CHENNAI SAHODAYA SCHOOL COMPLEX
COMMON EXAMINATION
CLASS 10- SET 3 – Marking Scheme
MATHEMATICS STANDARD (041)
SECTION A

1. c) - 16
2. b) 2 : 1
3. b) (-1,-2)
4. d) $(\sqrt{2}x + \sqrt{3})^2 = 2x^2 - 3x$
5. a) 0
6. d) x = 3 , y = 4
7. c) 6.5 cm
8. d) $3\sqrt{3}$ cm
9. a)12
10. a) 2
11. b) 0
12. (a) 39 and 13
13. c) 14 cm
14. c) 8 cm
15. a)18π cu.cm
16. a)386
17. c) $\frac{12}{15}$
18. b) 0
19. (c)Assertion is correct but Reason is incorrect.
20. (c)Assertion is correct but Reason is incorrect.

SECTION B

21. Write the coordinates of a point on the x-axis which is equidistant from points A(-2, 0) and B(6, 0).

Sol:Let the coordinates of point P on x-axis be $(x,0)$.

$AP=BP$

$\Rightarrow AP^2=BP^2$

$\Rightarrow (x-(-2))^2+(0-0)^2=(x+6)^2+(0-0)^2$ -------------------------(1/2 m)

$\Rightarrow x^2+4x+4=(x+6)^2$ ___________________(1/2m)

$\Rightarrow 8x=-32$

$\Rightarrow x = -4$ ----------------------(1/2m)

Hence, required point P is $(-4,0)$ ---------------------(1/2m)

22. Two dice are rolled together bearing numbers 4, 6, 7, 9, 11, 12. Find the probability that the product of numbers obtained is an odd number.

Sol :Total number of products = 6 × 6 = 36 ------------------(1/2 m)

Number of odd products = 9

Probability that product is odd = 9/36 = **1/4** -----------------(1 ½ m)

[OR]

How many positive three-digit integers have the hundredths digit 8 and unit's digit 5? Find the probability of selecting one such number out of all three digit numbers.

Sol : possible numbers (805, 815, 825, 835, 845, 855, 865, 875, 885, 895). -----(1/2 m)

The total number of three-digit numbers ranges from 100 to 999 = 900 possible numbers.

Probability = 10/900 = 1/90. ----------------------(1 ½ m)

23. A rectangular courtyard is 32m long and 16cm broad. It is to be paved with square tiles of the same size. Find the least possible number of such tiles

Sol :Length of the yard = 18 m 72 cm =1872 cm

Breadth of the yard =13 m 20 cm = 1320 cm

The size of the square tile of same size needed to the pave the rectangular yard is equal the HCF of the length and breadth of the rectangular yard.

$1872 = 2^4 \times 3^2 \times 13$

1320=24 × 32 × 13

HCF=$2^3 \times 3 = 24$ -------------- (1 m)

∴ Length of side of the square tile = 24 cm

Number of tiles required = Area of the courtyard /Area of each tile

= 1872 × 1320 /24 x 24 = 4290 ------(1/2 + ½)

24. Find the value of x if : $2\ \text{cosec}^2 30^o + x \sin^2 60^o - \frac{3}{4}\tan^2 30^o = 10$

$$2\text{cosec}^2 30^o + x\sin^2 60^o - \frac{3}{4}\tan^2 30 = 10$$

$$2 \times (2)^2 + x\left(\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4}\left(\frac{1}{\sqrt{3}}\right)^2 = 10$$

$$2 \times 4 + x\frac{3}{4} + \left(\frac{-3}{4}\right) \times \frac{1}{3} = 10$$

$$8 + \frac{3x}{4} - \frac{1}{4} = 10$$

$$\frac{3x}{4} = \frac{1}{4} + 2 \Rightarrow \frac{9}{4}$$

$$x = 3$$

25. Solve for x and y : $mx - ny = m^2 + n^2$; $x + y = m - n$

Sol : On mult eq 2 by n

$mx - ny = m^2 + n^2$

$nx + ny = mn - n^2$

On solving $(m+n)x = m(m+n) \Rightarrow x = m$ ---------- (1m)

$y = -n$ --------- (1 m)

SECTION C

26. If α and β are the zeros of the quadratic polynomial $25x^2 - 15x + 2$, find a quadratic polynomial whose zeros are $\frac{1}{2\alpha}$ and $\frac{1}{2\beta}$

Sol : $25x^2 - 15x + 2$

$= 25x^2 - 10x - 5x + 2$

$= 5x(5x - 2) - 1(5x - 2)$

$= (5x - 1)(5x - 2)$

$\alpha = 1/5$; $\beta = 2/5$ ---------- (1 m)

