frac{dy}{dx} + \log \cos y$ Or, $\frac{dy}{dx}(\log \sin x + x \tan y) = \log \cos y - y \cot x$ Or, $\frac{dy}{dx} = \frac{\log \cos y - y \cot x}{\log \sin x + x \tan y}$	1 1 1
27	$Given\ f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ Differentiating both sides w.r.t. x $f'(x) = 12x^3 - 12x^2 - 24x^1 = 12x(x^2 - x - 2) = 12x(x+1)(x-2)$	

		1
	$f'(x) < 0 \; if \; x \in (-\infty,-1) \cup (0,2)$	1
	$\therefore f(x) \; is \; strictly \; decreasing \; in (-\infty,-1) \cup (0,2)$	1
	$Similarly, f'(x) > 0 \; if \; x \in (-1,0) \cup (2,\infty)$	
	$\therefore f(x) \; is \; strictly \; increasing \; in \; (-1,0) \cup (2,\infty)$	
28	$\int_0^{\frac{\pi}{4}} \frac{\sin x \; + \cos x}{16 + 9\sin 2x} dx$	
	$= \int_0^{\frac{\pi}{4}} \frac{\sin x \; + \cos x}{16 + 9[1 - (1 - \sin 2x \;)]} dx$	
	$= \int_0^{\frac{\pi}{4}} \frac{\sin x \; + \cos x}{16 + 9[1 - (\sin x \; - \cos x \;)^2]} dx$	
	x: 0, $\frac{\pi}{4}$	
	z: -1, 0	1
	$= \int_0^{\frac{\pi}{4}} \frac{\sin x \; + \cos x}{25 - 9(\sin x \; - \cos x \;)^2} dx$	0.5
	$Let, \sin x \; - \cos x \; = z$	
	$\therefore (\cos x \; + \sin x \;)dx = dz$	
	$= \int_{-1}^{0} \frac{dz}{25 - 9z^2}$	
	$= -\int_{-1}^{0} \frac{dz}{9z^2 - 25}$	
	$= -\int_{-1}^{0} \frac{dz}{(3z)^2 - 5^2}$	1
	$= -\frac{1}{2.5.3}\left[log \; \left\lvert\frac{3z-5}{3z+5}\right\rvert\right]_{-1}^{0}$	

$$= -\frac{1}{30}[log\,1 - log\,4\ ]$$

0.5

$$= \frac{1}{30} log\,4$$

OR

$$\int_0^a f(a-x)dx$$

$$Let, a - x = z$$

x	0	a
z	a	0

$$\therefore dx = -dz$$

$$= -\int_a^0 f(z)dz$$

$$= \int_0^a f(z)dz$$

$$= \int_0^a f(x)dx$$

$$\therefore \int_0^a f(x)dx = \int_0^a f(a-x)dx\ (proved)$$

1

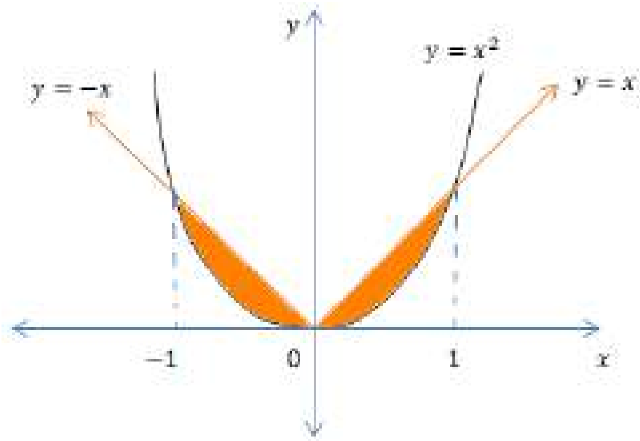
$$\int_0^1 x^2(1-x)^n dx$$

$$= \int_0^1 (1-x)^2[1-(1-x)]^n dx, \left(Applying \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right)$$

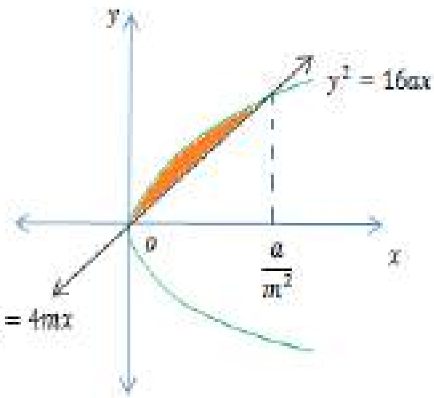
1

$$= \int_0^1 (1-x)^2 x^n dx$$

$$= \int_0^1 (1-2x+x^2)x^n dx$$

<table>
<tr><td></td><td>$= \int_0^1 (x^n - 2x^{n+1} + x^{n+2})dx$
$= \left[\frac{x^{n+1}}{n+1} - 2.\frac{x^{n+2}}{n+2} + \frac{x^{n+3}}{n+3}\right]_0^1$
$= \left[\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3}\right]$</td><td>1</td></tr>
<tr><td>29</td><td>$y = x^2 \dots (i)$
$y = |x| \dots (ii)$
$Solving (i) \ and \ (ii), we get$
$|x| = x^2$
$\Rightarrow x^2 = x^4$
$\Rightarrow x^2(1 - x^2) = 0$
$\Rightarrow x = -1,0,1$


$The \ required \ area \ is:$
$= 2\left[\int_0^1 y_l dx - \int_0^1 y_p dx\right]$
$= 2\left[\int_0^1 x dx - \int_0^1 x^2 dx\right]$
$= 2\left[\frac{1}{2}[x^2]_0^1 - \frac{1}{3}[x^3]_0^1\right]$
$= 2\left[\frac{1}{2}(1-0) - \frac{1}{3}(1-0)\right]$
$= 2\left[\frac{1}{2} - \frac{1}{3}\right]$
$= 2.\frac{1}{6}$</td><td>1.5
1
0.5
1</td></tr>
</table>

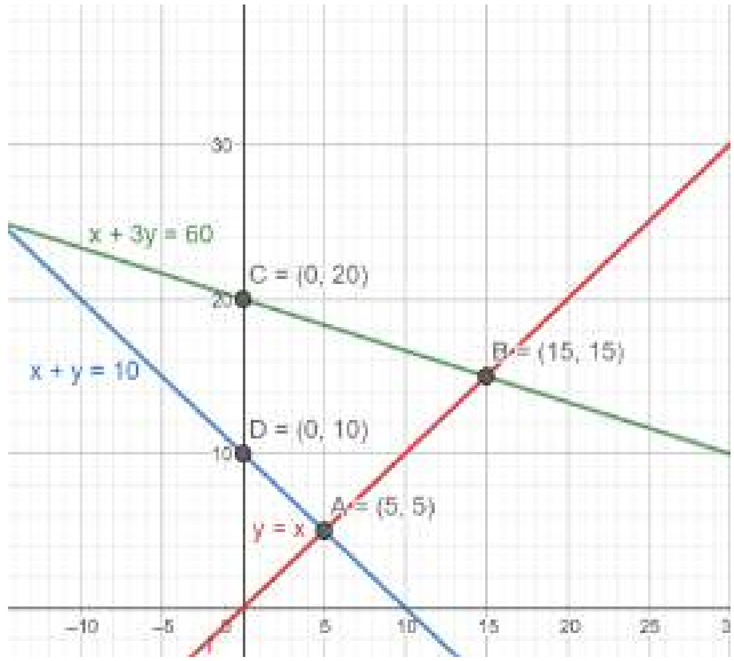
	$=\frac{1}{3} sq.units$ **OR** $$y^2 = 16ax \dots (i)$$ $$y = 4mx \dots (ii)$$ $$Solving (i)\ and\ (ii), we get$$ $$(4mx)^2 = 16ax$$ $$\Rightarrow 16m^2x^2 - 16ax = 0$$ $$\Rightarrow 16x(m^2x - a) = 0$$ $$\Rightarrow x = 0, \frac{a}{m^2}$$  $$The\ required\ area\ is:$$ $$= \int_0^{\frac{a}{m^2}} y_p dx - \int_0^{\frac{a}{m^2}} y_l dx$$ $$= \int_0^{\frac{a}{m^2}} 4\sqrt{a}\sqrt{x} dx - \int_0^{\frac{a}{m^2}} 4mx dx$$ $$= 4\sqrt{a}.\frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^{\frac{a}{m^2}} - \frac{4m}{2}[x^2]_0^{\frac{a}{m^2}}$$ $$= \frac{8\sqrt{a}}{3}.\frac{a}{m^2}.\sqrt{\frac{a}{m^2}} - 2m.\frac{a^2}{m^4}$$ $$= \frac{8a^2}{3m^3} - \frac{2a^2}{m^3}$$ $$= \frac{2a^2}{3m^3} sq.units$$ $$\therefore \frac{2a^2}{3m^3} = \frac{a^2}{12}$$ $$\Rightarrow m^3 = 8$$ $$\Rightarrow m = 2$$	0.5 0.5 0.5 0.5
30	$$\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$$	

	$$\Rightarrow \frac{dy}{dx} = (1+x^2) + y^2(1+x^2)$$ $$\Rightarrow \frac{dy}{dx} = (1+x^2)(1+y^2)$$ $$\Rightarrow \frac{dy}{1+y^2} = (1+x^2)dx$$	1
	$$\Rightarrow \int \frac{dy}{1+y^2} = \int (1+x^2)dx$$ $$\Rightarrow y = x + \frac{x^3}{3} + c$$ $$It\ is\ given\ that\ y = 0\ when\ x = 1$$	1
	$$\therefore 0 = 1 + \frac{1}{3} + c$$ $$\Rightarrow c = -\frac{4}{3}$$ $$Hence, the\ particular\ sol^n\ is:$$ $$y = x + \frac{x^3}{3} - \frac{4}{3}$$ **OR** $$x\frac{dy}{dx} sin\ sin\ \left(\frac{y}{x}\right) + x - y\ sin\ sin\ \left(\frac{y}{x}\right) = 0$$	1
	$$\Rightarrow \frac{dy}{dx} = \frac{y\ sin\ sin\ \left(\frac{y}{x}\right) - x}{x\ sin\ sin\ \left(\frac{y}{x}\right)} \ldots(i)$$ $$Let, f(x,y) = \frac{y\ sin\ sin\ \left(\frac{y}{x}\right) - x}{x\ sin\ sin\ \left(\frac{y}{x}\right)}$$ $$\therefore\ f(\lambda x, \lambda y) = \frac{\lambda y\ sin\ sin\ \left(\frac{\lambda y}{\lambda x}\right) - \lambda x}{\lambda x\ sin\ sin\ \left(\frac{\lambda y}{\lambda x}\right)} = \frac{y\ sin\ sin\ \left(\frac{y}{x}\right) - x}{x\ sin\ sin\ \left(\frac{y}{x}\right)} = f(x,y)$$ $Hence, f\ is\ a\ homogeneous\ function\ of\ degree\ 0.\ Let, y = vx$ $$Differentiating\ both\ sides\ w.r.t.x$$	1

	$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$ $$Hence,\ eq^n(i)\ becomes$$ $$v + x\frac{dv}{dx} = \frac{vx\ sin\ sin\ v\ - x}{x\ sin\ sin\ v}$$ $$\Rightarrow vx\frac{dv}{dx} = \frac{vx\ sin\ sin\ v\ - x}{x\ sin\ sin\ v} - v$$ $$\Rightarrow x\frac{dv}{dx} = \frac{vx\ sin\ sin\ v\ - x - vx\ sin\ sin\ v}{x\ sin\ sin\ v}$$ $$\Rightarrow x\frac{dv}{dx} = \frac{-x}{x\ sin\ sin\ v}$$ $$\Rightarrow x\frac{dv}{dx} = -\frac{1}{sin\ sin\ v}$$ $$\Rightarrow sin\ sin\ v\ dv = -\frac{dx}{x}$$ $$\Rightarrow \int\ sin\ sin\ v\ dv = -\int\ \frac{dx}{x}$$ $$\Rightarrow -\ cos\ cos\ v\ = -\ log\ log\ \|x\| + c$$ $$\Rightarrow -\ cos\ cos\ \left(\frac{y}{x}\right)\ = -\ log\ log\ \|x\| + c$$ $$It\ is\ given\ that\ y = \frac{\pi}{2}\ when\ x = 1$$ $$\therefore -\ cos\ cos\ \frac{\pi}{2}\ = -\ log\ log\ 1\ + c$$ $$\Rightarrow -0 = -0 + c$$ $$\Rightarrow c = 0$$ $$Hence, the\ complete\ sol^n\ is:$$ $$-\ cos\ cos\ \left(\frac{y}{x}\right)\ = -\ log\ log\ \|x\|$$ $$\Rightarrow cos\ cos\ \left(\frac{y}{x}\right)\ = log\ log\ \|x\|$$	0.5 0.5 0.5 0.5

31

The region bounded by the points A, B, C and D is the feasible region.



1.5

Corner Points	$Maximize\ Z = 3x + 9y$
$A(5, 5)$	60
$B(15, 15)$	180
$C(0, 20)$	180
$D(0, 10)$	90

1

0.5

Maximum value of Z is 180 and which is at any point on the line segment joining B and C.

SECTION-D

32

<u>**For Reflexive:**</u>

$ab = ba$, for all $a, b \in N$.

$\therefore\ (a, b)\ R\ (a, b)$, for all $a, b \in N$.

Hence, R is reflexive.

1.5

<u>**For Symmetric:**</u>

Let, $a, b, c, d \in N$ such that $(a, b)\ R\ (c, d)$.

$$(a, b)\ R\ (c, d)$$

$$\Rightarrow ad = bc$$

$$\Rightarrow da = cb$$

$$\Rightarrow cb = da$$

Hence, $(c, d)\ R\ (a, b)$.

$\therefore (a, b)\ R\ (c, d) \Rightarrow (c, d)\ R\ (a, b)$, for all $a, b, c, d \in N$.

Hence, R is symmetric.

1.5

<u>For Transitive:</u>

Let, $a, b, c, d, e, f \in N$ such that $(a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f)$.

$(a, b)\ R\ (c, d)$; $(c, d)\ R\ (e, f)$

$$\Rightarrow ad = bc\ ; cf = de$$

$$\Rightarrow \frac{c}{d} = \frac{a}{b}\ ; \frac{c}{d} = \frac{e}{f}$$

$$\Rightarrow \frac{a}{b} = \frac{e}{f}$$

$$\Rightarrow af = be$$

Hence, $(a, b)\ R\ (e, f)$.

$\therefore (a, b)\ R\ (c, d)\ ;\ (c, d)\ R\ (e, f) \Rightarrow (a, b)\ R\ (e, f)$, for all $a, b, c, d, e, f \in N$.

Hence, R is transitive.

Hence, R is an equivalence relation on $N \times N$.

OR

2

<u>For one-one:</u>

Let, $x_1, x_2 \in N\ (Domain)$ such that $f(x_1) = f(x_2)$

$$\Longrightarrow {x_1}^2 + x_1 + 1 = {x_2}^2 + x_2 + 1$$

$$\Longrightarrow {x_1}^2 - {x_2}^2 + x_1 - x_2 = 0$$

$$\Longrightarrow (x_1 + x_2)(x_1 - x_2) + (x_1 - x_2) = 0$$

$$\Longrightarrow (x_1 - x_2)(x_1 + x_2 + 1) = 0$$

$$\Longrightarrow (x_1 - x_2) = 0\ (\because x_1, x_2 \in N\ \Longrightarrow x_1 + x_2 + 1 \neq 0)$$

$$\Longrightarrow x_1 = x_2$$

$\therefore f(x_1) = f(x_2) \Longrightarrow x_1 = x_2\ for\ all\ x_1, x_2 \in N\ (Domain).$

Hence, f is one-one.

<u>For onto:</u> 2.5

Let us take 1 from $N\ (Co - domain)$ such that $f(x) = 1$

$$\Longrightarrow x^2 + x + 1 = 1$$

$$\Longrightarrow x^2 + x = 0$$

$$\Longrightarrow x(x + 1) = 0$$

$$\Longrightarrow x = 0, -1$$

But, $0, -1 \notin N\ (Domain)$

Hence, for 1 in co-domain, we do not get its pre image in domain. 2.5

$\therefore f$ is not onto.

33

$$A = [1\ 1\ 1\ 0\ 1\ 3\ 1\ \ -2\ 1\ ]$$

$$|A| = 1(1 + 6) - 1(0 - 3) + 1(0 - 1) = 7 + 3 - 1 = 9(\neq 0)$$ 1

$$\therefore A^{-1}\ exists.$$

$$adjA = [|1\ 3\ \ -2\ 1\ |\ \ -|0\ 3\ 1\ 1\ |\ |0\ 1\ 1\ \ -2\ |\ \ -|1\ 1\ \ -2\ 1\ |\ |1\ 1\ 1\ 1\ |$$
$$-|1\ 1\ 1\ \ -2\ |\ |1\ 1\ 1\ 3\ |\ \ -|1\ 1\ 0\ 3\ |\ |1\ 1\ 0\ 1\ |\ ]^t$$
$$= [7\ 3\ \ -1\ \ -3\ 0\ 3\ 2\ \ -3\ 1\ ]^t = [7\ \ -3\ 2\ 3\ 0\ \ -3\ \ -1\ 3\ 1\ ]$$

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$$\therefore A^{-1} = \frac{adjA}{|A|} = \frac{1}{9}[7\ \ -3\ 2\ 3\ 0\ \ -3\ \ -1\ 3\ 1\ ]$$

$x + y + z = 6,\ \ y + 3z = 7, x - 2y + z = 0$

The above system of equations can be written as: $AX = B$,

where,

$$A = [1\ 1\ 1\ 0\ 1\ 3\ 1\ \ -2\ 1\ ], X = [x\ y\ z\ ], B = [6\ 7\ 0\ ]$$

$$\therefore X = A^{-1}B = \frac{1}{9}[7\ \ -3\ 2\ 3\ 0\ \ -3\ \ -1\ 3\ 1\ ][6\ 7\ 0\ ]$$

$$= \frac{1}{9}[42 - 21 + 0\ 18 + 0 + 0\ \ -6 + 21 + 0\ ] = \frac{1}{9}[21\ 18\ 15\ ]$$

$$= [21/9\ 18/9\ 15/9\ ] = \left[\frac{7}{3}\ 2\ \frac{5}{3}\right]$$

$$\therefore x = \frac{7}{3}, y = 2, z = \frac{5}{3}$$

(OR)

Let $\frac{1}{x} = p;\ \frac{1}{y} = q; and\ \frac{1}{z} = r,$ then the given equations become

2p+3q+10r=4; 4p-6q+5r=1; 6p+9q-20r=2

|A|=1200

Thus A is non singular matrix. Therefore, its inverse exist.

adjA= $\begin{pmatrix} 75 & 150 & 75 \\ 110 & -110 & 30 \\ 72 & 0 & -24 \end{pmatrix}$

$$A^{-1} = \frac{1}{1200}\begin{pmatrix} 75 & 150 & 75 \\ 110 & -110 & 30 \\ 72 & 0 & -24 \end{pmatrix}$$

Simplifying, we get $x = 2; y = 3; and\ z = 5$

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$$\int_1^4 [|x-1| + |x-2| + |x-3|]dx$$

$$= \int_1^4 |x-1|dx + \int_1^4 |x-2|dx + \int_1^4 |x-3|dx$$

Now,

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$$\int_1^4 |x-1|dx$$

$$= \int_1^4 (x-1)dx$$

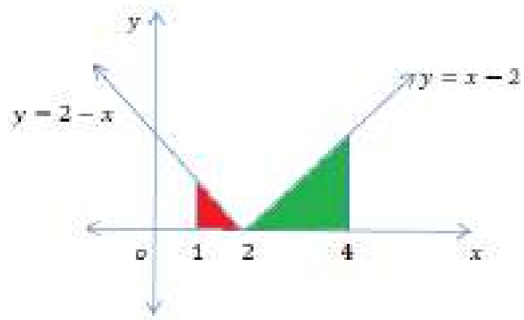
$$= \left[\frac{x^2}{2} - x\right]_1^4$$

$$= \left[(8-4) - \left(\frac{1}{2} - 1\right)\right] = 4 + \frac{1}{2} = 4.5$$

1.5

Now,

$$\int_1^4 |x-2|dx$$



$$= \int_1^2 (2-x)dx + \int_2^4 (x-2)dx$$

$$= \left[2x - \frac{x^2}{2}\right]_1^2 + \left[\frac{x^2}{2} - 2x\right]_2^4$$

$$= \left[(4-2) - \left(2 - \frac{1}{2}\right)\right] + [(8-8) - (2-4)]$$

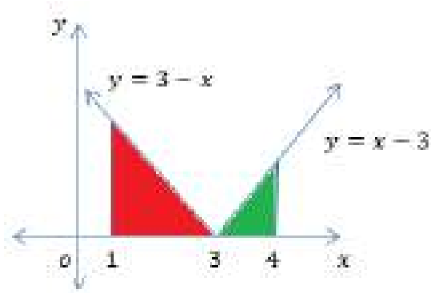
$$= 2 - 1.5 + 2$$

$$= 2.5$$

Now,

$$\int_1^4 |x-3|dx$$

1.5



$$= \int_1^3 (3-x)dx + \int_3^4 (x-3)dx$$

$$= \left[3x - \frac{x^2}{2}\right]_1^3 + \left[\frac{x^2}{2} - 3x\right]_3^4$$

$$= \left[\left(9 - \frac{9}{2}\right) - \left(3 - \frac{1}{2}\right)\right] + \left[(8-12) - \left(\frac{9}{2} - 9\right)\right]$$

$$= 4.5 - 2.5 - 4 + 4.5 = 2.5$$

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$$\therefore \int_1^4 \quad [|x-1|+|x-2|+|x-3|]dx$$

$$= 4.5 + 2.5 + 2.5$$

$$= 9.5$$

</td><td>1.5

0.5</td></tr>
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The equation of a line joining the points B (0, – 1, 3) and C (2, – 3, – 1) is

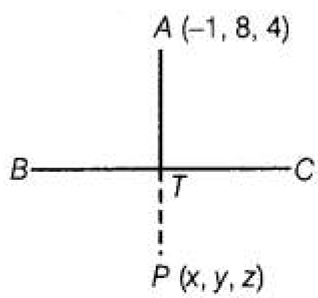
$\vec{r} = (0\hat{\imath} - \hat{\jmath} + 3\hat{k}) + \lambda[(2\hat{\imath} - 3\hat{\jmath} - 1\hat{k}) - (0\hat{\imath} - \hat{\jmath} + 3\hat{k})]$

$$\Rightarrow \vec{r} = (-\hat{\jmath} + 3\hat{k}) + \lambda(2\hat{\imath} - 2\hat{\jmath} - 4\hat{k})$$

$$\Rightarrow \vec{r} = (2\lambda)\hat{\imath} + (-2\lambda - 1)\hat{\jmath} + (-4\lambda + 3)\hat{k}, \lambda \in R$$

So any point on line BC is to the form (2λ, – 2λ – 1, – 4λ + 3)

Let foot of the perpendicular drawn from point A to the line BC be T(2λ, – 2λ – 1, – 4λ + 3).



Now, DR's of line AT is (2λ + 1, – 2λ – 1 – 8, – 4λ + 3 – 4) or (2λ + 1, 2λ – 9, – 4λ – 1).

Since, AT is perpendicular to BC.

∴ 2 × (2λ + 1) + (- 2) × (- 2λ– 9) + (- 4) (- 4λ– 1) = 0

[∵ $a_1a_2 + b_1b_2 + c_1c_2 = 0$]

⇒ 4λ + 2 + 4λ + 18 + 16λ + 4 = 0

⇒ 24λ + 24 = 0 , λ = -1

∴ Coordinates of foot of perpendicular is

T (2 × (- 1)), – 2 × (- 1) – 1, – 4 × (- 1) + 3) or T(- 2, 1, 7)

Let P(x, y, z) be the image of a point A with respect to the line BC. So, point T is the mid-point of AP.

∴ Coordinates of T = Coordinates of mid-point of AP

⇒ (- 2, 1, 7) = [(x–1)/2,(y+8)/2,(z+4)/2]

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	On equating the corresponding coordinates, we get $-2 = (x-1)/2$, $1 = (y+8)/2$ and $7 = (z+4)/2$ $\Rightarrow x = -3$, $y = -6$ and $z = 10$	2
	SECTION-E	
36	Let A, B and C be the events that calculation is done by Jayant, Sonia and Olivia respectively. Let D be the event that there is an error in the calculation. Then, $P(A) = \frac{50}{100}$, $P(B) = \frac{20}{100}$, $P(C) = \frac{30}{100}$ and $P(D/A) = 0.06$,$P(D/B) = 0.04$,$P(D/C) = 0.03$ (i)The probability that Sonia processed the calculation and committed a mistake $= \frac{P(D/B)\,P(B)}{P(D/A)\,P(A) + P(D/B)\,P(B) + P(D/C)\,P(C)}$ $= \frac{0.04 \times \frac{20}{100}}{0.06 \times \frac{50}{100} + 0.04 \times \frac{20}{100} + 0.03 \times \frac{30}{100}} = \frac{8}{47}$	1
	(ii)The total probability of committing a mistake in processing the calculation $= P(D/A)\,P(A) + P(D/B)\,P(B) + P(D/C)\,P(C)$ $=0.06 \times \frac{50}{100} + 0.04 \times \frac{20}{100} + 0.03 \times \frac{30}{100}$ $=.047$	1
	(iii) If the form selected at random has a mistake, the probability that the form is not processed by Jayant is =1-probability that the form has a mistake and is processed by Jayant $= 1 - \frac{P(D/A)\,P(A)}{P(D/A)\,P(A) + P(D/B)\,P(B) + P(D/C)\,P(C)}$ $= 1 - \frac{0.06 \times \frac{50}{100}}{0.06 \times \frac{50}{100} + 0.04 \times \frac{20}{100} + 0.03 \times \frac{30}{100}}$ $= \frac{17}{47}$	2
37	(i) Projection of $\overrightarrow{AB}$ along $\hat{\imath} = \lvert\overrightarrow{AB}\rvert \cos 60° = 5 \times \frac{1}{2} = 2.5$	1

	Projection of $\overrightarrow{AB}$ along $\hat{j} = \left|\overrightarrow{AB}\right| \sin 60° = 5 \times \frac{\sqrt{3}}{2} = 2.5\sqrt{3}$ So, scalar components of $\overrightarrow{AB}$ are 2.5, $2.5\sqrt{3}$. (ii) Unit vector along $\overrightarrow{AB} = \frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} = \frac{\frac{5}{2}(\hat{\imath}+\sqrt{3}\hat{\jmath})}{\sqrt{\frac{25}{4}+\frac{75}{4}}} = \frac{1}{2}\left(\hat{\imath} + \sqrt{3}\hat{\jmath}\right)$. (iii) $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = -3\hat{\imath} + \left(\frac{5}{2}\hat{\imath} + \frac{5\sqrt{3}}{2}\hat{\jmath}\right) = -\frac{1}{2}\hat{\imath} + \frac{5\sqrt{3}}{2}\hat{\jmath}$.	1 2
38	(i) C = 5000 x^2 + 40000 h^2 ....(1) $x^2h = 250 \Rightarrow h = 250/x^2$ ....(2) From (1) and (2), C= 5000 x^2 + 40000 $(250/x^2)^2$ =5000 x^2 + 2500000000/x^4 (ii)(a)For C to be minimum , $\frac{dC}{dx}$=0 ⇒10000x – 10000000000/x^5=0 ⇒ x = 10 OR (b) $\frac{dC}{dx}$= 10000x – 100000 00000/x^5 Which is < 0 for 0<x<10. So, the cost function C(x) is not increasing where 0<x<10	2 2

KENDRIYA VIDYALAYA SANGATHAN: HYDERABAD REGION
MODEL QUESTION PAPER - 3 (2025 - 26)
SUBJECT: MATHEMATICS (041)
MARKING SCHEME

SECTION- A

Q.NO	ANSWER	Q.NO	ANSWER	Q.NO	ANSWER
1.	B	2.	B	3.	C
4.	A	5.	B	6.	A
7.	D	8.	C	9.	B
10.	C	11.	D	12.	C
13.	D	14.	A	15.	A
16.	A	17.	D	18.	A
19.	A	20.	D		

SECTION-B

21.	$S=\{1,2,3,4,5,6\}$ $A=\{2,4,6\}$ $B=\{1,2,3\}$ $A\cap B=\{2\}$ $P(A)=\frac{3}{6}=\frac{1}{2}$ $P(B)=\frac{3}{6}=\frac{1}{2}$ $P(A\cap B)=\frac{1}{6}$ $P(A).P(B)=\frac{1}{2}.\frac{1}{2}=\frac{1}{4}$	1½

	$\therefore P(A \cap B) \neq P(A).P(B)$ $Hence, A\ and\ B\ are\ not\ independent$	½
22.	Given a relation $R = \{(a,b): a\ \leq b\}$ on IR (the set of real numbers). **Reflexivity** Since, $a \leq a$ is true for all value of a $\in IR$ $(a,a)\ \in R\forall a\ \in IR$ Hence, the given relation is reflexive. **Transitivity** Let $(a,b) \in R$ be any arbitrary elements. Then, we have $a \leq b$ and $b \leq c$ $a \leq b \leq c$ $a \leq c\ (a,c) \in R$ Hence, the given relation is transitive. **Symmetricity** Note that $(2,3) \in R$ as $2 < 3$ But $(3,2) \notin R$ as $3 \nless 2$ Hence, the given relation is not symmetric.	½ ½ ½ 1/2
23.	We have I = $\int \frac{2-3sinsin\,x}{cos^2x}dx$ $= \int \left(\frac{2}{cos^2x} - \frac{3sinx}{cos^2x}\right)dx$ $= \int (2sec^2x - x\ \ tan\,tan\,x\,)dx$ $= 2\int sec^2xdx - 3\int secx\ tanxdx$ $= 2tanx - 3secx + C$ Or We have I = $\int secx(secx + tanx)dx$ $= \int (sec^2x + sec\,sec\,x\ \ tan\,tan\,x\,)dx$ $= \int sec^2x\,dx + \int\ sec\,sec\,x\ \ tan\,tan\,x\,dx$	½ ½ 1 ½ ½

	$=\tan \tan x \ +\sec \sec x \ + C$	1
24.	Let θ be the angle between $\vec{a}$ and $\vec{b}$. We have $\lvert \vec{a} \rvert = 8, \lvert \vec{b} \rvert = 3$ and $\lvert \vec{a} \times \vec{b} \rvert = 12$ If θ is the angle between $\vec{a}$ and $\vec{b}$, then we have $$\lvert \vec{a} \times \vec{b} \rvert = \lvert \vec{a} \rvert \lvert \vec{b} \rvert sin\ \theta$$ $$12 = \lvert \vec{a} \rvert \lvert \vec{b} \rvert sin\ \theta$$ $$sin\ \theta = \frac{12}{\lvert \vec{a} \rvert \lvert \vec{b} \rvert} = \frac{12}{8 \times 3}$$ $$sin\ \theta = \frac{1}{2} \Longrightarrow \theta = \frac{\pi}{6}$$ Hence, the required angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$. OR We have $\vec{a} = 2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$ And $\vec{b} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ Since, vectors are perpendicular. $\vec{a}\ .\ \vec{b} = 0$ $$\left(2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}\right).\left(\hat{\imath} - 2\hat{\jmath} + 3\hat{k}\right) = 0$$ $$2 - 2\lambda + 3 = 0$$ $$5 - 2\lambda = 0 \Longrightarrow \lambda = 5/2$$	½ ½ 1 ½ 1 ½
25.	Given equation of a line is $5x - 3 = 15y + 7 = 3 - 10z$ … (1) Here coefficients of x, y and z are 5, 15 and 10. LCM (5, 15, 10) = 30. Thus, dividing by 30 we have eq. (1) becomes $$\frac{5x-3}{30} = \frac{15y+7}{30} = \frac{3-10z}{30}$$	½

	$$\frac{5(x-\frac{3}{5})}{30} = \frac{15(y+\frac{7}{15})}{30} = \frac{-10(z-\frac{3}{10})}{30}$$	½
	$$\frac{x-\frac{3}{5}}{6} = \frac{y+\frac{7}{15}}{2} = \frac{z-\frac{3}{10}}{-3} \quad \text{.... (2)}$$	
	The standard form of equation is given as	
	$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} \quad \text{.... (3)}$$	
	Comparing the above standard equation with Eq. (2), we get 6, 2, -3 are the direction ratios of the given line.	½
	Now $\sqrt{6^2+2^2+(-3)^2} = \sqrt{49} = 7$	½
	Now, the direction cosines of given line are $\frac{6}{7}, \frac{2}{7}, \frac{-3}{7}$.	

SECTION – C

26.	We have	
	$$tan\left\{\frac{\pi}{4}+\frac{1}{2}cos^{-1}\frac{a}{b}\right\} + tan\left\{\frac{\pi}{4}-\frac{1}{2}cos^{-1}\left(\frac{a}{b}\right)\right\} = \frac{2b}{a}$$	
	Let $\cos^{-1}\frac{a}{b} = \theta$ then $cos\,\theta = \frac{a}{b}$. Now substituting in LHS we have	½
	LHS $= tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right) + tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$	
	$$= \frac{tan\frac{\pi}{4}+tan\frac{\theta}{2}}{1-tan\frac{\pi}{4}tan\frac{\theta}{2}} + \frac{tan\frac{\pi}{4}-tan\frac{\theta}{2}}{1+tan\frac{\pi}{4}tan\frac{\theta}{2}}$$	1/2
	$$= \frac{1+tan\frac{\theta}{2}}{1-tan\frac{\theta}{2}} + \frac{1-tan\frac{\theta}{2}}{1+tan\frac{\theta}{2}}$$	
	$$= \frac{\left(1+tan\frac{\theta}{2}\right)^2+\left(1-tan\frac{\theta}{2}\right)^2}{\left(1-tan\frac{\theta}{2}\right)\left(1+tan\frac{\theta}{2}\right)}$$	1

	$= 2\left(\frac{1+tan^2\frac{\theta}{2}}{1-tan^2\frac{\theta}{2}}\right)$ $\qquad cos\,\theta = \frac{1-tan^2\frac{\theta}{2}}{1+tan^2\frac{\theta}{2}}$ $= \frac{2}{cos\,\theta} = \frac{2}{\frac{a}{b}} = \frac{2b}{a}$ $= RHS$ Hence proved.	1
27.	We have $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$ $2x^2 - 40 = 18 - (-14)$ $2x^2 - 40 = 32$ $2x^2 = 72$ $x^2 = 36$ $x = \pm 6$	1 1 1
28.	We have $y = log\,(1+x) - \frac{2x}{2+x}$. Differentiating both sides w.r.t. x, we get $\frac{dy}{dx} = \frac{1}{1+x}(1) - \frac{(2+x).2-2x.1}{(2+x)^2}$ $= \frac{1}{1+x} - \frac{4+2x-2x}{(2+x)^2}$ $= \frac{(2+x)^2-4(1+x)}{(1+x)(2+x)^2}$ $= \frac{4+x^2+4x-4-4x}{(1+x)(2+x)^2}$ $= \frac{x^2}{(1+x)(2+x)^2}$ … (i) Now, $x^2, (2+x)^2$ are always positive, also $1+x > 0$ for $x > -1$.	1 1 1

Thus $\frac{dy}{dx} > 0$ for $x > -1$. Hence, function increases for $x > -1$

Or

We have $f(x) = \sin x + \cos x$.

1

Differentiating both sides w.r.t. x, we get

$$f(x) = \cos x - \sin x$$

Now, substituting $f(x) = 0$ we have

$$\cos x - \sin x = 0$$

$$\tan x = 1,$$

$$x = \frac{\pi}{4}, \ \frac{5\pi}{4}, \text{ as } 0 \le x \le 2\pi$$

1

Now, we find the intervals in which $f(x)$ is strictly increasing or strictly decreasing.

For interval $0 < x < \frac{\pi}{4}$ at $x = \frac{\pi}{6}$,

$$f(x) = \cos x - \sin x$$

$$= \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{\sqrt{3}-1}{2}$$ +ve

For interval $\frac{\pi}{4} < x < \frac{5\pi}{6}$ at $x = \frac{\pi}{2}$,

½

$$f(x) = \cos x - \sin x$$

$$= 0 - 1 = -1$$ - ve

For interval $\frac{5\pi}{4} < x < 2\pi$ at $x = \frac{3\pi}{2}$,

$$f(x) = \cos x - \sin x$$

$$= 0 - 1 = -1$$ + ve

Here $f(x) > 0$ in $\left(0, \frac{\pi}{4}\right)$, $f(x) < 0$ in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ and $f(x) > 0$ in $\left(\frac{5\pi}{4}, 2\pi\right)$.

