APPLIED MATHEMATICS

CLASS 12

TOP 5 PAPERS

MR. AMIT MITTAL

PGT Mathematics, St. Francis School, Baraut

Time to Achieve Excellence

APPLIED MATHEMATICS

CLASS 12

TOP 5 SAMPLE QUESTION PAPERS

(Based on the Latest CBSE Syllabus 2025–26)

Written By

MR. AMIT MITTAL

PGT Mathematics
St. Francis School, Baraut

Edition: 2025–26

PREFACE

Applied Mathematics at the senior secondary level is designed to help students develop analytical thinking, logical reasoning, financial literacy, and real-life problem-solving skills. In alignment with the latest CBSE 2025–26 syllabus, this book, *"Applied Mathematics Class 12 – Top 5 Sample Question Papers,"* aims to provide students with focused practice that reflects the competency-based approach now emphasized in board examinations.

CBSE's modern assessment framework encourages students to progress through different levels of Bloom's Taxonomy—from remembering and understanding to applying, analysing, evaluating, and creating. The sample papers in this book are structured to support this learning journey. They include problem types that test conceptual clarity, mathematical reasoning, data interpretation, and application of mathematics in financial, statistical, and real-world contexts.

The goal of this book is not only to help students score well, but also to build their confidence to approach unfamiliar questions with clarity and accuracy. Each paper has been created to resemble the actual CBSE pattern, ensuring that students experience the correct difficulty level, marking scheme, and time pressure.

I believe that consistent practice, along with smart time management and thorough revision, can transform a student's performance. I hope this book becomes a valuable resource for every learner preparing for the Class 12 Applied Mathematics examination.

Mr. Amit Mittal
PGT Mathematics
St. Francis School, Baraut

The fear of the Lord is the beginning of wisdom.

ST Francis School
Baraut

ESTD. 2016

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
- **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।**
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।
- *Please check that this question paper contains 23 printed pages.*
- *Please check that this question paper contains 38 questions.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
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व्यावहारिक गणित
APPLIED MATHEMATICS

निर्धारित समय : 3 घण्टे

अधिकतम अंक : 80

Time allowed : 3 hours

Maximum Marks : 80

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General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains 38 questions. All questions are compulsory.

(ii) This Question paper is divided into five Sections - A, B, C, D and E.

(iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.

(iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.

(v) In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.

(vi) In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.

(vii) In Section E, Questions no. 36 to 38 are case study-based questions carrying 4 marks each.

(viii) Thre is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and one sub-part each in 2 questions of Section E.

(ix) Use of calculators is not allowed.

SECTION-A

This section comprises of 18 multiple choice questions and two assertion and reason type questions of 1 mark each.

1. In what ratio, water must be added to dilute honey costing 240 per litre so that the resulted syrup would be worth be worth ₹ 200/litre?

   (a) 2 : 3 (b) 1 : 5 (c) 4 : 5 (d) 5 : 6 [1]

2. A runs 3 times as fast as B. If A gives B a start of 40 metres, how far must the goal on the race course be so that A and B reach the goal at the same time?

   (a) 30 m (b) 20 m (c) 80 m (d) 60 m [1]

3. Find $\frac{dy}{dx}$, when $x^3 + y^3 = xy$.

   (a) $\frac{y-3x^2}{3y^2-x}$ (b) $\frac{y-5x^2}{3y^2-x}$ (c) $\frac{y-3x^2}{8y^2-x}$ (d) $\frac{8y-3x^2}{5y^2-x}$ [1]

4.

Types of data Analysis	Data collected
(i) Time series data	(a) data in a combination of time series data and cross-sectional data.
(ii) Cross-sectional data	(b) data of the variable is collected at distinct time intervals.
(iii) Pooled data	(c) data for one or more variables is collected at the same point.