$\frac{1}{2\alpha} = 5/2$; $\frac{1}{2\beta} = 5/4$

Sum of zeros = 15/4 ; Product = 25/8 -------- (1m)

New polynomial is $x^2 - \frac{15}{4}x - \frac{25}{8}$ ------- (1m)

27. A school has five houses A,B,C,D and E. In class X, House A has 4 students, 8 from house B, 5 from house C, 2 from house D and the rest from house E. If the total number of students in class X is 23 and if one student is chosen as class monitor, find the probability that the selected student is

Sol: (i) not from A, B and C = 6/23

(ii) Either from C or E = 9/23

(iii) Neither from A nor D = 17/23

28. A plane left 30 minutes late than its scheduled time and in order to reach the destination 1500 km away in time, it had to increase its speed by 100 km/hr from the usual speed. Find its usual speed.

Increased speed = (x + 100) km/h.
∴ Distance to cover = 1500 km.
Time taken by plane with usual speed = 1500x hr.
Time taken by plane with increased speed =1500(100+x)hrs.
According to the question,
$1500/x-1500/(100+x)=30/60=1/2$ ------- (½ m)
$1500[1/x-1/x+100]=1/2$
$1500[x+100-x/(x)(x+100)]=1/2$
$1500\times100/x^2+100x=12$

$x^2+100x=300000$
$x^2+100x-300000=0$ ------ (1 m)
$x2+600x-500x-300000=0$
$x(x+600)-500(x+600)=0$
$(x+600)(x-500)=0$ ------- (1 m)
Ether x + 600 = 0
x = -600 (Rejected)
x - 500 = 0
x = 500
∴ Usual speed of plane = 500 km/hr. ------ (½ m)

29. Prove that $\frac{1}{1+\sin^2\theta} + \frac{1}{1+\cos^2\theta} + \frac{1}{1+\sec^2\theta} + \frac{1}{1+\operatorname{cosec}^2\theta} = 2$

Sol : $\frac{1}{1+\sin^2\theta} + \frac{1}{1+\cos^2\theta} + \frac{\cos^2\theta}{1+\cos^2\theta} + \frac{\sin^2\theta}{1+\sin^2\theta}$ ------ (2 m)

$= \frac{1+\cos^2\theta}{1+\cos^2\theta} + \frac{1+\sin^2\theta}{1+\sin^2\theta} = 2$ -------- (1 m)

30. In the given fig, D and E are the midpoints of the sides BC and AC respectively of Δ ABC, where A(4 , -2) B(2, -2) and C(-6, 2) are the vertices of the triangle. Find the lengths of DE and AB and hence prove that DE = ½ BC

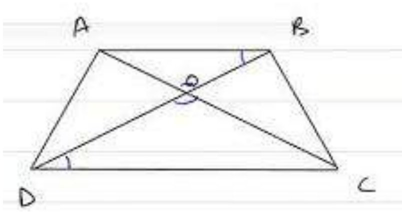
Sol : D = (3, -2) (using midpoint formula) ------- (½ m)
E = (-1, 0) ------- (½ m)
DE = √20 (using distance formula) ------- (1 m)
BC = √80 = 2√20 ------- (1 m)

[OR]

The line segment joining the points (3 , -4) and (1,2) is trisected at the points P and Q. If the coordinates of P and Q are (p, -2) and ($\frac{5}{3}$, q) respectively , find the values of p and q

Sol :P divides *AB* in the ratio 1:2
∴p=1(2) +2(3)/(1 + 2)
p=7/3 ------------ (1 ½ m)
Q divides AB in the ratio 2:1
q=2(2) +1×(−4)/(1 + 2)
q=0 ----------- (1 ½ m)

31. If one of the diagonals of a trapezium divides the other diagonal in the ratio 1: 2, prove that one of the parallel sides is twice the other



---------- (fig ½ m)

In ΔAOB and ΔCOD
∠AOB = ∠COD (Vertically opposite angles)
∠ABD = ∠ODC (Alternate angles)
∴ ΔAOB ~ ΔCOD (AA similarity criterion) -------- (1 m)
⇒ AB/CD = AO/OC = BO/OD [Corresponding sides of similar triangles are proportional.]
⇒ AB/CD = BO/OD -------(½ m)
⇒ AB/CD =2/1 ------- (½ m)
⇒ AB=2 CD ------ (½ m)

[OR]

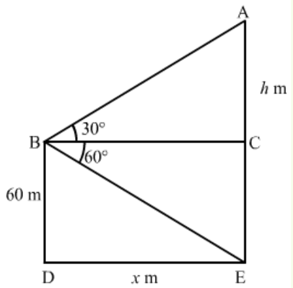
In a triangle PQR, L and M are two points on the base QR, such that $\angle LPQ = \angle$ QRP . Prove that $PQ^2 = QR \times QL$

In ΔLPQ and ΔPQR
∠Q=∠Q [Common]
∠QPL=∠PRQ [Given]
∴ΔLPQ~ΔPRQ [By A.A. axiom of similarity] --------- (2m)
⇒PQ/QR=QL/PQ
⇒PQ^2=QL×QR --------- (1 m)
Hence, proved.