	Since, $f(x)$ is a trigonometric function, so it is continuous at $x = 0, \frac{\pi}{4}, \frac{5\pi}{4}$ and 2π. Hence, the function is (i) increasing in $\left[0, \frac{\pi}{4}\right]$ and $\left[\frac{5\pi}{4}, 2\pi\right]$. (ii) decreasing in $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$.	1/2
29.	We have $y = (sin\ x)^{x} + \sqrt{x}$ … (1) Let $u = (sin\ x)^{x}$ … (2) Then, Eq. (1) becomes, $y = u + \sqrt{x}$ … (3) Taking log both sides of Eq. (2), we get $\frac{1}{u}\frac{du}{dx} = x\frac{d}{dx}(log\ sin\ x) + log\ sin\ x\ \frac{d}{dx}(x)$ $\frac{du}{dx} = u\left[x \times \frac{1}{sin\ x}\frac{1}{dx}(sin\ x) + log\ sin\ x(1)\ \right]$ $\frac{du}{dx} = (sin\ \ x\ )^{x}\left[\frac{x}{sin\ x} \times cos\ x\ + logsin\ x\ \right]$ [from Eq. (2)] $\frac{du}{dx} = (sin\ x\ )^{x}[x\ cot\ x + log\ sin\ x]$ … (4) Differentiating both sides of Eq. (4) w.r.t. x, we get $\frac{dy}{dx} = \frac{du}{dx} + \frac{1}{\sqrt{1-(\sqrt{x})^2}}\frac{d}{dx}(\sqrt{x})$ $\frac{dy}{dx} = (sin\ x)^{x}[x\ cot\ x\ + logsin\ x\ ]$ **(OR)** Here, $(\cos y)^x = (\sin x)^y$ Taking log both sides we get $\log (\cos y)^x = \log (\sin x)^y$ Or, x log cos y = y log sin x	1 1 1 1

	Or, y log sin x = x log cos y Differentiating both sides w.r.t x y X $\frac{\cos x}{\sin x}$ + log sin x $\frac{dy}{dx}$ = x X $\frac{-\sin y}{\cos y}$ $\frac{dy}{dx}$ + log cos y Or, y cot x + log sin x $\frac{dy}{dx}$ = - x tan y $\frac{dy}{dx}$ + log cos y Or, $\frac{dy}{dx}$ (log sin x + x tan y) = log cos y – y cot x Or, $\frac{dy}{dx}$ = $\frac{\log \cos y - y \cot x}{\log \sin x + x \tan y}$	 1 1
30.	We have $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ Here, $\lvert\vec{a}\rvert = \sqrt{1^2 + (-2)^2 + 3^2}$ $= \sqrt{1+4+9} = \sqrt{14}$ $\lvert\vec{b}\rvert = \sqrt{3^2 + (-2)^2 + 1^2}$ $= \sqrt{9+4+1} = \sqrt{14}$ and $\vec{a} \times \vec{b} = \lvert \hat{\imath}\ \hat{\jmath}\ \hat{k}\ 1\ -2\ 3\ 3\ -2\ 1 \rvert$ $= \hat{\imath}(-2+6) - \hat{\jmath}(1-9) + \hat{k}(-2+6)$ $= 4\hat{\imath} + 8\hat{\jmath} + 4\hat{k} = 4(\hat{\imath} + 2\hat{\jmath} + \hat{k})$ $\lvert\vec{a} \times \vec{b}\rvert = 4\sqrt{1^2 + 2^2 + 1^2}$ $= 4\sqrt{1+4+1} = 4\sqrt{6}$ Now $\lvert\vec{a} \times \vec{b}\rvert = \lvert\vec{a}\rvert\lvert\vec{b}\rvert \sin\theta$ $\sin\theta = \frac{\lvert\vec{a}\times\vec{b}\rvert}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert}$ $= \frac{4\sqrt{6}}{\sqrt{14}.\sqrt{14}}$	 1 1

	$= \frac{4\sqrt{6}}{14} = \frac{2\sqrt{6}}{7}$	1
31.	We have $I = \int_{-1}^{2} \lvert x^3 - x \rvert dx$ We observe that, Since $\lvert x^3 - x \rvert = \{(x^3 - x),\ when -1 < x < 0\ \ -(x^3 - x), when\ 0 < x < 1\ (x^3 - x),$ $when\ 1 \le x < 2$ Thus $I = \int_{-1}^{0} (x^3 - x)dx + \int_{0}^{1} -(x^3 - x)dx + \int_{1}^{2} (x^3 - x)dx$	1
	$= \int_{-1}^{0} (x^3 - x)dx - \int_{0}^{1} (x^3 - x)dx + \int_{1}^{2} (x^3 - x)dx$ $= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{2} - \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{0}^{2} + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{1}^{2}$ $= \left[0 - \left(\frac{1}{4} - \frac{1}{2}\right)\right] - \left[\left(\frac{1}{4} - \frac{1}{2}\right) - 0\right] + \left[\left(\frac{16}{4} - \frac{4}{2}\right) - \left(\frac{1}{4} - \frac{1}{2}\right)\right]$	1
	$= -\frac{1}{4} + \frac{1}{2} - \frac{1}{4} + \frac{1}{2} + 4 - 2 - \frac{1}{4} + \frac{1}{2}$ $= \frac{3}{4} + \frac{3}{2} + 2 = \frac{-3+6+8}{4} = \frac{11}{4}$	1
	OR We have $I = \int_{-2}^{2} \frac{x^2}{1+5^x} dx$ … (i) Using the fact $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$ we have $I = \int_{-2}^{2} \frac{(2-2-x)^2}{1+5^{2-2-x}} dx$ $I = \int_{-2}^{2} \frac{x^2}{1+5^{-x}} dx$	1

	or $\quad I = \int_{-2}^{2} \frac{5^{-x}}{5^{x}+1} x^2 dx \quad$ …(ii) Adding Eqs. (i) and (ii), we get $$2I = \int_{-2}^{2} \left(\frac{1+5^{x}}{5^{x}+1}\right) x^2 dx$$ $$= \int_{-2}^{2} x^2 dx$$ or $\quad 2I = 2\int_{0}^{2} x^2 dx \quad$ Even function $$I = \left[\frac{x^3}{3}\right]_0^3 = \frac{1}{3}(2^3 - 0) = \frac{8}{3}$$	1 1

SECTION – D

32.	We have $\quad cosecx\ log\ \lvert y \rvert\ \frac{dy}{dx} + x^2y^2 = 0 \quad$ … (1) $$cosecx\ log\ \lvert y \rvert\ \frac{dy}{dx} = -x^2y^2$$ Separating the variables, we get $$\frac{log\lvert y \rvert}{y^2} dy = \frac{-x^2}{cosec\ x} dx$$ Integrating both sides, we have $\int \frac{log\ \lvert y \rvert}{y^2} dy = -\int \frac{x^2}{cosecx} dx$ $I_1 = -I_2 \quad$ … (2) where, $\quad I_1 = \int \frac{log\ \lvert y \rvert}{y^2} dy$ and $\quad I_2 = \int \frac{x^2}{cosecx} dx = \int x^2\ sin\ x\ dx$ Now $\quad I_1 = \int \frac{log\ \lvert y \rvert}{y^2} dy$	1 1

	Substituting $log\, y = t \Longrightarrow y = e^t$, then $\frac{dy}{y} = dt$ we have $I_1 = \int \mathrm{t}\, e^{-t} \mathrm{dt}$ $= t \int e^{-t} dt - \left[\int \frac{d}{dt}(t) . \int e^{-t} dt\right] dt$ $= -te^{-t} - (-\int e^{-t} dt)$ $= -te^{-t} + \int e^{-t} dt$ $= -te^{-t} - e^{-t} + C_1$ $= -\frac{log\|y\|}{y} - \frac{1}{y} + C_1$ … (3) and $I_2 = \int x^2\, sinx\, dx$ $= x^2 \int sin\, x\, dx - \int \left[\frac{d}{dx}(x)^2 \int sin\, x\, dx\right] dx$ $= x^2(-\,cos\, x\,) - \int [2x(-\,cos\, x\,)]\, dx$ $= -x^2\, cos\, x\, + 2 \int x\, cosx\, dx$ $= -x^2\, cos\, x\, + 2\left[x \int cos\, x\, dx - \int \left\{\frac{d}{dx}(x) \int cos\, x\, dx\right\} dx\right]$ $= -x^2\, cos\, x\, + 2[x\, sin\, x\, - \int sin\, x\, dx]$ $= -x^2\, cos\, x\, + 2x\, sin\, x\, + 2\, cos\, x\, + C_2$ Substituting the values of I_1 and I_2 from Eqs. (3) and (iv) in Eq. (2), we get $-\frac{log\, \|y\|}{y} - \frac{1}{y} + C_I = x^2\, cos\, x\, - 2x\, sin\, x\, - 2\, cos\, x\, - C_2$ $-\frac{(1+log\, \|y\|\,)}{y} = x^2\, cos\, x\, - 2x\, sin\, x\, - 2\, cos\, x\, - C_2 - C_I$ $-\frac{(1+log\, \|y\|\,)}{y} = x^2\, cos\, x\, - 2x\, sin\, x\, - 2\, cos\, x\, + C$ where, $C = -C_2 - C_1$ which is the required solution of given differential equation. **OR**	1 1 1

	We have $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$ … (1) Which is a homogeneous differential equation as $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$ Substituting $y = vx \Longrightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ in eq (1) we have $$x\cos v\left[v + x\frac{dv}{dx}\right] = vx\cos v + x$$ $$vx\cos v + x^2\cos v\frac{dv}{dx} = vx\cos v + x$$ $$x^2\cos v\frac{dv}{dx} = x$$ $$\cos v\, dv = \frac{dx}{x}$$ Integrating both sides, we have $$\int \cos v\, dv = \int \frac{dx}{x}$$ $$\sin v = \log\lvert x\rvert + C$$ $$\sin\left(\frac{y}{x}\right) = \log\lvert x\rvert + C \qquad [\text{put } v = \frac{y}{x}]$$ which is the required solution of given differential equation.	½ ½ 1 1 1 1
33.	Writing the given line in standard form as $$\frac{(x-1)}{-3} = \frac{y-2}{\frac{p}{7}} = \frac{z-3}{2} = r_i(let) \qquad …. (1)$$ and $$\frac{x-1}{\frac{-3p}{7}} = \frac{y-5}{1} = \frac{z-6}{-5} = r_2(let) \qquad …. (2)$$ Two lines with DR's a_1, b_1, c_1 and a_2, b_2, c_2 are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ Thus line (1) and (2) will intersect at right angle, if	½

	$-3\left(\frac{-3p}{7}\right)+\frac{p}{7}(1)+2(-5)=0$ $\frac{9p}{7}+\frac{p}{7}=10$ $\frac{10p}{7}=10 \Longrightarrow p=7$ Which is the required value of p. Also, we know that the equation of a line which passes through the point (x_1, y_1, z_1) with direction ratios $a\ , b, c$ is given by $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$ Since, required line is parallel to line l_1. So, $a=-3, b=\frac{7}{7}=1\ and\ c=2$ Now, equation of line passing through the point (3,2,-4) and having direction ratios (-3,1,2) is $\frac{x-3}{-3}=\frac{y-2}{1}=\frac{z+4}{2}$ $\frac{3-x}{3}=\frac{y-2}{1}=\frac{z+4}{2}$	½ 1 1 1

34.

Maximise $Z = 5x + 3y$ subject to the constraints: $3x + 5y \leq 15$; $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$.

or

Minimize $Z = 3x + 5y$ such that $x + 3y \geq 3$, $x + y \geq 2$, x, $y \geq 0$.

Sol :

Maximize $\quad Z = 5x + 3y$

Subject to $\quad 3x + 5y \leq 15 \quad$ (i)

$\quad 5x + 2y \leq 10 \quad$ (ii)

and $\quad x \geq 0$, $y \geq 0$

(i) Region corresponding to $3x + 5y \leq 15$:

Line $3x + 5y = 15$			
x	0	5	Point (0,0) is true for $3x + 5y \leq 15$. So, the region is towards the origin.
y	3	0	

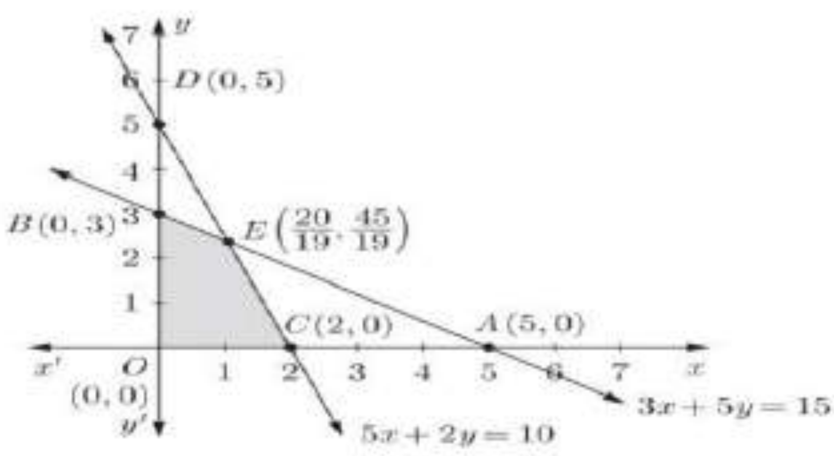
1

(ii) Region corresponding to $5x + 2y \leq 10$:

Line $5x + 2y = 10$			
x	0	2	Point (0,0) is true for $5x + 2y \leq 10$. So, the region is towards the origin.
y	5	0	

Since $x \geq 0$, $y \geq 0$ the feasible region lies in the first quadrant.

Now we draw all line on the graph and find the common area. Clearly, feasible region is $OBECO$ which is bounded. Solving equations $3x + 5y = 15$ and $5x + 2y = 10$, we get $E(\frac{20}{19}, \frac{45}{19})$. The corner points of the feasible region are $O(0,0)$, $B(0,3)$, $E(\frac{20}{19}, \frac{45}{19})$ and $B(2,0)$. 1



Since, the feasible region is a bounded region, we can check the objective function at all the corner to

1

find the maxima. The values of objective function Z at these points are as follows. 1

Corner	$Z=5x+3y$
$O(0,0)$	$Z=0+0=0$
$B(0,3)$	$Z=5\times0+3\times3=9$
$E(\frac{20}{19},\frac{45}{19})$	$Z=5\times\frac{20}{19}+3\times\frac{45}{19}=12.37$
$C(2,0)$	$=5\times2+3\times0=10$

Hence the maximum value of Z is 12.37 is at $E(\frac{20}{19},\frac{45}{19})$ and the optimal solution is $x=\frac{20}{19}$ and $y=\frac{45}{19}$.

or 1

Minimize $\quad Z=3x+5y$

Subject to $\quad x+3y\geq 3 \quad$ (i)

$\quad x+y\geq 2 \quad$ (ii)

and $\quad x\geq 0,\ y\geq 0$

(i) Region corresponding to $x+3y\geqslant 3$:

Line $x+3y=3$			
x	0	3	Point $(0,0)$ is false for $x+3y\geqslant 3$. So, the region is away from the origin.
y	1	0	

1

(ii) Region corresponding to $x+y\geqslant 2$:

Line $x+y=2$			
x	0	2	Point $(0,0)$ is false for $x+y\geqslant 2$. So, the region is away from the origin.
y	2	0	

Since $x\geq 0$, $y\geq 0$ the feasible region lies in the first quadrant.

Now we draw all line on the graph and find the common area. Clearly, feasible region is ABC which is open and unbounded region. Solving equations $x+3y=3$ and $x+y=2$, we get $B(\frac{3}{2},\frac{1}{2})$. The corner points of the feasible region are $A(3,0)$, $B(\frac{3}{2},\frac{1}{2})$ and $C(0,2)$. 1

The values of objective function Z at these points are as follows.

Corner points	$Z=3x+5y$
$A(3,0)$	$Z=3\times3+0=9$
$B(\frac{3}{2},\frac{1}{2})$	$Z=3\times\frac{3}{2}+5\times\frac{1}{2}=7$
$C(0,2)$	$Z=3\times0+5\times2=10$

Hence the minimum value of Z is 7 at $B(\frac{3}{2},\frac{1}{2})$.

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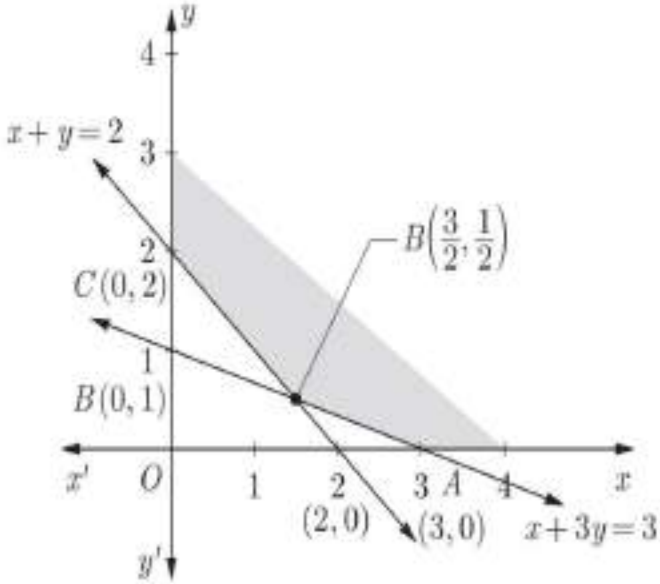

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We have $I = \int_0^{\pi/2} \frac{xsinx\, cos\, x}{sin^4 x + cos^4 x} dx$ … (1)

Using the fact $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ we have

$$I = \int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2} - x\right) \sin\left(\frac{\pi}{2} - x\right) \cos\left(\frac{\pi}{2} - x\right)}{sin^4\left(\frac{\pi}{2} - x\right) + cos^4\left(\frac{x}{2} - x\right)} dx$$

$$= \int_0^{\pi/2} \frac{\left(\frac{\pi}{2} - x\right) cosx\, sin\, x}{cos^4 x + sin^4 x} dx$$

Adding Eqs. (1) and (2), we get

$$2I = \frac{\pi}{2} \int_0^{\pi/2} \frac{cos\, x\, sin\, x}{sin^4 x + cos^4 x} dx$$

or

$$I = \frac{\pi}{4} \int_0^{\pi/2} \frac{sinx\, cos\, x}{(sin^2 x)^2 + (1 - sin^2 x)^2} dx$$

Substituting

$$sin^2 x = t$$

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	$$2sinx\ cosx\ dx\ = dt\ sinx\ cosx\ dx\ = \frac{dt}{2}$$ When $x = 0$, then $t = sin\,0 = 0$ and when $x = \frac{\pi}{2}$, then $t = sin^2\frac{\pi}{2} = 1$ Thus $$I\ = \frac{\pi}{4}\int_0^1 \frac{1}{t^2+(1-t)^2}\frac{dt}{2}\ = \frac{\pi}{8}\int_0^1 \frac{1}{t^2+(1+t^2-2t)}dt\ = \frac{\pi}{8}\int_0^1 \frac{1}{2t^2-2t+1}dt$$ $$= \frac{\pi}{16}\int_0^1 \frac{1}{t^2-t+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+\frac{1}{2}}dt\ = \frac{\pi}{16}\int_0^1 \frac{1\ dt}{\left(t-\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2}$$ $$= \frac{\pi}{16}\frac{1}{\frac{1}{2}}\left[tan^{-1}\left(\frac{t-\frac{1}{2}}{\frac{1}{2}}\right)\right]_0^1\ \int \frac{1}{x^2+a^2}dx\ = \frac{1}{a}tan^{-1}\frac{x}{a}+C$$ $$= \frac{\pi}{8}\left[tan^{-1}2\left(1-\frac{1}{2}\right)-tan^{-1}2\left(0-\frac{1}{2}\right)\right]$$ $$= \frac{\pi}{8}[tan^{-1}(1)-tan^{-1}(-1)]$$ $$= \frac{\pi}{8}\left[\frac{\pi}{4}+\frac{\pi}{4}\right] = \frac{\pi^2}{16}$$	1 1 1 1

SECTION - E

36.	(i) Required probability, $$= P(A)\ = P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)+P(E_3)P\left(\frac{A}{E_3}\right)$$ $$= 0.5\times 0.06+0.2\times 0.4+0.3\times 0.3\ = 0.030+0.008+0.009 = 0.047$$	2

	(ii) Required probability, $P\left(\frac{E_1}{A}\right) = 1 - P\left(\frac{\bar{E}_1}{A}\right) = 1 - [\frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}]$ $= 1 - \left[\frac{0.5 \times 0.6}{0.5 \times 0.06 + 0.2 \times 0.04 + 0.3 \times 0.03}\right] = 1 - \frac{0.030}{0.047} = 1 - \frac{30}{47} = \frac{17}{47}$	1 1
37.	We have $P(x) = -6x^2 + 120x + 25000$ $P'(x) = -12x + 120$ (i) At $x = 3$ we have, $P(3) = -6(3)^2 + 120(3) + 25000 = -54 + 360 + 25000 = 25306$ $(ii) P'(5) = -12 \times 5 + 120 = -60 + 120 = 60$ (iii) For strictly increasing, $P'(x) > 0$ $-12x + 120 > 0$ $120 > 12x$ $x < 10\,;\ x \in (0, 10)$ or For maximum profit we put $P'(x) = 0$, i.e $0 = -12x + 120\ \ x = 10$ Now $p''(x) = -12 < 0$ At $x = 10$, profit function is maximum.	1 1 ½ 1½
38.	(i)We can write $\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 20 \\ 29 \end{bmatrix}$; where A $= \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \end{bmatrix}$, B = $\begin{bmatrix} 20 \\ 29 \end{bmatrix}$ Also $\lvert A \rvert = -2$ (ii)Solving to get x=8, y=1	$\frac{1}{2}$ $\frac{1}{2}$ 1

KENDRIYA VIDYALAYA SANGATHAN: HYDERABAD REGION

MODEL QUESTION PAPER - 4 (2025 - 26)

SUBJECT: MATHEMATICS (041)

MARKING SCHEME

....

SECTION-A Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of **1 mark each.**									
1. a)	2. b)	3.b)	4.b)	5.c)	6.d)	7.b)	8.c)	9.d)	10.d)
11.c)	12.b)	13.a)	14.c)	15.d)	16.d)	17.b)	18.a)	19. d)	20. b)

Q No	Expected Answers/Value Points	Marks
21.	$\left(Sin(13\ \frac{\pi}{7}\right) = (sin(2Л - Л/7)$ $(-sin\ Л/7\ )$ $(\sin^{-1}(-sin\ Л/7\ )\ = -\ \frac{Л}{7}$ OR Given, $\tan^{-1} 1\ + \cos^{-1}(-\frac{1}{2}) + \sin^{-1}(-\frac{1}{2})$ $\Rightarrow \frac{\pi}{4} + \left(\pi - \frac{\pi}{3}\right) - \frac{\pi}{6}$ On solving$\Rightarrow \frac{3\pi}{4}$	½ ½ ½ ½ 1½ ½

22	Let the numbers be x and $\frac{9}{x}$, required sum is $x^2+\frac{81}{x^2}=f(x)$ $f'(x)=2x-\frac{162}{x^3}$ $f'(x)=0 \Longrightarrow x=3$ $f''(x)=2+\frac{486}{x^4}>0$, $f(x)$ is minimum when $x=3$So the numbers are 3,3	½ ½ ½ ½
23	$=\int e^x\left\{\frac{1+x-1}{(1+x)^2}\right\}dx$ $=\int e^x\left\{\frac{1}{1+x}-\frac{1}{(1+x)^2}\right\}dx \therefore \int\frac{xe^x}{(1+x)^2}\,dx=\frac{e^x}{1+x}+C$	1 1
24	$f'(x)=\frac{16[4+\cos x]\cos x+16\sin^2 x}{(4+\cos x)^2}-1$ $=\frac{\cos x\,(56-\cos x)}{(4+\cos x)^2}$ in $(\frac{\pi}{2},\pi)$, $\cos x<0 \Rightarrow f'(x)<0$ $\therefore$ f(x) in strictly decreasing in $(\frac{\pi}{2},\pi)$ (OR)	1 ½ ½
24	$f'(x)=3x^2-\frac{3}{x^4}$ $f'(x)=0\Rightarrow\frac{3(x^6-1)}{x^4}=0\Rightarrow\frac{3(x^3-1)(x^3+1)}{x^4}=0$ $\Rightarrow x=-1,1 \quad (\because x\neq 0)$ $\therefore f(x)$ is decreasing when $x\in[-1,1]-\{0\}$	½ 1 ½

25	Here, $\frac{dx}{dt} = \frac{dy}{dt}$ Given $y^2 = 8x$ gives $2y.\frac{dy}{dt} = 8\frac{dx}{dt}$ $\Rightarrow 2y = 8$ or $y = 4$ Also, $y = 4$ gives $x = 2$. Thus, the point $(2, 4)$	½ ½ ½ ½
26	It is given that, $y = e^{a\cos^{-1}x}$ Taking logarithm on both the sides, we obtain $\log y = a\cos^{-1}x \log e$ $\log y = a\cos^{-1}x$ Differentiating both sides with respect to x, we obtain $\frac{1}{y}\frac{dy}{dx} = a \times \frac{-1}{\sqrt{1-x^2}}$ $\Rightarrow \frac{dy}{dx} = \frac{-ay}{\sqrt{1-x^2}}$ By squaring both the sides, we obtain $\left(\frac{dy}{dx}\right)^2 = \frac{a^2y^2}{1-x^2}$ $\Rightarrow (1-x^2)\left(\frac{dy}{dx}\right)^2 = a^2y^2$	1 1½ ½
27	Let $I = \int e^{x^2}(x^5 + 2x^3)dx$ Put $x^2 = t$, so that $2xdx = dt$ $I = \frac{1}{2}\int e^t(t^2 + 2t)dt$ $= \frac{1}{2}\ e^t.t^2 + c = \frac{1}{2}\ e^{x^2}.x^4 + c$	½ 1 1+½
28 a)	$$\frac{dy}{dx} + y\ cotx = 4x\ cosecx$$ $I.F = \int e^{cotx\ dx} = e^{logsinx} = sinx$ Solution is given by $$y.(sinx) = \int 4x.cosecx.sinx\ dx = \int 4xdx = 2x^2 + c$$ Now $y = 0$ when $x = \frac{\pi}{2}$ gives $c = -\frac{\pi^2}{2}$	1 ½ +½ ½ ½

Required solution is $y.(sinx) = 2x^2 - \frac{\pi^2}{2}$

28. b)

$$\int \frac{dy}{1+y^2} = \int (1+x^2)dx$$ 1

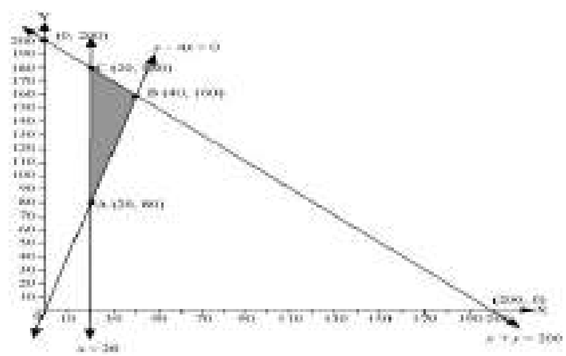
$$\Rightarrow \tan^{-1} y = \int dx + \int x^2 dx$$ 1

$$\Rightarrow \tan^{-1} y = x + \frac{x^3}{3} + C$$ 1

29 a)

Maximize $z = 1000x + 600y$ ... (1) subject to the constraints, ½ mark each (1½M)

$$x + y \le 200 \quad ...(2)$$
$$x \ge 20 \quad ...(3)$$
$$y - 4x \ge 0 \quad ...(4)$$
$$x, y \ge 0 \quad ...(5)$$



The c

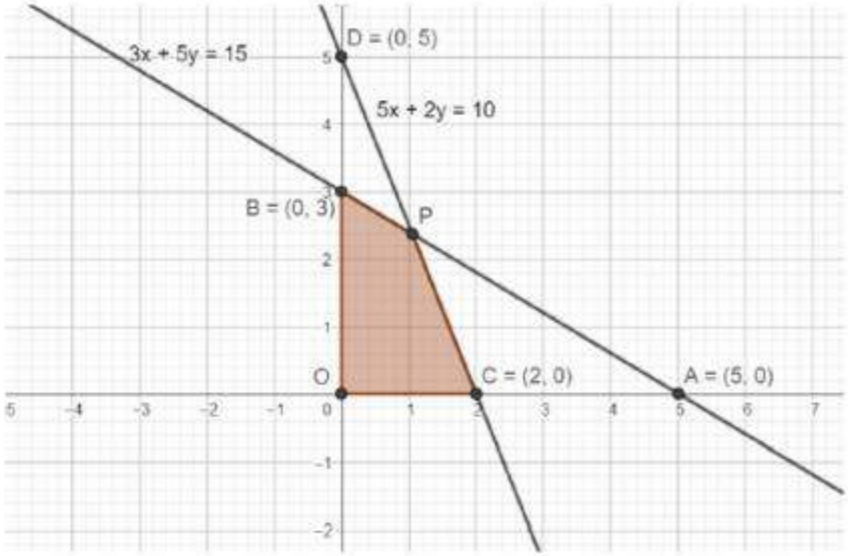
½

Corner points of the feasible region are A (20, 80), B (40, 160), andC (20, 180).

The values of z at these corner points are as follows.

Corner point	z = 1000*x* + 600*y*	
A (20, 80)	68000	
B (40, 160)	136000	→ Maximum
C (20, 180)	128000	

The maximum value of z is 136000 at (40, 160). 1

<table>
<tr><td>29
b)</td><td>

<table>
<tr><th>Corner Points</th><th>Value of Z</th></tr>
<tr><td>O(0,0)</td><td>0</td></tr>
<tr><td>B(0,3)</td><td>9</td></tr>
<tr><td>C(2,0)</td><td>10</td></tr>
<tr><td>P(20/19,45/19)</td><td>$\frac{235}{19}$ → Max</td></tr>
</table></td><td>1 ½ for correct graph

1+½ for maximum</td></tr>
<tr><td>30</td><td>$S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$

$P(A) = \frac{6}{12} = \frac{1}{2}$, $P(B) = \frac{2}{12} = \frac{1}{6}$, $P(A \cap B) = \frac{1}{12}$ $P(A \cap B) = \frac{1}{12} = P(A).P(B)$

A and B are independent events</td><td>1

1½

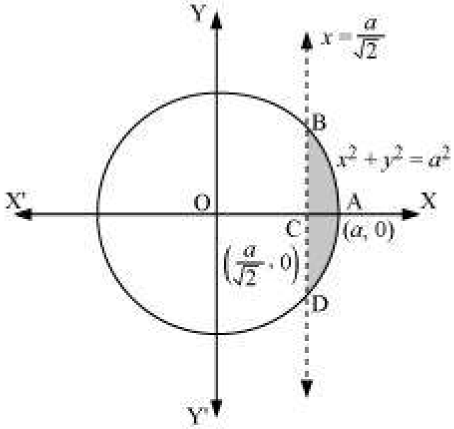
½</td></tr>
<tr><td>31
a)</td><td>f(x) = x$\sin^2$x is odd as f(-x)= -f(x)

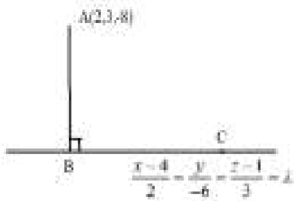
therefore $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)\ dx = 0$

(OR)</td><td>2

1</td></tr>
<tr><td>31
b)</td><td>$\int \frac{2}{(1-x)(1+x^2)} dx = \int \frac{1}{1-x} dx + \int \frac{x+1}{1+x^2} dx$
$= -\log|1-x| + \frac{1}{2}\log(1+x^2) + \tan^{-1} x + C$</td><td>½+½+½</td></tr>
</table>

		½+½+½
32 a)	Let (a,b) ∈ N x N ⟹ ab = ba ⟹ (a,b) R (a,b) Therefore R is reflexive Let (a,b), (c,d) ∈ N x N s.t. (a,b) R (c,d). Then ad = bc ⟹ cb = da ⟹ (c,d) R (a,b) . Therefore R is symmetric Let (a,b), (c,d),(e,f) ∈ N x N s.t. (a,b) R (c,d) and (c,d) R (e,f). Then (a,b) R (c,d) ⟹ ad = bc (c,d) R (e,f) ⟹ cf = de ⟹ (ad)(cf)= (bc)(de) ⟹ af = be ⟹ a,b) R (e,f) Therefore R is Transitive. Hence R is an equivalane relation on N x N	½ 1 1 ½ 1/2 1
32 b)	For one-one Let $f(x_1) = f(x_2)$, for some $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$ $\frac{4x_1}{3x_1+4} = \frac{4x_2}{3x_2+4}$ $\Rightarrow x_1 = x_2$ f is one one For onto let $y \in R$, and for some x , let $y = \frac{4x}{3x+4}$ $\Rightarrow x = -\frac{4y}{3y-4}$ or $x = \frac{4y}{4-3y}$ x is real if $y \neq \frac{4}{3}$, So $R_f = R - \left\{\frac{4}{3}\right\} \neq$codomain($f$) f is not onto	½ 1 ½ 1 1 1

33	 $Area\ of\ ABC = \int_{\frac{a}{\sqrt{2}}}^{a} y\,dx$ $= \int_{\frac{a}{\sqrt{2}}}^{a} \sqrt{a^2 - x^2}\,dx$ $= \left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_{\frac{a}{\sqrt{2}}}^{a}$ $= \left[\frac{a^2}{2}\left(\frac{\pi}{2}\right) - \frac{a}{2\sqrt{2}}\sqrt{a^2 - \frac{a^2}{2}} - \frac{a^2}{2}\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]$ $= \frac{a^2\pi}{4} - \frac{a}{2\sqrt{2}} \cdot \frac{a}{\sqrt{2}} - \frac{a^2}{2}\left(\frac{\pi}{4}\right)$ $= \frac{a^2\pi}{4} - \frac{a^2}{4} - \frac{a^2\pi}{8}$ $= \frac{a^2}{4}\left[\pi - 1 - \frac{\pi}{2}\right]$ $= \frac{a^2}{4}\left[\frac{\pi}{2} - 1\right]$ $\Rightarrow Area\ ABCD = 2\left[\frac{a^2}{4}\left(\frac{\pi}{2} - 1\right)\right] = \frac{a^2}{2}\left(\frac{\pi}{2} - 1\right)$	1 mark for correct figure 1 1 ½ ½ 1
34	Given sysytem is $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix}$ A X = B ⇒ X = A^{-1}B $\lvert A \rvert$= 35 ≠ 0 A_{11} = 0 , A_{12} = 7 , A_{13}= 7 A_{21} = 10 , A_{22} = –11, A_{23} = 4	½ 1 1 ½

	$A_{31} = 5$, $A_{32} = 5$, $A_{33} = -5$ $\therefore A^{-1} = \frac{1}{35}\begin{bmatrix} 0 & 10 & 5 \\ 7 & -11 & 5 \\ 7 & 4 & -5 \end{bmatrix}$ $X = \frac{1}{35}\begin{bmatrix} 0 & 10 & 5 \\ 7 & -11 & 5 \\ 7 & 4 & -5 \end{bmatrix}\begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ Therefore x = 1, y =1, z =1	½ ½ + ½ + ½
35 a)	 Any point on line is $(2\lambda+4, -6\lambda, 3\lambda+1)$ for some λ. Let $B(2\lambda+4, -6\lambda, 3\lambda+1)$ d.r. of $AB = <2\lambda+2, -6\lambda-3, 3\lambda+9>$ d.r. of $BC = <2, -6, 3>$ $AB \perp BC \Rightarrow 2(2\lambda+2) - 6(-6\lambda-3) + 3(3\lambda+9) = 0$ $\Rightarrow \lambda = -1$ $\therefore B(2, 6, -2)$ Now, $AB = \sqrt{(2-2)^2 + (6-3)^2 + (-2+8)^2} = \sqrt{45}$ or $3\sqrt{5}$ units	1 ½ 1 ½ 1 1
35 b)	Vector equation of required line through (1, 2, – 4) is $\vec{r} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ and the cartesian equation is $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$ Equation of line through A(3, 3, – 5) and B(1, 0, – 11) is $\vec{r} = 3\hat{\imath} + 3\hat{\jmath} - 5\hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ Distance between parallel lines is given by $d = \frac{\lvert(\vec{a_2} - \vec{a_1}) \times \vec{b}\rvert}{\lvert\vec{b}\rvert}$ Here $\vec{b} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$, $\vec{a_1} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k}$, $\vec{a_2} = 3\hat{\imath} + 3\hat{\jmath} - 5\hat{k}$ $(\vec{a_2} - \vec{a_1}) = 2\hat{\imath} + \hat{\jmath} - \hat{k}$ $(\vec{a_2} - \vec{a_1}) \times \vec{b} = 9\hat{\imath} - 14\hat{\jmath} + 4\hat{k}$ $d = \frac{\sqrt{293}}{7}$	1 1 ½ ½ 1 1

36 (i)	Let A, B and C be the events that calculation is done by Jayant, Sonia and Olivia respectively. Let D be the event that there is an error in the calculation. Then, P(A) = $\frac{50}{100}$, P(B) = $\frac{20}{100}$, P(C) = $\frac{30}{100}$ and P(D/A) = 0.06, P(D/B) =0.04, P(D/C) =0.03 The probability that Sonia processed the calculation and committed a mistake $$=\frac{P(D/B)\,P(B)}{P(D/A)\,P(A)\;+\;P(D/B)\,P(B)+\;P(D/C)\,P(C)}$$ $=\frac{0.04\times\frac{20}{100}}{0.06\times\frac{50}{100}\;+0.04\times\frac{20}{100}+0.03\times\frac{30}{100}}\quad=\frac{8}{47}$ (ii)The total probability of committing a mistake in processing the calculation = $P(D/A)\,P(A)\;+\;P(D/B)\,P(B)+\;P(D/C)\,P(C)$ =0.06 x $\frac{50}{100}$+ 0.04 x $\frac{20}{100}$ + 0.03 x $\frac{30}{100}$ =.047 (iii) If the form selected at random has a mistake, the probability that the form is not processed by Jayant is =1-probability that the form has a mistake and is processed by Jayant $$=1-\frac{P(D/A)\,P(A)}{P(D/A)\,P(A)\;+\;P(D/B)\,P(B)+\;P(D/C)\,P(C)}$$ $$=1-\frac{0.06\;\times\frac{50}{100}}{0.06\;\times\frac{50}{100}\;+0.04\;\times\frac{20}{100}+0.03\;\times\frac{30}{100}}=\frac{17}{47}$$	 1 1 2
37 (i)	$$\overrightarrow{AB}=\hat{\imath}+2\hat{\jmath}+2\hat{k}$$	1
37 (ii)	$$\left\|\overrightarrow{AB}\right\|=3$$	1
37 (iii)	a) $\overrightarrow{AB}+\overrightarrow{BC}=(\hat{\imath}+2\hat{\jmath}+2\hat{k}\;)+\left(2\hat{\imath}+2\hat{\jmath}+2\hat{k}\right)=3\hat{\imath}+4\hat{\jmath}+4\hat{k}$	½

	Unit vector in the direction of $\overrightarrow{AB}+\overrightarrow{BC}=\frac{3\hat{\imath}+4\hat{\jmath}+4\hat{k}}{\sqrt{41}}$ $\overrightarrow{AB}-\overrightarrow{BC}=(\hat{\imath}+2\hat{\jmath}+2\hat{k}\ )-\left(2\hat{\imath}+2\hat{\jmath}+2\hat{k}\right)=-\hat{\imath}$ Unit vector in the direction of $\overrightarrow{AB}-\overrightarrow{BC}=-\hat{\imath}$	½ ½ ½
37 (iii)	b)Area of triangle = $\frac{1}{2}\left\|\overrightarrow{AB}\times\overrightarrow{AC}\right\|=\frac{1}{2}\left\|\hat{\imath}\ \hat{\jmath}\ \hat{k}\ 1\ 2\ 2\ 3\ 4\ 4\ \right\|=\frac{1}{2}\left\|2\hat{\jmath}\ -2\hat{k}\right\|=\left\|\hat{\jmath}-\hat{k}\right\|=\frac{1}{\sqrt{2}}$	1 ½ ½
38 (i)	$f'(x)=x^2-8x+15=(x-3)(x-5)f'(x)=0\Rightarrow x=3{,}5$ are the critical points.	1 1
38 (ii	$f''(x)=2x-8$ $f''(3)<0$ and $f''(5)>0$ So minimum value of $f(x)$ is at $x=5$ Min.value of $=f(5)=\frac{56}{3}$	½ ½ 1

KENDRIYA VIDYALAYA SANGATHAN: HYDERABAD REGION

MODEL QUESTION PAPER - 5 (2025 - 26)

SUBJECT: MATHEMATICS (041)

MARKING SCHEME

....