   (a) (i)-(c); (ii)-(a) ;(iii)-(b) (b) (i)-(a); (ii)-(c); (iii)-(b) (c) (i)-(b); (ii)-(c); (iii)-(a) (d) (i)-(b); (ii)-(a); (iii)-(c) [1]

5. At 6% converted quarterly, find the present value of a perpetuity of ₹ 600 payable at the end of each quarter

   (a) ₹ 20000 (b) ₹ 32000 (c) ₹ 25000 (d) ₹ 40000 [1]

6. If $p > q$ and $r < 0$, then which of the following is true?

   (a) $pr < qr$ (b) $p - r < q - r$ (c) $p + r < q + r$ (d) None of the above [1]

7. Let $Z = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. The only correct statement about the matrix Z is:

   (a) Diagonal Matrix (b) Scalar Matrix (c) Identity Matrix (d) Singular Matrix [1]

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8. Evaluate: $\int e^{4-5x}dx$

(a) $-\frac{1}{5}e^{4-5x}+C$ (b) $\frac{2}{5}e^{2-5x}+C$ (c) $-\frac{1}{5}e^{2-8x}+C$ (d) $-\frac{1}{6}e^{8-8x}+C$ [1]

9. For the purpose of $t-$ test of significance, a random sample of size (n) 2025 is drawn from a normal population, then the degree of freedom (v) is

(a) 2025 (b) 2024 (c) 2020 (d) 2021 [1]

10. Find the critical t values for $\alpha = 0.10$ with d.f. = 18 for a two-tailed t test.

(a) $t = \pm 1.734$ (b) $t = \pm 1.34$ (c) $t = \pm 1.4$ (d) $t = \pm 1.42$ [1]

11. Find k, if $A = \begin{bmatrix} -2 & 3 \\ k & 4 \end{bmatrix}$ is a singular matrix

(a) $\frac{5}{2}$ (b) $\frac{8}{3}$ (c) $-\frac{8}{3}$ (d) $\frac{5}{6}$ [1]

12. At what rate converted semi-annually will the present value of a perpetuity of ₹ 450 payable at the end of each 6 months be ₹ 20,000?

(a) 4.5% (b) 3.2% (c) 5.2% (d) 5.6% [1]

13. If $A = \begin{bmatrix} 1 & 2a \\ -8 & b+1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -6 \\ -8 & 13 \end{bmatrix}$ are equal matrices, find the values of a and b.

(a) $a = -3\,b = 12$ (b) $a = 3\,b = -12$ (c) $a = -3\,b = 1$ (d) $a = -5\,b = 12$ [1]

14. What is the order and degree of the differential equation obtained by eliminating the arbitrary constants a and b from $y = ae^{2x} + be^{-2x}$? [1]

(a) Order = 2, Degree = 1 (b) Order = 1, Degree = 2 (c) Order = 2, Degree = 2 (d) Order = 1, Degree =1

15. If a fair coin is tossed 9 times, find the probability of exactly five tails.

(a) $\frac{29}{353}$ (b) $\frac{63}{256}$ (c) $\frac{48}{552}$ (d) $\frac{5}{625}$ [1]

16. The constraints of a linear programming problem along with their graphs are shown below:

$$x + 2y \geq 3$$
$$x \geq 10$$
$$y \geq 0$$

Which of the following inequality may be removed so that the feasible region remains the same as above graph?

(a) $x + 2y \geq 3$ (b) $x \geq 10$ (c) $y \geq 0$ (d) None of these [1]

17. If $A = \begin{pmatrix} 3 & 5 \\ -4 & 6 \end{pmatrix}$, find $|A|$

(a) 23 (b) 32 (c) 38 (d) 56 [1]

18. Mr. X takes a loan of ₹ 2,00,000 with 10% annual interest rate for 5 years. Calculate EMI under Flat Rate system.

(a) 5000 (b) 3000 (c) 4500 (d) 5500 [1]

Instructions: Questions number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R).

Select the correct answer from the options (A), (B), (C) and (D) as given below:

(a) Both Assertion (A) and Reason (R) are true and (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) but (R) is not the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

19. Assertion (A): For any two square matrices A and B, multiplication of their determinant values is same as determinant of their product. $|A| \times |B| = |A \times B|$

Reason (R): For matrix $A = [a]$ of order 1×1, det $A = a$ [1]

20. Assertion (A): In Normal distribution, mean, median and mode of the sample space are exactly the same.

Reason (R): Normal distribution is perfectly symmetrical around its center, which is both the mean, median and mode. [1]

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SECTION-B

This section comprises of 5 very short answer (VSA) type questions of 2 marks each.