SECTION D

32. The angles of elevation and depression of the top and the bottom of a tower from the top of a building, 60 m high, are 30° and 60° respectively. Find the difference between the heights of the building and the tower and the distance between them. (use √3 = 1.732)

Let AE be the light house and BD be the building of 60 m height



-------- (fig 1 m)

In right triangle ABC
tan30°=AC/BC⇒1/√3=h/x⇒x=√3h ----- (1 1/2 m)
Now, In right triangle BDE
tan60°=BD/DE⇒√3=60/x⇒x=60/√3=20√3m ------ (1 ½ m)
Now,x=√3h⇒20√3=√3h⇒h=20m ------- (1 /2 m)
(i) The difference between the heights of the light-house and the building is h = 20 m

(ii) The distance between the light-house and the building is x = 20√3 = 34.64 m (1/2 m)

33. Find the value of p, if the mean of the given data is 15.45

Class	0-6	6-12	12-18	18-24	24-30

Frequency	6	8	p	9	7
x	3	9	15	21	27
fx	18	72	15p	189	189

15.45=(468+15*p)/(30 +p)*

$\frac{1545}{100}=\frac{468+15p}{30+p} \Rightarrow 46350 + 1545p = 46800 + 1500p$ -----------*(2 1/2m)*

$450 = 45\ p \Rightarrow$ p = 10 -------- (½ m)

Mode

$12 + \left(\frac{10-8}{20-8-9}\right)$ x 6

= 12 + 4 = 16 ------ (2 m)

34. A solid spherical ball of the metal is divided into two hemispheres and joined as shown in the fig. The solid is placed in a cylindrical tub full of water in such a way that the whole solid is submerged in water. The radius and height of cylindrical tub are 4 cm and 11 cm respectively and the radius of spherical ball is 3 cm. Find the volume of the water left in the cylindrical tub (use $\pi = 3.14$)

Sol : Volume of cylinder = 3.14 x 4 x 4 x 11 = 552.64 cu cm --------- (2 m)

Volume of 2 hemispheres = 2 x $\frac{2}{3}$ x 3 x 3x 3 = 113.04 cu cm ---------- (2 m)

Volume of water left = 552.64 – 113.04 = 439.6 cu. Cm ------ (1 m)

[OR]

A woman runs a small-scale industry in a shed made out of metal, which is in the shape of a cuboid surmounted by a half cylinder as shown in the figure. If the base of the shed is of dimensions 7m x 15 m and the height of the cuboidal portion is 8m, find the volume of the shed. Also find the amount of metal required to construct the shed

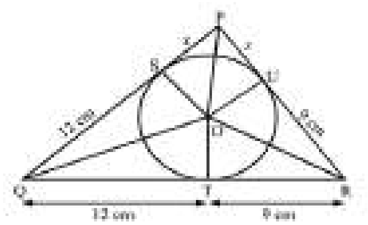
Sol : the diameter of the half cylinder is 7 m and its height is 15 m.

required volume = volume of the cuboid +1/2 volume of the cylinder
= 15×7×8+1/2×22/7×7/2×7/2×15=1128.75 ------ (2 m)
=1128.75m^3 -------- (½ m)

Amount of metal required = πrh + πr^2 + 2h(l + b) + lb
= 11 x 15 + 38.5 + 16 x 22 + 15 x 7
= 165 + 38.5 + 352 + 85 ------- (2m)
= 640.5 sq cm ------- (½ m)

35. In the adjoining fig, a triangle PQR is drawn to circumscribe a circle of radius 6cm such that the segments QT and QR into which QR is divided by the point of contact T, are of

lengths 12 cm and 9 cm respectively. If the area of ΔPQR is 189 sq cm, then find the lengths of PQ and PR