Q.No	Answer	Marks
1	D	1
2	C	1
3	D	1
4	D	1
5	B	1
6	C	1
7	B	1
8	A	1
9	B	1
10	D	1
11	A	1
12	D	1
13	B	1
14	A	1
15	A	1
16	C	1
17	B	1
18	A	1
19	C	1
20	A	1
21	Check whether the function f(x) = x$\lvert x \rvert$ is differentiable at x = 0 or not. Calculation of LHD = -1 …. 1 mark Calculation of RHD = 1 …… 1 mark Since LHD ≠ RHD f is not differentiable at x = 0 OR If y = $\sqrt{tan\sqrt{x}}$, prove that $\sqrt{x}$ dy/dx = $\frac{1+y^2}{4y}$. Dy/dx = $\frac{1}{2\sqrt{tan\sqrt{x}}}$ sec$^2\sqrt{x}\,\frac{1}{2\sqrt{x}}$ …… 1 mark On simplifying we get $\sqrt{x}$ dy/dx = $\frac{1+y^2}{4y}$ …… 1 mark	2
22	Let two numbers are x and y ,xy=9,S = x^3+y^3 to be minimum $S^1(x)$ = 2x-$\frac{162}{x^3} = 0$ ….. 1 mark Show that $S^{11}(x)$ is positive when x = 3 X = 3,y = 3 are the required numbers ……. 1 mark	2
23	If α, β, γ are the angles made by the lines with x,y and z axes then show that cos2 α +cos2 β +cos2 γ = -1	2

	If α, β, γ are the angles made by the lines with x,y and z axes then we have $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ ……. 1 mark By writing $\cos 2x = 2\cos^2 x - 1$ we get the result …. 1 mark	
24	Let $\vec{a} = 2\hat{i}-4\hat{j}+5\hat{k}$, $\vec{b} = 3\hat{i}-6\hat{j}+2\hat{k}$ and $\vec{c} = 3\hat{i}+\hat{j}$ are such that $(\vec{a}+\mu\vec{b})$ is perpendicular to $\vec{c}$,then find the value of μ. For finding $(\vec{a}+\mu\vec{b})$ ……. 1 mark Finding calculating $(\vec{a}+\mu\vec{b}) \cdot \vec{c} = 0$ we get the value of μ = -2/3 ……. 1 mark OR If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $(\vec{a}+\vec{b})$ is perpendicular to $\vec{a}$ and $(\overrightarrow{2a}+\vec{b})$ is perpendicular to $\vec{b}$,then prove that $\lvert\vec{b}\rvert = \sqrt{2}\lvert\vec{a}\rvert$ If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $(\vec{a}+\vec{b})$ is perpendicular to $\vec{a}$ then (a+b).a = a.a+a.b = 0 ….. 1mark and $(\overrightarrow{2a}+\vec{b})$ is perpendicular to $\vec{b}$, $(\overrightarrow{2a}+\vec{b}) \cdot \vec{b} = 0$,2a.b+b.b =0 ….. 1 mark On simplifying above we get $\lvert\vec{b}\rvert = \sqrt{2}\lvert\vec{a}\rvert$	2
25	Find the principal value of $\cos^{-1}(1/2) - 2\sin^{-1}(-1/2)$. $\frac{\pi}{3} + \frac{2\pi}{6}$ …. 1 mark $= 2\pi/3$ … 1 mark	2
26	Find the intervals in which the function $f(x) = 2x^3-9x^2+12x+15$ is strictly increasing or strictly decreasing. $F^1(x) = 0, 6x^2-18x+12 = 0, x=2,1$ ….. 1 mark Function strictly increasing in $(-\infty,1)$ and $(2,\infty)$ …. 1 mark Function strictly decreasing in (1,2) …. 1 mark	3
27	The volume of cube is increasing at the rate of 9 cm^3/sec. How fast is surface area is increasing when the length of the edge is 10 cm Dv/dt = 9 implies that $3x^2 dx/dt = 9$ ….. 1 mark $Ds/dt = 36/x^2 = 36/10 = 9/25\ cm^2/s$ …… 2 mark	3
28	Find $\int \frac{1}{\cos(x-a)\cos(x-b)}$ dx Multiply and divided by sin (a-b) $= \frac{1}{\sin(a-b)} \int \frac{\sin(a-b)}{\cos(x-a)\cos(x-b)}$ dx …..1 mark $= \frac{1}{\sin(a-b)} \int \frac{\sin(a-x+x-b)}{\cos(x-a)\cos(x-b)}$ dx $= \frac{1}{\sin(a-b)} \int \frac{\sin[(x-b)-(x-a)]}{\cos(x-a)\cos(x-b)}$ dx $= \frac{1}{\sin(a-b)} \int [\tan(x-b)\,dx - \tan(x-a)]dx$ ….. 1 mark $= \frac{1}{\sin(a-b)} \log[\frac{\sec(x-b)}{\sec(x-a)}$ +c ….. 1 mark OR	3

	Evaluate $\int \frac{cosx}{(\sin x-1)(sinx-2)}$ dx Let sinx = t,cosxdx dt Since it is proper fraction so we can apply partial fraction $\frac{1}{(t-1)(t-2)} = \frac{A}{t-1} + \frac{B}{t-2}$ …… 1 mark Finding the values A = -1 ,B = 1 ….. 1 mark On putting A,B values and getting the result after integration = log$\frac{(sinx-2)}{(sinx-1)}$ +c …… 1 mark	
29	Using vectors find the area of triangle with vertices are A (1,1,2), B (2,3,5), C(1,5,5). Finding vector AB,vector AC ….. 1mark Finding ABx AC ….. 1 mark Finding the area of triangle = ½ $\left\|\overrightarrow{AB} \times \overrightarrow{AC}\right\|$ =$\sqrt{61}/2$ sq u….. 1 mark OR Show that the points A, B, C with position vectors $2\hat{\imath}$-$\hat{\jmath}$+$\hat{k}$, $\hat{\imath}$-3$\hat{\jmath}$-5$\hat{k}$ and 3$\hat{\imath}$-4$\hat{\jmath}$-4$\hat{k}$ respectively, are the vertices of a right-angled triangle. Hence find the area of the triangle. Find vectors AB,BC,CA …. 1 mark Find their magnitudes, and verify Pythagoras theorem ….. 1 mark Finding the area of triangle = ½ $\left\|\overrightarrow{AB} \times \overrightarrow{AC}\right\|$ =$\sqrt{61}/2$ sq u .. 1 mark	3
30	Maximise z = 600x+400y, Subject to constraints x+2y≤ 12, 2x+y ≤ 12, x+1.25y ≥5, x ≥0 ,y ≥ 0 Draw the correct graphs for above constraints ……. 1 mark For deciding the corner points of feasible region (6,0),(5,0),(0,4),(4,4),(0,6) …. 1 mark For finding the maximum z = 4000 at (4,4) …… 1 mark	3
31	Given P(A)=0.6 P(not B)=0.3 P(B)= 1-0.3=0.7 ....1 P(both A and B occurs)= $P(A).P(B)$ = 0.6x 0.7= 0.42 ....1 P(exactly one of A or B occurs)=$P(A)P(B^I) + P(A^I)P(B)$ = 0.6 x 0.3 + 0.4 x 0.7 = 0.18 + 0.28 = 0.46 1	3
32	If A =$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$,find A^{-1} and use it to solve the system of following equations: x-2y= 10, 2x-y-z = 8, -2y+z = 7 for finding A^{-1} ……. 2 mark For writing A,X and B ….. 1 mark Finding det A ≠ 0 and A is non singular ……. 1 mark	5

	Solving $X = A^{-1}B$ …… 1 mark OR $A = \begin{bmatrix} -1 & a & 2 \\ 1 & 2 & x \\ 3 & 1 & 1 \end{bmatrix}$ and $A^{-1} = = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ b & y & 3 \end{bmatrix}$, find the value of (a+x) – (b+y). Finding det A $\neq 0$ and A is non singular ……. 1 mark Finding adjA …….. 2 Marks Writing $A^{-1} = \frac{adjA}{detA}$ …… I mark On comparing given A^{-1} and obtained A^{-1} we get the values a,b,x and y and calculate (a+x)-(b+y) ….. 1 mark	
33	Using integration, find the area of the region bounded by the curves y = x^2, y = x+2 and x- axis. For drawing correct figure and shading the region ….. 2 marks For finding the point of intersections….. 1 mark For writing the area of required region = $\int_{-1}^{2}(x+2-x^2)$dx …. 1 mark For obtaining the area 11/2 sq units…. 1 mark	5
34	Find the shortest distance between the following two lines: $\vec{r} = (4\hat{\imath} - \hat{\jmath}) + \lambda(\hat{\imath} + 2\hat{\jmath} - 3\hat{k})$, $\vec{r} = (\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \mu(\widehat{2\imath} + 4\hat{\jmath} - 5\hat{k})$. For writing vectors a_1, a_2, b_1, b_2 …… 1 mark Calculating a_2-a_1 ,b_1xb_2 …… 1 mark Finding magnitude of ,b_1xb_2 ……..1 mark For calculating shortest distance = $\frac{(a_2-a_1).\text{b1xb2}}{\text{b1xb2}}$ = $6\sqrt{5}/5$ …… 2 marks OR Show that the lines $\vec{r} = (3\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + \lambda(\hat{\imath} + 2\hat{\jmath} + \widehat{2k})$, $\vec{r} = (\widehat{5\imath} - 2\hat{\jmath}) + \mu(\widehat{3\imath} + 2\hat{\jmath} + 6\hat{k})$ are intersecting. Hence find their point of intersection. For writing vectors a_1, a_2, b_1, b_2 …… 1 mark Calculating a_2-a_1 ,b_1xb_2 …… 1 mark Finding magnitude of ,b_1xb_2 ……..1 mark For calculating shortest distance = $\frac{(a_2-a_1).\text{b1xb2}}{\text{b1xb2}}$ = 0 …… 1 mark For finding point of intersection (-1,-6,-12) ……. 1 mark	5
35	If x = a(cost +t sint),y = a(sint-tcost) find $\frac{d^2y}{dt^2}, \frac{d^2x}{dt^2}, \frac{d^2y}{dx^2}$ Dx/dt = atcost,dy/dt =atsint ……. 1 mark Dy/dx =tant …….. 1 mark $\frac{d^2y}{dt^2}$ = a(tcost+sint) ……. 1 mark $\frac{d^2x}{dt^2}$ = a(-tsint+cost) …… 1mark $\frac{d^2y}{dx^2}$ = sec^2t.dt/dx = sec^3t/a ….. 1 mark OR	5

	If $\sqrt{1-x^2}+\sqrt{1-y^2}$ = a(x-y), prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$ Put x = sinA,y =sin B … 1 mark CosA +CosB =a(sinA-sinB) …. 1 mark 2cos(A+B/2) cos(A-B/2) = a2cos(A+B/2) sin(A-B/2) …. 1 mark Tan(A-B/2) = 1/a A-B = 2 $\tan^{-1}$(1/a) implies that $\sin^{-1}$x-$\sin^{-1}$y = 2 $\tan^{-1}$(1/a) …. 1 mark Diff w.r.t.x we get $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$ …. 1 mark	
36	(i) $2^{3x2} = 2^6 = 64$ (ii) {(g_1,g_1),(g_2,g_2) (iii) (A) so the minimum number of elements to be added are (b1,b1),(b2,b2),(b3,b3),(b2,b3) (B) so the minimum number of elements to be added are (b1,b1),(b2,b2),(b3,b3),(b2,b3),(b3,b2) OR One-one and onto function: F(x) = $\frac{x^2}{4}$ clearly one - one function in [0,20$\sqrt{2}$] For any arbitrary element in [0,200] the preimage of y exists in [0,20$\sqrt{2}$] Hence f is onto	1 1 2 2
37	(i) $\tan^{-1}$(5/x) (ii) -5/(x^2+25) (iii) -4/101 OR -15 m/s	1 1 2
38	Let E1 = severe, E2 = moderate, E3 = Light Turbulence (i) P(E1) =P(E2) = P(E3) = 1/3 P(airplane reached destination late) = P(E1)P(A/E1)+ P(E2)P(A/E)+ P(E3)P(A/E3) =1/3(55/100) + 1/3(37/100) +1/3(17/100) = 109/300 (ii) P(If the airplane reached its destination late, find the probability that it was due to moderate turbulence) P(E3/A) = P(E3).P(A/E3)/P(A) =17/109	2 2

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KENDRIYA VIDYALAYA SANGATHAN: HYDERABAD REGION

MODEL QUESTION PAPER - 6 (2025 - 26)

SUBJECT: MATHEMATICS (041)

MARKING SCHEME

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प्र.क्र. Q.No.	ASNWERS	अंक Marks
	SECTION – A (1 x 20 = 20) **(*This section comprises of multiple choice questions (MCQs) of 1 mark each*)**	
01 - 20	(see table below)	Each Carry 1 Mark

1	B) I	6	C) 3	11	C) 8	16	D) q = 3p
2	B) $A^2 = I$	7	A) 1	12	D) 3√2	17	C) $\left(\frac{2}{7}, \frac{3}{7}, \frac{-6}{7}\right)$
3	C) 0	8	A) y(x *cot* x + log*sin*x)	13	D	18	B) $\cos^2 x$
4	D) 1/12	9	B) 3/8	14	C) $\frac{5}{\sqrt{6}}$	19	D
5	C) ±√3	10	A) $A=\begin{bmatrix} 2 & 2 & -4 \\ 2 & 3 & 4 \\ -4 & 4 & 2 \end{bmatrix}$	15	B) 2	20	A

	SECTION B (2m x 5 =10m)	
21	dy/ dx = 4x (x-1) (x-2) critical points x = 0,1,2 dy/dx ≥0 =>interval x ∈ [0,1]U[2,∞) **OR** dy/dx = cosx + sinx critical point x = 3π/4 dy/dx ≥0 => interval x ∈ [0,3π/4]	½ ½ 1 ½ ½ 1
22	$\cos^{-1}\{\cos\frac{13\pi}{6}\} \neq \frac{13\pi}{6}$ as $\frac{13\pi}{6} \notin [0, \pi]$ $\cos^{-1}\{\cos\frac{13\pi}{6}\} = \cos^{-1}\{\cos(2\pi+\pi/6)\}$ $= \cos^{-1}\{\cos\pi/6\} = \pi/6$	½ ½ 1
23	$y = e^x + e^{-x}$ $\frac{dy}{dx} = e^x - e^{-x}$ $= \sqrt{(e^x + e^{-x})^2 - 4} = \sqrt{y^2 - 4}$	1 1
24	$\int \frac{1-\tan x}{1+\tan x} dx = \int \tan\left(\frac{\pi}{4} - x\right) dx$ $= -\log\left\|\sec\left(\frac{\pi}{4} - x\right)\right\| + c$ **OR** $\int e^x \left(f(x) + f'(x)\right) dx$ $= e^x f(x) + x$	1 1 ½

	$I=\int e^x\left(\frac{1}{x+5}-\frac{1}{(x+5)^2}\right)dx$ = e^x / (x+ 5) + c	1 ½
25	V = 4/3 π r^3 dv/dt = 4πr^2(dr/dt) dr/dt = 3/16π S= 4πr^2 dS/dt = 8πr (dr/dt) = 3 cm^2/s	½ ½ ½ ½
	SECTION C (3m x 6 =18m)	
26	Let the events be: A: Raju is selected B: Ravi is selected Then according to the question, A and B are independent events and $P(A)=0.4,\quad P(A\cap\bar{B})+P(B\cap\bar{A})=0.5$ Let $P(B)=x$ Then $P(A\cap\bar{B})+P(B\cap\bar{A})=0.5$ $\Rightarrow P(A)P(\bar{B})+P(B)P(\bar{A})=0.5$ $\Rightarrow 0.4(1-x)+x(1-0.4)=0.5$ $\Rightarrow 0.4-0.4x+0.6x=0.5$ $\Rightarrow 0.2x=0.5-0.4=0.1$ $\Rightarrow x=\frac{0.1}{0.2}=\frac{1}{2}=0.5$ So, probability of selection of Rashi = 0.5 Probability of selection of at least one of them $=1-P(\bar{A}\cap\bar{B})$ $=1-P(\bar{A})P(\bar{B})$ $=1-0.6\times 0.5$ $=1-0.3=0.7$ **OR**	 1 ½ 1 ½

Let P(A) = Probability that A solves the problem

P(B) = Probability that B solves the problem

P(Ā) = Probability that A does not solve the problem

and P(Ā) = Probability that B does not solve the problem.

According to the question, we have

$$P(A) = \frac{1}{2}$$

then $$P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$[\because P(A) + P(\bar{A}) = 1]$$

and $$P(B) = \frac{1}{3}$$

then $$P(\bar{B}) = 1 - P(B) = 1 - \frac{1}{3} = \frac{2}{3}$$

P (problem is solved)

= 1 − P (none of them solve the problem)

$= 1 - P(\bar{A} \cap \bar{B})$

$= 1 - P(\bar{A}) \cdot P(\bar{B}) = 1 - \left(\frac{1}{2} \times \frac{2}{3}\right)$

$\because P(\bar{A}) = \frac{1}{2}$ and $P(\bar{B}) = \frac{2}{3}$

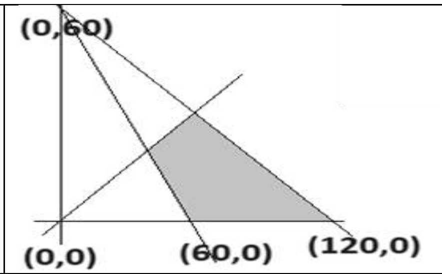
$= 1 - \frac{1}{3} = \frac{2}{3}$

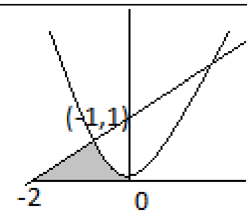
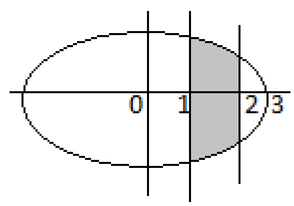
(ii) P (exactly one of them solve the problem)

$= P(A) + P(B) - 2P(A \cap B)$

$= P(A) + P(B) - 2P(A) \times P(A)$

$= \frac{1}{2} + \frac{1}{3} - 2 \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{2}$

27	 Graph + shade Table 1 Max Z = 600 at line segment joining (120,0) & (40,40)	½ + 1 1 ½
28	$\frac{dx}{dy}+\frac{2xy}{\left(y^2+1\right)}=\frac{1}{\left(1+y^2\right)\left(y^2+1\right)}$ $IF=e^{\int\frac{2ydy}{1+y^2}}=1+y^2$ Solution x(IF) = $\int Q(IF)dy+c$ $x\left(y^2+1\right)=\tan^{-1}y+C$ (OR) Let y= Vx → $\frac{dy}{dx}$ = V + x $\frac{dV}{dx}$ $y+x\sin\left(\frac{y}{x}\right)=x\frac{dy}{dx}\Rightarrow\ \frac{dx}{x}=\frac{dV}{\sin V}$ Solving => cx = (cosecx – cotx) Particular solution: x (√2 – 1)/2= (cosecx – cotx)	½ 1 ½ 1 ½ 1 ½ + ½ ½
29	$I=\int_{-5}^{5}\frac{x^2}{1+e^x}dx$ ….. (i), using property $\int_{-5}^{5}f(x)dx=\int_{-5}^{5}f(-x)dx$ $I=\int_{-5}^{5}\frac{x^2}{1+e^{-x}}dx$ …..(ii) From i and ii , adding $2I=\int_{-5}^{5}x^2dx\Rightarrow\ \ I=\frac{125}{3}$ (OR) $\int_0^4\left(\lvert x-1\rvert+\lvert x-2\rvert\right)dx$ $=\int_0^1(3-2x)dx+\int_1^2dx+\int_2^4(2x-3)dx$ =2+1+6 = 9	½ ½ ½ 1 + ½ ½ + ½ + ½ 1+½
30	$I=\int\frac{2x}{\left(x^2+1\right)\left(x^2+2\right)}dx$ let x^2 = t => 2xdx = dt $\frac{1}{(t+1)(t+2)}=\frac{A}{t+1}+\frac{B}{t+2}$ Solving A = 1, B = -1	½ ½ ½ + ½ ½ + ½

	Correct integration and solution $I = \log\left(\frac{x^2+1}{x^2+2}\right)$	
31	For correct differentiation of LHS 1 marks Right hand side with simplification 1 marks Proving 1 marks	1 1 1
	SECTION D (5m x 4 = 20m)	
32	Proving reflexive Symmetric Transitive Equivalence class = (1,5)(2,6),(3,7)(4,8)(5,9) **(OR)** Proving one to one with all steps correct $f(x_1) = f(x_2)$ $(x_1 - x_2)(x_1 + x_2 + 3) = 0 \Rightarrow x_1 = x_2$ Proving Onto with range = $[-5, \infty)$ Final statement	1 1 2 1 ½ 1½ 2 ½ ½
33	Correct graph and shade Point of intersection and value of corner points Correct integration Ans = 5/6 **OR** Correct graph and shade Value of y for integration Correct integration and limits Ans = $\frac{4}{3}\left[\sqrt{5} + \frac{9}{2}\sin^{-1}\frac{2}{3} - \sqrt{2} - \frac{9}{2}\sin^{-1}\frac{1}{3}\right]$   	1 1 2 1 1 1 2 1
34	Given A = $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}$ B = $\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ finding AB = $\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ Given system of equations : $x + 3z = 9$; $-x + 2y - 2z = 4$; $2x - 3y + 4z = -3$. Writing in Matrix form $\begin{bmatrix}1 & 0 & 3\\ -1 & 2 & -2\\ 2 & -3 & 4\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}9\\ 4\\ -3\end{bmatrix}$ A^{-1} = transpose of $\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ as $A^T B^T = I$ solving we get x = 0, y = 5 and z = 3	2 Mark 1 Mark 2 Mark
35	(i) Finding vector d, perpendicular to both d = λ (a x b) $\vec{d} = \lambda\left(-21\hat{i} + 21\hat{j} + 21\hat{k}\right)$ Value of $\lambda = 3$, from the given condition c . d = 21 Ans: $\vec{d} = \left(-7\hat{i} + 7\hat{j} + 7\hat{k}\right)$ ii) $\sin\theta = \left\lvert\frac{a \times b}{\lvert a \times b\rvert}\right\rvert$	½ 1 ½ ½ ½ 1 1

	$\theta = 60^0$	
SECTION E (4m x 3 = 12m)		
36		
i	$2\pi rh + \pi r^2 = 75\pi$ Volume (V) = $\left(\frac{\pi}{2}\right)(75r - r^3)$ cm^3	1M
ii	Find $\frac{dV}{dr} = \left(\frac{\pi}{2}\right)(75 - 3r^2)$	1M
iii	When volume is maximum $\frac{dV}{dr} = \left(\frac{\pi}{2}\right)(75 - 3r^2) = 0$ r = ± 5 by using second derivative test (OR) For maximum volume, h = r. Hence the statement is false.	2M
Case Study-2		
37		
i	paths are parallel and non-intersecting	1M
ii	$\vec{r} = \lambda\left(9\hat{i} - 6\hat{j} - 2\hat{k}\right)$ = point + λ direction unit vector = $\hat{b} = \frac{9i-6j-2k}{11}$	1M
iii	formula SD = $\left\|(a2 - a1) \times \frac{\vec{b}}{\|\vec{b}\|}\right\| = \left\|(200i + 200j - 100k) \times \frac{(9i-6j-2k)}{11}\right\|$ cross product calculation (or) determinant method Ans = $(500\sqrt{41}) / 11$ units	2M
Case Study- 3		
38		
i	P(A1) = $\frac{4}{10}$ P(A2) = $\frac{4}{10}$ P(A3) = $\frac{2}{10}$ let G denote the event of germination P(G/A1) = 45% P(G/A2)= 60% P(G/A3) = 35% P(G) = $\frac{49}{100}$ = 49%	2M
ii	P(A2/G) = $\frac{P(A2).P(G/A2)}{P(G)} = \frac{24}{49}$	2M

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KENDRIYA VIDYALAYA SANGATHAN: HYDERABAD REGION

MODEL QUESTION PAPER - 7 (2025 - 26)

SUBJECT: MATHEMATICS (041)

MARKING SCHEME

Sections - A

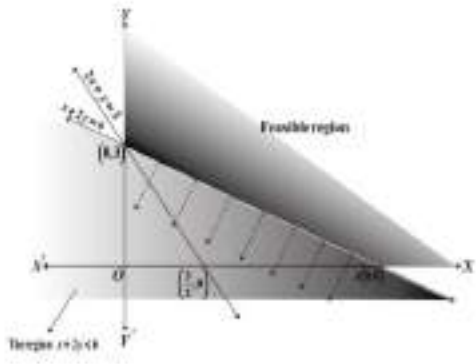
Q.NO	ANSWER	Q.NO	ANSWER	Q.NO	ANSWER
1.	**d**	**2.**	**c**	**3.**	**c**
4.	**c**	**5.**	**a**	**6.**	**c**
7.	**d**	**8.**	**d**	**9.**	**a**
10.	**d**	**11.**	**b**	**12.**	**b**
13.	**b**	**14.**	**a**	**15.**	**d**
16.	**a**	**17.**	**a**	**18.**	**d**
19.	**a**	**20.**	**b**		

Q.NO	Solution	Marks
	Section - B	
21.	The marginal cost function is $C'(x) = 0.00039x^2 + 0.004x + 5$ $C'(150)$ =Rs 14.375	2
22.	We have $$I = \int_0^1 \frac{e^x}{1+e^{2x}}dx = \int_0^1 \frac{e^x}{1+(e^x)^2}dx$$	

	Substituting $e^x = t \Rightarrow e^x dx = dt$ Also, when $x = 0$, then $t = 1$ and when $x = 1$, then $$t = e$$ Now, $$I = \int_1^e \frac{dt}{1+t^2} = [\tan^{-1}(t)] = \tan^{-1} e - \tan^{-1}(1) = \tan^{-1}\left(\frac{e-1}{1+e}\right)$$	1 1
23.	We have $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 7\hat{k}$ Sum of two vectors, $$\vec{a} + \vec{b} = (2\hat{i} + 2\hat{j} - 5\hat{k}) + (2\hat{i} + \hat{j} - 7\hat{k}) = 4\hat{i} + 3\hat{j} - 12\hat{k}$$ Required unit vector $$\frac{\vec{a}+\vec{b}}{\lvert\vec{a}+\vec{b}\rvert} = \frac{4\hat{i} + 3\hat{j} - 12\hat{k}}{\sqrt{4^2+3^2+12^2}} = \frac{4\hat{i} + 3\hat{j} - 12\hat{k}}{\sqrt{169}} = \frac{4\hat{i} + 3\hat{j} - 12\hat{k}}{31}$$ $$= \frac{1}{13}(4\hat{i} + 3\hat{j} - 12\hat{k})$$ or We have $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$ Then, $\vec{a} + \vec{b} = (\hat{i} - \hat{j} + 7\hat{k}) + (5\hat{i} - \hat{j} + \lambda\hat{k})$ $$= 6\hat{i} - 2\hat{j} + (7+\lambda)\hat{k}$$ and $$\vec{a} - \vec{b} = (\hat{i} - \hat{j} + 7\hat{k}) - (5\hat{i} - \hat{j} + \lambda\hat{k}) = -4\hat{i} + (7-\lambda)\hat{k}$$ Since, $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are perpendicular vectors, we have $$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$$ $$[6\hat{i} - 2\hat{j} + (7+\lambda)\hat{k}] \cdot [-4\hat{i} + (7-\lambda)\hat{k}] = 0$$ $$-24 + (7+\lambda)(7-\lambda) = 0$$	1/2 1/2 1

	$49-\lambda^2 \quad = 24$ $\lambda^2 \quad = 25$ $\lambda \ = \pm 5$	
24.	Vector equation of a line passing through a point with position vector $\vec{a}$ and parallel to a given vector $\vec{b}$ is $$r^{\to} = a^{\to} + \lambda b^{\to},\ where\ \lambda \in R$$ Vector for point $(1,-1,2)$ is $$a^{\to} = i^{\wedge} - j^{\wedge} + 2k^{\wedge}$$ and vector for line $\frac{(x-3)}{1} = \frac{(y-1)}{2} = \frac{(z+1)}{-2}$ is $$b^{\to} = i^{\wedge} + 2j^{\wedge} - 2k^{\wedge}.\ DR's\ are\ 1,2\ and\ -2$$ Required vector equation of line is $$r^{\to} = (i^{\wedge} - j^{\wedge} + 2k^{\wedge}) + \lambda(i^{\wedge} + 2j^{\wedge} - 2k^{\wedge}), where\ \lambda \in R$$	1 1
25.	Let E_1 and E_2 denote the events that first and second group will win. Then, $$P(E_1) = 0.6\ and\ P(E_2) = 0.4$$ Let E be the event of introducing the new product. Then, $P(E/E_1) = 0.7$ and $P(E/E_2) = 0.3$ Now, we have to find the probability that new product is introduced by second group. $$P\left(\frac{E_2}{E}\right) = \frac{P(E_2)P\left(\frac{E}{E_2}\right)}{P(E_1)P\left(\frac{E}{E_1}\right) + P(E_2)P\left(\frac{E}{E_2}\right)} = \frac{0.4 \times 0.3}{0.6 \times 0.7 + 0.4 \times 0.3} = \frac{0.12}{0.42 + 0.12} = \frac{0.12}{0.54}$$ $$= 0.22$$	1 1
	Section - C	
26.	The feasible region determined by the constraints,	

2x+ y ≥ *3*, x+2 y ≥ *6*, x ≥0, y ≥0   is as shown.



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The corner points of the unbounded feasible region are A(6,0) and B(0, 3) . The values of Z at these corner points are as follows:

Corner point	Value of the objective function Z=x+2 y
A(6, 0)	6
B(0,3)	6

1

We observe the region x +2 y < 6 have no points in common with the unbounded feasible region. Hence the minimum value of z = 6.

It can be seen that the value of Z at points A and B is same. If we take any other point on the line x +2y= 6 such as (2,2) on line x + 2y = 6 then Z = 6.

Thus, the minimum value of Z occurs for more than 2 points, and is equal to 6.

27.

First, find the vector $2\vec{a} - \vec{b} + 3\vec{c}$, then find a unit vector in the direction of $2\vec{a} - \vec{b} + 3\vec{c}$. After this, the unit vector is multiplying by 6 .
We have

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$$

and

$$\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$$

1

Now

$$\vec{d} = 2\vec{a} - \vec{b} + 3\vec{c}$$

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$$= 2(\hat{i} + \hat{j} + \hat{k}) - (4\hat{i} - 2\hat{j} + 3\hat{k}) + 3(\hat{i} - 2\hat{j} + \hat{k})$$

$$= 2\hat{i} + 2\hat{j} + 2\hat{k} - 4\hat{i} + 2\hat{j} - 3\hat{k} + 3\hat{i} - 6\hat{j} + 3\hat{k}$$

Now, unit vector $\hat{d}$ in the direction of $\vec{d}$ is $\frac{\vec{d}}{|\vec{d}|}$

$$\hat{d} = \frac{2\vec{a} - \vec{b}3 + \vec{c}}{|2\vec{a} - \vec{b} + 3\vec{c}|} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{(1)^2 + (-2)^2 + (2)^2}} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{9}} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$$

$$= \frac{1}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$$

Hence, vector of magnitude 6 units parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$ is given by

$$6\hat{d} = 6(1/3\,\hat{i} - 2/3\,\hat{j} + 2/3\,\hat{k}) = 2\hat{i} - 4\hat{j} + 4\hat{k}$$

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28.	We have $I=\int_0^{\frac{\pi}{2}} x^2 \sin x dx$ Using integration by parts technique $$\int uv\, dx=\left[u\int vdx-\int\left\{\frac{d}{dx}u\cdot\int vdx\right\}dx\right]$$ and choosing its function with the help of ILATE procedure we have $$=-x^2\cos x+2\left[x(\sin x)-\int 1\cdot(\sin x)dx\right]$$ $$=-x^2\cos x+2(x\sin x+\cos x)$$ $$=[-x^2\cos x+2(x\sin x+\cos x)]_0^{\pi/2}$$ $$=\left[-\left(\frac{\pi}{2}\right)^2\cos\left(\frac{\pi}{2}\right)+2\left(\frac{\pi}{2}\sin\frac{\pi}{2}+\cos\frac{\pi}{2}\right)-2(0+cos0)\right.$$ $$=-\frac{\pi^2}{4}\times 0+2\left(\frac{\pi}{2}+0\right)-2(0+1)$$ $$=\pi-2$$ **Or** We have $I=\int_{\pi/4}^{\pi/2}\cos 2x\log(\sin x)dx$ Using integration by parts technique $\int \underset{\mathrm{I}}{u}\cdot \underset{\mathrm{II}}{v}dx=\left[u\int vdx-\int\left\{\frac{d}{dx}u\cdot\int vdx\right\}dx\right]$ and choosing its function with the help of ILATE procedure we have $$I=\left[\log(\sin x)\frac{\sin 2x}{2}-\int\frac{1}{\sin x}\cos x\cdot\frac{\sin 2x}{2}dx\right]_{\pi/4}^{\pi/2}$$ $$=\left[\frac{\log(\sin x)\cdot\sin 2x}{2}\right]_{\pi/4}^{\pi/2}-\int_{\pi/4}^{\pi/2}\frac{\cos x\cdot(2\sin x\cos x)}{2\sin x}dx$$ $$=\frac{1}{2}\left[\log\left(\sin\frac{\pi}{2}\right)\cdot\sin\pi-\log\left(\sin\frac{\pi}{4}\right)\cdot\sin\frac{\pi}{2}\right]-\int_{\pi/4}^{\pi/2}\cos^2 xdx$$ $$=\frac{1}{2}\left[0-\log\frac{1}{\sqrt{2}}\right]-\int_{\pi/4}^{\pi/2}\frac{1+\cos 2x}{2}dx$$ $$=\frac{1}{4}\log 2-\frac{1}{2}\left[x+\frac{\sin 2x}{2}\right]_{\pi/4}^{\pi/2}$$ $$=\frac{1}{4}\log 2-\frac{1}{2}\left[\left(\frac{\pi}{2}+\frac{\sin\pi}{2}\right)-\left(\frac{\pi}{4}+\frac{\sin\pi/2}{2}\right)\right]$$ $$=\frac{1}{4}\log 2-\frac{1}{2}\left(\frac{\pi}{2}-\frac{\pi}{4}-\frac{1}{2}\right)$$ $$=\frac{1}{4}\log 2-\frac{1}{2}\left(\frac{\pi}{4}-\frac{1}{2}\right)$$ $$=\frac{1}{4}\log 2-\frac{\pi}{8}+\frac{1}{4}$$	1 1 1 1 ½ ½ ½ ½

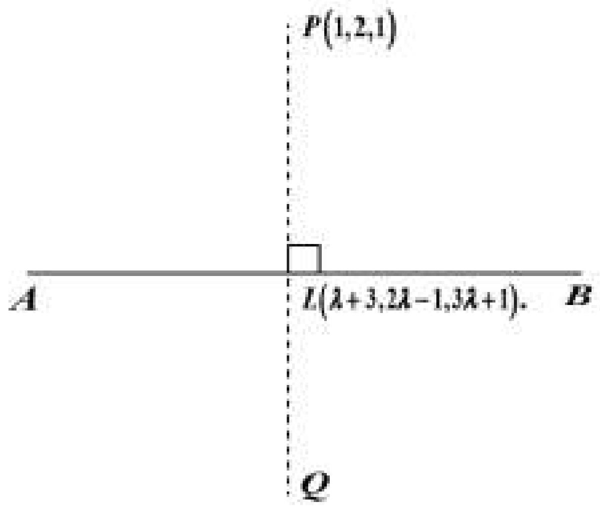
29.	We have $\cos^{-1} x + \sin^{-1}\left(\frac{x}{2}\right) = \frac{\pi}{6}$ $$\cos^{-1} x = \frac{\pi}{6} - \sin^{-1}\frac{x}{2}x \quad => x = \cos\left(\frac{\pi}{6} - \sin^{-1}\frac{x}{2}\right)$$ Using $\cos(x-y) = \cos x \cos + \sin x \sin y$ we have $$x = \cos\frac{\pi}{6}\cos\left(\sin^{-1}\frac{x}{2}\right) + \sin\frac{\pi}{6}\sin\left(\sin^{-1}\frac{x}{2}\right)$$ $$x = \frac{\sqrt{3}}{2}\cos\left(\cos^{-1}\sqrt{1-\frac{x^2}{4}}\right) + \frac{x}{4}$$ $$x = \frac{\sqrt{3}}{2}\left(\sqrt{1-\frac{x^2}{4}}\right) + \frac{x}{4}$$ $$x - \frac{x}{4} = \frac{\sqrt{3}}{2}\left(\sqrt{1-\frac{x^2}{4}}\right)$$ $$\frac{3x}{4} = \frac{\sqrt{3}}{2}\left(\sqrt{1-\frac{x^2}{4}}\right)$$ $$\frac{9x^2}{16} = \frac{3}{4}\left(1-\frac{x^2}{4}\right)$$ $$\frac{3}{4}x^2 = 1 - \frac{x^2}{4}$$ $$\frac{3}{4}x^2 + \frac{x^2}{4} = 1$$ $$\frac{4x^2}{4} = 1$$ $$x^2 = 1 \Rightarrow x = \pm 1$$ But $x = -1$, does not satisfy the given equation. Hence, $x = 1$ satisfy the given equation.	1 ½ ½ ½ ½