21. (A) Verify that $(14 + 8) \bmod 5 = (14 \bmod 5 + 8 \bmod 5) \bmod 5$ [2]

OR

(B) Insert the appropriate sign of inequality:

$\sqrt{3}(\sqrt{50} - \sqrt{32})$ ____ $3\sqrt{64} + 2\sqrt{24}$ [2]

22. Let X denote the number of hours a class XII student studies during a randomly selected school day. The probability that X can take the values x_i for an unknown constant K is given by:

$$P(X = x_i) = \begin{cases} 0.1, & \text{if } x_i = 0 \\ kx_i, & \text{if } x_i = 1 \text{ or } 2 \\ k(5 - x_i), & \text{if } x_i = 3 \text{ or } 4 \end{cases}$$

Find the value of k. [2]

23. The present value of a perpetual income of ₹ x at the end of each six months is ₹ 40000. Find the value of x if money is worth 6% compounded semi-annually. [2]

24. (A) For a Poisson distribution model, if arrival rate of passengers at an airport is recorded as 30 per hour on a given day. Find the expected number of arrivals in the first 10 minutes of an hour [2]

OR

(B) It is given that 3% defective electric bulb are manufactured by a company. Using Poisson distribution, find the probability of 100 bulbs will contain no defective bulbs. (Use $e^{-3} = 0.05$) [2]

25. On 1ˢᵗ April, 2020, Ram purchased a machinery costing ₹ 40,000 and spent ₹ 5,000 on its erection. The estimated effective life of the machinery is 10 years with a scrap value of ₹ 5,000. Calculate the depreciation using the Linear/ Straight line method with accounting year ending on 31ˢᵗ March, 2021. [2]

SECTION-C

This section comprises of 6 short answer (SA) type questions of 3 marks each.

26. A person can row a boat 5 km an hour in still water. It takes him thrice as long to row upstream as to row downstream. Find the rate at which the stream is flowing. [3]

27. (A) The price–demand relation is $x = \frac{1}{2} p^2 - 2p + 3$.

(i) Find the revenue function R.

(ii) Find price intervals where revenue increases/decreases. [3]

OR

(B) The total cost function of a manufacturing company is given by $C(x) = 2x\left(\dfrac{x+4}{x+3}\right) + 3$. Show that MC (Marginal Cost) falls continuously as the output 'x' increases. [3]

28. (A) A recent accounting graduate opened a new business and installed a computer system that costs ₹ 45,200. The computer system will be depreciated linearly over 3 years and will have a scrap value of ₹ 0.

(i) What is the rate of depreciation?

(ii) Give a linear equation that describes the computer system's book value at the end of ith year, where $0 \le i \le 3$.

(iii) What will be the computer system's book value at the end of the first year and a half? [3]

OR

(B) Find the effective rate which is equivalent to normal rate of 10% p.a. compounded:

(i) semi-annually.

(ii) quarterly.

[Given $(1.05)^2 = 1.1025$, $(1.025)^4 = 1.1038$] [3]

29. A cistern can be filled by two pipes A and B in 12 minutes and 15 minutes respectively. Another tap C can empty the full tank in 20 minutes. If the tap C is opened 5 minutes after the pipes A and B are opened, find when the cistern becomes full? [3]

30. Two batches of the same product are tested for their mean life. Assuming that, the lives of the product follow a normal distribution with an unknown variance; test the hypothesis that the mean life is the same for both the branches, given the following information:

Batch	Sample Size	Mean Life (in hours)	Standard Deviation (in hours)
Batch I	10	750	12
Batch II	8	820	14

[Given $\sqrt{4.4444} = 2.1081$ and $t_{16}(0.05) = 2.120$]

[3]

31. A firm is engaged in breeding pigs. The pigs are fed on various products grown on the farm. In view of the need to ensure certain nutrients constituents (call them X, Y and Z), It is necessary to buy two additional products, say A and B. One unit of product A contains 36 units of nutrient X, 3 units of nutrient Y and 20 units of nutrient Z. One unit of product B contains 6 units of nutrient X, 12 units of nutrient Y and 10 units of nutrient Z. The minimum requirement of nutrients X, Y and Z is 108 units, 36 units and 100 units respectively. Product A costs 20 per unit and product B costs 40 per unit. Formulate the above as a linear programming problem to minimize total cost.   [3]

SECTION-D

This section comprises of 4 long answer (LA) type questions of 5 marks each

32. (A) A school plans to award ₹ 6000 in total to its students to reward for certain values - honesty, regularity and hard work. When three times the award money for hard work is added to the award money given for honesty amounts to ₹ 11000. The award money for honesty and hard work together is double the award money for regularity. Use matrix method to find the prize money for each category of award.   [5]

OR

(B) Solve the following system of equations using Cramer's rule $x + y + z = 10, 2x + y = 13, x + y - 4z = 0$ [5]

33. A firm knows that the price per unit 'p' for one of its product is linear. It also knows that it can sell 1400 units when the price is 4 per unit, and it can sell 1800 units at a price of 2 per unit. Find the price per unit if 'x' units are sold (or demanded). Also find the revenue function and the marginal revenue function.   [5]

34. (A) Given below are the consumer price index numbers (CPI) of the industrial workers.