We have, OS = OT = OU = 6 cm (Radii of the circle)
QT = 12 cm and TR = 9 cm
QR = QT + TR = 12 cm + 9 cm = 21 cm
Now, QT = QS = 12 cm (Tangents from the same point)
TR = RU = 9 cm
Let PS= PU = x cm
Then, PQ = PS + SQ = (12 + x) cm and PR = PU + RU = (9 + x) cm
It is clear that
ar (ΔOQR) + ar (ΔOPR) + ar (ΔOPQ) = ar (ΔPQR)

$$Ar(PQR) = Ar(POR) + Ar(QOR) + Ar(POQ)$$

$$\Rightarrow 189 = \frac{1}{2} \times OU \times PR + \frac{1}{2} \times OT \times QR + \frac{1}{2} \times OS \times PQ$$

$$\Rightarrow 189 = \frac{1}{2}\left[(6)\times(x+9)+(6)\times(12+9)+(6)\times(12+x)\right]$$

$$\Rightarrow 189\times 2 = \left[(6)\times(x+9)+(6)\times(12+9)+(6)\times(12+x)\right]$$

$$\Rightarrow 378 = 6\times(x+9)+6\times(21)+6\times(12+x)$$

$$\Rightarrow 63 = x+9+21+x+12$$

--------- (3 m)

X = 21/2 = 10.5 --------- (1 m)

Thus, PQ = (12 + 10.5) cm = 22.5 cm ------ (½ m)

PR = 9 + 10.5 = 19.5 cm ------ (½ m)

[OR]

Length of two tangents drawn from the same point to the circle are equal.
∴ CF = CD = 6 cm
∴ BE = BD = 8 cm
∴ AE = AF = x

AB = AE + EB = x + 8
BC = BD + DC = 8 + 6 = 14
CA = CF + FA = 6 + x
Let perimeter of triangle ABC be s.
⇒ 2s = AB + BC + CA
= x + 8 + 14 + 6 + x
= 28 + 2x
⇒ s = 14 + x
Area of Δ ABC = $\sqrt{s(s-a)(s-b)(s-c)}$
$=\sqrt{(14+x)(14+x-14)(14+x-x-6)(14+x-x-8)}$

$=\sqrt{(14+x)(x)(8)(6)}$

$=\sqrt{(14+x)48x}$...(i)

Also, area of Δ ABC = 2 × (area of ΔAOF+ΔCOD+ΔDOB)

=2×[(1/2×OF×AF)+(1/2×CD×OD)+(1/2×DB×OD)]

=2×1/2(4x+24+32)=56+4x...(ii)

Equating equations (i) and (ii) we get,

$\sqrt{(14+x)48x}=56+4x$

Squaring both sides,

48x(14+x)=(56+4x)2

⇒48x=[4(14+x)]2(14+x)

⇒48x=16(14+x)

⇒48x=224+16x

⇒32x=224

⇒ x = 7 cm

So, AB = x + 8 = 7 + 8 = 15 cm

CA = 6 + x = 6 + 7 = 13 cm.

Hence, AB=15 cm , CA = 13 cm

SECTION E

36. The Ferris wheel had equally spaced seats such that the central angle formed was 20^0. The diameter of the Ferris wheel is 42 m

(i) How many passenger carrying components were there?

Sol :360/20 = 18

ii. Into how many equal sectors will the circle be divided if the angle of a sector is to 12°

Sol :360/12 = 30

iii. How far apart along the circle are two adjacent seats.

Sol : Length of arc = $\frac{20}{360}$ 2 x$\frac{22}{7}$x 21 = 7$\frac{1}{3}$ cm

[OR]

What is the area of the sector between two consecutive rims?

Area = $\frac{20}{360}$ x $\frac{22}{7}$x 21 x 21 = 77

37. A teacher ask her class student to make an irregular polygon with 31 sides, using cardboard. The student made the polygon is such a way that the lengths of which, starting from the smallest are in AP. If the perimeter of the polygon is 527 cm and the length of the largest side is sixteen times the smallest

a,a+d,a+2d,..,a+30d=527

⇒Sn=527

⇒3122a+30d=527

⇒31a+15d=527

⇒a+15d=17 ……..(1)

As the length of the largest side is sixteen times the smallest. So,

a+30d=16a
⇒30d=15a
⇒a=2d …..(2)
Substitute (2) in (1) to determine the value of d.
(2d)+15d=17
⇒17d=17
⇒d=1
Substitute the value of d in (2) to determine the value of a.
a=2(1)
⇒a=2
Therefore, the smallest side is of length 2 cm and the common difference is 1 cm.

Find the sum of the lengths of the smallest side and the largest side of the polygon
Smallest side = 2 cm
Largest side = 32 cm
Sum = 34 cm

[OR]

Find the ratio of the length of 5^{th} side and the length of 20^{th} side

6: 22 = 3 : 11

38. A farmer was asked to design a rectangular field whose length is 10 m less than twice its breadth and the area is 600 sq m. If the breadth of the field is 'x' metre

Answer the following questions based on the information:

[i] Find the length of the field in terms of x . length = 2x - 10

[ii] Find the equation obtained $x^2 - 5x - 300 = 0$

[iii] Find the area of the field breadth = 20 m ; length = 30 m

Area = 600 sq m

[OR]

Find the perimeter of the field = 2(20 + 30) = 100 m