30.	We have I $= \int \frac{x^2+x-1}{(x^2+1)(x+2)}\,\mathrm{d}x$ Now $\frac{x^2+x-1}{(x^2+1)(X+2)} = \frac{A}{(x+2)} + \frac{Bx+c}{x^2+1}$ $x^2+x+1 = A(x^2+1)+(Bx+C)(x+2)$ …. (1) $x^2+x+1 = x^2(A+B)+x(2B+C)+(A+2C)$ …..(2) Substituting $x=-2$ in eq (1) we have $4-2+1 = A(4+1)+0 \Rightarrow A = 3/5$ Comparing the coefficients of x^2, x and constant terms both sides in eq (2), we get A + B = 1 …..(3) 2B + C = 1 ……(4) A + 2C = 1 ……(5) and Substituting A = 3/5 in (3) and (5) we get C = 1/5 and B = 2/5. Thus$\int \frac{x^2+x+1}{(x^2+1)(x+2)}dx = \frac{3}{5}\int \frac{1}{(x+2)}\mathrm{d}x + \frac{1}{5}\int \frac{(2x+1)}{(x^2+1)}dx$ Now $= 3/5\, log\,\lvert x+2\rvert + 1/5\, log\,\lvert x^2+1\rvert + 1/5\ \tan^{-1} x)\, x + C$	½ ½ ½ 1½

31.	We have, $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$ Here, $\|A\| = \begin{vmatrix} 2 & -3 \\ -4 & 7 \end{vmatrix}$ $= 14 - 12 = 2$ Since $\|A\| \neq 0$, therefore A^{-1} exists. Now, $\text{adj}(A) = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ $A^{-1} = \frac{1}{\|A\|}\text{adj}(A)$ $= \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} \dots\dots (i)$ Now $2A^{-1} = 9I - A$ $\text{RHS} = 9I - A$ $= 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$ $= \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ $= 2A^{-1}$ [using Eq. (i) or We have, $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ $\|A\| = \begin{vmatrix} 2 & 3 \\ 5 & -2 \end{vmatrix}$ $= -4 - 15 = -19$ Since $\|A\| \neq 0$, matrix A is non-singular and A^{-1} exists. Now adjA= $\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$ $A^{-1} = \frac{1}{\|A\|}$ adj A $A^{-1} = \frac{-1}{19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$ $= \frac{1}{19}\begin{bmatrix} 2 & 3 \\ 5 & 2 \end{bmatrix}$ $= \frac{1}{19}A \;\; => k = \frac{1}{19}$	1 1 ½ ½ ½ ½ 1 1

Section D

32.	**(A)** Given that equation of lines are $$\vec{r} = (-\hat{\imath} - \hat{\jmath} - \hat{k}) + \lambda(7\hat{\imath} - 6\hat{\jmath} + \hat{k}) \ldots (i) \ and \ \vec{r} = (3\hat{\imath} + 5\hat{\jmath} + 7\hat{k}) + \mu(\hat{\imath} - 2\hat{\jmath} + \hat{k}) \ldots\ldots (ii)$$ The given lines are non-parallel lines as vectors $7\hat{\imath} - 6\hat{\jmath} + \hat{k}$ and $\hat{\imath} - 2\hat{\jmath} + \hat{k}$ are not parallel. There is a unique line segment PQ (P lying on line (i) and Q on the other line (ii)), which is at right angles to both the lines PQ & is the shortest distance between the lines.	½
	Hence, the shortest possible distance between the lines $= PQ$. Let the position vector of the point P lying on the line $\vec{r} = (-\hat{\imath} - \hat{\jmath} - \hat{k}) + \lambda(7\hat{\imath} - 6\hat{\jmath} + \hat{k})$ where' λ' is a scalar, is $(7\lambda - 1)\hat{\imath} - (6\lambda + 1)\hat{\jmath} + (\lambda - 1)\hat{k}$, for some λ and the position vector of the point Q lying on the line $\vec{r} = (3\hat{\imath} + 5\hat{\jmath} + 7\hat{k}) + \mu(\hat{\imath} - 2\hat{\jmath} + \hat{k})$ where ' μ ' is a scalar, is $(\mu + 3)\hat{\imath} + (-2\mu + 5)\hat{\jmath} + (\mu + 7)\hat{k}$, for some μ. Now, the vector	½
	$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$$ $$\overrightarrow{PQ} = (\mu + 3 - 7\lambda + 1)\hat{\imath} + (-2\mu + 5 + 6\lambda + 1)\hat{\jmath} + (\mu + 7 - \lambda + 1)k$$ i.e., $\vec{(PQ)} = (\mu - 7\lambda + 4)\hat{\imath} + (-2\mu + 6\lambda + 6)\hat{\jmath} + (\mu - \lambda + 8)\hat{k}$; (where ' O ' is the origin), is perpendicular to both the lines, so the vector $\overrightarrow{PQ}$ is perpendicular to both the vectors $7\hat{\imath} - 6\hat{\jmath} + \hat{k}$ and $\hat{\imath} - 2\hat{\jmath} + \hat{k}$. $$\Rightarrow (\mu - 7\lambda + 4) \cdot 7 + (-2\mu + 6\lambda + 6) \cdot (-6) + (\mu - \lambda + 8) \cdot 1 = 0$$ $$\&(\mu - 7\lambda + 4) \cdot 1 + (-2\mu + 6\lambda + 6) \cdot (-2) + (\mu - \lambda + 8) \cdot 1 = 0 \quad \Rightarrow 20\mu - 86\lambda = 0 => 10\mu - 43\lambda = 0 \& 6\mu - 20\lambda = 0 \Rightarrow 3\mu - 10\lambda = 0$$ On solving the above equations, we get $\mu = \lambda = 0$ So, the position vector of the points P and Q are $-\hat{\imath} - \hat{\jmath} - \hat{k}$ and $3\hat{\imath} + 5\hat{\jmath} + 7\hat{k}$ respectively. $$\vec{(PQ)} = 4\hat{\imath} + 6\hat{\jmath} + 8\hat{k} \ and \quad \lvert\vec{(PQ)}\rvert = \sqrt{(4^2 + 6^2 + 8^2)} = \sqrt{116} = 2\sqrt{29} \ units.$$	½
	(B)	



½

½

Let P(1,2,1) be the given point and L be the foot of the perpendicular from P to the given line AB (as shown in the figure above).

½

Let's put $\frac{x-3}{1}=\frac{y+1}{2}=\frac{z-1}{3}=\lambda$. Then, $x=\lambda+3, y=2\lambda-1, z=3\lambda+1$

Let the coordinates of the point L be $(\lambda+3, 2\lambda-1, 3\lambda+1)$.

So, direction ratios of PL are $(\lambda+3-1, 2\lambda-1-2, 3\lambda+1-1)$ *i.e.*, $(\lambda+2, 2\lambda-3, 3\lambda)$ ½

Direction ratios of the given line are *1*,*2* and *3*, which is perpendicular to PL. Therefore, we have,

$$(\lambda+2)\cdot 1+(2\lambda-3)\cdot 2+3\lambda\cdot 3=0 \Rightarrow 14\lambda=4 \Rightarrow \lambda=2/7$$

Then, $\lambda+3=\frac{2}{7}+3=\frac{23}{7};\ 2\lambda-1=2\left(\frac{2}{7}\right)-1=-\frac{3}{7};\ 3\lambda+1=3\left(\frac{2}{7}\right)+1=\frac{13}{7}$

Therefore, coordinates of the point L are $\left(\frac{23}{7},-\frac{3}{7},\frac{13}{7}\right)$. ½

Let $Q(x_1, y_1, z_1)$ be the image of P(1,2,1) with respect to the given line. Then, L is the mid-point of PQ. ½

Therefore, $\frac{1+x_1}{2}=\frac{23}{7}, \frac{2+y_1}{2}=-\frac{3}{7}, \frac{1+z_1}{2}=\frac{13}{7} \Rightarrow x_1=\frac{39}{7}, y_1=-\frac{20}{7}, z_1=\frac{19}{7}$

Hence, the image of the point P(1,2,1) with respect to the given line $Q\left(\frac{39}{7},-\frac{20}{7},\frac{19}{7}\right)$.

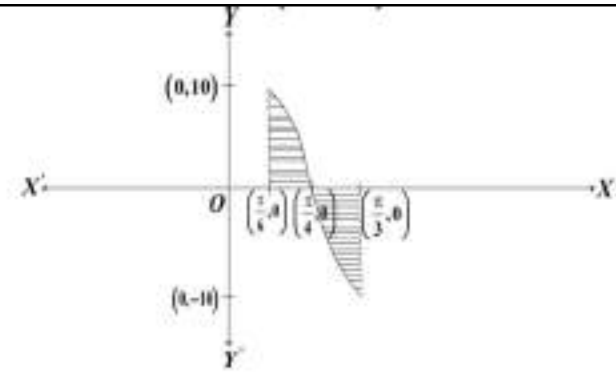
The equation of the line joining P(1,2,1) and $Q\left(\frac{39}{7},-\frac{20}{7},\frac{19}{7}\right)$ is

Page 12 of

$$\frac{x-1}{32/7}=\frac{y-2}{-34/7}=\frac{z-1}{12/7} \Rightarrow \frac{x-1}{16}=\frac{y-2}{-17}=\frac{z-1}{6}$$

½

33.	$y=20\cos 2x; \{\pi/6 \le x \le \pi/3\}$	1



$$Required\ area = 20\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos 2x\, dx + \left|20\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos 2x dx\right|$$ 1

$$= 20\left[\frac{sin2x}{2}\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}} + \left|20\left[\frac{\sin 2x}{2}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}\right|$$ 1

$$= 10\left(1-\frac{\sqrt{3}}{2}\right) + 10\left(1-\frac{\sqrt{3}}{2}\right)$$ 1

1

$$= 20(1-\sqrt{3}/2)\ sq.units.$$

34. We have $A = \begin{bmatrix}1 & -1\\2 & -1\end{bmatrix}$ and $B = \begin{bmatrix}a & 1\\b & -1\end{bmatrix}$

Now, $A + B = \begin{bmatrix}1 & -1\\2 & -1\end{bmatrix} + \begin{bmatrix}a & 1\\b & -1\end{bmatrix}$

1

$$= \begin{bmatrix}1+a & -1+1\\2+b & -1-1\end{bmatrix} = \begin{bmatrix}1+a & 0\\2+b & -2\end{bmatrix}$$
$$(A+B)^2 = \begin{bmatrix}1+a & 0\\2+b & -2\end{bmatrix}\cdot\begin{bmatrix}1+a & 0\\2+b & -2\end{bmatrix}$$
$$= \begin{bmatrix}1+a^2+2a & 0\\2+2a+b+ab-4-2b & 4\end{bmatrix}$$
$$= \begin{bmatrix}a^2+2a+1 & 0\\2a-b+ab-2 & 4\end{bmatrix}$$

	Also, $A^2 + B^2 = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} \cdot \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ $= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} a^2+b & a-1 \\ ab-b & b+1 \end{bmatrix}$ $= \begin{bmatrix} a^2+b-1 & a-1 \\ ab-b & b \end{bmatrix}$ Now, $(A+B)^2 = A^2 + B^2$ $\begin{bmatrix} a^2+2a+1 & 0 \\ 2a-b+ab-2 & 4 \end{bmatrix} = \begin{bmatrix} a^2+b-1 & a-1 \\ ab-b & b \end{bmatrix}$ Equating the corresponding elements, we get $a^2+2a+1 = a^2+b-1$ $2a-b = -2$ $a-1 = 0 \Rightarrow a = 1$ $2a-b+ab-2 = ab-b$ $2a-2 = 0 \Rightarrow a = 1$ And. $b = 4$ Since, $a = 1$ and $b = 4$ also satisfy Eq. (i), therefore $a = 1$ and $b = 4$. **(OR)** We have $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ Let A_{ij} be the cofactor of an element a_{ij} of $\|A\|$. Then, cofactors of elements of $\|A\|$ are	1 1

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$$A_{11} = (-1)^{1+1}\begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} = (1-4) = -3$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 2 & -2 \\ 2 & 1 \end{vmatrix} = -(2+4) = -6$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 2 & 1 \\ 2 & -2 \end{vmatrix} = (-2-4) = -6$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} -2 & -2 \\ -2 & 1 \end{vmatrix} = -(-2-4) = 6$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} = (-1+4) = 3$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = -(2+4) = -6$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} -2 & -2 \\ 1 & -2 \end{vmatrix} = (4+2) = 6$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = -(2+4) = -6$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} -2 & -2 \\ 1 & -2 \end{vmatrix} = (-1+4) = 3$$

Adjoint of the matrix A is given by

$$\mathrm{adj}A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

Now,

$$|A| = \begin{vmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{vmatrix}$$
$$= -1(1-4) + 2(2+4) - 2(-4-2)$$
$$= -(-3) + 2(6) - 2(-6)$$
$$= 3 + 12 + 12 = 27$$

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	$$A \cdot (\mathrm{adj}A) = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}\begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$ $$= \begin{bmatrix} 3+12+12 & -6-6+12 & -6+12-6 \\ -6-6+12 & 12+3+12 & 12-6-6 \\ -6+12-6 & 12-6-6 & 12+12+3 \end{bmatrix}$$ $$= \begin{bmatrix} 27 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 27 \end{bmatrix}$$ $$= 27\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ $= 27I_3 = \|A\|I_3$ Hence proved.	½ ½
35.	Given that, $$y = \sin t$$ $$dy/dt = \cos t \quad [differentiate\ w.r.t.\ t] \ldots (i)$$ $$\frac{d^2y}{dt^2} = -\sin t \ \ [differentiate\ w.r.t.\ t]$$ $$\left[\frac{d^2y}{dt^2}\right]_{t=\frac{\pi}{4}} = -\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$$ Again, $x = \cos t + \log\tan t/2$ $$\frac{dx}{dt} = -\sin t + \frac{1}{\tan\frac{t}{2}} \cdot \sec^2\frac{t}{2} \cdot \frac{1}{2} \quad [\text{differentiate w.r.t. } \square]$$ $$= -\sin t + \frac{\cos\frac{t}{2}}{2 \cdot \sin\frac{t}{2}} \cdot \frac{1}{\cos^2\frac{t}{2}}$$ $$= -\sin t + \frac{1}{\sin 2 \times \frac{t}{2}} \quad \because 2\sin a\cos a = \sin 2a]$$ $$= -\sin t + \operatorname{cosec} t$$ Now, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos t}{\operatorname{cosec} t - \sin t}$ $[using\ Eqs. (i)\ and\ (ii)]$ $$= \frac{\cos t}{1-\sin^2 t} \cdot \sin t = \frac{\sin t \cdot \cos t}{\cos^2 t}$$ $$\frac{dy}{dx} = \tan t$$	½ ½ ½ 1 ½

	$$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{\frac{d}{dx}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}=\frac{sec^2t}{cosect-sint}$$ $$=\frac{sec^2t\cdot sint}{cos^2t}=sec^3t\cdot tant$$ $$\left[\frac{d^2y}{dx^2}\right]_{t=\frac{\pi}{4}}=sec^3\frac{\pi}{4}\cdot tan\frac{\pi}{4}=2\sqrt{2}\times 1=2\sqrt{2}$$ **(OR)** Given, $\begin{cases}x^2+3x+a, x\le 1\\ bx+2, x>1\end{cases}$ is differentiable at $x=1$. $$Lf'(1)=Rf'(1)\ \ldots.\text{(i)}$$ Here, $Lf'(1)=lim_{h\to 0}\frac{f(1-h)-f(1)}{-h}$ $$=\lim_{h\to 0}\frac{(1-h)^2+3(1-h)+a-(4+a)}{-h}\quad=\lim_{h\to 0}\frac{1+h^2-2h+3-3h+a-4-a}{-h}$$ $$=\lim_{h\to 0}\frac{h^2-5h}{-h}$$ and $$Rf'(1)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$ $$=\lim_{h\to 0}\frac{b(1+h)+2-(4+a)}{h}$$ $$=\lim_{h\to 0}\frac{b+bh+2-4-a}{h}$$ $$=\lim_{h\to 0}\frac{bh+b-a-2}{h}$$ Clearly, for $Rf(1)$ to be exist $b-a-2$ should be equal to 0 , i.e. $$b-a-2=0\quad \ldots\ldots\text{(ii)}$$	½ ½ ½ ½ ½ ½ ½ ½ ½

	Now, $$Rf(1) = \lim_{h\to 0}\frac{bh}{h} = \lim_{h\to 0} b = b$$ From Eq. (i), we have $$Lf(1) = Rf'(1) \Rightarrow 5 = b$$ Now, on substituting b = 5 in Eq. (ii), we get $$5 - a - 2 = 0 \Rightarrow a = 3$$ Hence, $a = 3$ and $b = 5$.	½ 1 1
	Section E	
36.	(i) Number of relations is equal to the number of subsets of the set $B \times G = 2^{n(B\times G)}$ $$= 2^{n(B)\times n(G)} = 2^{3\times 2} = 2^6$$ (Where n(A) denotes the number of the elements in the finite set A) (ii) Smallest Equivalence relation on G is $\{(g_1, g_1), (g_2, g_2)\}$ (iii) (a) (A) reflexive but not symmetric = $$\{(b_1, b_2), (b_2, b_1), (b_1, b_1), (b_2, b_2), (b_3, b_3), (b_2, b_3)\}$$ So the minimum number of elements to be added are $$(b_1, b_1), (b_2, b_2), (b_3, b_3), (b_2, b_3)$$ {Note: it can be any one of the pair from, $(b_3, b_2), (b_1, b_3), (b_3, b_1)$ in place of (b_2, b_3) also } (B) reflexive and symmetric but not transitive = $$\{(b_1, b_2), (b_2, b_1), (b_1, b_1), (b_2, b_2), (b_3, b_3), (b_2, b_3), (b_3, b_2)\}.$$ So the minimum number of elements to be added are $$(b_1, b_1), (b_2, b_2), (b_3, b_3), (b_2, b_3), (b_3, b_2)$$ OR (iii) (b) One-one and onto function $$x^2 = 4y.\ \text{let } y = f(x) = \frac{x^2}{4}$$ Let $x_1, x_2 \in [0, 20\sqrt{2}]$	½ ½ ½ ½

	such that $f(x_1) = f(x_2) \Rightarrow \frac{{x_1}^2}{4} = \frac{{x_2}^2}{4}$ $\Rightarrow {x_1}^2 = {x_2}^2 \Rightarrow (x_1 - x_2)(x_1 + x_2) = 0 \Rightarrow x_1 = x_2$ as $x_1, x_2 \in [0,20\sqrt{2}]$ ∴ f is one-one function Now, $0 \leq y \leq 200$ hence the value of y is non-negative and $f(2\sqrt{y}) = y$ ∴ for any arbitrary $y \in [0,200]$, the pre-image of y exists in $[0,20\sqrt{2}]$ hence f is onto function.	1 1
37.	Since rate is proportional to the square of the amount of the first substance present at any time t, we are led to the differential equation $$\frac{dQ}{dt} = kQ^2$$ The differential equation is separable. Separating the variables and integrating, we obtain $$\int \frac{dQ}{Q^2} = \int kdt$$ and $$-\frac{1}{Q} = kt + C$$ Therefore, $$Q = -\frac{1}{kt + C}$$ Now, $Q = 50$ when $t = 0$, therefore, $50 = -\frac{1}{C}$ and $C = -\frac{1}{50}$. Therefore, $$Q = -\frac{1}{kt - \frac{1}{50}}$$ Since $Q = 10$ when $t = 1$, $$10 = -\frac{1}{k - \frac{1}{50}}$$ $$10\left(k - \frac{1}{50}\right) = -1, 10k - \frac{1}{5} = -1, 10k = -1 + \frac{1}{5} = -\frac{4}{5}, k = -\frac{4}{50} = -\frac{2}{25}$$	½ ½

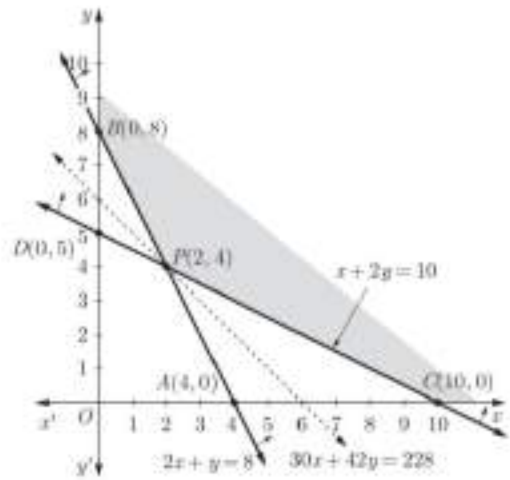
Therefore, $Q(t) = \frac{1}{\frac{2}{25}t+\frac{1}{50}}$

$$= \frac{1}{\frac{4t+1}{50}} = \frac{50}{4t+1}$$ ½

and

$$Q(2) = \frac{50}{8+1} \approx 5.56\ grams$$

½



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The values of □ at corner points are given below.

Corner Points	$Z = 30x + 42y$
$C(10,0)$	$Z = 30 \times 10 + 0 = 300$
$P(2,4)$	$Z = 30 \times 2 + 42 \times 4 = 228$
$B(0,8)$	$Z = 0 + 42 \times 8 = 336$

½

½

From table, the minimum value of Z is 228 . As the feasible region is unbounded, therefore 228 may or may not be the minimum value of □. For this, we draw a dotted graph of the inequality $30x + 42y < 228$ and check graph of the inequality plane has point in common with the feasible region or not. 1

It can seen that the feasible region has no common point with $30x + 42y < 228$. Hence, the minimum cost Rs. 38, when $x = 2$ and $y = 4$.

38.	Consider the following events E_1 = The policy holder is accident prone E_2 = The policy holder is not accident prone E = The new policy holder has an accident within a year of purchasing a policy. $P(E_1) = \frac{20}{100}$ and $P(E_2) = \frac{80}{100}$ $P\left(\frac{E}{E_1}\right) = 0.6 = \frac{6}{10}$ and $P(E) = P(E_1) \times P(E/E_1) + P(E_2) \times P(E/E_2)$ $= \frac{20}{100} \times \frac{6}{10} + \frac{80}{100} \times \frac{2}{10}$ $= \frac{280}{1000} = \frac{7}{25}$ (ii) By Bayes' theorem, $P(E_1/E) = \frac{P(E_1) \times P(E/E_1)}{P(E)}$ $= \frac{\frac{20}{100} \times \frac{6}{10}}{\frac{7}{25}} = \frac{\frac{3}{25}}{\frac{7}{25}} = \frac{3}{7}$	 ½ ½ 1 1 1

KENDRIYA VIDYALAYA SANGATHAN: HYDERABAD REGION

MODEL QUESTION PAPER - 8 (2025 - 26)

SUBJECT: MATHEMATICS (041)

MARKING SCHEME

Q.No.	ANSWERS	Marks
	SECTION – A (1 x 20 = 20)	
1	A	1M
2	C	1M
3	B	1M
4	C	1M
5	D	1M
6	C	1M
7	C	1M
8	B	1M
9	A	1M
10	B	1M
11	A	1M
12	D	1M
13	D	1M
14	D	1M
15	C	1M
16	B	1M
17	A	1M
18	C	1M
19	B	1M
20	A	1M
	SECTION B (2 x 5 =10) ***(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)***	
21	Given $f(x)=\sin^{-1}\left(x^2-4\right)$ $-1\le x^2-4\le 1$ $3\le x^2\le 5$ $x\in\left[-\sqrt{5},-\sqrt{3}\right]\cup\left[\sqrt{3},\sqrt{5}\right]$ Domain of $f(x)$ *is* $\left[-\sqrt{5},-\sqrt{3}\right]\cup\left[\sqrt{3},\sqrt{5}\right]$ Range $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$	1M 1/2M 1/2M
22	Given $sin^2y+\cos xy=K$ Differentiating both side w.r.t to y we get $2\sin y\cos y-\sin xy\left(y\frac{dx}{dy}+x\right)=0$ Given $x=1, y=\frac{\pi}{4}$	1M

	$2\times\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\left(\frac{\pi}{4}\times\frac{dx}{dy}+1\right)=0$ $1=\frac{1}{\sqrt{2}}\left(\frac{\pi}{4}\times\frac{dx}{dy}+1\right)$ $\left(\sqrt{2}-1\right)\frac{4}{\pi}=\frac{dx}{dy}$ $\therefore \frac{dy}{dx}=\frac{\pi(\sqrt{2}+1)}{4}$	1M
23	$f(x)=2+3x-x^3$ $f'(x)=3-3x^2=3(1+x)(1-x)$ for decreasing $f'(x)<0$ f is decreasing in $(-\infty,-1]U[1,\infty)$ (OR) $f(x)=x^3-18x^2+96x$ $f'(x)=3x^2-36x+96$ $=3(x^2-12x+32)$ $f'(x)=3(x-4)(x-8)$ $f'(x)=0\Rightarrow x=4,\ 8\in[0,9]$ $f(4)=4^2-18(4)^2+96(4)$ $=64-288+384$ $=160$ $f(8)=8^3-18(8)^2+96(8)$ $=522-1152+768$ $=128$ $f(9)=9^2-18\times9^2+96\times9$ $=729-1458+864$ $=135$ $f(0)=0$. Minimum value of $f(x)$ is 0.	1M 1M 1M 1M
24	Given vectors $\vec{a}=\hat{\imath}-\hat{\jmath}$ and $\vec{b}=\hat{\imath}+\hat{k}$ $\cos\theta=\frac{\vec{a}\cdot\vec{b}}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert}$ $\cos\theta=\frac{(\hat{\imath}-\hat{\jmath})\cdot(\hat{\imath}+\hat{k})}{\sqrt{1+1}\sqrt{1+1}}$ $\cos\theta=\frac{1}{2}$ $\therefore\theta=\frac{\pi}{3}$	1M 1M
25	Here $\lvert\vec{a}\rvert=\sqrt{3},\lvert\vec{a}\times\vec{b}\rvert=\sqrt{2}$ As $\lvert\vec{a}\times\vec{b}\rvert^2+(\vec{a}.\vec{b})^2=\lvert\vec{a}\rvert^2\lvert\vec{b}\rvert^2$ $\left(\sqrt{2}\right)^2+1=\left(\sqrt{3}\right)^2\lvert\vec{b}\rvert^2$ $\lvert\vec{b}\rvert^2=1$ $\lvert\vec{b}\rvert=1$ (OR) Given diagonals $\vec{a}=4\hat{\imath}-\hat{\jmath}-3\hat{k}$ *and* $\vec{b}=-2\hat{\imath}+\hat{\jmath}-2\hat{k}$ Area of parallelogram = $\frac{1}{2}\lvert\vec{a}\times\vec{b}\rvert$ $=\frac{1}{2}\begin{vmatrix}\hat{\imath}&\hat{\jmath}&\hat{k}\\4&-1&-3\\-2&1&-2\end{vmatrix}$	1M 1M 1/2M 1M 1/2M

	$=\frac{1}{2}\left\|5\hat{\imath}+14\hat{\jmath}+2\hat{k}\right\|$ = 15/2 sq.units	
SECTION C (3 x 6 =18) ***(This section comprises of 6 short answer (SA) type questions of 3 marks each.)***		
26	: $\frac{dy}{d\theta}=\ (2+cos\theta)(4cos\theta)-4sin\theta.(-sin\theta)\ \ 2+cos\theta 2\ \ -1$ $=\ 8cos\theta+4cos^2\theta+4sin^2\theta-(2+COS\theta)^2$ $(2+\cos\theta)2$ $=\frac{8COS\theta+4COS\theta^2+4sin^2\theta-(2+cos\theta)^2}{(2+cos\theta)^2}$ $=\frac{4cos\theta-cos^2\theta}{(2+cos\theta)^2}$ $\frac{dy}{d\theta}=\frac{cos\theta(4-cos\theta)}{(2+\cos\theta)^2}$ ________ (i) In $[\ 0,\frac{\pi}{2}]\Rightarrow cos\theta>0$ 4- $\cos\theta \Rightarrow$+ve, $(2+\cos\Theta)^2>0$ Therefore $\frac{dy}{d\theta}>0$ Y= $\frac{4sin\theta}{2+cos\theta}-\Theta$ is increasing function in $[0,\frac{\pi}{2}]$	2M 1M
27	Let the side of square to be cut off be 'x' cm. Length of box l = 45-2x, breadth b = 24-2x, height h = x cm Volume v = l bh = (45-2x) (24-2x) x = x (1080 -90x – 48x +4x^2) = 4x^3-138x^2+1080x $\frac{dv}{dx}$ = 12x^2 – 276x + 1080 = 12 (x^2-23x+90) = 12 (x-18) (x-5) $\frac{dv}{dx}$ = 0 ⇒ x = 5, x = 18 x = 18 is not possible, at x = 5, $\frac{d^2v}{dx^2}<0$ Length of side of square to be cut off is 5 cm. (OR) y =- x^3+3x^2+9x -27 $\frac{dy}{dx}$ = -3x^2+6x +9= slope of the curve. Let f(x)= -3x^2+6x +9 f'(x)= -6x +6 = -6(x -1) f'(x)= 0 ⇒ x =1. f''(x) <0, the maximum slope of given curve is at x =1. f (1)= -3(1)^2+6(1)+9=12	1M 1M 1M 1M 1M 1M
28	Given require line is perpendicular to the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3},\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$ d.r's of line = $\begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ -3 & 2 & 5 \end{vmatrix}$ d.r's = $4i-14j+8k$ d.r's = (2,-7, 4) cartesian equation line equation is $\frac{x+1}{2}=\frac{y-3}{-7}=\frac{z+2}{4}$ vector equation of line is $\vec{r}=-\hat{\imath}+3\hat{\jmath}-2\hat{k}+\mu(=2\hat{\imath}-7\hat{\jmath}+4\hat{k})$	1M 1M 1M
29	Let $I=\int\frac{dx}{x\sqrt{x^4-1}}$ Put $x^2=sec\theta\Rightarrow\theta=\sec^{-1}x^2$	1M

$\Rightarrow \quad 2\,xdx = sec\theta.tan\,\theta\, d\theta$

$\therefore \quad I = \dfrac{1}{2}\displaystyle\int \dfrac{sec\theta.tan\theta}{sec\theta\, tan\theta}\, d\theta = \dfrac{1}{2}\displaystyle\int d\theta = \dfrac{1}{2}\theta + C$ 2M

$= \frac{1}{2}\sec^{-1}(x^2) + C$

(OR)

Let $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{tanx}} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{cosx}dx}{\sqrt{cosx}+\sqrt{sinx}}$ ….(1) 1M

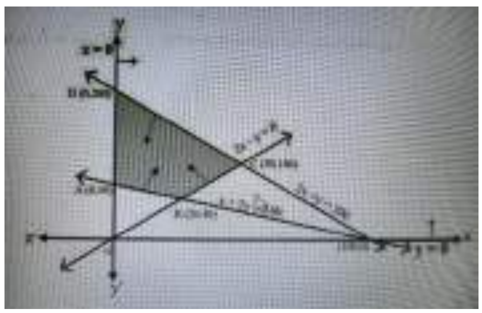
Then, by P_3 $\quad I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)dx}}{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}+\sqrt{\sin\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}$ 1M

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{sinx}}{\sqrt{sinx}+\sqrt{cosx}}\, dx$ ….(2) 1M

Adding (1) and (2), we get

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} dx = [x]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}.\ Hence\ I = \frac{\pi}{12}$$

30 The feasible region determined by the constraints,
$x + 2y \geq 100, 2x - y \leq 0, 2x + y \leq 200, x, y \geq 0$ is as shown 1M



The corner points of feasible region are A(0, 50), B(20, 40), C(50,100) and D(0, 200) 1M

The value of Z at these corner points as follows 1M

Corner points	Value of the objective function Z = x + 2y
A(0, 50)	100
B(20, 40)	100
C(50,100)	250
D(0, 200)	400

The minimum value of Z is 100 at all the points on the line segment joining the points A(0,50) and (20,40).

31 Let E_1 = Selecting Box -1 1M

E_2 = Selecting Box – II

$P(E_1) = \frac{1}{2}$, $P(E_2) = \frac{1}{2}$ 1M

A = getting a red ball from the selected box 1M

$P\left(\frac{A}{E_1}\right) = \frac{3}{9} = \frac{1}{3}$ 1M

$P\left(\frac{A}{E_2}\right) = \frac{5}{10} = \frac{1}{2}$

By using Baye s theorem.

$P\left(\frac{E_2}{A}\right). = \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1) + P(E_2)\,P(A/E_2)}$ 1M

	$= \frac{\frac{1}{2}\frac{1}{2}}{\frac{1}{2}\times\frac{1}{3}+\frac{1}{2}\times\frac{1}{2}}$ $P\left(\frac{E_2}{A}\right). = \frac{\frac{1}{4}}{\frac{1}{6}+\frac{1}{4}} = \frac{\frac{1}{4}}{\frac{10}{24}} = \frac{3}{5}$ (OR) Let B and b represent elder and younger boy child. Also, G and g represent elder and younger girl child. If a family has two children, then all possible cases are S = {Bb, Bg, Gg, Gb} $\therefore$ n(S) = 4 Let us define event A : Both children are girls, then A = {Gg} → n(A) = 1 (i) Let E_1 : The event that youngest child is a girl. Then, E_1 = {Bg, Gg} and $n(E_1) = 2$ so $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{2}{4} = \frac{1}{2}$ and $A \cap E_1 = \{Gg\} \Rightarrow n(A \cap E_1) = 1$ so $P(A \cap E_1) = \frac{n(A \cap E_1)}{n(S)} = \frac{1}{4}$ Now, $P\left(\frac{A}{E_1}\right) = \frac{P(A \cap E_1)}{P(E_1)} = \frac{1/4}{1/2} = \frac{1}{2}$ $\therefore$ **Required probability** $= \frac{1}{2}$	1M

(ii) Let E_2: The event that atleast one is girl.