Year	2014	2015	2016	2017	2018	2019	2020
Index Number	145	140	150	190	200	220	230

Find the best fitted trend line by the method of least squares and tabulate the trend values.   [5]

(B) The quarterly profits of a small-scale industry (₹ in thousands) are as follows.

Year	Quarter 1	Quarter 2	Quarter 3	Quarter 4
2020	39	47	20	56
2021	68	59	66	72
2022	88	60	60	67

Calculate 4-quarterly moving averages.   [5]

35. Amrita buys a car for which she makes a down payment of ₹ 2,50,000 and the balance is to be paid in 2 years by monthly installments of ₹ 25,448 each. If the financer charges interest at the rate of 20% p.a, find the actual price of the car. [Given $\left(\frac{61}{60}\right)^{-24} = 0.67253$]   [5]

SECTION-E

This section comprises of 3 case-study/passage-based questions of 4 marks each with sub parts.
The first two case study questions have three sub parts (i), (ii), (iii) of marks 1, 1, 2 respectively.
The third case study question has two sub parts of 2 marks each

36. A data analyst at a manufacturing plant is studying the distribution of the daily production output (in units) of a specific product, denoted by the random variable X. After analysing several months of data, it is concluded that X follows a normal distribution. The analysis shows that the mean daily production (μ) is 12 units with a standard deviation (σ) of 4 units.

Based on this information, answer the following questions:

(i) Calculate the Z-Score for a day where the production output was observed to be 20 units.   [1]

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(ii) On a day of peak efficiency, the production output had a Z-Score of $+5$. What was the actual production output for that day? [1]

(iii) (A) During a day with significant machinery issues, the production output had a Z-Score of -2.

(a) Determine the actual number of units produced that day.

(b) State how many standard deviations this output is below the mean. [2]

OR

(B) If an observation from this distribution is found to have a Z-Score of 1.5, what can you conclude about its value relative to the mean? Calculate the value of this observation. Interpretate your findings.

37. Suppose the demand for a good is given by the equation $x_d = 500 - 50P$, where x_d is the quantity demanded and P is the price of the good. The supply of the good is described by the equation $x_s = 50 + 25P$, where x_s is the quantity supplied.

(i) What is the equilibrium price in the market? [1]

(ii) What is the equilibrium quantity in the market? [1]

(iii) (A) Using integration, calculate the consumer surplus at the equilibrium price. [2]

OR

(B) If the price of the good increases by 10%, how will the equilibrium price and quantity change? [2]

38. A company has two groups of inspectors namely, group A and B, who are assigned to do a quality inspection work. It is required that at least 1800 pieces are inspected for 8-hour day. It is known that inspectors of group A can check pieces at the rate of 25 per hour with an accuracy of 98%, while inspectors of group B can check at the rate of 15 pieces per hour with an accuracy of 95%. The inspectors of group A and B are paid ₹ 40 and ₹ 30 per hour respectively to do the work. Each time an error is caused by the any inspector, it costs a loss of ₹ 20 to the company. The company has 8 inspectors in group A and 10 in group B.

(i) Formulate an LPP. [2]

(ii) Determine the optimal assignment of Inspectors to minimise total inspection cost. [2]

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Candidates must write the Q.P Code on the title page of the Answer book.

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APPLIED MATHEMATICS

निर्धारित समय : 3 घण्टे

Time allowed : 3 hours

अधिकतम अंक : 80

Maximum Marks : 80

General Instructions :

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(i)   This question paper contains **38** questions. **All** questions are **compulsory**.

(ii)   This question paper is divided into **five** Sections – **A, B, C, D** and **E**.

(iii) In **Section A**, Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.

(iv)   In **Section B**, Questions no. **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.

(v)   In **Section C**, Questions no. **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.

(vi)   In **Section D**, Questions no. **32** to **35** are Long Answer (LA) type questions carrying **5** marks each.