Then, $E_2 = \{Eg, Gg, Gb\} \Rightarrow n(E_2) = 3$,

so $\quad P(E_2) = \dfrac{n(E_2)}{n(S)} = \dfrac{3}{4}$

and $\quad (A \cap E_2) = \{Gg\}$

$\Rightarrow \quad n(A \cap E_2) = 1$

so $\quad P(A \cap E_2) = \dfrac{n(A \cap E_2)}{n(S)} = \dfrac{1}{4}$

Now, $\quad P\left(\dfrac{A}{E_2}\right) = \dfrac{P(A \cap E_2)}{P(E_2)} = \dfrac{1/4}{3/4} = \dfrac{1}{3}$

$\therefore$ Required probability $= \dfrac{1}{3}$

SECTION D (5 x 4 =20)

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32

Given A = $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ 1M

$|A| = 2(-4+4) + 3(-6+4) + 5(3-2) = -1$

$adj(A) = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$ 2M

$A^{-1} = \frac{1}{|A|} adj(A)$

$A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$

Again given system of equations $2x - 3y + 5z = 11,\ 3x + 2y - 4z = -5, x + y - 2z = -3$ 2M

Writing the above equations in matrix form AX = B

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

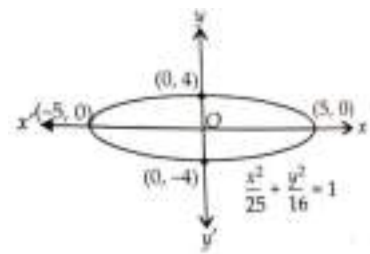
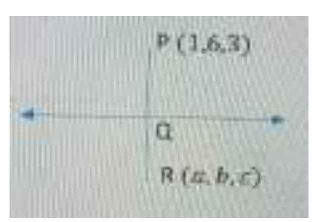
AX = B $\Rightarrow X = A^{-1}B$

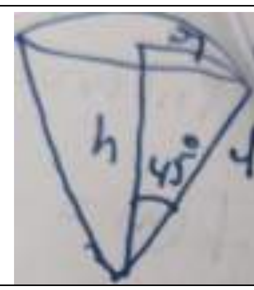
$$X = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence x = 1, y = 2, z = 3 1M

(OR)

	Given $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$	1M
	$A^2 = A.A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$	1M 1M
	$= \begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix}$	1M
	$A^3 = A^2.A = \begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$	
	$= \begin{bmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{bmatrix}$	
	So, by taking L.H.S:	
	$A^3 - 4A^2 - 3A + 11I = \begin{bmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{bmatrix} - 4 \begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix} - 3 \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix} +$ $11 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	
	$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	
	Hence $A^3 - 4A^2 - 3A + 11I = O$	
	Now, by multiplying the above equation by A^{-1}	
	$A^{-1}A^3 - 4A^{-1}A^2 - 3A^{-1}A + 11A^{-1}I = O$	
	$11A^{-1} = 4A + 3I - A^2$	
	$11A^{-1} = 4 \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix} + 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix}$	
	$A^{-1} = \frac{1}{11} \begin{bmatrix} -2 & 8 & 6 \\ 10 & -1 & -2 \\ -1 & 2 & 6 \end{bmatrix}$	
33	Given $y = x^x$ Taking log on both sides	
	Log y = x log x	1M
	Differentiating both side w.r.t x. we get $\frac{1}{y}\frac{dy}{dx} = x \times \frac{1}{x} + \log x$ ……….(i)	
	$\frac{dy}{dx} = y(1 + \log x)$	2M
	Again Differentiating both side w.r.t x. we get $\frac{d^2y}{dx^2} = y\left(\frac{1}{x}\right) + (1 + \log x)\frac{dy}{dx}$	

	$\frac{d^2y}{dx^2} = y\left(\frac{1}{x}\right) + \frac{1}{y}\frac{dy}{dx} \times \frac{dy}{dx}$ (from (i)) $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$	2M
34	We have $\frac{x^2}{25} + \frac{y^2}{16} = 1$ Here, $a = \pm 5$ and $b = \pm 4$ and $\frac{y^2}{4^2} = 1 - \frac{x^2}{5^2}$ $\Rightarrow y^2 = 16\left(1 - \frac{x^2}{25}\right)$ $\Rightarrow y = \sqrt{\frac{16}{25}(25 - x^2)}$ $\Rightarrow \frac{4}{5}\sqrt{(5^2 - x^2)}$  Thus the area of the shaded region $= 2.\frac{4}{5}\int_{-5}^{5}\sqrt{5^2 - x^2}\,dx = 2.\frac{8}{5}\int_{0}^{5}\sqrt{5^2 - x^2}\,dx$ $= 2.\frac{8}{5}\left[\frac{x}{2}\sqrt{5^2 - x^2} + \frac{5^2}{2}\sin^{-1}\frac{x}{5}\right]_0^5$ $= 2.\frac{8}{5}\left[\frac{5}{2}\sqrt{5^2 - 5^2} + \frac{5^2}{2}\sin^{-1}\frac{5}{5} - 0 - \frac{25}{2}.0\right]$ $= 2.\frac{8}{5}\left[\frac{25}{2}.\frac{\pi}{2}\right]$ $= 20\pi\ sq.units$	1M 1M 2M 1M
35	$\vec{r} = (3\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + \lambda(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$ and $\vec{r} = (5\hat{\imath} - \hat{\jmath}) + \mu(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$ $\overrightarrow{a_1} = 3\hat{\imath} + 2\hat{\jmath} - 4\hat{k}, \overrightarrow{b_1} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ $\overrightarrow{a_2} = 5\hat{\imath} - \hat{\jmath}, \overrightarrow{b_2} = 3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$ Shortest distance between the above lines = $\left\|\frac{(\overrightarrow{a_2} - \overrightarrow{a_1}).(\overrightarrow{b_1} \times \overrightarrow{b_2})}{\|\overrightarrow{b_1} \times \overrightarrow{b_2}\|}\right\|$ $(\overrightarrow{a_2} - \overrightarrow{a_1}) = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ $\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 2 \\ 3 & 2 & 6 \end{vmatrix} = 8\hat{\imath} + 0\hat{\jmath} - 4\hat{k}$ $\|\overrightarrow{b_1} \times \overrightarrow{b_2}\| = \sqrt{80}$ Now, S.D = $\left\|\frac{(2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}).(8\hat{\imath} + 0\hat{\jmath} - 4\hat{k})}{3\sqrt{80}}\right\|$ $= \frac{0}{\sqrt{80}} = 0$ If Shortest distance is zero then the lines are intersecting (OR) Let Q is the foot of the perpendicular from P Therefore, Q is (λ , 2λ +1, 3λ +2) d.r's of PQ is (λ – 1, 2λ – 5, 3λ – 1) d.r's given is (1, 2, 3) 	1M 1M 1M 2M 1M 2M 1M

	PQ is perpendicular to the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$, so we have $(\lambda-1)1+(2\lambda-5)2+(3\lambda-1)3=0$ $\lambda=1$ *so that* Q = (1,3,5) Using midpoint formula $\frac{a+1}{2}=1, \frac{b+6}{2}=3, \frac{c+3}{2}=5 \Rightarrow a=1, b=0, c=7$ ∴ image is (1,0,7) Required distance PR = $2\sqrt{13}$	1M
SECTION D (4 x 3 =12) **(*This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)***		
Case Study-1		
36		
i	Given $f(x)=x^2$ and $f: N \to R$ f(1) = 1, f(2) = 4, f(3) = 9 …… range of f {1, 4, 9, ….}	 1/2M 1/2M
ii	f: N → N is given by, $f(x) = x^2$ It is seen that for x, y ∈N, f(x) = f(y) ⇒ $x^2 = y^2$ ⇒ x = y. ∴f is injective	 1/2M 1/2M
iii	f: R → R is given by, $f(x) = x^2$ It is seen that f(−1) = 1, f(1) =1 but −1 ≠ 1. ∴ f is not on-one. (OR) f: R → R is given by, $f(x) = x^2$ let f(x) = y $x^2 = y$ $x = \sqrt{y}$ Now,−2 ∈ R. But, there does not exist any element x ∈ R such that $f(x) = x^2 = -2$. ∴ f is not onto.	 1M 1M 1M 1M
Case Study-2		
37		
i	Tan $45^0=\frac{r}{h} \Rightarrow l=\frac{r}{h} \Rightarrow$ r=h $l^2 = h^2+r^2 = r^2+r^2 = 2r^2$ $l=\sqrt{2}\,r$ Volume V $=\frac{1}{3}\pi r^2 h=\frac{1}{3}\pi r^2 \times r=\frac{1}{3}\pi r^3$ 	1M
ii	Given $\frac{dv}{dt}$ = -2 cm³/s. $V=\frac{1}{3}\pi r^3; \frac{dv}{dt}=\frac{1}{3}\pi 3r^2\frac{dr}{dt}$ $-2=\pi r^2\frac{dr}{dt}$ $=-2=\pi \times 2\sqrt{2}\times 2\sqrt{2}\times\frac{dr}{dt}$	1M

	$\frac{dr}{dt} = \frac{-1}{4\pi}$ cm/s	
iii	$S = \pi rl = \pi r\sqrt{2}r = \sqrt{2}\pi r^2$ $\frac{ds}{dt} = \sqrt{2}\pi \times 2r\times\frac{dr}{dt} = \sqrt{2}\,\pi \times 2\times 2\sqrt{2} \times\frac{-1}{4\pi}$ $\frac{ds}{dt} = -2$ cm^2/sec (OR) Given $l = 4$ $l^2 = r^2 + h^2$ $h^2 = 8$ Volume $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times h^2 \times h = \frac{1}{3}\pi h^3$ $\frac{dv}{dt} = \frac{1}{3}\pi 3h^2\frac{dh}{dt} \Rightarrow -2 = \pi h^2\frac{dh}{dt}$ $\Rightarrow -2 = \pi\, 8\frac{dh}{dt}$ $\frac{dh}{dt} = \frac{-1}{4\pi}$ cm/s	2M 1M 1M
Case Study- 3		
38		
i	If Tiki reaches college by bus 1 , the probability that she caught bus B is $= \frac{\frac{4}{7}\times\frac{1}{3}}{\frac{3}{7}\times\frac{2}{5}+\frac{4}{7}\times\frac{1}{3}}$ $= \frac{20}{38}$ $= \frac{10}{19}$	1M 1M
ii	The probability that she reaches college by bus 2 $\frac{4}{7} \times \frac{3}{5} = \frac{12}{35}$	2M

KENDRIYA VIDYALAYA SANGATHAN: HYDERABAD REGION

MODEL QUESTION PAPER - 9 (2025 - 26)

SUBJECT: MATHEMATICS (041)

MARKING SCHEME

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1. D	2. D	3. B	4. C	5. C	6. B	7. A	8. B	9. A	10. C
11. A	12. B	13. A	14. C	15. D	16. A	17. C	18. C	19. B	20. D

21. If $\cos(\tan^{-1} x) = \sin\left(\cot^{-1}\left(\frac{3}{4}\right)\right)$

Simplification of RHS = $\frac{4}{5}$

1m

For getting $x = \pm\frac{3}{4}$

1m

(OR)

$\tan^{-1}\left[2\cos\left\{2\sin^{-1}\left(\frac{1}{2}\right)\right\}\right]$

$=\tan^{-1}\left[2\cos\left(2*\frac{\pi}{6}\right)\right]$

1m

$=\frac{\pi}{4}$

1m

22. $2x + 2x\frac{dy}{dx} + 2y + 3y^2\frac{dy}{dx} = 0$

1m

For getting $\frac{dy}{dx} = -\frac{2x+2y}{2x+3y^2}$

1m

23. For getting $\frac{dv}{dt} = 4\pi r^2\frac{dr}{dt}$

1m

For getting $\frac{dr}{dt} = \frac{1}{\pi}\ \ cm/sec$

1m

(OR)

For writing $A = xy\ \ and\ \frac{dA}{dt} = x.\frac{dy}{dt} + y.\frac{dx}{dt}$

1m

Substitution of values and finding $\frac{dA}{dt} = 2\, cm^2/minute$

1m

24. For getting sum and difference of vectors

1m

For getting required vector $-2\,\hat{\imath} + 4\,\hat{\jmath} - 2\,\hat{k}$

1m

25. For writing formula for angle between two vectors

1m

For getting angle $\theta = \frac{\pi}{3}$

1m

(OR)

For finding cross product $5\,\hat{\imath} + 10\,\hat{\jmath} + 5\,\hat{k}$

1m

For writing area formula and finding modulus = $5\sqrt{6}$

1m

26. For getting $f'(x) = cotx$

1m

For proving strictly increasing on $\left(0, \frac{\pi}{2}\right)$

1m

For proving strictly decreasing on $\left(\frac{\pi}{2}, \infty\right)$.

1m

27. For finding $P'(x)$ and solving for x=21

1m For finding second derivative and confirmation of maxima

1m

For finding Maximum value=513

1m

28. $\int \frac{\sin^6 x + cos^6 x}{\sin^2 x . cos^2 x} dx = \int \frac{(\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x - \sin^2 x . \cos^2 x)}{\sin^2 x . \cos^2 x}$

1m

$= \int (\text{Sec}^2 x - 1 + \cot^2 x - 1 - 1) dx$

1m

$= tanx - cotx - 3x + c$

1m

(OR)

For rewriting $\int \frac{(sinx+cosx)}{\sqrt{sinx.cosx}}dx$

1m

For taking substitution $sinx - cosx = t$ and rewriting the integration as $\int \sqrt{2}\frac{dt}{\sqrt{1-t^2}}$

1m

Evaluating and getting answer $\boldsymbol{\sqrt{2}sin^{-1}(sin\,x - cos\,x) + c}$

1m

29. For writing $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$ & $(\vec{a}+\vec{b}+\vec{c}).(\vec{a}+\vec{b}+\vec{c}) = 0$

1m

For writing $2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})$=$-\left(|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2\right)$

1m

For getting $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a} = -\frac{3}{2}$

1m

(OR)

For finding cross product and writing required vector $\gamma(32\,\hat{i} - \hat{j} + 5\,\hat{k})$

1m

For getting $\gamma = \frac{5}{3}$

1m

For getting vector $\frac{5}{3}(32\,\hat{i} - \hat{j} + 5\,\hat{k})$

1m

30. For getting and shading feasible region

1m

For getting corner points of the feasible region

(15,0), (40,0),(4,18),(6,12)

1m

Zmin = 240 at x = 6, y = 12

1m

31. (i) The probability that India wins the second match after losing the first one is 0.3.

1m

(ii) The probability that India loses the third match after losing the first two matches is 0.7. (Since the probability of winning the next match after a loss is 0.3, the probability of losing is complementary, which is 1 - 0.3 = 0.7.)

1m

(iii) The probability that India loses the first two matches is the product of the probabilities of losing the first match and then losing the second match, which is 0.4 * 0.7 = 0.28.

1m

32. For findinging $A^{-1} = \begin{pmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{pmatrix}$

2m

For expressing given system of equations in matrix equation

1m

For getting $x = 1, y = 2, z = 3.$

2m

(OR)

For finding AB=6I

1m

For writing given equation as matrix equation

1m

For writing $X = A^{-1}B$

1m

For substituting matrices

1m

For solving $x = 2, y = -1, z = 4$

1m

33. For finding derivative of $(tanx)^{cotx}$

2m

For finding derivative of $(cotx)^{tanx}$

2m

For finding $\frac{dy}{dx} = (tanx)^{cotx}\,(\cot^2 x\,.\,sec^2x - \log(tanx)\,.\,cosec^2x) +$

$(cotx)^{tanx}(-\tan^2 x.\,cosec^2x + \log(cotx).\sec^2 x)$ 1m

34. For writing Area = $\int_0^5 \sqrt{25 - x^2}dx$

1m

For evaluating integration

2m

For substitution of limits

1m

Simplification, area $\frac{25\pi}{4}$ *square units*

1m

(OR)

For rough shape of the curve

1m

For applying symmetry and writing integral $4\int_0^5 \frac{5}{8}\sqrt{64 - y^2}dy$

2m

For evaluating integral and final answer 40π

2m

35. For finding Foot of the perpendicular $\left(\frac{16}{21}, \frac{74}{21}, \frac{92}{21}\right)$

3m

For finding imabe $\left(-\frac{10}{21}, \frac{85}{21}, \frac{100}{21}\right)$

1m

For finding distance from origin $\frac{131}{21}$

1m

36. (i) 0.44

1m

(ii) 0.38

1m

(iii) $\frac{14}{38}$

2m

(iii) $\frac{24}{38}$

2m

37. (i) For getting area= $\frac{4b}{a}x\sqrt{a^2 - x^2}$

1m

(ii) For critical point $x = \frac{a}{\sqrt{2}}$

1m

(iii)For the values of x less than $\frac{a}{\sqrt{2}}$ and close to $\frac{a}{\sqrt{2}}$, $\frac{dA}{dx} > 0$ and

For the values of x greater than $\frac{a}{\sqrt{2}}$ and close to $\frac{a}{\sqrt{2}}$, $\frac{dA}{dx} < 0$

2m

(OR)

(iii)For proving $\frac{d^2A}{dx^2} < 0$, and concluding

2m

38. R_1 is reflexive, symmetric and Transitive.

2m

R_2 is only symmetric.

2m

KENDRIYA VIDYALAYA SANGATHAN: HYDERABAD REGION

MODEL QUESTION PAPER - 10 (2025 - 26)

SUBJECT: MATHEMATICS (041)

MARKING SCHEME

Instruction:

All alternative methods are accepted and marks will be given according to the procedure.

MARKING SCHEME:

1	2	3	4	5	6	7	8	9	10
a	b	a	c	d	c	b	a	b	b
11	12	13	14	15	16	17	18	19	20
b	a	d	a	a	d	b	b	c	a

Q. No	Value points	Marks	Total Marks
21	For finding $\lim_{x\to 2} f(x) = 7$ Since f is continuos, $\lim_{x\to 2} f(x) = f(2)$ For writing k = 2 **OR** For writing, $\frac{dy}{dx} = \frac{ysinxy}{sin2y - xsinxy}$ $\left(\frac{dy}{dx}\right)_{x=1,\ y=\frac{\pi}{4}} = \frac{\pi}{4(\sqrt{2}-1)}$	1M ½ M ½ M 1M 1 M	2 M
22	$(\vec{a} + \vec{b} + \vec{c}).(\vec{a} + \vec{b} + \vec{c}) = 0$ $\lvert\vec{a} + \vec{b} + \vec{c}\rvert^2 = 0$ $\lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2 + \lvert\vec{c}\rvert^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$ 49+576+625+$2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$ For simplifying and writing, $(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = -625$	½ M ½ M 1M	2 M
23	Let X be the random variable representing the defective bulbs and X = 0, 1, 2 $P(X=0) = \frac{28}{45}, P(X=1) = \frac{16}{45}, P(X=2) = \frac{1}{45}$	½ M 1½ M	2 M

24	$y = (\sin^{-1} x)^2$ $\frac{dy}{dx} = \frac{2\sin^{-1} x}{\sqrt{1-x^2}}$ $\sqrt{1-x^2}\frac{dy}{dx} = 2\sin^{-1} x$ D.w.r.t x $\sqrt{1-x^2}\ \frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}}\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$ $(1-x^2)\ \frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$ **OR** $Let\ y = x^{sinx}$ Applying log on both sides $logy = sinx\ logx$ D. w.r.t. x $\frac{1}{y}\frac{dy}{dx} = cosx\ logx + \frac{sinx}{x}$ $\frac{dy}{dx} = y\left(cosx\ logx + \frac{sinx}{x}\right)$ $\frac{dy}{dx} = x^{sinx}\left(cosx\ logx + \frac{sinx}{x}\right)$	½ M 1M ½ M ½ M 1M ½ M	2M
25	$Let\ \tan x = t$ D.w.r.t. x $\sec^2 x\ dx = dt$ $\int(\tan^2 x \sec^4 x)\ dx = \int \tan^2 x \sec^2 x\ \sec^2 x\ dx$ $= \int t^2\,(1+t^2)dt$ $= \int(t^2 + t^4)dt$ $= \frac{t^3}{3} + \frac{t^5}{5} + c$ $= \frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} + c$	½ M ½ M ½ M ½ M	2M

26	Forshowing, $\mathrm{I}+\mathrm{A}=\begin{pmatrix}1 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 1\end{pmatrix}$ For showing, $I-\boldsymbol{A}=\begin{pmatrix}1 & \tan\frac{\alpha}{2}\\ -\tan\frac{\alpha}{2} & 1\end{pmatrix}$ $(I-A)\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}=\begin{pmatrix}1 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 1\end{pmatrix}$ Therefore, $\boldsymbol{I}+\boldsymbol{A}=(I-A)\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}$	½ M ½ M 1½M ½ M	3M
27	$f(x)=3x^4+4x^3-12x^2+12$ $f'(x)=12x^3+12x^2-24x$ $=12x(x-1)(x+2)$ Take $f'(x)=0$ $12x(x-1)(x+2)$=0 $x=-2,0,1$ For showing local maxima at x=0 and local maximum value of f = f(0) = 12 For showing local minima at x= -2 and f(-2) =20 and local minimum at x=1 and f(1)=7 **OR** Let r be the radius, h be the height and V be the volume of the cone. $\frac{dv}{dt}=12cm^3/s$ $h=\frac{1}{6}r \quad i.e, r=6h$ Volume of the cone = V = $\frac{1}{3}\pi r^2 h$ For writing, $V=12\pi h^3$ $\frac{dv}{dt}=36\pi h^2\frac{dh}{dt}$ For writing, $\frac{dh}{dt}=\frac{1}{3\pi h^2}$ $\left(\frac{\mathrm{dh}}{\mathrm{dt}}\right)_{\mathrm{h}=4}=\frac{1}{48\pi}cm/s$	½ M ½ M ½ M ½ M 1M ½ M ½ M ½ M ½ M ½ M ½ M	3M
28	$Let\ 5x+3=A\frac{d}{dx}(x^2+4x+10)+B$ For finding A=5/2 and B = -7	½ M	3M

	For finding integral as $5\sqrt{x^2+4x+10}-7log\left\|(x+2)+\sqrt{x^2+4x+10}\right\|+c$ **OR** $Let\ sinx = t$ $cosx\ dx = dt$ $\int \frac{cosx}{(1+sinx)(2+sinx)}dx = \int \frac{dt}{(1+t)(2+t)}$ $Let\ \frac{1}{(1+t)(2+t)} = \frac{A}{1+t}+\frac{B}{2+t}$ For finding, A= -1, B=1 For finding integral as $\log\left\|\frac{2-sinx}{1-sinx}\right\|+c$	½ M 2M ½ M ½ M ½ M ½ M 1M	
29	For writing, $\frac{dy}{dx}+2xy = \frac{1}{(1+x^2)^2}$ Here, $P(x) = \frac{2x}{(1+x^2)}\ and\ Q(x) = \frac{1}{(1+x^2)^2}$ $I.F = e^{\int p(x)dx} = e^{log(1+x^2)} = (1+x^2)$ For writing general solution:$y(1+x^2) = \tan^{-1}x + c$ $when\ y = 0, x = 1,\ \ c = -\frac{\pi}{4}$ The particular solution is $y(1+x^2) = \tan^{-1}x - \frac{\pi}{4}$	½ M ½ M ½ M ½ M ½ M ½ M	3M

30	Let $\vec{d} = \lambda(\vec{a} X \vec{b})$ $\lambda(\vec{a} X \vec{b}) = \lambda(32\hat{\imath} - \hat{\jmath} - 14\hat{k})$ $Given\ \vec{c}.\vec{d} = 21$ For simplifying, $\lambda = \frac{5}{3}$ Therefore, $\vec{d} = \lambda(\vec{a} X \vec{b}) = \frac{5}{3}(32\hat{\imath} - \hat{\jmath} - 14\hat{k})$ $\vec{d} = \frac{1}{3}(160\hat{\imath} - 5\hat{\jmath} - 70\hat{k})$ **OR** For writing, $\overrightarrow{AB} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}, \qquad \overrightarrow{AC} = \widehat{0\imath} + 4\hat{\jmath} + 3\hat{k},$ $\lvert\overrightarrow{AB} X \overrightarrow{AC}\rvert = -\mathbf{6}\hat{\imath} - 3\hat{\jmath} + 4\widehat{k}$ Area of triangle = $\frac{1}{2}\lvert\overrightarrow{AB} X \overrightarrow{AC}\rvert$ $= \frac{\sqrt{61}}{2}$ square units	½ M 1M 1M ½ M 1M 1M ½ M ½ M	3 M
31	For correct graph The corner points of the feasible region are (0, 11/2), (8/3, 0), and (2, 1/2) For finding Min Z = 10 at the point (2, ½)	1½ M ½ M 1M	3M
32	For showing reflexive For showing symmetric For showing transitive Therefore, R is equivalence relation **OR** For showing f is one- one For showing f is onto Therefore, f is bijective	1M 1M 2M 1M 2 M 2 M 1M	5M

33	Let $I = \int_0^{\pi} \frac{x\tan x}{\sec x + \tan x}$ ....(1) Apply formula $\int_0^a f(x)dx = f(a-x)dx$ Then $I = \int_0^{\pi} \frac{\pi tanx - x\tan x}{\sec x + \tan x}$ ....(2) Add (1) and (2) we get $I = \frac{\pi}{2}\int_0^{\pi} \frac{\tan x}{\sec x + \tan x}$ $I = \frac{\pi}{2}\int_0^{\pi} \left(1 - \frac{1}{1+sinx}\right) dx$ For finding integral, $\frac{\pi}{2}(\pi - 2)$	½ M ½ M 1M 1M 1M 1M	5 M
34	For rough sketch $Given\ y^2 = 4x$ and the lines x = 0 and x = 3 The point of intersection of $y^2 = 4x$ and x = 3 are $(3, 2\sqrt{3})\ and\ (3, -2\sqrt{3})$ Required area = A = $\int_0^3 2\sqrt{x}\,dx$ = $2.\frac{2}{3}\left(x^{3/2}\right)_0^3$ = $8\sqrt{3}$ square units	1M 1 M 1M 1M 1M	5M
35	For writing the vectors $\vec{a}_1, \vec{b}_1, \vec{a}_2 and\ \vec{b}_2$ from given lines. For finding $\vec{b}_1 X\, \vec{b}_2 = 2\hat{\imath} + 3\hat{\jmath} - 13\hat{k}$ $\left\|\vec{b}_1 X\, \vec{b}_2 = \sqrt{182}\right\|$ $\vec{a}_2 - \vec{a}_1 = -2\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$ For writing S. D formula For writing S. D = $\frac{-21}{\sqrt{182}}$ **OR** For showing lines are intersecting The point on first line is P = $(3\lambda - 1,\ 5\lambda - 3, 7\lambda - 5)$ **T**he point on second line is Q = $(5\lambda + 21,\ 3\lambda + 4, 5\lambda + 6)$ For intersecting, $(3\lambda - 1,\ 5\lambda - 3, 7\lambda - 5) = (5\lambda + 21,\ 3\lambda + 4, 5\lambda + 6)$ For finding $\lambda = \frac{1}{2}\ \ and\ \mu = -\frac{-3}{2}$ Finding the point of intersection	1M 1M ½ M ½ M 1M 1M 2M 1M ½ M ½ M 1M	5M
36	(i) 4x + 5y + 2z = 8700, 3x + 4y+ 5z =8500, x +y + z =2200	1M	4M

	(ii) for finding $adjA = \begin{pmatrix} -1 & -3 & 17 \\ 2 & 2 & -14 \\ -1 & 1 & 1 \end{pmatrix}$	1M	
	(iii) for finding x= 800, y = 900, z = 500 **OR** for verifying, $A.adjA = \|A\|I$	2M	
37	(i) $A(x) = \pi \left(\frac{28-x}{2\pi}\right)^2 + \frac{x^2}{16}$	1M	4M
	(ii) $\frac{x-28}{\pi} + \frac{x}{8}$	1M	
	(iii) $x = \frac{112}{4+\pi}$ **OR** Total area is minimum at $x = \frac{112}{4+\pi}$	2M	
38	(i) By applying Baye's theorem, When the doctor arrives late, the probability that he comes by train = 1/2	2M	4 M
	(ii) By applying Baye's theorem, When the doctor arrives late, the probability that he comes by scooter = 1/18	2M	

KENDRIYA VIDYALAYA SANGATHAN: HYDERABAD REGION

MODEL QUESTION PAPER - 11 (2025 - 26)

SUBJECT: MATHEMATICS (041)

MARKING SCHEME

1	A	2	C	3	D	4	A	5	C	6	C	7	D	8	B	9	C	10	A
11	D	12	B	13	B	14	C	15	D	16	C	17	B	18	B	19	A	20	A

Q.No	Value Points	Marks
21	$2\tan^{-1}(\cos x) = \tan^{-1}(2\csc x)$ $\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-cos^2 x}\right) = \tan^{-1}(2\csc x)$ $\Rightarrow \frac{2\cos x}{sin^2 x} = \frac{2}{\sin x}$ $\Rightarrow \cos x = \sin x$ $\Rightarrow$ x = π/4	½ M ½ M ½ M ½ M
22	LHL = - λ RHL = 5 Continuos ⇒ LHL = RHL ⇒ λ = -5 OR $x = \sqrt{a^{\sin^{-1} t}},\ \ y = \sqrt{a^{\cos^{-1} t}}$ $\Rightarrow xy = \sqrt{a^{\sin^{-1} t + \cos^{-1} t}}$ $\Rightarrow xy = \sqrt{a^{\pi/2}}$ Diff. w.r.t.x $x\frac{dy}{dx} + y.1 = 0$ $\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$	1M ½ M ½ M 1M 1M 1M
23	Marginal Cost (MC) = $\frac{dC}{dx} = 0.015x^2 - 0.04x + 30$ When x = 3, MC = $0.015(3)^2 - 0.04(3) + 30$ = 30.015 = 30.02 nearly OR P = 2(x + y) dP/dt = 2(dx/dt + dy/dt) = - 2 cm/min A = x x y dA/dt = x dy/dt + y dx/dt	1M 1M 1M

	when x=8cm, y = 6 cm, dA/dt = 2 cm/min	1M
24	$\vec{a} = 2\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$ and $\vec{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$. $\vec{c} = \vec{a} + \vec{b} = 4\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$ $\lvert\vec{c}\rvert = \sqrt{29}$ $\hat{c} = \frac{4}{\sqrt{29}}\hat{\imath} + \frac{3}{\sqrt{29}}\hat{\jmath} - \frac{2}{\sqrt{29}}\hat{k}$	1M 1M
25	$\vec{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{b} = 2\hat{\imath} - 7\hat{\jmath} + \hat{k}$ $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$ $= \hat{\imath}(20) - \hat{\jmath}(-5) + \hat{k}(-5)$ $= 20\hat{\imath} + 5\hat{\jmath} - 5\hat{k}$ Area of the parallelogram = $\lvert\vec{a} \times \vec{b}\rvert = \sqrt{400 + 25 + 25} = \sqrt{450}$ sq units	1M 1M
26	$\frac{dy}{dx} - \frac{y}{x} + \csc\left(\frac{y}{x}\right) = 0$, given that y = 0 when x = 1. $\frac{dy}{dx} = \frac{y}{x} - \csc\left(\frac{y}{x}\right) = 0$ Put y/x = v $\frac{dy}{dx} = v + x\frac{dv}{dx}$ Given DE becomes $v + x\frac{dv}{dx} = v - \csc v$ $\Rightarrow \int sinv\, dv = -\int \frac{1}{x} dx$ $\Rightarrow \cos\left(\frac{y}{x}\right) = \log x + c$	1M 1M 1M

	OR $x\log x.\frac{dy}{dx}+y=\frac{2}{x}\log x.$ $\Rightarrow\frac{dy}{dx}+\frac{y}{x\log x}=\frac{2}{x^2}.$ I.F. = $e^{\int Pdx}$ = logx G.S. $y\log x=2\int\frac{1}{x^2}\log x\,dx$ $\Rightarrow y\log x=-\frac{1}{x}(1+\log x)+c$	1M 1M 1M
27	$y'=\frac{1}{1+x}-\frac{4}{(2+x)^2}$ $=\frac{x^2}{(1+x)(2+x)^2}$ y is increasing if y' > 0 then 1 + x > 0 then x > -1 Hence proved OR f'(x) = cos x – sin x f'(x) = 0 gives x = π/4, 5π/4 f'(x) > 0, if x € (0, π/4) and (5π/4, 2π) Therefore f is increasing on (0, π/4) and (5π/4, 2π) f'(x) < 0, if x € (π/4, 5π/4) Therefore f is decreasing on (π/4, 5π/4)	1M 1M 1M
28	$I=\int_0^{\pi}\frac{x}{1+sinx}dx=\int_0^{\pi}\frac{\pi-x}{1+sinx}dx$ $\pi\int_0^{\pi}\frac{1}{1+sinx}dx-I$ $2I=\pi\int_0^{\pi}\frac{1-\sin x}{cos^2x}dx$ $=\pi\int_0^{\pi}\left(\frac{1}{cos^2x}-\frac{\sin x}{cos^2x}\right)dx$ $=\pi\int_0^{\pi}(sec^2x-\sec x\tan x)\,dx$	1M 1M 1M

	$= \pi(\tan x - \sec x)_0^{\pi}$ = 2π $\Rightarrow$ I = π OR Let $\sin^{-1} x = t \Rightarrow$ x = sint $\Rightarrow$ dx = cost dt $\int (\text{Sin}^{-1} x)^2 \, dx = \int t^2 \cos t \, dt$ $= t^2 \int cost\, dt - 2\int t \ \sin t\, dt$ $= t^2 \sin t - 2\left\{t(-\cos t) - \int (-\cos t)\, dt\right\}$ $= t^2 \sin t + 2t \cos t - \sin t + c$	1M 1M 1M
29	$\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ $\Rightarrow \frac{x-1}{-3} = \frac{y-2}{2p/7} = \frac{z-3}{2}$ and $\frac{x-1}{3p/7} = \frac{y-5}{1} = \frac{z-6}{-5}$ Two lines are perpendicular $\Rightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0$ $\Rightarrow (-3)\left(\frac{3p}{7}\right) + \left(\frac{2p}{7}\right)(1) + (2)(-5) = 0$ $\Rightarrow -\frac{9p}{7} + \frac{2p}{7} - 10 = 0$ $\Rightarrow$ p = - 10	1M 1M 1M
30	For correct graph Points z = 2x + 3y (0, 0) 0 (7, 0) 14 (6, 2) 18 (0, 5) 15 $Max\ Z = 18\ at\ x = 6, y = 2$	1M 1M 1M
31	Let E1 , E2 and E3 be the events that boxes I, II and III are chosen, respectively. Then P(E1) = P(E2) = P(E3) = 1/3 Also, let A be the event that 'the coin drawn is of gold' P(A/E1) = P(a gold coin from bag I) = 2/2 = 1 P(A/E1) = P(a gold coin from bag II) = 0 P(A/E1) = P(a gold coin from bag III) = ½	1M 1M

	Now, the probability that the other coin in the box is of gold = the probability that gold coin is drawn from the box I = P(E1/A) By Bayes' theorem, we know that $P(E1/A) = \frac{P(E1)P(A/E1)}{P(E1)P\left(\frac{A}{E1}\right) + P(E2)P\left(\frac{A}{E2}\right) + P(E3)P(A/E3)}$ = 2/3	1M
32	$A^2 = \begin{pmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{pmatrix}$ $A^3 = \begin{pmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{pmatrix}$ For Showing $A^3 - 4A^2 - 3A + 11I = 0$ For finding A^{-1}	1M 1M 1M 2M
33	x = $\frac{\cos y}{\cos(a+y)}$ diff. w.r.t. y $\frac{dx}{dy} = \frac{-\cos(a+y)\sin y + \cos y \sin(a+y)}{cos^2(a+y)}$ $\frac{dx}{dy} = \frac{\sin(a+y-y)}{cos^2(a+y)}$ $\frac{dy}{dx} = \frac{\sin a}{cos^2(a+y)}$ OR $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$ $\sqrt{1-x^2}\frac{dy}{dx} = 1$ $\sqrt{1-x^2}y'' - y'\frac{x}{\sqrt{1-x^2}} = 0$ $(1-x^2)y'' - xy' = 0$	1M 2M 1M 1M 1M 1M 2M 1M

34	For correct graph Area = $\int_0^{\frac{\pi}{2}} \cos x \, dx + \left| \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos x \, dx \right| + \int_{\frac{3\pi}{2}}^{2\pi} \cos x \, dx$ $= [\sin x]_0^{\frac{\pi}{2}} + \left| [\sin x]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \right| + [\sin x]_{\frac{3\pi}{2}}^{2\pi}$ = 1 + 2 + 1 = 4 OR For graph $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \rightarrow y = \pm \frac{b}{a}\sqrt{a^2 - x^2}$ $A = 4\int_0^a \frac{b}{a}\sqrt{a^2 - x^2}\, dx$ $= \frac{4b}{a}\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$ =πab	1 ½ M 1 ½ M 1M 1M 1M 1M 1M 1M 1M
35	$\overrightarrow{a_1} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}, \quad \overrightarrow{a_2} = \hat{\imath} - \hat{\jmath} - \hat{k}$ $\overrightarrow{b_1} = -\hat{\imath} + \hat{\jmath} - 2\hat{k}, \quad \overrightarrow{b_2} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$ $\overrightarrow{a_1} - \overrightarrow{a_2} = \hat{\jmath} - 4\hat{k}, \ \overrightarrow{b_1} \times \overrightarrow{b_2} = 2\hat{\imath} - 4\hat{\jmath} - 3\hat{k}$ $(\overrightarrow{a_1} - \overrightarrow{a_2}).(\overrightarrow{b_1} \times \overrightarrow{b_2}) = 8$ $\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \sqrt{29}$ $S.D. = \left| \frac{(\overrightarrow{a_1} - \overrightarrow{a_2}).(\overrightarrow{b_1} \times \overrightarrow{b_2})}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|} \right| = \frac{8}{\sqrt{29}}$	1M 1M 1M 1M 1M
36 (i)	R = { (a, b)/ | a - b| is divisible by 3 }	1M
(ii)	{ 2, 5, 8,11,14, 17, 20, 23, 26, 29}	1M
(iii)	{3,6,9,12,15, 18, 21, 24, 27, 30}	2M
37(i)	1/7 x 1/5 = 1/35	1M
(ii)	1/7 x 4/5 + 6/7 x 1/5 = 2/7	1M
(iii)	6/7 X 4/5 = 24/35 OR	2M

	1 – 24/35 = 11/35	
38(i)	dx/dt = 300m/s tan θ = 1000/x x = 1000 cot θ dx/dt = - 1000 $\csc^2\theta$ dθ /dt.	2M
(ii)	When x = 500, Tan θ = 2 dx/dt = 1000 x 5/4 x dθ/dt d θ/dt = -300/1250 = -6/25 rad/sec	2M

KENDRIYA VIDYALAYA SANGATHAN: HYDERABAD REGION

MODEL QUESTION PAPER - 12 (2025 - 26)

SUBJECT: MATHEMATICS (041)

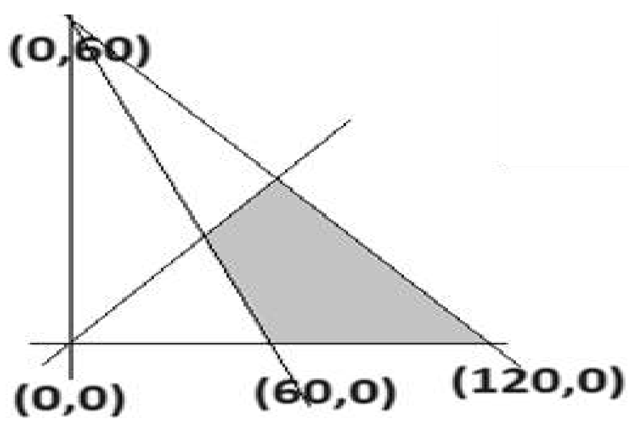
MARKING SCHEME

MARKING SCHEME :

1	2	3	4	5	6	7	8	9	10
b	d	a	d	d	d	b	b	c	c
11	12	13	14	15	16	17	18	19	20
c	d	b	c	a	c	b	a	d	a

Q.No	Value points	Marks	Total Marks
21	A can be expressed as $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$ $\frac{1}{2}(A + A^T) = \begin{bmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{bmatrix}$ which is symmetric $\frac{1}{2}(A - A^T) = \begin{bmatrix} 0 & -5/2 & -3/2 \\ 5/2 & 0 & -3 \\ 3/2 & 3 & 0 \end{bmatrix}$ which is skew symmetric	1M ½ M ½ M	2 M
22	Given y = $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$ $= \tan^{-1}[2cos\left(\frac{\pi}{3}\right)]$ $= \tan^{-1}(1)$ $= \frac{\pi}{4}$	1M ½ M ½ M	2 M
23	For showing L.H.L =10 For showing R.H.L =10 And $f(2) = 10$ LHL = RHL = $f(2)$ Hence, f is continuous at x=2 **OR** At x =1, For showing L.H.D = 2 For showing R.H.D = -1 $L.H.D \neq R.H.D$ Hence, f is not derivable at x=1	½ M ½ M ½ M ½ M ½ M ½ M ½ M ½ M	2 M

<table>
<tr><td>24</td><td>Let $\overrightarrow{OP} = 2\vec{a} + \vec{b}$ and $\overrightarrow{OQ} = \vec{a} - 3\vec{b}$
For writing section formula, $\overrightarrow{OR} = \frac{1X\overrightarrow{OQ} - 2X\overrightarrow{OP}}{1-2}$
For simplifying and writing $\overrightarrow{OR} = 3\vec{a} + 5\vec{b}$
OR
Let $\vec{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k},\ \vec{b} = 2\hat{\imath} - 7\hat{\jmath} + \hat{k}$
For finding, $\vec{a}X\vec{b} = 20\hat{\imath} + 5\hat{\jmath} - 5\hat{k}$
For finding, $\left|\vec{a}X\vec{b}\right| = 15\sqrt{2}$
Hence, the area of the parallelogram is $\left|\vec{a}X\vec{b}\right| = 15\sqrt{2}$ sq. units</td><td>1M
1M

1M
½ M
½ M</td><td>2M</td></tr>
<tr><td>25</td><td>(i) P(the problem solved) = $P(A \cup B)$
$= P(A) + P(B) - P(A \cap B)$
$= P(A) + P(B) - P(A).P(B)$
$= \frac{5}{6}$
(ii) P(Exactly one of them solve theproblem)
$= P(A).P(B') + P(B).P(A')$
$= \frac{1}{2}$</td><td></td><td>2M</td></tr>
<tr><td>26</td><td>For correct graph including shading area


For writing all the corner points (30,30),(60,0) and (120,0)of the feasible region and finding Z values at those points

Max Z= 600 at every point on the line segment joining (120, 0) and (40,40)</td><td>1 ½ M
1M
½ M</td><td>3M</td></tr>
<tr><td>27</td><td>$Let\ I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ ……………..(1)
For applying formula, $\int_a^b f(x)\,dx = f(a+b-x)dx$
For writing, $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\cot x}}$
$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{tanx}}{1+\sqrt{\tan x}} dx$ ……………..(2)</td><td>½ M
½ M</td><td>3M</td></tr>
</table>

	By adding (1) and (2) and for getting, $2I = \int_{\pi/6}^{\pi/3} dx$ For finding definite integral, $I = \frac{\pi}{12}$ **OR** $I = \int_1^3 [\lvert x-1\rvert + \lvert x-2\rvert]\ dx$ $= \int_1^2 [\lvert x-1\rvert + \lvert x-2\rvert] dx + \int_2^3 [\lvert x-1\rvert + \lvert x-2\rvert]\ dx$ $= \int_1^2 (x-1) - (x-2) dx + \int_2^3 (x-1) + (x-2)\ dx$ $= \int_1^2 dx + \int_2^3 (2x-3)\ dx$ For integration and substituting upper and lower limits Simplification and for getting 3	½ M ½ M 1M ½ M ½ M ½ M 1½ M	
28	For writing integration by parts formula By applying formula and simplifying, we get $\int x \sin^{-1} x\ dx = \left(\frac{2x^2-1}{4}\right) \sin^{-1} x + \frac{x}{4}\sqrt{1-x^2} + C$	1M 2M	3M
29	Given $(1+x^2)\,dy + 2xy\ dx = \cot x\ dx$ $\frac{dy}{dx} = \frac{\cot x - 2xy}{1+x^2}$ For writing, $\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{\cot x}{1+x^2}$ Comparing with $\frac{dy}{dx} + p(x)y = Q(x)$ P= $\frac{2x}{1+x^2}$ Q= $\frac{\cot x}{1+x^2}$ I.F $= e^{\int p\,dx} = e^{\int \frac{2x}{1+x^2}} = 1 + x^2$ The general solution is $y.I.F = \int Q.IF\ dx + c$ $(1+x^2).y = \log\lvert\sin x\rvert + c$ **OR** $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$ …………….. (1) For substituting in (1) , $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ After simplification for getting, $x\frac{dv}{dx} = \frac{-v^3}{1+v^2}$ For writing, $\frac{1+v^2}{v^3} = \frac{-1}{x} dx$ After integrating on both sides, for getting $\frac{-1}{2v^2} + \log v = -\log\lvert x\rvert + c$ After substituting $v = \frac{y}{x}$, for getting $\frac{-x^2}{2y^2} + \log\lvert y\rvert = c$	1M ½ M ½ M 1M ½ M ½ M ½ M 1M ½ M	3M
30	For writing, $\vec{a} + \lambda\vec{b} = (2-\lambda)\hat{\imath} + (2+2\lambda)\hat{\jmath} + (3+\lambda)\hat{k}$ For writing, $(\vec{a} + \lambda\vec{b}).\vec{c} = 0$	1M	3 M

	For simplifying and for getting $\lambda = 8$	½ M 1½ M	
31	$f(x) = \sin x + \cos x,\ \ 0 \le x \le 2\pi,$ $f'(x) = cosx - sinx$ For writing $f'(x) = 0$ For writing possible intervals, $\left(0, \frac{\pi}{4}\right), \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ & $(\frac{5\pi}{4}, 2\pi)$ f is strictly increasing in $\left(0, \frac{\pi}{4}\right)$ f is strictly decreasing in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ f is strictly decreasing in $(\frac{5\pi}{4}, 2\pi)$ **OR** Given $f(x) = 2x^3 - 15x^2 + 36x + 1$ $f'(x) = 6x^2 - 30x + 36$ = 6(x-2)(x-3) $f'(x) = 0$ and getting x= 2 and 3 $f(1) = 24, f(2) = 29, f(3) = 28\ f(5) = 56$ The absolute maximum value is 56 The absolute minimum value is 24	 ½ M ½ M ½ M ½ M ½ M ½ M ½ M 1M ½ M ½ M ½ M	3M
32	$A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ $BA = 6I$ $B^{-1} = \frac{A}{6} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ $x - y = -5$,$2x + 3y + 4z$ =18 and $y + 2z = 8$ $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ 18 \\ 8 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}^{-1}\begin{bmatrix} -5 \\ 18 \\ 8 \end{bmatrix}$ $= \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} -5 \\ 18 \\ 8 \end{bmatrix}$ $= \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix}$ X=-1 y=4 z=2	 1M 1M ½ M 1M 1½ M	5M
33	For finding, $\frac{dx}{dt} = atcost$ For finding, $\frac{dy}{dt} = atsint$ For finding, $\frac{d^2x}{dt^2} = a(cost - tsint)$ For finding, $\frac{d^2y}{dt^2} = a(sint + tcost)$ For finding, $\frac{d^2y}{dx^2} = \frac{sec^2t}{atcost}$	1M 1M 1M 1M 1M	5 M

	OR $(cosx)^y = (cosy)^x$ Apply logarithms on both sides $log(cosx)^y = log(cosy)^x$ $ylogcosx = xlogcosy$ Differentiate w.r.t 'x' on both sides and simplifying then $(logcosx)\frac{dy}{dx} - y.tanx = (logcosy) - xtany.\frac{dy}{dx}$ For simplifying and writing $\frac{dy}{dx} = \frac{logcosy + ytanx}{logcosx + xtany}$	1M 2M 2M	
34	Let a, b, c are the direction ratios of the required and passing through the point (1, 2,-4) then the line equation is $\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+4}{c}$ The required line is perpendicular to both $\vec{r} = \hat{\imath} + \hat{\jmath} - \hat{k} + \lambda(2\hat{\imath} - 2\hat{\jmath} + \hat{k})$ and $\vec{r} = 2\hat{\imath} - \hat{\jmath} - 3\hat{k} + \mu(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$ Then $2a - 2b + c = 0$ and $a + 2b + 2c = 0$ Solving these equations, we get a =-2, b = -1 and c= 2 The vector equation of line is $\vec{r} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \mu(2\hat{\imath} + \hat{\jmath} - 2\hat{k})$ The Cartesian form of line equation is $\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z+4}{-2}$ **OR** $\vec{r} = (8 + 3\lambda)\hat{\imath} - (9 + 16\lambda)\hat{\jmath} + (10 + 7\lambda)\hat{k}$ $\vec{r} = (8\hat{\imath} - 9\hat{\jmath} + 10\hat{k}) + \lambda(3\hat{\imath} - 16\hat{\jmath} + 7\hat{k})$ $\vec{a}_1 = (8\hat{\imath} - 9\hat{\jmath} + 10\hat{k})$ and $\vec{b}_1 = (3\hat{\imath} - 16\hat{\jmath} + 7\hat{k})$ $\vec{r} = (15\hat{\imath} + 29\hat{\jmath} + 5\hat{k}) + \mu(3\hat{\imath} + 8\hat{\jmath} - 5\hat{k}$ $\vec{a}_2 = (15\hat{\imath} + 29\hat{\jmath} + 5\hat{k})$ and $\vec{b}_2 = (3\hat{\imath} + 8\hat{\jmath} - 5\hat{k})$ $\vec{b}_1 \times \vec{b}_2 = (24\hat{\imath} - 36\hat{\jmath} + 72\hat{k})$ $\|\vec{b}_1 \times \vec{b}_2\| = 84$ $\vec{a}_2 - \vec{a}_1 = (7\hat{\imath} + 38\hat{\jmath} - 5\hat{k})$ S.D $= \left\|\frac{(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1)}{\|\vec{b}_1 \times \vec{b}_2\|}\right\|$ $= \frac{(24\hat{\imath} - 36\hat{\jmath} + 72\hat{k}).(7\hat{\imath} - 38\hat{\jmath} - 5\hat{k})}{81}$ $= \left\|\frac{168 + 1368 - 360}{84}\right\|$ $= \frac{1176}{84}$ =14 units	½ M 2½ M 1M 1M 1M 1M ½ M ½ M ½ M ½ M 1M	5M

35	For correct sketch Given ellipse is $\frac{x^2}{9}+\frac{y^2}{4}$=1 Here, a=3 and b=2 For writing, $y=\frac{2}{3}\sqrt{9-x^2}$ Required area = $4\int_0^3 \frac{2}{3}\sqrt{9-x^2}$ $=\frac{8}{3}\int_0^3 \frac{2}{3}\sqrt{9-x^2}$ For applying formula, $\int \sqrt{a^2-x^2}\,dx$ After simplification for getting, $6\pi\ square\ units$	1M 1M 1M 1M 1M	5M
36	(i) 64 (ii) 9 (iii) B={(b_1,b_1), (b_1, b_2), (b_2,b1), (b_2,b_2)}For checking equivalence relation **OR** f is not bijective because two elements of domain $g_1\ and\ g_3$ have same image $b_1$in the codomain.	1M 1M 2M	4M
37	(iv) V= $4x(x^2-18x+81)$	1M	4M
	(v) $\frac{dV}{dx}=0\ and\ \frac{d^2V}{dx^2}<0$	1M	
	(vi) x = 3 **OR** 432 sq. units	2M	
38	(iii) P(E1) = 0.25 P(E/E1) = 4/15 P(E2) = 0.35 P(E/E2) = 5/15 P(E3) = 0.40 P(E/E3) = 6/15 P(E1/E) = P(E1) P(E/E1) / P(E) = 20/103 (iv) $\sum_{i=1}^{3} P(\frac{E}{E_i})$ = P(E1) P(E/E1) / P(E) + P(E2) P(E/E2) / P(E) + P(E3)P(E/E3) / P(E) = P(E) / P(E) = 1	 2M 2M	4 M

KENDRIYA VIDYALAYA SANGATHAN: HYDERABAD REGION

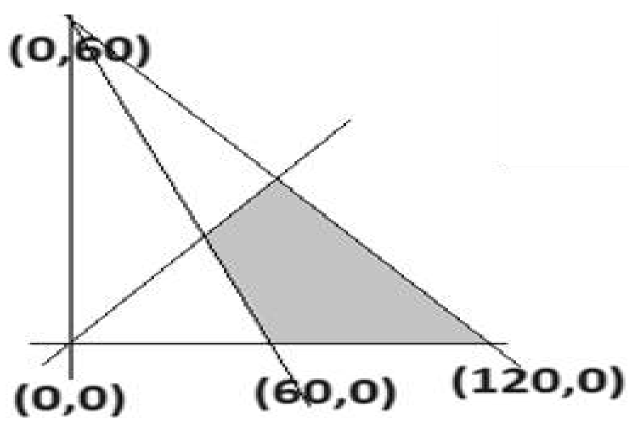
MODEL QUESTION PAPER - 13 (2025 - 26)

SUBJECT: MATHEMATICS (041)

MARKING SCHEME

1	2	3	4	5	6	7	8	9	10
B	D	A	D	D	D	B	B	C	C
11	12	13	14	15	16	17	18	19	20
C	D	B	C	A	C	B	A	D	A

Q.No	Value points	Marks	Total Marks
21	A can be expressed as $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$ $\frac{1}{2}(A + A^T) = \begin{bmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{bmatrix}$ which is symmetric $\frac{1}{2}(A - A^T) = \begin{bmatrix} 0 & -5/2 & -3/2 \\ 5/2 & 0 & -3 \\ 3/2 & 3 & 0 \end{bmatrix}$ which is skew symmetric	1M ½ M ½ M	2 M
22	Given y = $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$ $= \tan^{-1}[2cos\left(\frac{\pi}{3}\right)]$ $= \tan^{-1}(1)$ $= \frac{\pi}{4}$	1M ½ M ½ M	2 M
23	For showing L.H.L =10 For showing R.H.L =10 And $f(2) = 10$ LHL = RHL = $f(2)$ Hence, f is continuous at x=2 **OR** At x =1, For showing L.H.D = 2 For showing R.H.D = -1 $L.H.D \neq R.H.D$ Hence, f is not derivable at x=1	½ M ½ M ½ M ½ M ½ M ½ M ½ M ½ M	2 M

24	Let $\overrightarrow{OP} = 2\vec{a} + \vec{b}$ and $\overrightarrow{OQ} = \vec{a} - 3\vec{b}$ For writing section formula, $\overrightarrow{OR} = \frac{1X\overrightarrow{OQ} - 2X\overrightarrow{OP}}{1-2}$ For simplifying and writing $\overrightarrow{OR} = 3\vec{a} + 5\vec{b}$ **OR** Let $\vec{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k},\ \vec{b} = 2\hat{\imath} - 7\hat{\jmath} + \hat{k}$ For finding, $\vec{a}X\vec{b} = 20\hat{\imath} + 5\hat{\jmath} - 5\hat{k}$ For finding, $\lvert\vec{a}X\vec{b}\rvert = 15\sqrt{2}$ Hence, the area of the parallelogram is $\lvert\vec{a}X\vec{b}\rvert = 15\sqrt{2}$ sq. units	1M 1M 1M ½ M ½ M	2M
25	(iii) P(the problem solved) = $P(A \cup B)$ $= P(A) + P(B) - P(A \cap B)$ $= P(A) + P(B) - P(A).P(B)$ $= \frac{5}{6}$ (iv) P(Exactly one of them solve theproblem) $= P(A).P(B') + P(B).P(A')$ $= \frac{1}{2}$		2M
26	For correct graph including shading area  For writing all the corner points (30,30),(60,0) and (120,0)of the feasible region and finding Z values at those points Max Z= 600 at every point on the line segment joining (120, 0) and (40,40)	1 ½ M 1M ½ M	3M
27	$Let\ I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ ……………..(1) For applying formula, $\int_a^b f(x)\,dx = f(a+b-x)dx$ For writing, $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\cot x}}$ $= \int_{\pi/6}^{\pi/3} \frac{\sqrt{tanx}}{1+\sqrt{\tan x}} dx$ ……………..(2)	 ½ M ½ M	3M

	By adding (1) and (2) and for getting, $2I = \int_{\pi/6}^{\pi/3} dx$ For finding definite integral, $I = \frac{\pi}{12}$ **OR** $I = \int_1^3 [\lvert x-1\rvert + \lvert x-2\rvert]\ dx$ $= \int_1^2 [\lvert x-1\rvert + \lvert x-2\rvert] dx + \int_2^3 [\lvert x-1\rvert + \lvert x-2\rvert]\ dx$ $= \int_1^2 (x-1) - (x-2) dx + \int_2^3 (x-1) + (x-2)\ dx$ $= \int_1^2 dx + \int_2^3 (2x-3)\ dx$ For integration and substituting upper and lower limits Simplification and for getting 3	½ M ½ M 1M ½ M ½ M ½ M 1½ M	
28	For writing integration by parts formula By applying formula and simplifying, we get $\int x\, sin^{-1} x\, dx = \left(\frac{2x^2-1}{4}\right) sin^{-1}x + \frac{x}{4}\sqrt{1-x^2} + C$	1M 2M	3M
29	Given $(1 + x^2)\, dy + 2xy\ dx = \cot x\ dx$ $\frac{dy}{dx} = \frac{\cot x - 2xy}{1+x^2}$ For writing, $\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{\cot x}{1+x^2}$ Comparing with $\frac{dy}{dx} + p(x)y = Q(x)$ P= $\frac{2x}{1+x^2}$ Q= $\frac{\cot x}{1+x^2}$ I.F $= e^{\int p\, dx} = e^{\int \frac{2x}{1+x^2}} = 1 + x^2$ The general solution is $y.I.F = \int Q.IF\ dx + c$ $(1+x^2).y = log\ \lvert\sin x\rvert + c$ **OR** $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$ …………….. (1) For substituting in (1) , $y = vx\ and\ \frac{dy}{dx} = v + x\frac{dv}{dx}$ After simplification for getting, $x\frac{dv}{dx} = \frac{-v^3}{1+v^2}$ For writing, $\frac{1+v^2}{v^3} = \frac{-1}{x} dx$ After integrating on both sides, for getting $\frac{-1}{2v^2} + logv = -log\lvert x\rvert + c$ After substituting $v = \frac{y}{x}$, for getting $\frac{-x^2}{2y^2} + log\lvert y\rvert = c$	1M ½ M ½ M 1M ½ M ½ M ½ M 1M ½ M	3M
30	For writing, $\vec{a} + \lambda\vec{b} = (2-\lambda)\hat{\imath} + (2+2\lambda)\hat{\jmath} + (3+\lambda)\hat{k}$ For writing, $(\vec{a} + \lambda\vec{b}).\vec{c} = 0$	1M	3 M

	For simplifying and for getting $\lambda = 8$	½ M 1½ M	
31	$f(x) = \sin x + \cos x, \ \ 0 \le x \le 2\pi,$ $f'(x) = cosx - sinx$ For writing $f'(x) = 0$ For writing possible intervals, $\left(0, \frac{\pi}{4}\right), \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ & $(\frac{5\pi}{4}, 2\pi)$ f is strictly increasing in $\left(0, \frac{\pi}{4}\right)$ f is strictly decreasing in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ f is strictly decreasing in $(\frac{5\pi}{4}, 2\pi)$ **OR** Given $f(x) = 2x^3 - 15x^2 + 36x + 1$ $f'(x) = 6x^2 - 30x + 36$ = 6(x-2)(x-3) $f'(x) = 0$ and getting x= 2 and 3 $f(1) = 24, f(2) = 29, f(3) = 28\ f(5) = 56$ The absolute maximum value is 56 The absolute minimum value is 24	 ½ M ½ M ½ M ½ M ½ M ½ M ½ M 1M ½ M ½ M ½ M	3M
32	$A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ $BA = 6I$ $B^{-1} = \frac{A}{6} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ $x - y = -5$,$2x + 3y + 4z$ =18 and $y + 2z = 8$ $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ 18 \\ 8 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}^{-1}\begin{bmatrix} -5 \\ 18 \\ 8 \end{bmatrix}$ $=\frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} -5 \\ 18 \\ 8 \end{bmatrix}$ $=\begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix}$ X=-1 y=4 z=2	 1M 1M ½ M 1M 1½ M	5M
33	For finding, $\frac{dx}{dt} = atcost$ For finding, $\frac{dy}{dt} = atsint$ For finding, $\frac{d^2x}{dt^2} = a(cost - tsint)$ For finding, $\frac{d^2y}{dt^2} = a(sint + tcost)$ For finding, $\frac{d^2y}{dx^2} = \frac{sec^2t}{atcost}$	1M 1M 1M 1M 1M	5 M

	OR $(cosx)^y = (cosy)^x$ Apply logarithms on both sides $log(cosx)^y = log(cosy)^x$ $ylogcosx = xlogcosy$ Differentiate w.r.t 'x' on both sides and simplifying then $$(logcosx)\frac{dy}{dx} - y.tanx = (logcosy) - xtany.\frac{dy}{dx}$$ For simplifying and writing $$\frac{dy}{dx} = \frac{logcosy + ytanx}{logcosx + xtany}$$	1M 2M 2M	
34	Let a, b, c are the direction ratios of the required and passing through the point (1, 2,-4) then the line equation is $\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+4}{c}$ The required line is perpendicular to both $\vec{r} = \hat{i} + \hat{j} - \hat{k} + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ and $\vec{r} = 2\hat{i} - \hat{j} - 3\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$ Then $2a - 2b + c = 0$ and $a + 2b + 2c = 0$ Solving these equations, we get a =-2, b = -1 and c= 2 The vector equation of line is $$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \mu(2\hat{i} + \hat{j} - 2\hat{k})$$ The Cartesian form of line equation is $$\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z+4}{-2}$$ **OR** $\vec{r} = (8 + 3\lambda)\hat{i} - (9 + 16\lambda)\hat{j} + (10 + 7\lambda)\hat{k}$ $\vec{r} = (8\hat{i} - 9\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$ $\vec{a}_1 = (8\hat{i} - 9\hat{j} + 10\hat{k})$ and $\vec{b}_1 = (3\hat{i} - 16\hat{j} + 7\hat{k})$ $\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k}$ $\vec{a}_2 = (15\hat{i} + 29\hat{j} + 5\hat{k})$ and $\vec{b}_2 = (3\hat{i} + 8\hat{j} - 5\hat{k})$ $\vec{b}_1 \times \vec{b}_2 = (24\hat{i} - 36\hat{j} + 72\hat{k})$ $\|\vec{b}_1 \times \vec{b}_2\| = 84$ $\vec{a}_2 - \vec{a}_1 = (7\hat{i} + 38\hat{j} - 5\hat{k})$ S.D $= \left\|\frac{(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1)}{\|\vec{b}_1 \times \vec{b}_2\|}\right\|$ $= \frac{(24\hat{i}-36\hat{j}+72\hat{k}).(7\hat{i}-38\hat{j}-5\hat{k})}{81}$ $= \left\|\frac{168+1368-360}{84}\right\|$ $= \frac{1176}{84}$ =14 units	½ M 2½ M 1M 1M 1M 1M ½ M ½ M ½ M ½ M 1M	5M

35	For correct sketch Given ellipse is $\frac{x^2}{9}+\frac{y^2}{4}=1$ Here, a=3 and b=2 For writing, $y=\frac{2}{3}\sqrt{9-x^2}$ Required area = $4\int_0^3 \frac{2}{3}\sqrt{9-x^2}$ $=\frac{8}{3}\int_0^3 \frac{2}{3}\sqrt{9-x^2}$ For applying formula, $\int \sqrt{a^2-x^2}\,dx$ After simplification for getting, 6π *square units*	1M 1M 1M 1M 1M	5M
36	(iv) $2^{4\times3}=2^{12}$ (v) Since, S_2 and S_3 have been assigned the same judge J_2, the function is not one-one. Hence, it is not bijective. **(vi)** There cannot exist any one-one function from S to J as n(S) > n(J). Hence, the number of one-one functions from S to J is 0. **OR** To make $R1$ reflexive and not symmetric we need to add the following ordered pairs: $(S1,S1),(S2,S2),(S3,S3),(S4,S4)$	1M 1M 2M	4M
37	(vii) V= $4x(x^2-18x+81)$	1M	4M
	(viii) $\frac{dV}{dx}=0$ *and* $\frac{d^2V}{dx^2}<0$	1M	
	(ix) x = 3 **OR** 432 sq. units	2M	
38	(v) $P(E_1) = 0.25$ $P(A/E_1) = 4/15$ $P(E_2) = 0.35$ $P(A/E_2) = 5/15$ $P(E_3) = 0.40$ $P(A/E_3) = 6/15$ $P(E_1/A) = P(E_1)\,P(A/E_1) / P(A)$ = 20/103 (vi) $\sum_{i=1}^{3} P(\frac{E_i}{A})$ = $P(E_1)\,P(A/E_1) / P(A) + P(E_2)\,P(A/E_2) / P(A) + P(E_3)P(A/E_3) / P(A)$ = $P(A) / P(A)$ = 1	2M 2M	4 M

KENDRIYA VIDYALAYA SANGATHAN: HYDERABAD REGION

MODEL QUESTION PAPER - 14 (2025 - 26)

SUBJECT: MATHEMATICS (041)

MARKING SCHEME

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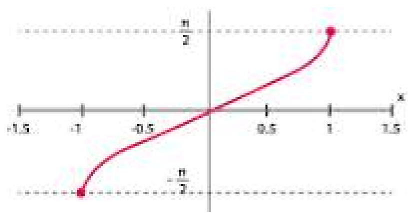
1. B	2. B	3. C	4. C	5. D	6. D	7. C	8. A	9. B	10. A
11. A	12. C	13. C	14. A	15. B	16. B	17. B	18. B	19. A	20. A



21. Graph 1m

Domain: $[-1,1]$ ½ m

Range: $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ ½ m

22. For finding limit as $\frac{k}{2}$ 1½ m

For finding $k = 10$ ½ m

(OR)

For getting 2a+b=5 ½ m

For getting 10a+b=21 ½ m

For finding a=2 ½ m

For finding b=1 ½ m

23. For finding $\frac{dC}{dx} = (0.005)(3x^2) - (0.02)(2x) + 30$ $=0.015(x^2) - 0.04(x) + 30$

1m

Marginal Cost= 30.015 ≅ 30.02 1m

24. For writing formula ½ m

For finding dot product =3 ½ m

For finding modulus=$\sqrt{6}$ ½ m

Final answer = $\frac{3}{\sqrt{6}}$ ½ m

(OR)

For finding $\bar{a} + \gamma\bar{b}$ ½ m

For finding dot product = 8-γ ½ m

For getting $\gamma = 8$ 1m

25. For writing formula ½ m

For finding cross product $5\hat{\imath} + \hat{\jmath} - 4\hat{k}$ 1 m

For finding modulus $\sqrt{42}$ ½ m

(OR)

For answering No 1m

For counter example 1m

26. For finding $f'(x)$=$6x^2 - 6x - 36$ ½ m

Solving $x = -2,3$ ½ m

For fining strictly increasing in $(-\infty, -2) \cup (3, \infty)$ 1m

For finding strictly decreasing in $(-2,3)$ 1m

27. For finding $\frac{dx}{dt} = \frac{3}{x^2}$ 1m

For finding $\frac{dS}{dt} = \frac{36}{x}$ 1m

For finding $\frac{dS}{dt} = 3.6\ cm^2/sec$ 1m

28. For writing structure of partial fractions $\frac{a}{x-1} + \frac{b}{x-2} + \frac{c}{x-3}$ ½ m

For finding a=1 , b=-5 , c=4 1 ½ m

For finding solution: $ln \mid x - 1 \mid -5ln \mid x - 2 \mid +4ln \mid x - 3 \mid +C$ 1m

(OR)

For writing formula $\int_0^a f(x)\, dx = \int_0^a f(a - x)\, dx$ ½ m

For getting $I = \int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1+tanx}\right) dx$ 1m

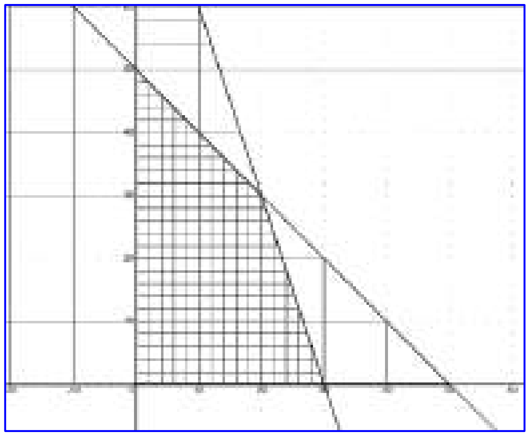
For getting $2I = \int_0^{\frac{\pi}{4}} log2\ dx$ ½ m

For getting $\frac{\pi}{8} log2$ 1m

29.

For drawing graph with feasible region

1+1/2m



For getting corner points $(0,0),(30,0),(20,30),(0,50)$ 1/2m

For conclusion the maximum value of Z is 120 at the point (30,0) 1m

30. For **Line 1**, the direction ratios are $\langle 3,-16,7\rangle$.

For **Line 2**, the direction ratios are $\langle 3,8,-5\rangle$.

1m

For finding cross product $24\hat{\imath}+36\,\hat{\jmath}+72\hat{k}$

1m

For finding vector and cartesian equations

$Cartesian\ equation{:}\dfrac{x-1}{24}=\dfrac{y-2}{36}=\dfrac{z+4}{72}$

Vector equation: $\bar{r}=\ \hat{\imath}+2\,\hat{\jmath}-4\,\hat{k}+\ \alpha\left(24\,\hat{\imath}+36\,\hat{\jmath}+72\,\hat{k}\right)$

1m

(OR)

For writing Dr's as $-3,\frac{2p}{7},2\ and\ -\frac{3p}{7},1\,,-5$ 1m

Applying condition for perpendicular of Dr's

1m

For getting $p=\frac{70}{11}$

1m

31. For writing $P(A^c)=\frac{1}{2},P(B^c)=\frac{2}{3}$

1m

The probability that the problem is solved: $\frac{2}{3}$

1m

The probability that exactly one of them solves the problem: $\frac{1}{2}$

1m

(OR)

The probability that the student reads neither Hindi nor English newspapers: 0.20
1m
The probability that the student reads English newspaper given that she reads Hindi newspaper: $\frac{1}{3}$
1m
The probability that the student reads Hindi newspaper given that

she reads English newspaper: 0.50

1m

32. For Evaluating $A^{-1} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ $where\ |A| = 67$

2 m
Expressing equations in matrix equation form AX=B, and $X = A^{-1}B$
1 m
Using A^{-1} and multiplying $\frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$
1m
For finding $x = 3, y = -2, z = 1$

1m

33. For redefining function

1m

Proof of continuity at x=1 and x=2

2m

Proof of not differentiable at x=1 and x=2

2m

(OR)

For writing $y = \sqrt{sinx + y}$

1m

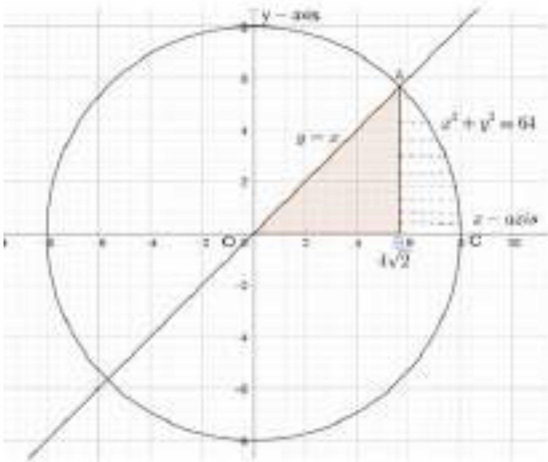
For squaring and then solving for $y = \frac{1+\sqrt{1+4sinx}}{2}$

2m

Finding $\frac{dy}{dx} = \frac{cosx}{\sqrt{1+4sinx}}$

2m

34. For correct graph 1m



Equation of circular table top $x^2 + y^2 = 64$ ½ m

Equation of the line $y = x$

½ m

The line and circle intersect at

$(4\sqrt{2}, 4\sqrt{2})$ ½ m

Area of shaded region

$=\int_0^{4\sqrt{2}} x\ dx + \int_{4\sqrt{2}}^{8} \sqrt{64 - x^2}\, dx$

1m

Evaluation of integration and substitution of limits

1

And final answer= $8\pi cm^2$ ½ m

35. Equation of the given line in standard form

$L_1: \frac{x}{2} = \frac{y-3}{2} = \frac{z-1}{1}$

½ m

Equation of the line parallel to L_1 & passing through $(4, 0, -5)$ is

$L_2: \frac{x-4}{2} = \frac{y}{2} = \frac{z+5}{1}$

½ m

Vector equation of Lines are $L_1: \vec{r} = \left(0\hat{\imath} + 3\hat{\jmath} + \hat{k}\right) + \lambda(2\hat{\imath} + 2\hat{\jmath} + \hat{k})$

$L_2: \vec{r} = \left(4\hat{\imath} + 0\hat{\jmath} - 5\hat{k}\right) + \mu(2\hat{\imath} + 2\hat{\jmath} + \hat{k})$

1m

Now,$\overrightarrow{a_2} - \overrightarrow{a_1} = 4\hat{\imath} - 3\hat{\jmath} - 6\hat{k}$

1½ m

$(\overrightarrow{a_2} - \overrightarrow{a_1}) \times \vec{b} = 9\hat{\imath} - 16\hat{\jmath} + 14\hat{k}$, $\vec{b} = \sqrt{4 + 4 + 1} = 3$

1m

Thus, distance between the lines is

S.D.= $\frac{|(\vec{a_2}-\vec{a_1})\times\vec{b}|}{|\vec{b}|} = \frac{\sqrt{81+256+196}}{3} = \frac{\sqrt{533}}{3} units$

1m

(OR)

For finding line equation BC $\bar{r} = -\hat{j} + 3\hat{k} + \alpha\left(2\hat{\imath} - 2\hat{j} - 4\hat{k}\right)$

1m

For writing general point on the line BC (2λ, – (2λ + 1), (3 – 4λ))

1m

For finding Dr's of the line Perpendicular to BC and passing through the point (1,8,4) are 1 – 2λ, 2λ + 9, 1 + 4λ

1m

Finding λ=$-\frac{5}{6}$

1m

Finding foot of perpendicular $\left(-\frac{5}{3},\frac{2}{3},\frac{19}{3}\right)$

1m

36. (i) $2^{12} = 4096$

1m

(ii) $6^2 = 36$

1m

(iii) (a) Neither symmetric nor transitive

(b) Not symmetric but transitive

2m

37. (i) P(A/E2)=0.04

1m

(ii) P(A) =P(E1).P(A/E1)+ P(E1).P(A/E1)+ P(E1).P(A/E1)=

=0.5x0.06+ 0.2x0.04+0.3x0.03=0.04

1m

(iii) (a) $\frac{17}{47}$

(b) $\frac{8}{47}$

2m

38. (i) $A = \frac{px-2x^2}{2}$

2m

(ii) $\frac{p^2}{16}$

2m

KENDRIYA VIDYALAYA SANGATHAN: HYDERABAD REGION

MODEL QUESTION PAPER - 15 (2025 - 26)

SUBJECT: MATHEMATICS (041)

MARKING SCHEME

<table>
<tr><th>Q.No.</th><th>ANS</th><th>Marks</th></tr>
<tr><td colspan="3">SECTION – A (1 x 20 = 20)</td></tr>
<tr><td>1</td><td>A</td><td>1M</td></tr>
<tr><td>2</td><td>D</td><td>1M</td></tr>
<tr><td>3</td><td>A</td><td>1M</td></tr>
<tr><td>4</td><td>C</td><td>1M</td></tr>
<tr><td>5</td><td>C</td><td>1M</td></tr>
<tr><td>6</td><td>A</td><td>1M</td></tr>
<tr><td>7</td><td>B</td><td>1M</td></tr>
<tr><td>8</td><td>D</td><td>1M</td></tr>
<tr><td>9</td><td>D</td><td>1M</td></tr>
<tr><td>10</td><td>A</td><td>1M</td></tr>
<tr><td>11</td><td>B</td><td>1M</td></tr>
<tr><td>12</td><td>C</td><td>1M</td></tr>
<tr><td>13</td><td>D</td><td>1M</td></tr>
<tr><td>14</td><td>B</td><td>1M</td></tr>
<tr><td>15</td><td>B</td><td>1M</td></tr>
<tr><td>16</td><td>B</td><td>1M</td></tr>
<tr><td>17</td><td>D</td><td>1M</td></tr>
<tr><td>18</td><td>C</td><td>1M</td></tr>
<tr><td>19</td><td>A</td><td>1M</td></tr>
<tr><td>20</td><td>D</td><td>1M</td></tr>
<tr><td colspan="3">SECTION B (2 x 5 =10)
(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)</td></tr>
<tr><td>21</td><td>$\sin^{-1}\left(\sin\left(\frac{3\pi}{4}\right)\right)+\cos^{-1}(\cos\pi)+\tan^{-1}(1)$
$=\sin^{-1}\left(\sin\left(\pi-\frac{\pi}{4}\right)\right)+\cos^{-1}(\cos\pi)+\tan^{-1}(\tan\pi/4)$
$=\sin^{-1}(\sin(\pi/4))+\cos^{-1}(\cos\pi)+\tan^{-1}(\tan(\pi/4))$
$=\frac{\pi}{4}+\pi+\frac{\pi}{4}=\frac{3\pi}{2}$</td><td>1M

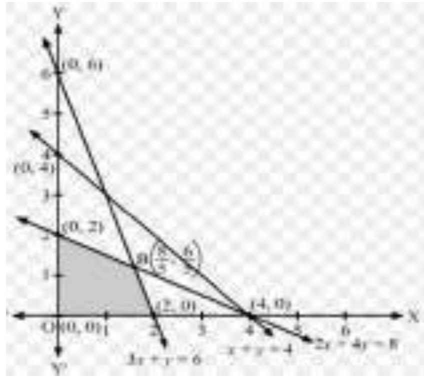
1M</td></tr>
<tr><td>22</td><td>$f(x)=\begin{cases}\frac{\sin(a+1)x+2sinx}{x}, & x<0\\ 2, & x=0\\ \frac{\sqrt{1+bx}-1}{x}, & x>0\end{cases}$
Given f is continuous at x = o then

$\lim_{x\to0^-} f(x) = \lim_{x\to0^+} f(x) = f(0)$</td><td>1M</td></tr>
</table>

	$\lim_{x\to 0^-} \frac{\sin(a+1)x+2sinx}{x} = \lim_{x\to 0^+} \frac{\sqrt{1+bx}-1}{x} = 2$ a + 3 = b/2 = 2 a = - 1, b = 4	1M
23	Volume of cone V = $\frac{1}{3}\pi r^2$ h, h= 4 cm Given $\frac{dV}{dt}$ = 12cm^3/s, h= $\frac{1}{6}$ $\Rightarrow r = 6h$ V= $\frac{1}{3}\pi r^2$ h = $\frac{1}{3}\pi \times 36h^2 \times h = 12\pi h^3$ $\frac{dV}{dt} = 12\pi. 3h^2 \frac{dh}{dt}$ $12 = 12\pi. 3\times 4\times 4\times \frac{dh}{dt}$ I = $48\pi \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{48\pi}$ cm/s. (OR) Surface area of balloon A = $4\pi r^2$, Given $\frac{dA}{dt}$ = 2cm/s. r = 8cm $\frac{dA}{dt} = 4\pi.2r. \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{4\pi r}$ $2 = 4\pi. 2x. \frac{dr}{dt}$ Volume v = $\frac{4}{3}\pi r^3$ $\frac{dv}{dt} = \frac{4}{3}\pi 3r^2. \frac{dr}{dt} = 4\pi r^2. \frac{dr}{dt}$ $\frac{dv}{dt} = 4\pi \times 8 \times 8 \times \frac{1}{4\pi\times 8}$ = 8cm^3/s. Volume of balloon is increasing at the rate of 8cm^3/s.	1/2M 1/2M 1M 1M 1M
24	Projection of $\vec{a}$ on $\vec{b}$ = 4 units $\frac{\vec{a}\cdot\vec{b}}{\|\vec{b}\|} = 4$ $\frac{(\lambda\hat{\imath}+\hat{\jmath}+4\hat{k})\cdot(2\hat{\imath}+6\hat{\jmath}+3\hat{k})}{\sqrt{4+36+9}} = 4$ $\frac{2\lambda+6+12}{7} = 4$ $\lambda = 5$	1M 1M
25	Consider $\|\vec{a}+\vec{b}\|^2 = \|\vec{b}\|^2$ $\|\vec{a}\|^2 + \|\vec{b}\|^2 + 2\vec{a}\cdot\vec{b} = \|\vec{b}\|^2$ $\vec{a}.\vec{a} + 2\vec{a}.\vec{b} = 0$ $\vec{a}.(\vec{a}+2\vec{b}) = 0$ $\therefore\ \vec{a} \perp (\vec{a}+2\vec{b})$ (OR) $\overrightarrow{AB} = -\hat{\imath} - 2\hat{\jmath} - 6\hat{k}$ and $\overrightarrow{AC} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ Area of triangle ABC = $\frac{1}{2}\|\overrightarrow{AB}\times\overrightarrow{AC}\|$ $= \frac{1}{2}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & -2 & -6 \\ 1 & -3 & -5 \end{vmatrix}$ $= \frac{1}{2}\|-8\hat{\imath} - 11\hat{\jmath} + 5\hat{k}\|$	1M 1M 1/2M 1M 1/2M

	$=\frac{1}{2}\sqrt{210}$ sq.units	
SECTION C (3 x 6 =18) ***(This section comprises of 6 short answer (SA) type questions of 3 marks each.)***		
26	: $f(x) = \log(1+x) - \frac{2x}{2+x}$ $f'(x) = \frac{1}{1+x} - \frac{(2+x)2-2x.1}{(2+x)^2} = \frac{1}{1+x} - \frac{4}{(2+x)^2}$ $= \frac{4+x^2+4x-4-4x}{(2+x)^2}$ $= \frac{x^2}{(2+x^2)}$, $x^2 \geq 0$,$(2+x)^2 \geq 0$ $= f'(x) \geq 0$ f is an increasing function on its domain	2M 1M
27	$f'(x) = x^3 - 3x^2 - 10x + 24$ $= (x-2)(x-4)(x+3)$ $f'(x) = 0 \Rightarrow x = -3,2,4.$ f is strictly increasing on $(-3,2) \cup (4,\infty)$ f is strictly decreasing on $(-\infty,-3) \cup (2,4)$ (OR) Given x+y=60, y= 60-x $f(x) = xy^3$ $f(x) = x(60-x)^3$ $f^1(x) = x.3(60-x)^2(-1) + (60-x)^3.1$ $= (60-x)^2(60-x-3x)$ $= (60-x)^2(60-4x)$ $f^1(x) = 0 \quad x = 60, x = 15$ at x = 15, $f^{11}(x) < 0$, f(x) has maximum x =15, y=60-x = 60-15=45 Two positive numbers are 15, 45	1M 1M 1M 1M 1M 1M
28	$(x+3)/3 = (y-1)/5 = (z+3)/4$ $(x+1)/1 = (y-4)/1 = (z-5)/2$ The direction ratios of the first line are: $a_1 = 3, b_1 = 5, c_1 = 4$ The direction ratios of the second line are: $a_2 = 1, b_2 = 1, c_2 = 2$ $COS\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1{}^2+b_1{}^2+c_1{}^2}\sqrt{a_2{}^2+b_2{}^2+c_2{}^2}}$ $\cos\theta = \frac{16}{\sqrt{50}\sqrt{6}}$ $\cos\theta = \frac{8\sqrt{3}}{15}$ $\theta = cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)$	1M 1M
29	Let $I = \int \frac{x^2}{x^4-x^2-12}\,dx$ $= \int \frac{x^2}{x^4-4x^2+3x^2-12}\,dx$ $= \int \frac{x^2dx}{x^2(x^2-4)+3(x^2-4)}$	

	$= \int \frac{x^2 dx}{(x^2-4)(x^2+3)}$ Now, $\frac{x^2}{(x^2-4)(x^2+3)}$ $\Rightarrow \quad \frac{t}{(t-4)(t+3)} = \frac{A}{t-4} + \frac{B}{t+3}$ $\Rightarrow \quad t = A(t+3) + B(t-4)$ On comparing the coefficient of t on both sides, we get $\Rightarrow \quad A + B = 1$ $\Rightarrow \quad 3A - 4B = 0$ $\Rightarrow \quad 3(1-b) - 4B = 0$ $\Rightarrow \quad 3 - 3B - 4B = 0$ $\Rightarrow \quad 7B = 3$ $\Rightarrow \quad B = \frac{7}{3}$ If $B = \frac{3}{7}, then\ A + \frac{3}{7} = 1$ $\Rightarrow \quad A = 1 - \frac{3}{7} = \frac{4}{7}$ $\frac{x^2}{(x^2-4)(x^2+3)} = \frac{4}{7(x^2-4)} + \frac{3}{7(x^2+3)}$ $I = \frac{4}{7}\int \frac{1}{x^2-(2)^2} dx + \frac{3}{7}\int \frac{1}{x^2+(\sqrt{3})^2} dx$ $= \frac{4}{7} . \frac{1}{2.2} \log\left\|\frac{x-2}{x+2}\right\| + \frac{3}{7} . \frac{1}{\sqrt{3}} \tan^{-1}\frac{x}{\sqrt{3}} + C$ $= \frac{1}{7} log \left\|\frac{x-2}{x+2}\right\| + \frac{\sqrt{3}}{7} \tan^{-1}\frac{x}{\sqrt{3}} + C$ (OR) Let $\quad I = \int_0^{\pi} \frac{x}{1+sinx}$ dx …(i) And $\quad I = \int_0^{\pi} \frac{\pi - x}{1+\sin(\pi-x)} dx = \int_0^{\pi} \frac{\pi-x}{1+sinx} dx$ …(ii) On adding Eqs. (i) and (ii) we get $2I = \pi \int_0^{\pi} \frac{1}{1+sinx} dx$ $= \pi \int_0^{\pi} \frac{(1-sinx)dx}{(1+sinx)(1-sinx)}$ $= \pi \int_0^{\pi} \frac{(1-sinx)dx}{\cos^2 x}$ $= \pi \int_0^{\pi} (\sec^2 x - tanx.secx)\, dx$ $= \pi \int_0^{\pi} \sec^2 x dx - \pi \int_0^{\pi} sec\, x\, x.tanx\, dx$ $= \pi[tanx]_0^{\pi} - \pi[secx]_0^{\pi}$ $= \pi[tanx - secx]_0^{\pi}$ $= \pi[tanx - sec\pi - tan0 - sec0]$ $2I = \pi[0 + 1 - 0 + 1]$ $\Rightarrow \quad 2I = 2\pi$ $\therefore \quad I = \pi$	1M 1M 1M 1M 1M 1M
30	The feasible region determined by the constraints, $2x + 4y \le 8, 3x + y \le 6, x + y \le 4, x \ge 0, y \ge 0$ is as shown	1M



The corner points of feasible region are O(0, 0), A(0, 2)), B(8/5, 6/5) and C(2, 0)
The value of Z at these corner points as follows 1M

Corner points	Value of the objective function $Z = 2x + 5y$
O(0, 0)	0
A(0, 2)	10
B(8/5, 6/5)	46/5
C(2, 0)	4

1M

Maximum value of Z is 10 at x = 0, y = 2

31 Let the events to be

Let E_1 = transferring a red ball from A to B

E_2 = transferring a black ball from A to B

A=Getting a red ball from bag B 1M

$P(E_1) = \frac{3}{5}$, $P(E_2) = \frac{2}{5}$

$P\left(\frac{A}{E_1}\right) = \frac{1}{2}$ $P\left(\frac{A}{E_2}\right) = \frac{1}{3}$

By using Baye s theorem. 1M

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)\,P(A/E_2)}$$

$$== \frac{\frac{1}{2}\frac{3}{5}}{\frac{1}{2}\times\frac{3}{5}+\frac{2}{5}\times\frac{1}{3}} = \frac{9}{13}$$ 1M

(OR)

A fair coin and unbiased dice is tossed.

S = {(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1)(T, 2) (T, 3)(T, 4) (T, 5)(T, 6)}

n(S) = 12

A = {(H,1)(H,2)(H,3)(H,4)(H,5)(H,6)} 1M

n(A) = 6

$P(A) = \frac{n(A)}{n(S)} = \frac{6}{12} = \frac{1}{2}$

B = {(H, 3)(3, T)}

n(B) = 2

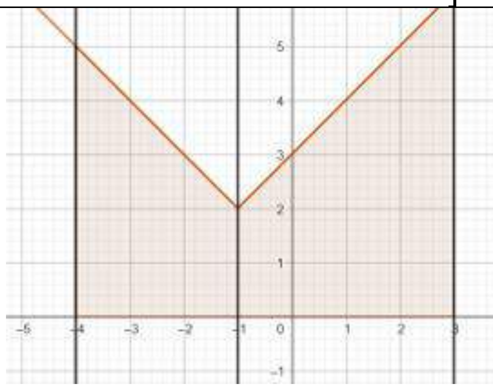
$P(B) = \frac{n(B)}{n(S)} = \frac{2}{12} = \frac{1}{6}$ 1M

A∩B = {(H,3)}

n(A∩B) = 1

	$P(A\cap B) = \frac{n(A\cap B)}{n(S)} = \frac{1}{12}$ $P(A)\,P(B) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$ $P(A\cap B) = \frac{1}{12}$ $P(A\cap B) = P(A)\,P(B)$ These are independent events	1M
	SECTION D (5 x 4 =20) ***(This section comprises of 4 long answer (LA) type questions of 5 marks each)***	
32	$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ $AB = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$ Thus, AB = I B = A^{-1} -------------- (1) Given $x + 3z = 9\,, -x + 2y - 2z = 4, 2x - 3y + 4z = \ -3$ Writing the equation in the matrix form we get $\begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & -2 \\ 2 & -3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix}$ $A^T X = C$ $X = (A^T)^{-1}C$ $X = B^T C$ (from 1) $X = \begin{bmatrix} -2 & 9 & 6 \\ 0 & 2 & 1 \\ 1 & -3 & -2 \end{bmatrix}\begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$ Thus, x = 0, y = 5, z = 3 (OR) Given equations can be written as AX = B ⇒ $X = A^{-1}B$ Where $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix}, B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$	2M 1M 2M 1M

	$\|A\| = 2(120 - 45) - 3(-80 - 30) + 10(36 + 36) = 1200 \neq 0$ A^{-1} exists $adj(A) = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$ $A^{-1} = \frac{1}{\|A\|} adj(A)$ $= \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$ $X = A^{-1}B$ $X = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$ $\begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$ x = 2, y = 3, z = 4	1M 2M 1M
33	Let $x = \sin A, \quad y = \sin B \Rightarrow A = \sin^{-1} x, B = \sin^{-1} y$ $\therefore \sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$ $\Rightarrow \cos A + \cos B = a(\sin A - \sin B)$ $\Rightarrow 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = 2a\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ $\Rightarrow \cot\left(\frac{A-B}{2}\right) = a \Rightarrow A - B = 2\cot^{-1} a$ $=> \sin^{-1} x - \sin^{-1} y = 2\cot^{-1} a$ Differentiate both sides w.r.t x, $\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$ $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$ OR $x = a(\cos\theta + \log\tan\frac{\theta}{2})$ $=> \frac{dx}{d\theta} = a(-\sin\theta + \frac{1}{\tan\frac{\theta}{2}}\sec^2\frac{\theta}{2} \times \frac{1}{2})$ $= a(-\sin\theta + \frac{1}{\sin\theta}) = a(\frac{1 - \sin^2\theta}{\sin\theta})$ $\frac{dx}{d\theta} = a\cot\theta\cos\theta$	1 1 1 ½ 1 ½ ½ ½ ½

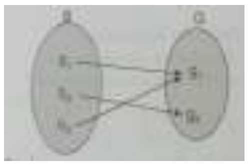
	Also, $y = \sin\theta => \frac{dy}{d\theta} = \cos\theta$ $$\therefore \frac{dy}{dx} = \frac{\tan\theta}{a}$$ Differentiating w.r.t x, $$\frac{d^2y}{dx^2} = \frac{\sec^2\theta}{a} \times \frac{d\theta}{dx}$$ $$= \frac{\sec^3\theta \tan\theta}{a^2}$$ $$\left.\frac{d^2y}{dx^2}\right\rvert\ at\ \theta = \frac{\pi}{4} = \frac{2\sqrt{2}}{a^2}$$	½ 1 1 1
34	 Correct graph Required area = $\int_{-4}^{-1}(2 + \lvert x+1 \rvert)\, dx + \int_{-1}^{3}(2 + \lvert x+1 \rvert) dx$ $=\int_{-4}^{-1}(2-(x+1)\, dx + \int_{-1}^{3}(2+x+1)dx$ $=\int_{-4}^{-1}(1-x)\, dx + \int_{-1}^{3}(3+x)dx$ $= -\left.\frac{(1-x)^2}{2}\right]_{-4}^{-1} + \left.-\frac{(3+x)^2}{2}\right]_{-1}^{3}$ $=\frac{21}{2}+16=\frac{53}{2}$	1M 1M 1M 1M 1M
35	The general point on the line $(3\lambda-2, 2\lambda-1, 2\lambda+3)$ is Q, from some $\lambda \in R$ PQ= $3\sqrt{2} \Rightarrow (PQ)^2 = 18 \Rightarrow (3\lambda-3)^2 + (2\lambda-3)^2 + (2\lambda)^2 = 18$ $17\lambda^2 - 30\lambda = 0 \Rightarrow \lambda = 0\ or\ \lambda = \frac{30}{17}$ Thus, the point is $Q(-2,-1,3)$ or $Q(\frac{56}{17}, \frac{43}{17}, \frac{111}{17})$ (OR) $L_1: \frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2}$ and $L_2: \frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ Direction ratio's of $L_1$is (-3, $\frac{\lambda}{7}$, 2) Direction ratio's of $L_2$is ($\frac{-3\lambda}{7}, 1, -5)$ Given lines are perpendicular to each other $\therefore (-3)\left(-\frac{3\lambda}{7}\right) + \frac{\lambda}{7} \times 1 + 2 \times -5 = 0$ $\lambda = 7$ $\therefore\ L_1: \frac{1-x}{3} = \frac{7y-14}{7} = \frac{z-3}{2}, L_2: \frac{7-7x}{21} = \frac{y-5}{1} = \frac{6-z}{5}$ $L_1: \frac{x-1}{-3} = \frac{y-2}{1} = \frac{z-3}{2}, L_2: \frac{x-1}{-3} = \frac{y-5}{1} = \frac{z-6}{-5}$ Two lines are intersect or not then we check $\begin{vmatrix} (1-1) & (5-2) & (6-3) \\ -3 & 1 & 2 \\ -3 & 1 & -5 \end{vmatrix} = \begin{vmatrix} 0 & 3 & 3 \\ -3 & 1 & 2 \\ -3 & 1 & -5 \end{vmatrix}$ = -63 Which is not equal to zero. Therefore, lines do not intersect	2M 1M 1M 1M 1M 1M 1M 2M

SECTION D (4 x 3 =12)

(*This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)*

Case Study-1

36	Given B = $\{b_1, b_2, b_3\}$ and G = $\{g_1, g_2\}$	
i	Number of relations from B to G = $2^{n(B)n(G)} = 2^{3\times 2} = 64$	1M

ii	Number of functions from B to G = $n(G)^{n(B)} = 2^3 = 8$	1M
iii	$R: B \to B$ will be $R = \{(b_1, b_1), (b_1, b_2), (b_1, b_3), (b_2, b_1), (b_2, b_2), (b_2, b_3), (b_3, b_1), (b_3, b_2), (b_3, b_3)\}$ As b_1, b_2, b_3 are all boys As $\forall (b_i, b_i)$ are present $\therefore$ R is reflexive $\forall (b_i, b_j)$ there exist (b_j, b_i) $\therefore$ R is symmetric As $\forall (b_i, b_j), (b_j, b_k)$ there is (b_i, b_k) $\therefore$ R is transitive Hence R is an equivalence relation (OR) $f: B \to G$ has mapping diagram as below  Clearly range of f = codomain of f But element g_1 has two pre-images $\therefore$ f is onto but not one-one hence f is not bijective	1/2 M 1/2M 1/2M 1/2M 1 M 1 M
Case Study-2		
37		
i	$C = \frac{250 \times 50}{h} + 400 \times h^2$ $C = \frac{12500}{h} + 400\, h^2$	1M
ii	$C = \frac{12500}{h} + 400\, h^2$ $\frac{dc}{dh} = \frac{-12500}{h^2} + 800\, h$ $\frac{dc}{dh} = 0 \Rightarrow 800\, h^3 = 12500 \Rightarrow h^3 = \frac{125}{8} \Rightarrow h = \frac{5}{2} = 2.5$	1M
iii	$\frac{dc}{dh} = \frac{-12500}{h^2} + 800\, h$ $\frac{d^2 c}{dh^2} = \frac{-(-2) \times 12500}{h^3} + 800 = \frac{25000}{h^3} + 800$ (OR) $\frac{dc}{dh} = 0$, we get h = 2.5 h = 2.5, $\frac{d^2 c}{dh^2} > 0$ Value of x at minimum cost $x = \frac{400 \times (2.5)^2}{250} = \frac{2500}{250} = 10$ mt	2M 2M
Case Study- 3		
38		
i	P(A) = 2/10, P(B) = 3/10, P(C) = 5/10 E = defective item P(E/A) = 3/100, P(E/B) = 4/100. P(E/C) = 2/100 $P(C/E) = \frac{\frac{5}{10} \times \frac{2}{100}}{\frac{2}{10} \times \frac{3}{100} + \frac{3}{10} \times \frac{4}{100} + \frac{5}{10} \times \frac{2}{100}}$ $= \frac{10}{6+12+10}$	1M 1M

	$= \frac{5}{14}$	
ii	P(A) = 2/10, P(B) = 3/10, P(C) = 5/10 E = defective item P(E/A) = 3/100, P(E/B) = 4/100. P(E/C) = 2/100 P(B/E) = $\frac{\frac{3}{10}\times\frac{4}{100}}{\frac{2}{10}\times\frac{3}{100}+\frac{3}{10}\times\frac{4}{100}+\frac{5}{10}\times\frac{2}{100}}$ $= \frac{12}{6+12+10}$ $= \frac{3}{7}$	1M 1M

KENDRIYA VIDYALAYA SANGATHAN: HYDERABAD REGION

MODEL QUESTION PAPER - 16 (2025 - 26)

SUBJECT: MATHEMATICS (041)

MARKING SCHEME

<table>
<tr><td>Q. No.</td><td>ASNWERS</td><td>Marks</td></tr>
<tr><td colspan="3">SECTION – A (1 x 20 = 20)
(This section comprises of multiple choice questions (MCQs) of 1 mark each)</td></tr>
<tr><td>01 - 20</td><td>
<table>
<tr><td>1 - c</td><td>2 - b</td><td>3 - a</td><td>4 - b</td><td>5 - c</td><td>6 - b</td><td>7 - a</td><td>8 - b</td><td>9 - d</td><td>10– a</td></tr>
<tr><td>11- d</td><td>12- b</td><td>13- d</td><td>14- c</td><td>15- b</td><td>16- d</td><td>17- c</td><td>18- b</td><td>19- a</td><td>20- a</td></tr>
</table>
</td><td>Each Carry 1 Mark</td></tr>
<tr><td></td><td>SECTION B (2m x 5 =10m)</td><td></td></tr>
<tr><td>21</td><td>For the given relation S = { (a,b): a$\leq$ b^2, where a,b $\in$ R} taking any real value for ‘a’ between 0 and 1 disprove S is not reflexive
Similarly by taking a counter example disproving S is not symmetric
(OR)
For the given R = { (x,y) : $3x^2 - 7xy + 4y^2 = 0$, x,y $\in$ N} Proving R is reflexive by replacing y=x
Proving R is not symmetric: (4,3)$\in$ R but (3, 4) not in R</td><td>1 Mark

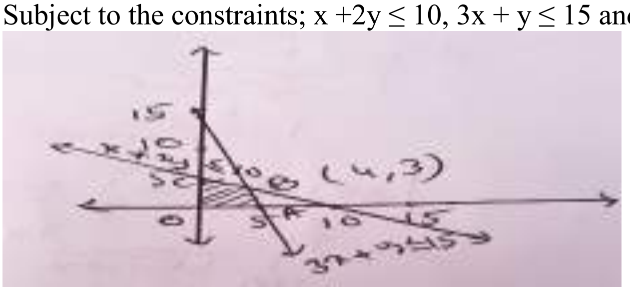
1 Mark
(OR)
1 Mark
1 Mark</td></tr>
<tr><td>22</td><td>Finding $\frac{d\,tan^{-1}x}{dx} = \frac{1}{1+x^2}$
and $\frac{d\,logx}{dx} = 1/x$ then $\frac{d\,tan^{-1}x}{d\,logx} = \frac{x}{1+x^2}$
(OR)
For finding left hand derivative = -1, finding right hand derivative = 1
Proving LHD $\neq$ RHD hence the given function f(x) = |x – 2| at x = 2 is not derivable</td><td>½ + ½
1 Mark

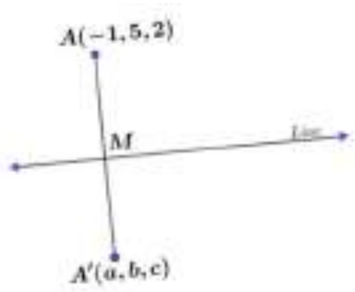
(OR)

½ + ½
1 Mark</td></tr>
<tr><td>23</td><td>Finding $f^1(x) = 4x^3 - 4x^2$ and for finding critical points x= 0, x = 1
Checking $f^1(x) > 0$,when x ϵ [1 , ∞)</td><td>1 Mark
1 Mark</td></tr>
<tr><td>24</td><td>$|3\vec{a} - 2\vec{b} + 2\vec{c}| = \sqrt{(3\vec{a} - 2\vec{b} + 2\vec{c}).(3\vec{a} - 2\vec{b} + 2\vec{c})}$
Given $\vec{a}\,.\,\vec{b} = \vec{a}\,.\,\vec{c}$ and $\vec{b}\,.\,\vec{c} = 0$ applying the dot product
$|3\vec{a} - 2\vec{b} + 2\vec{c}| = \sqrt{61}$</td><td>1 Mark

1 Mark</td></tr>
<tr><td>25</td><td>The unit perpendicular vector to both $3\vec{\imath} + \vec{\jmath} + 2\vec{k}$ and $2\vec{\imath} - 2\vec{\jmath} + 4\vec{k}$ is
$\frac{(3\vec{\imath} + \vec{\jmath} + 2\vec{k})\,x\,(2\vec{\imath} - 2\vec{\jmath} + 4\vec{k})}{|(3\vec{\imath} + \vec{\jmath} + 2\vec{k})x\,(2\vec{\imath} - 2\vec{\jmath} + 4\vec{k})|} = \frac{8(\vec{\imath} - \vec{\jmath} - \vec{k})}{8\sqrt{3}}$
$= \frac{1}{\sqrt{3}}\vec{\imath} - \frac{1}{\sqrt{3}}\vec{\jmath} - \frac{1}{\sqrt{3}}\vec{k}$</td><td>1 Mark

1 Mark</td></tr>
<tr><td></td><td>SECTION C (3m x 6 =18m)</td><td></td></tr>
<tr><td>26</td><td>For proving f: Z → $[\frac{-1}{2}, \frac{1}{2}]$ defined by f(x) = $\frac{x}{1+x^2}$ is one-one
For proving f: Z → $[\frac{-1}{2}, \frac{1}{2}]$ defined by f(x) = $\frac{x}{1+x^2}$ is on-to</td><td>1 ½ Mark
1 ½ Mark</td></tr>
</table>

27	$\frac{dx}{dt}$ = a e^t(sint + cos t) - a e^t(sint - cos t), $\frac{dy}{dt}$ = a e^t(sint + cos t) + a e^t(sint - cos t) $\frac{dy}{dx} = (\frac{dy}{dt}) \div \frac{dx}{dt} = \frac{x+y}{x-y}$ (OR) Given y = x^x taking log on both sides We get (1/y) $\frac{dy}{dx}$ = 1 + log x → $\frac{dy}{dx}$ = y(1+log x) Further deriving we get $\frac{d^2y}{dx^2} - \frac{1}{y}(\frac{dy}{dx})^2 - \frac{y}{x} = 0$	1Mark 1Mark 1 Mark 1Mark 1Mark 1Mark
28	$\int \frac{2x+1}{(x+1)^2(x-1)}\,dx$ Consider $\frac{2x+1}{(x+1)^2(x-1)} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x-1)}$ using partial fractions Find A = -3/4 B = 1/2 and C = 3/4 $\frac{-3}{2}$ log|x+1| + $\frac{1}{2(x+1)}$ + $\frac{3}{2}$ log|x-1| +C (OR) $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}$ dx substituting e^x = t $\int \frac{1}{\sqrt{5-4t-t^2}}$ dt = $\sin^{-1}(\frac{t+2}{3})$ +C = $\sin^{-1}(\frac{e^x+2}{3})$ +C	1 Mark 1 Mark 1 Mark 1 Mark 2 Mark
29	I = $\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\cot x}}$ dx applying the property $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$ We get I = $\int_{\pi/6}^{\pi/3} \frac{\sqrt{\cot x}}{1+\sqrt{\cot x}}$ dx adding above two I s We get 2I = $\int_{\pi/6}^{\pi/3} 1\,dx = \frac{\pi}{6}$ hence I = $\frac{\pi}{12}$ sq.units (OR) $\int_0^{3/2} \|x\cos\pi x\|$ dx = $\int_0^{1/2} x\cos\pi x$ dx + $\int_{1/2}^{3/2} -x\cos\pi x$ dx By using by parts method for integrating $\int_0^{1/2} x\cos\pi x$ dx + $\int_{1/2}^{3/2} -x\cos\pi x$ dx $\int_0^{3/2} \|x\cos\pi x\|$ dx = $\frac{5}{2\pi} - \frac{1}{\pi^2}$	1 Mark 1 Mark 1 Mark 1 Mark 1mark 1mark
30	Given Objective function Z = 3x + 2y Subject to the constraints; x +2y ≤ 10, 3x + y ≤ 15 and x ≥ 0, y≥0.  For graph corner points of feasible region are O (0,0) A(5,0) B(4,3) and C(0,5) Max Z = 18 obtained at B (4,3)	1 Mark 1 Mark 1 Mark

31	P(A) = 65%, P($\overline{A}$) = 35% , P(B) = 85%, P($\overline{B}$) = 15% P(A⋂$\overline{B}$) + P($\overline{A}$ ⋂ B) = P(A). P($\overline{B}$) + P($\overline{A}$). P(B) = 65% x 15% + 35% x 85% = 39.5%	1 Mark 2 Mark
	SECTION D (5m x 4 = 20m)	
32	Finding $A^{-1} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ where $\lvert A\rvert = 1 \neq 0$ Writing given system of equation in matrix equation form and expressing it as $A^T X = B$ Expressing X= $(A^{-1})^T B$ Finding X value as $x = 0, y = -5, z = -3$ OR Let the number of chairs , tables and beds produced be $x, y\ and\ z\ respectively$ $x + y + z = 45, -x + 0.y + z = 8, x - 2y + z = 0$ Finding $\lvert A\rvert = 6 \neq 0$ AX=B ⇒ $X = A^{-1}B$ finding $A^{-1} = \frac{1}{6}\begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 15 \\ 19 \end{bmatrix}$	2 Mark 1 Mark 1 Mark 1Mark 1 ½ ½ ½ 1 ½ Mark 1Mark
33	Solving 2x + y = 4, 3x – 2y = 6 and x – 3y + 5 = 0 we get the vertices A(1,2) B(2,0) and C (4, 3) Area of Triangle ABC = $\int_1^2 \frac{(x+5)}{3} - (4 - 2x)dx\ + \int_2^4 \frac{(x+5)}{3} - \frac{(3x-6)}{2}dx$ Solving we get Area of ΔABC = $\frac{7}{2}$ sq.units	1 Mark 2 Mark 2 Mark
34	Equation of diagonal PR : $\frac{x-4}{8} = \frac{y-2}{2} = \frac{z+6}{11}$ Equation of diagonal QS: $\frac{x-5}{6} = \frac{y+3}{12} = \frac{z-1}{-3}$ General points on PR& QS are $(8k + 4, 2k + 2, 11k - 6)$ and $(6t + 5, 12t - 3, -3t + 1)$ for real numbers ‘k’ and ‘t’ respectively For point of intersection of PR and QS : $8k + 4 = 6t + 5, 2k + 2 = 12t - 3$ Solving, we get $k = \frac{1}{2}, t = \frac{1}{2}$ ∴ The point of intersection is $(8, 3, -\frac{1}{2})$ (OR)  The Line in the standard form is $\frac{x-2}{1} = \frac{y}{2} = \frac{z-2}{-3}$, then M is the point $(\lambda + 2, 2\lambda, -3\lambda + 2)$, for some $\lambda \in R$ Direction ratios of AM are $\lambda + 3, 2\lambda - 5, -3\lambda$ AM ⊥ $Line$, ∴ $1(\lambda + 3) + 2(2\lambda - 5) - 3(-3\lambda) = 0 \Rightarrow \lambda = \frac{1}{2}$ $M\left(\frac{5}{2}, 1, \frac{1}{2}\right) = M\left(\frac{a-1}{2}, \frac{b+5}{2}, \frac{c+2}{2}\right) \Rightarrow a = 6, b = -3, c = -1$ ∴The image of A in the line is $A'(6, -3, -1)$ And, $AA' = \sqrt{49 + 64 + 9} = \sqrt{122}$	½ M ½ M 1M 1M 2M 1 M ½ M ½ M 1M 1M ½ M ½ M

35	$(x^2-1)\frac{dy}{dx} + 2xy = \frac{2}{x^2-1}$ Integral factor is $e^{\int \frac{2x}{x^2-1}dx} = (x^2 - 1)$ Multiplying the I.F. with the given differential equation $y(x^2 - 1) = 2\int \frac{1}{x^2-1}dx$ $y(x^2 - 1) = \log \frac{\lvert x-1\rvert}{\lvert x+1\rvert} + C$ (OR) $\{x \sin^2(\frac{y}{x}) - y\}dx + x\, dy = 0$ $\frac{dy}{dx} = \frac{y - x\, sin^2(\frac{y}{x})}{x}$ is a homogeneous differential equation Substituting y = v/x And solving we get $\cot(\frac{y}{x}) = \log\lvert x\rvert + C$ and substituting $y=\frac{\pi}{4}$, x = 1 we get C = 1 $\therefore \cot(\frac{y}{x}) = \log\lvert x\rvert + 1$ is the required solution	2 Mark 1 Mark 2 Mark 2 Mark 1 Mark 2 Mark
	SECTION E (4m x 3 = 12m)	
36		
i	$2\pi rh + \pi r^2 = 75\pi$ Volume (V) = $\left(\frac{\pi}{2}\right)(75r - r^3)\ cm^3$	1M
ii	Find $\frac{dV}{dr} = \left(\frac{\pi}{2}\right)(75 - 3r^2)$	1M
iii	When volume is maximum $\frac{dV}{dr} = \left(\frac{\pi}{2}\right)(75 - 3r^2) = 0$ $r = \pm 5$ by using second derivative test (OR) For maximum volume, h = r=5. Hence the statement is false.	2M
Case Study-2		
37		
i	$5\vec{i} + 3\vec{j} - 4\vec{k}$	1M
ii	$2\vec{j} \cdot 2\vec{k}$	1M
iii	$\theta = \cos^{-1}(\frac{18}{5\sqrt{15}})$ (OR) The point where 1/3 of the fuel runs out is the point of division in 1:2 ratio internally is $\frac{-1}{3}\vec{i} + 2\vec{j} + \frac{5}{3}\vec{k}$	2M
Case Study- 3		
38		
i	$P(A1) = \frac{4}{10}$ $P(A2) = \frac{4}{10}$ $P(A3) = \frac{2}{10}$ let G denote the event of germination P(G/A1) = 45% P(G/A2)= 60% P(G/A3) = 35% $P(G) = \frac{49}{100} = 49\%$	2M
ii	$P(A2/G) = \frac{P(A2).P(G/A2)}{P(G)} = \frac{24}{49}$	2M

KENDRIYA VIDYALAYA SANGATHAN: HYDERABAD REGION

MODEL QUESTION PAPER - 17 (2025 - 26)

SUBJECT: MATHEMATICS (041)

MARKING SCHEME

1	C	2	A	3	D	4	C	5	C	6	C	7	D	8	D	9	A	10	A
11	C	12	C	13	D	14	B	15	C	16	B	17	C	18	A	19	A	20	D

Q.No	Value Points	Marks
21	Put x = tan θ $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) = \tan^{-1}\left(\frac{\sqrt{1+tan^2\theta}-1}{\tan\theta}\right)$ $= \tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$ $= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$ $= \tan^{-1}(\tan\theta/2)$ = θ/2 $= \frac{1}{2}\tan^{-1}x$	½ M ½ M ½ M ½ M
22	LHL = k/2 RHL = 3 K = 6 OR $\frac{d}{dx}\left(sec\left(tan\sqrt{x}\right)\right) = sec\left(tan\sqrt{x}\right)tan\left(tan\sqrt{x}\right)\frac{d}{dx}\left(tan\sqrt{x}\right)$ $= \frac{1}{2\sqrt{x}}sec\left(tan\sqrt{x}\right)tan\left(tan\sqrt{x}\right)sec^2\sqrt{x}$	1M ½ M ½ M 1M
23	$\frac{dr}{dt} = 4cm/s$ A =π r^2 $\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$ $\left(\frac{dA}{dt}\right)_{r=10cm} = 2\pi(10)4 = 80\pi\ cm^2/s$ OR $x^2 + y^2 = 25$ When x = 4cm, y = 3cm dx/dt = 2cm/s 2x dx/dt + 2y dy/dt = 0 dy/dt = -8/3 cm/s	½ M 1 M ½ M 1M 1M
24	$\lvert\vec{a}\rvert = \sqrt{5}$	½ M

	$\hat{a} = \frac{\vec{a}}{\lvert\vec{a}\rvert} = \frac{1}{\sqrt{5}}(\hat{\imath} - 2\hat{\jmath})$ Required vector = $\frac{7}{\sqrt{5}}\hat{\imath} - \frac{14}{\sqrt{5}}\hat{\jmath}$	1M ½ M
25	Projection of $\hat{\imath} + 3\hat{\jmath} + 7\hat{k}$ on the vector $7\hat{\imath} - \hat{\jmath} + 8\hat{k} = \frac{(\hat{\imath}+3\hat{\jmath}+7\hat{k}).(7\hat{\imath}-\hat{\jmath}+8\hat{k})}{\lvert 7\hat{\imath}-\hat{\jmath}+8\hat{k}\rvert}$ $= \frac{7-3+56}{\sqrt{49+1+64}}$ = 60/√114	1M 1M
26	For showiing homogeneous $\frac{dy}{dx} = \frac{x^2+y^2}{x^2+xy}$ Put y/x = v $\frac{dy}{dx} = v + x\frac{dv}{dx}$ Given DE becomes $v + x\frac{dv}{dx} = \frac{1+v^2}{1+v}$ $\Rightarrow \int \frac{1+v}{1-v}dv = \int \frac{1}{x}dx$ $\Rightarrow (x-y)^2 = Cx\, e^{-\frac{y}{x}}$ OR $\frac{dy}{dx} + \left(\frac{1+x\cot x}{x}\right)y = 1$ I.F. = $e^{\int P dx} =$ x sinx G.S. yx sinx = -x cosx + sinx + c	1/2M 1/2M 1M 1M 1M 1M 1M
27	f'(x) = $12x^2$ – 12x -- 72 = 12((x - 3)(x + 2) x = -2, 3 increasing on (-∞, -2) and (3, ∞) decreasing on (-2, 3)	1M 1M 1M

	OR $y' = \frac{1}{1+x} - \frac{4}{(2+x)^2}$ $= \frac{x^2}{(1+x)(2+x)^2}$ y is increasing if y' > 0 then 1 + x > 0 then x > -1 Hence proved	2M 1M
28	$1 - \tan x = t \Rightarrow -\sec^2 x\, dx = dt$ $\int \frac{1}{\cos^2 x(1-\tan x)^2} = \int \frac{1}{t^2} dt$ $= \frac{1}{t} + c = \frac{1}{1-\tan x} + c$ OR $2 - x = t \Rightarrow dx = -dt$ $\int_0^2 x(2-x)^5 dx = -\int_2^0 (2-t)t^5\, dt$ $= \int_2^0 (t^6 - 2t^5)\, dt$ $= \left[\frac{t^7}{7} - \frac{t^6}{3}\right]_2^0$ = 64/21	1M 1M 1M 1M 1M 1M
29	$\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ $\Rightarrow \frac{x-1}{-3} = \frac{y-2}{2p/7} = \frac{z-3}{2}$ and $\frac{x-1}{3p/7} = \frac{y-5}{1} = \frac{z-6}{-5}$ Two lines are perpendicular $\Rightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0$ $\Rightarrow (-3)\left(\frac{3p}{7}\right) + \left(\frac{2p}{7}\right)(1) + (2)(-5) = 0$ $\Rightarrow -\frac{9p}{7} + \frac{2p}{7} - 10 = 0$ $\Rightarrow p = -10$	1M 1M 1M
30	For correct graph Points z = 2x + 3y (0, 0) 0 (7, 0) 14 (6, 2) 18 (0, 5) 15 MAX Z = 18 at X=6 , Y=2	1M 1M 1M
31	n(S) = 36 E : sum of the numbers is 4	

	F : numbers are different P(E) = 3/36 P(F) = 30/36 P(E∩F) = 2/36 P(E/F) = 1/15	1M 1M 1M
32	$\begin{pmatrix}1 & -1 & 0\\2 & 3 & 4\\0 & 1 & 2\end{pmatrix}\begin{pmatrix}2 & 2 & -4\\-4 & 2 & -4\\2 & -1 & 5\end{pmatrix} = 6I$ $A^{-1} = 1/6\ B$ $\begin{pmatrix}1 & -1 & 0\\2 & 3 & 4\\0 & 1 & 2\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}3\\17\\7\end{pmatrix}$ AX = B $X = A^{-1}B$ x = 2, y = -1 and z = 4	2M 1M 1M 1M
33	$y = \tan^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$ $y = \tan^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}} \times \frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}\right)$ $y = \tan^{-1}\left(\frac{\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)^2}{2\sin x}\right)$ $y = \tan^{-1}\left(\frac{2+2\cos x}{2\sin x}\right)$ $y = \tan^{-1}\left(\cot\left(\frac{x}{2}\right)\right)$ $y = \frac{\pi}{2} - \frac{x}{2}$ $\frac{dy}{dx} = -\frac{1}{2}$ OR $\frac{d}{dx}(y^x) = y^x\left(\frac{x}{y}.\frac{dy}{dx} + \log y\right)$ $\frac{d}{dx}(x^y) = x^y\left(\frac{y}{x} + \log x\frac{dy}{dx}\right)$ $\frac{d}{dx}(x^x) = x^x(1 + \log x)$ $\frac{d}{dx}(a^b) = 0$	 1M 1M 1M 1M 1M 1M 1M 1M 1M

	$\frac{dy}{dx}=\frac{-[y^x\log y+yx^{y-1}+x^x(1+\log x)]}{xy^{x-1}+x^y\log x}$	1M
34	For correct graph $\int_{-6}^{0}\lvert x+3\rvert\,dx=\int_{-6}^{-3}\lvert x+3\rvert\,dx+\int_{-3}^{0}\lvert x+3\rvert\,dx$ $-\int_{-6}^{-3}(x+3)\,dx+\int_{-3}^{0}(x+3)\,dx$ $=-\left(\frac{x^2}{2}+3x\right)_{-6}^{-3}+\left(\frac{x^2}{2}+3x\right)_{-3}^{0}$ = 9 sq. units For graph $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\rightarrow y=\pm\frac{b}{a}\sqrt{a^2-x^2}$ $A=4\int_0^a\frac{b}{a}\sqrt{a^2-x^2}\,dx$ $=\frac{4b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$ $=\pi ab$	1 ½ M 1M ½ M 1M 1M 1M 1M 1M 1M 1M
35	d.rs of the required line = 24, 36, 72 required line equation = $\frac{x-1}{24}=\frac{y-2}{36}=\frac{z+4}{72}$	2M 2M
36(i)	2^9	1M
(ii)	3^3	1M
(iii)	6 OR 6	2M
37(i)	P (Rating Medium) = 250/500 =1/2	1M
(ii)	P(Rating 2 stars) = 120/500=6/25	1M
(iii)	P (Rating one star/selecting the price high) = 20/100 OR P (price high /does not rating 4 stars) =60/420	2M
38(i)	8m	2M
(ii)	Height = 4m and T.S.A = 192 m^2	2M

KENDRIYA VIDYALAYA SANGATHAN: HYDERABAD REGION

MODEL QUESTION PAPER - 18 (2025 - 26)

SUBJECT: MATHEMATICS (041)

MARKING SCHEME

Q.No	Answer	Marks
1	D	1
2	C	1
3	D	1
4	D	1
5	B	1
6	C	1
7	C	1
8	B	1
9	B	1
10	C	1
11	A	1
12	B	1
13	C	1
14	A	1
15	A	1
16	C	1
17	B	1
18	A	1
19	C	1
20	A	1
21	Check whether the function f(x) = $\lvert x\rvert$ is differentiable at x = 0 or not. Calculation of LHD = -1 …. 1 mark Calculation of RHD = 1 …… 1 mark Since LHD ≠ RHD f is not differentiable at x = 0 OR If y = $\sqrt{tan\sqrt{x}}$, prove that $\sqrt{x}$ dy/dx = $\frac{1+y^2}{4y}$. dy/dx = $\frac{1}{2\sqrt{tan\sqrt{x}}}$ sec$^2\sqrt{x}\,\frac{1}{2\sqrt{x}}$ …… 1 mark On simplifying we get $\sqrt{x}$ dy/dx = $\frac{1+y^2}{4y}$ …… 1 mark	2
22	Show that the function f(x) = x^3+ x^2+ x +1 has neither maxima nor minima F^1(x) = 3x^2+2x +1 ≠0 for any real value of x ….. 1 mark Therefore, no maxima minima exist	2
23	If α, β, γ are the angles made by the lines with x,y and z axes then show that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$	2

	If α, β, γ are the angles made by the lines with x,y and z axes then we have $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$ ……. 1 mark By writing $\cos^2 x$ = 1- $\sin^2 x$ we get the result …. 1 mark	
24	Find the area of a parallelogram whose adjacent sides are $2\hat{\imath}$-$4\hat{\jmath}$+$5\hat{k}$ and $3\hat{\imath}$-$6\hat{\jmath}$+$2\hat{k}$. Let $\vec{a} = 2\hat{\imath}-4\hat{\jmath}+5\hat{k}$, $\vec{b} = 3\hat{\imath}-6\hat{\jmath}+2\hat{k}$ Finding a × b we get 22i+11j ……. 1 mark Finding area of parallelogram magnitude of (a×b) = $11\sqrt{5}$ sq units … 1 mark OR If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $(\vec{a} + \vec{b})$ is perpendicular to $\vec{a}$ and $(\overrightarrow{2a} + \vec{b})$ is perpendicular to $\vec{b}$,then prove that $\lvert\vec{b}\rvert = \sqrt{2}\lvert\vec{a}\rvert$ If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $(\vec{a} + \vec{b})$ is perpendicular to $\vec{a}$ then (a+b).a = a. a+ a. b = 0 ….. 1mark and $(\overrightarrow{2a} + \vec{b})$ is perpendicular to $\vec{b}$, $(\overrightarrow{2a} + \vec{b})$. $\vec{b}$ = 0 =>2a. b + b. b =0 ….. 1 mark On simplifying above we get $\lvert\vec{b}\rvert = \sqrt{2}\lvert\vec{a}\rvert$	2
25	Find the principal value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$. $\frac{\pi}{3} - \frac{2\pi}{3}$ …. 1 mark $= -\frac{\pi}{3}$ … 1 mark	2
26	Find the intervals in which the function f(x) = x^3-12 x^2+36 x +17 is strictly increasing or strictly decreasing. $f'(x) = 0$, => 3 x^2-24 x +36 = 0, x =2,6 ….. 1 mark Function strictly increasing in (-∞,2) and (6,∞) …. 1 mark Function strictly decreasing in (2,6) …. 1 mark	3
27	The volume of sphere is increasing at the rate of 8 cm^3/sec. Find the rate at which the surface area is increasing when the radius of the sphere is 12 cm. dv/dt = 8 implies that πr^2dr/dt = 2 ….. 1 mark ds/dt = 16/r = 16/12 = 4/3 cm^2/s …… 2 mark	3
28	Find $\int \frac{1}{\cos(x-a)\cos(x-b)}$ dx Multiply and divided by sin (a-b) $= \frac{1}{\sin(a-b)} \int \frac{\sin(a-b)}{\cos(x-a)\cos(x-b)}$ dx …..1 mark $= \frac{1}{\sin(a-b)} \int \frac{\sin(a-x+x-b)}{\cos(x-a)\cos(x-b)}$ dx $= \frac{1}{\sin(a-b)} \int \frac{\sin[(x-b)-(x-a)]}{\cos(x-a)\cos(x-b)}$ dx $= \int [\tan(x-b)\,dx$ -tan(x-a)]dx ….. 1 mark = log[$\frac{\sec(x-b)}{\sec(x-a)}$ +c ….. 1 mark	3

	OR Evaluate $\int \frac{1}{(x-1)(x-2)(x-3)}$ dx Since it is proper fraction so we can apply partial fraction $\frac{1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$ …… 1 mark Finding the values A = ½, B =-1 , C = ½ On putting A, B, C values and getting the result after integration = ½ log$\frac{(x-1)(x-3)}{x-2}$ +c	
29	Using vectors find the area of triangle with vertices are A (1,1,2), B (2,3,5), C (1,5,5). Finding vector AB, vector AC ….. 1mark Finding AB × AC ….. 1 mark Finding the area of triangle = ½ $\left\|\overrightarrow{AB} \times \overrightarrow{AC}\right\|$ =$5\sqrt{10}/2$ squ.. 1 mark OR Show that the points A, B, C with position vectors 2$\hat{\imath}$-$\hat{\jmath}$+$\hat{k}$,$\hat{\imath}$-3$\hat{\jmath}$-5$\hat{k}$ and 3$\hat{\imath}$-4$\hat{\jmath}$-4$\hat{k}$ respectively, are the vertices of a right-angled triangle. Hence find the area of the triangle. Find vectors AB, BC, CA …. 1 mark Find their magnitudes, and verify Pythagoras theorem ….. 1 mark Finding the area of triangle = ½ $\left\|\overrightarrow{AB} \times \overrightarrow{AC}\right\|$ =210 sq u .. 1 mark	3
30	Solve the following linear programming problem graphically: Maximise z = 500x+300y, Subject to constraints x+2y≤ 12 2x+y ≤ 12 4x+5y ≥20 x ≥0 , y ≥ 0 Draw the correct graphs for above constraints ……. 1 mark For deciding the corner points of feasible region (6,0), (5,0),(0,4),(4,4),(0,6) …. 1 mark For finding the maximum z =3200 at (4,4) …… 1 mark	3
31	E and F are independent events such that such that P ($\overline{E}$) = 0.6 and P (E∪ F) = 0.6. Find P(F) and P ($\overline{E}$ ∩ $\overline{F}$). P (E∪ F) = P(F)+ P(E)- P(E) P(F) = 0.6 ….. 1 mark = P(F)+0.4 – 0.4 P(F) = 0.6 …….. 1 mark P(F) = 1/3 Now P ($\overline{E}$ ∩ $\overline{F}$) =P($\overline{E \cup F}$) = 1-0.6 = 0.4 ……. 1 mark	3
32	Solve the system of equations by matrix method: $x + 2y + 3z = 6, 2x - y + z = 2, 3x + 2y - 2z = 3$ For writing A,X and B ….. 1 mark Finding det A ≠ 0 and A is non singular ……. 1 mark Finding adjA …….. 2 Marks Solving X =A^{-1}B …… 1 mark OR $\|A\|$=7…..1mark adjA= $\begin{bmatrix} 0 & -7 & -7 \\ 1 & 7 & 5 \\ 2 & -7 & -4 \end{bmatrix}$ …….2marks	5

	X =A^{-1}B ……1mark. $x = 1, y = -5$ $z = -5$…..1mark	
33	Using integration, find the area of the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$, included between the lines x = -2 and x = 2. For drawing correct figure and shading the region ….. 2 marks Writing the value of y = ½ $\sqrt{16 - x^2}$ ….. 1 mark For writing the area of required region = 2 $\int_{-2}^{2}$ ½ $\sqrt{16 - x^2}$ dx …. 1 mark For obtaining the area 8π …. 1 mark	5
34	Find the shortest distance between the following two lines: $\vec{r} = (\hat{\imath}+2\hat{\jmath}+\hat{k}) +\lambda(\hat{\imath}-\hat{\jmath}+\hat{k})$, $\vec{r} = (\widehat{2\imath}-\hat{\jmath}-\hat{k}) +\mu(\widehat{2\imath}+\hat{\jmath}+2\hat{k})$. For writing vectors a_1, a_2, b_1, b_2 ...... 1 mark Calculating a_2-a_1 ,$b_1 \times b_2$ ...... 1 mark Finding magnitude of ,$b_1 \times b_2$ ....1 mark For calculating shortest distance = $\frac{(a_2-a_1).(b1\times b2)}{\lvert b1\times b2\rvert}$ = $3\sqrt{2}$ /2 …… 2 marks OR Show that the lines $\vec{r} = (3\hat{\imath}+2\hat{\jmath} - 4\hat{k}) +\lambda(\hat{\imath} + 2\hat{\jmath}+\widehat{2k})$, $\vec{r} = (\widehat{5\imath}-2\hat{\jmath}) +\mu(\widehat{3\imath}+2\hat{\jmath}+6\hat{k})$ are intersecting. For writing vectors a_1, a_2, b_1, b_2 ...... 1 mark Calculating a_2-a_1 ,b_1xb_2 ...... 1 mark Finding magnitude of ,b_1xb_2 ....1 mark For calculating shortest distance = $\frac{(a_2-a_1).b1xb2}{b1xb2}$ = 0 …… 1 mark For finding point of intersection ……. 1 mark	5
35	If x^y = e^{x-y}, prove that dy/dx = $\frac{logx}{(1+logx)^2}$ x^y= e^{x-y} taking log on both sides we get ylog x = x -y …… 1mark ylog x +y = x ……1 mark y = $\frac{x}{1+logx}$ ……… 1 mark diff w.r.t. x we get dy/d x = $\frac{logx}{(1+logx)^2}$ …… 2 marks OR If y =$(\tan^{-1} x)^2$ prove that $(x^2+1)^2 \frac{d^2y}{dx^2}$+2 x $(x^2+1)\frac{dy}{dx}$ = 2 Diff w..r.t x dy/d x = 2$\tan^{-1} x$ /1+ x^2 ……. 1 mark (1+ x^2) dy/d x = 2$\tan^{-1} x$ ……. 1 mark Again diff w.r.t x we get $(x^2+1)^2 \frac{d^2y}{dx^2}$+2 x $(x^2+1)\frac{dy}{dx}$ = 2 ……..3 marks	5
36	(iv) $2^{3x2} = 2^6 = 64$ (v) {(g_1,g_1),(g_2,g_2) (vi) (A) so the minimum number of elements to be added are (b1,b1),(b2,b2),(b3,b3),(b2,b3) (B) so the minimum number of elements to be added are (b1,b1),(b2,b2),(b3,b3),(b2,b3),(b3,b2) OR	1 1 2

	One-one and onto function $f(x) = \frac{x^2}{4}$ clearly one - one function in $[0,20\sqrt{2}]$ For any arbitrary element in [0,200] the preimage of y exists in $[0,20\sqrt{2}]$ Hence f is onto	2
37	(iv) 1/3 πr^3 (v) $\frac{1}{4\pi}$ (vi) 2 OR $\frac{1}{4\pi}$	1 1 2
38	Let E1 = severe, E2 = moderate , E3 = Light Turbulence (iii) P(E1) =P(E2) = P(E3) = 1/3 P(airplane reached destination late) = P(E1)P(A/E1)+ P(E2)P(A/E)+ P(E3)P(A/E3) =1/3(55/100) + 1/3(37/100) +1/3(17/100) = 109/300 (iv) P(If the airplane reached its destination late, find the probability that it was due to moderate turbulence) P(E3/A) = P(E3).P(A/E3)/P(A) =17/109	2 2

KENDRIYA VIDYALAYA SANGATHAN: HYDERABAD REGION

MODEL QUESTION PAPER - 19 (2025 - 26)

SUBJECT: MATHEMATICS (041)

MARKING SCHEME

SECTION: A (Solution of MCQs of 1 Mark each)

1	2	3	4	5	6	7	8	9	10
A	A	D	C	C	B	A	B	C	B
11	12	13	14	15	16	17	18	19	20
D	C	A	B	A	B	C	C	A	A

SECTION-B

Q.No.	**HINTS/SOLUTION**	Marks
21.	Area of triangle ABC = $\frac{1}{2}\left\|\overrightarrow{BC}\times\overrightarrow{BA}\right\|$ $Now\,,\overrightarrow{BC}\times\overrightarrow{BA} = 9i + 7j + 12k$ $\left\|\overrightarrow{BC}\times\overrightarrow{BA}\right\|$ = $\sqrt{274}$ Therefore, Area of triangle ABC = $\frac{1}{2}\sqrt{274}sq.units$	½ 1 ½
22.	Given $A = \begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$ $A^2 = \begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} = \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix} = 2\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$ $A^2 = 2^1A$ Similarly, $A^3 = 2^2A$ Therefore, $A^{100} = 2^kA => k = 100 - 1 = 99$	1 ½ ½
23.	Let $y = 5cosx - 3sinx$ $\frac{dy}{dx} = -5sinx - 3cosx$ $\frac{d^2y}{dx^2} = -5cosx + 3sinx = -y$ $\frac{d^2y}{dx^2} + y = 0$	½ 1 ½
24.	$\int e^x\left(\frac{x-3}{(x-1)^2}\right)dx$ = $\int e^x\left(\frac{x-1-2}{(x-1)^2}\right)dt$ =$\int e^x\left(\frac{x-1}{(x-1)^2} - \frac{2}{(x-1)^2}\right)dt$ Therefore, $\int e^x\left(\frac{x-3}{(x-1)^2}\right)dx = \left(\frac{e^x}{(x-1)}\right) + C$	1 1
25.	Given line is $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2} = \mu$ then, General point on the line is R($3\mu - 2, 2\mu - 1, 2\mu + 3$) Distance from R to P(1, 3, 3) is5 units $\therefore \mu = 0, 2.$ ∴ R(-2, -1, 3) or R(4 , 3 , 7)	1 1

SECTION-C

26.	V $= \frac{4}{3}\pi r^3$ => $\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$ $\frac{dr}{dt} = \frac{900}{4\pi r^2}$ => $\frac{dr}{dt} = \frac{1}{4\pi} cm/s$ Now , S=$4\pi r^2$ => $\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$ => $\frac{dS}{dt} = 8\pi r \frac{900}{4\pi r^2}$ at r=30 $\frac{dS}{dt} = 60cm^2/s$	1 1 1
27.	$\int_0^2 \lvert x^2+2x-3\rvert dx = -\int_0^1 (x^2+2x-3)dx + \int_1^2 (x^2+2x-3)dx$ $\therefore \int_0^2 \lvert x^2+2x-3\rvert dx = 4$	2 1
28.	Given $\vec{a} = i + j + k$ and $\vec{b} = i + 2j + 3k$. $\vec{a} + \vec{b} = 2i + 3j + 4k$ and $\vec{a} - \vec{b} = -j - 2k$. Now, $\vec{r} = \overrightarrow{(a + b)} \times \overrightarrow{(a - b)} = -2i + 4j - 2k$ Vector of magnitude 6 and perpendicular to $\vec{a} + \vec{b}$ *and* $\vec{a} - \vec{b}$ is $6\hat{r} = 6\left(\frac{-2i+4j-2k}{\sqrt{4+16+4}}\right) = -\sqrt{6}\, i + 2\sqrt{6}\, j - \sqrt{6}\, k$.	1 1 1
29.	Given differential equation (x+y)dy + (x- y)dx=0 $\frac{dy}{dx} = \frac{y-x}{y+x}$ ……..(i) homogeneous. Put y=vx => $\frac{dy}{dx} = v + x\frac{dv}{dx}$ $v + x\frac{dv}{dx} = \frac{vx-x}{vx+x} = \frac{v-1}{v+1}$ Sampling, we get $tan^{-1}\frac{y}{x} + \frac{1}{2}\log\left\lvert 1 + \frac{y^2}{x^2}\right\rvert = C \ldots (ii)$ Given when x=1 and y=1 C=$\frac{\pi}{4} + \frac{1}{2}log2$ $tan^{-1}\frac{y}{x} + \frac{1}{2}\log\left\lvert 1 + \frac{y^2}{x^2}\right\rvert = \frac{\pi}{4} + \frac{1}{2}log2$ as required solution. (OR) $\frac{dy}{dx} + sec^2x\, y = sec^2xtanx.$ I.F= $e^{\int sec^2xdx} = e^{tanx}$ Solution is $e^{tanx}.y = tanx.e^{tanx} - e^{tanx} + C$	1 1 1 1 2
30.	Our problem is to Maximize Z = 8x + 9y. Subject to the constraints; $2x + 3y \leq 6;\ 3x - 2y =\leq 6\ ;\ y \leq 1\ ; x, y \geq 0$ For appropriate diagram,	1

	Possible points for maximum Z are A(2, 0), B$\left(\frac{30}{13},\frac{6}{13}\right)$, $C(\frac{3}{2}$,1) and D(0, 1) Therefore the maximum value of Z is 22.62. at B$\left(\frac{30}{13},\frac{6}{13}\right)$	1 1
31.	A:getting six ; P(A)=1/6 B:not getting six; P(B)=5/6 E:reports six P(E/A)=4/5 ; P(E/B)=1/5 Using Baye's Theorem P(A/E)=4/9	1 1 1

SECTION-D

32.	Let equation of line through (1, 1, 1) be $\frac{x-1}{a}=\frac{y-1}{b}=\frac{z-1}{c}$………………(i) Line(i) perpendicular to the lines $\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4}$ and $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ Therefore a+2b+4c=0 and 2a+3b+4c=0 ∴ dr's are -4, 4, -1 => 4, -4, 1 ∴ Cartesian equation is $\frac{x-1}{4}=\frac{y-1}{-4}=\frac{z-1}{1}$ and vector equation is r=(i+j+k)+$\mu(4i-4j+k)$ and $\theta = cos^{-1}(\frac{24}{\sqrt{609}})$.	½ 1 1 1 ½ ½ ½
33.	Consider BA=$\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix} = 6\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} = 6I$ …(i) Given system of equations are $x-y=3;\ 2x+3y+4z=17;\ y+2z=7.$ Matrix equation is AX = C => X = $A^{-1}C$ From equation (i) we have BA=6I => B = 6IA^{-1} => $A^{-1}=\frac{1}{6}B$ $x=2, y=-1\ , z=4$	1 2 1 1
34.	$V=\frac{1}{3}\pi r^2 h\ and\ S=\pi rl\ then\ S^2=\frac{9V^2}{r^2}+\pi^2 r^4$ Now, $\frac{dS^2}{dr}=\frac{-18V^2}{r^3}+4\pi^2 r^3$ For minima, put $\frac{dS^2}{dr}=0 => 9V^2=2\pi^2 r^6$ For proving $\frac{d^2S^2}{dr^2}>0$ And proving h=$\sqrt{2}r$	1 2 1 1
35.	Area bounded by the ellipse = 4×(Area of shaded region in the first quadrant only) = 4× $\int_{x=a}^{y=b}\|y\|dx$ = 4× $\int_0^2\|y\|dx => 4\int_0^2\frac{3}{2}\sqrt{4-x^2}\ dx$	1 1

	$4\int_0^2 \frac{3}{2}\sqrt{2^2 - x^2}\ dx$ $=> 6[\frac{x}{2}\sqrt{4-x^2} + \frac{2^2}{2} sin^{-1}\frac{x}{2}]_0^2$ Required area $=6\pi\ sq.unit.$	2 1

SECTION-E

Case Study-1

36.	(i) (A) Equivalence (ii). (A) R is Symmetric but neither reflexive nor transitive (iii). (A) Bijective OR (iii). (A) R	1 1 2

Case Study-2

37.	(i)(A) Let x be the number of extra days after 1st July. Therefore Price= Rs(300-3×x) Quantity=80 quintals+x(1 quintal per day)= (80 + x) quintals (ii)(B) R(x)=Quantity × Price R(x)=24000+60x-3x^2 (iii)(A) We have , R(x)=24000+60x-3x^2 For R(x) to be maximum, $R'(x) = 0\ and\ R''(x) < 0$ Therfore x=10 OR (iii) (D) Shyam's father will attain maximum revenue after 10days.i.e. on 11th July.	 1 1 2

Case Study-3

38	P(H)=1/7 P(W)=1/5 (i)P(only one selected) $= \mathrm{P}(\mathrm{H}\overline{\mathrm{W}} \text{ or } \overline{\mathrm{H}}\mathrm{W}) = \frac{1}{7}\times\frac{4}{5}+\frac{6}{7}\times\frac{1}{5} = \frac{10}{35} = \frac{2}{7}$ (ii)P(none selected) $= P(\overline{\mathrm{HW}}) = \frac{6}{7}\times\frac{4}{5} = \frac{24}{35}$	2 2

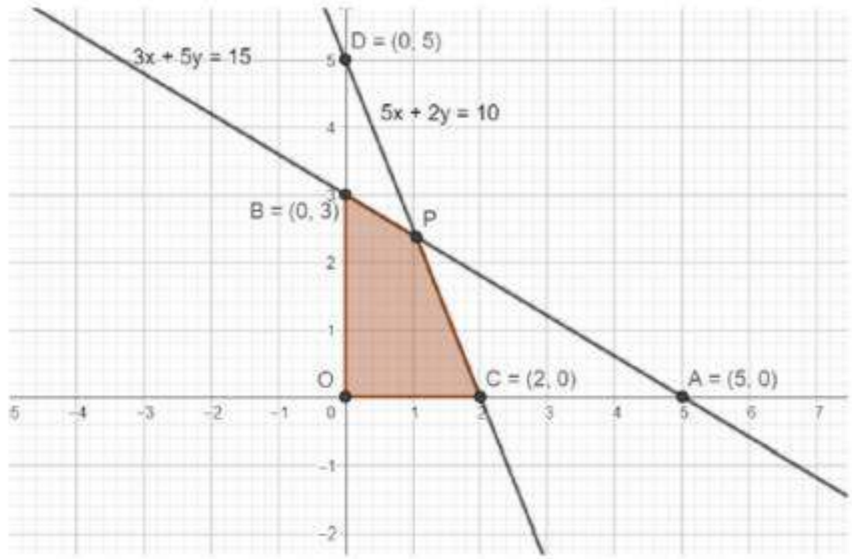
KENDRIYA VIDYALAYA SANGATHAN: HYDERABAD REGION
MODEL QUESTION PAPER – 20 (2025 - 26)
SUBJECT: MATHEMATICS (041)
MARKING SCHEME

MARKING SCHEME

Question No	Value Points	Marks
1 to 20	1) c) 16 2) d) $\pm\sqrt{7}$ 3) b) $\frac{2\pi}{3}$ 4) b) $\frac{7}{2}$ 5) b) $e^x \log\sqrt{x} + c$ 6) b) 3 ; 3 7) b) Half plane that neither contains the origin nor the points of the line 2x+3y=6 8) c) $\sqrt{209}$ 9) d) $\frac{\pi}{4}$ 10) c) 2 11) d) every point on the line segment CD 12) d) $\pm\sqrt{3}$ 13) b) -11 14) b)$\frac{2}{3}$ 15) d)$\frac{1}{x}$ 16) c) $-e^{y-x}$ 17) b) $\frac{1}{\sqrt{3}}$ 18) c) $(0,-3,0)$ 19) c) A is true but R is false. 20) a) Both A and R are true and R is the correct explanation of A.	1 Mark for each correct answer
21	$f(1) = f(2) = 1$, So f is not one one as f(x) takes only 3 values (1, 0, or −1) there does not exist any x in domain R such that f(x) = −2. ∴ f is not onto. **OR** $\sin^{-1}\left[\sin\left(\frac{13\pi}{7}\right)\right] = \sin^{-1}\left[\sin\left(2\pi - \frac{\pi}{7}\right)\right]$ $= \sin^{-1}\left[\sin\left(-\frac{\pi}{7}\right)\right] = -\frac{\pi}{7}$	1 ½ ½ 1 1
22	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$ $\frac{d^2y}{dx^2} = -\frac{1}{t^2}\frac{dt}{dx} = -\frac{1}{t^2}.\frac{1}{2at} = -\frac{1}{2at^3}$	1 1
23	$\frac{dr}{dt} = 2$ cm/s and $\frac{dh}{dt}$ =−8cm/s $V = \pi r^2 h \Rightarrow \frac{dV}{dt} = \pi r^2\frac{dh}{dt} + \pi h 2r\frac{dr}{dt}$ $\frac{dV}{dt}(at\ r = 3 \text{ and } h = 6) = 0$	½ 1 ½

24	$\overline{AB} = \hat{j} + 2\hat{k}$ $\overline{AC} = \hat{i} + 2\hat{j}$ $\overline{AB} \times \overline{AC} = -4\hat{i} + 2\hat{j} - \hat{k}$ Required area $= \frac{1}{2}\sqrt{21}$	½ ½ ½ ½
25.	$\cos\theta = \left\| \frac{(\hat{i} + 2\hat{j} + 2\hat{k}) \cdot (2\hat{i} + 3\hat{j} - 6\hat{k})}{\sqrt{3} \cdot \sqrt{7}} \right\|$ $\theta = \cos^{-1} \frac{4}{\sqrt{21}}$ **OR** The lines, $\frac{x}{1} = \frac{y}{-1} = \frac{z}{\frac{1}{k}}$ and $\frac{x-2}{1} = \frac{y+\frac{1}{2}}{\frac{1}{2}} = \frac{z-1}{-1}$ are perpendicular $\therefore\ 1 - \frac{1}{2} - \frac{1}{k} = 0 \Rightarrow k = 2$	1 1 1 1
26	$I = \int \sin^{-1} x \cdot 1\, dx$ $= \sin^{-1}x \cdot x - \int \frac{1}{\sqrt{1-x^2}} \cdot x\, dx$ $= x \cdot \sin^{-1} x + \frac{1}{2} \int \frac{-2x}{\sqrt{1-x^2}}\, dx$ $= x \cdot \sin^{-1} x + \frac{1}{2} \cdot 2\sqrt{1-x^2} + C$ or $x \sin^{-1}x + \sqrt{1-x^2} + C$	1 ½ ½ 1
27	$\int_{-4}^{4} \|x+2\|\, dx = \int_{-4}^{-2} -(x+2)\, dx + \int_{-2}^{4} (x+2)\, dx$ $= -\left[\frac{x^2}{2} + 2x\right]_{-4}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-2}^{4}$ $= 20$	1 ½ ½ 1

27**(OR)**	$I=\int_1^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4-x}}dx$ $=\int_1^3 \frac{\sqrt{4-x}}{\sqrt{4-x}+\sqrt{x}}dx$ (using property) $\Rightarrow 2I=\int_1^3 1\,dx = x]_1^3 = 2$ $\Rightarrow I=1$	1 1½ ½
28	$I=\int \frac{2x\cdot dx}{x^2+3x+2}=\int \frac{2x}{(x+1)(x+2)}dx$ $=\int\left(\frac{-2}{x+1}+\frac{4}{x+2}\right)dx$ using partial fraction $=-2\log\|x+1\|+4\log\|x+2\|+C$	½ 1 ½ 1
29	Given diff. equation is $\frac{dy}{dx}=\frac{2ye^{y/x}-x}{2xe^{y/x}}$ Put, $\frac{y}{x}=v \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$ $\therefore\quad v+x\frac{dv}{dx}=\frac{2ve^v-1}{2e^v}$ $\Rightarrow x\frac{dv}{dx}=\frac{2ve^v-1}{2e^v}-v=-\frac{1}{2e^v}$ $\Rightarrow 2e^v dv=-\frac{dx}{x}$ Integrating to get, $2e^v=-\log\|x\|+C$ $\Rightarrow 2e^{y/x}+\log\|x\|=C$ **OR** $\frac{dy}{dx}+y\cot x=4x\,\mathrm{cosec}\,x$ $I.F=e^{\int \cot x\,dx}=e^{\log\sin x}=\sin x$ Solution is given by $y.(\sin x)=\int 4x.\mathrm{cosec}x.\sin x\,dx=\int 4x dx=2x^2+c$ Now $y=0$ when $x=\frac{\pi}{2}$ gives $c=-\frac{\pi^2}{2}$ Required solution is $y.(\sin x)=2x^2-\frac{\pi^2}{2}$	½ ½ ½ ½ 1 1 1 ½ ½

30		Correct graph 1½
	<table><tr><th>Corner Points</th><th>Value of Z</th></tr><tr><td>O(0,0)</td><td>0</td></tr><tr><td>B(0,3)</td><td>9</td></tr><tr><td>C(2,0)</td><td>10</td></tr><tr><td>P(20/19,45/19)</td><td>$\frac{235}{19}$→Max</td></tr></table>	1 ½
31	Given ,A: First ticket drawn is shows an even number B: Second ticket drawn shows an even number Then P(A)=9/19 P(B/A)=8/18 P(A∩B)=P(A) P(B/A)= (9/19)×(8/18)= 4/19	 1 1 1
31(OR)	**(O**R) $S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$ $P(A) = \frac{6}{12} = \frac{1}{2}$, $P(B) = \frac{2}{12} = \frac{1}{6}$, $P(A \cap B) = \frac{1}{12}$ $P(A \cap B) = \frac{1}{12} = P(A).P(B)$ A and B are independent events	 ½ ½ +½+½ 1

32	$\|A\| = 2.$ co-factors of the elements of the matrix. $\left.\begin{array}{l} A_{11}=6 \;\; A_{12}=-3 \;\; A_{13}=-2 \\ A_{21}=-28 \;\; A_{22}=16 \;\; A_{23}=10 \\ A_{31}=-16 \;\; A_{32}=9 \;\; A_{33}=6 \end{array}\right\}$ $\therefore \; A^{-1}=\frac{1}{\|A\|}\cdot adj(A)=\frac{1}{2}\begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & 9 \\ -2 & 10 & 6 \end{bmatrix}$ The given system of equations can be written as $A \cdot X = B$ where, $X = A^{-1}.B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & 9 \\ -2 & 10 & 6 \end{bmatrix}\begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$ $\therefore \; x=-2,\; y=3,\; z=1$	½ 2 (1 mark for 4 correct cofactors) 1 ½ 1
33	Reflexive: For any $(a, b) \in N \times N$ $a \cdot b = b \cdot a$ $\therefore$ (a, b) R (a, b) thus R is reflexive Symmetric: For $(a, b), (c, d) \in N \times N$ $(a, b) R (c, d) \Rightarrow a \cdot d = b \cdot c$ $\Rightarrow c \cdot b = d \cdot a$ $\Rightarrow (c, d) R (a, b) \therefore$ R is symmetric Transitive : For any $(a, b), (c, d), (e, f), \in N \times N$ (a, b) R (c, d) and (c, d) R (e, f) $\Rightarrow a \cdot d = b \cdot c$ and $c \cdot f = d \cdot e$ $\Rightarrow a \cdot d \cdot c \cdot f = b \cdot c \cdot d \cdot e \Rightarrow a \cdot f = b \cdot e$ $\therefore$ (a, b) R (e, f), $\therefore$ R is transitive $\therefore$ R is an equivalance Relation **OR**	1 1 ½ 2 ½

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$(x - x) = 0$ is divisible by 3 for all $x \in z$. So, $(x, x) \in R$
$\therefore$ R is reflexive.
$(x - y)$ is divisible by 3 implies $(y - x)$ is divisible by 3.
So $(x, y) \in R$ implies $(y, x) \in R$, $x, y \in z$
$\Rightarrow$ R is symmetric.
$(x - y)$ is divisible by 3 and $(y - z)$ is divisble by 3.
So $(x - z) = (x - y) + (y - z)$ is divisible by 3.
Hence $(x, z) \in R \Rightarrow R$ is transitive
$\Rightarrow$ R is an equivalence relation
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1 ½
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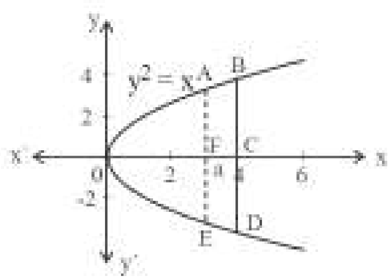

ar (OAEO) = ar(ABDEA)
$\Rightarrow$ 2 · ar (OAFO) = 2 · ar (ABCFA)
$\int_0^a \sqrt{x}\, dx = \int_a^4 \sqrt{x}\, dx$
$\frac{2}{3} \cdot a^{3/2} = \frac{2}{3}\left(4^{3/2} - a^{3/2}\right)$
$\Rightarrow \frac{2}{3} \cdot a^{3/2} = \frac{2}{3}\left(4^{3/2} - a^{3/2}\right)$
$\Rightarrow a^{3/2} = 4$, $\therefore a = 4^{2/3}$
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<td>Correct graph 1
1
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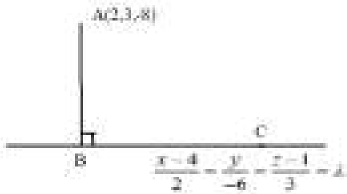
35	 Any point on line is $(2\lambda+4,-6\lambda,3\lambda+1)$ for some λ. Let $B(2\lambda+4,-6\lambda,3\lambda+1)$ d.r. of $AB=<2\lambda+2,-6\lambda-3,3\lambda+9>$ d.r. of $BC=<2,-6,3>$ $AB\perp BC\Rightarrow 2(2\lambda+2)-6(-6\lambda-3)+3(3\lambda+9)=0$ $\Rightarrow \lambda=-1$ $\therefore B(2,6,-2)$ Now, $AB=\sqrt{(2-2)^2+(6-3)^2+(-2+8)^2}=\sqrt{45}$ or $3\sqrt{5}$ units **0R** Vector equation of required line through (1, 2, – 4) is $\vec{r}=\hat{\imath}+2\hat{\jmath}-4\hat{k}+\lambda(2\hat{\imath}+3\hat{\jmath}+6\hat{k})$ and the cartesian equation is $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6}$ Equation of line through A(3, 3, – 5) and B(1, 0, – 11) is $\vec{r}=3\hat{\imath}+3\hat{\jmath}-5\hat{k}+\lambda(2\hat{\imath}+3\hat{\jmath}+6\hat{k})$ Distance between parallel lines is given by $d=\frac{\lvert(\vec{a_2}-\vec{a_1})\times\vec{b}\rvert}{\lvert\vec{b}\rvert}$ Here $\vec{b}=2\hat{\imath}+3\hat{\jmath}+6\hat{k}$, $\vec{a_1}=\hat{\imath}+2\hat{\jmath}-4\hat{k}$, $\vec{a_2}=3\hat{\imath}+3\hat{\jmath}-5\hat{k}$ $(\vec{a_2}-\vec{a_1})=2\hat{\imath}+\hat{\jmath}-\hat{k}$ $(\vec{a_2}-\vec{a_1})\times\vec{b}=9\hat{\imath}-14\hat{\jmath}+4\hat{k}$ $d=\frac{\sqrt{293}}{7}$	1 ½ 1 ½ 1 1 1 1 1/2 ½ 1 1
36	(i) $2x+4y+\pi x=40$ (ii) $40x-2x^2-\frac{\pi x^2}{2}$	1 1

	(iii) Area is maximum when $\frac{d}{dx}\left(40x - 2x^2 - \frac{\pi x^2}{2}\right) = 0$ That is $x = \frac{40}{(\pi+4)}$ Second derivative = $-4 - \pi < 0$ OR Using in the equation related to x and y $y = \frac{40}{2(\pi + 4)}$	½ 1 ½ 1 1
37	(i) $x = 12.5$ (ii) $(0,12.5)$ (iii) For differntiating and equation to 0 $m = 140$ OR For finding $P(0), P(14), P(16)$ Absolute maximum = 38480	1 1 1 1 1 ½ ½
38	Let E1 = The policy holder is accident prone. E2 = The policy holder is not accident prone. E = The new policy holder has an accident within a year of purchasing a policy. (i) $P(E) = P(E_1) \times P(E/E_1) + P(E_2) \times P(E/E_2)$ $= \frac{20}{100} \times \frac{6}{10} + \frac{80}{100} \times \frac{2}{10} = \frac{7}{25}$ (ii) By Bayes' Theorem, $P(E_1/E) = \frac{P(E_1) \times P(E/E_1)}{P(E)}$ $\frac{\frac{20}{100} \times \frac{6}{10}}{\frac{280}{1000}} = \frac{3}{7}$	 1 1 1 1