(vii) In **Section E**, Questions no. **36** to **38** are case study-based questions carrying **4** marks each.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E.

(ix)   Use of calculator is **not** allowed.

SECTION-A

This section comprises of 18 multiple choice questions and two assertion and reason type questions of 1 mark each.

1. A runs $1\frac{2}{3}$ times as fast as B. If A gives B a start of 50 m, how far must the winning post be so that they reach at the same time?

(a) 100 m   (b) 125 m   (c) 150 m   (d) 200 m   [1]

2. A solution of sugar syrup contains 15% sugar. How many grams of sugar must be added to 500 g of the solution to make the sugar content 20%?

(a) 25 g   (b) 30 g   (c) 31.25 g   (d) 35 g   [1]

3. The marginal cost function of manufacturing x units of a commodity is $MC = 6 + 10x - 6x^2$. The total cost of producing one unit of the commodity is ₹ 12. Find the total cost function $C(x)$.

(a) $6x + 5x^2 - 2x^3 + 7$ (b) $6x + 10x^2 - 6x^3 + 3$ (c) $6 + 10x - 6x^2 + 3$ (d) $6x + 5x^2 - 2x^3 + 3$ [1]

4. Match the components of time series in correct order:

I	Cyclic variation	(a)	Periodic with time span less than year
II	Seasonal variation	(b)	Long-term upward or downward trend
III	Secular trend	(c)	Recurrent but occurs over more than a year

(a) I-(c), II-(a), III-(b)   (b) I-(a), II-(b), III-(c)   (c) I-(b), II-(c), III-(a)   (d) I-(c), II-(b), III-(a)   [1]

5. If the coupon rate of a bond is 8% and the yield rate is 10%, the price of bond will:

(a) Be more than face value   (b) Be less than face value
(c) Equal face value   (d) Cannot be determined   [1]

6. For integer constraints $5 \le x \le 15, 2 \le y \le 6$, which inequality is always true?

(a) $\frac{x-y}{3} < 4$ (b) $\frac{x-y}{y} \le 3$ (c) $\frac{x+y}{2} \le 9$ (d) None of these   [1]

7. If matrix A satisfies $A^2 = -I$, then minimal polynomial of A is

(a) $x^2 + 1$ (b) $x^2 - 1$ (c) $x + 1$ (d) $x - 1$ [1]

8. If $\int \frac{3x+5}{x^2+5x+6}dx = Q\int \frac{1}{x^2+5x+6}dx + R\int \frac{x+2}{x^2+5x+6}dx$, Q equals:

(a) $\frac{1}{2}$ (b) -1 (c) $\frac{1}{4}$ (d) $\frac{1}{6}$ [1]

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9. The central limit theorem states that when sample size increases, the sampling distribution of the mean approaches:
   (a) Normal distribution
   (b) Binomial distribution
   (c) Exponential distribution
   (d) Uniform distribution   [1]

10. Two samples have size 15 and 18. Degrees of freedom for independent t-test is:
    (a) 32   (b) 31   (c) 33   (d) 30   [1]

11. If adjoint of matrix B is $\begin{bmatrix} 2 & \alpha \\ 5 & 4 \end{bmatrix}$ and $|B| = 4$, then α equals:

    (a) 7   (b) $\frac{4}{5}$ (c) 10   (d) 9   [1]

12. The order of the differential equation $\frac{d}{dx}\left[\left(\frac{dy}{dx}\right)^4\right] = 0$ is:

    (a) 1   (b) 2   (c) 4   (d) 3   [1]

13. For a perpetuity paying ₹ 600 quarterly, present value is ₹ 48,000. Rate of interest is:
    (a) 2.5% p.a.   (b) 6% p.a.   (c) 5% p.a.   (d) 3% p.a.   [1]

14. If matrices A, B satisfy $B = -A^{-1}BA$, then $(A + B)^2 =$
    (a) O (b) $A^2 + B^2$ (c) $A^2 + B^2 + 2AB$ (d) $A^2 + B^3 - 2AB$ [1]

15. Identify the redundant constraint in the LPP: Minimize $Z = 8x + 6y$ subject to
    $x \geq 4, y \geq 3, 3x + 2y \geq 20, x \geq 0, y \geq 0$
    (a) $x \geq 4$ (b) $y \geq 3$ (c) $3x + 2y \geq 20$ (d) None of these   [1]

16. In 8 coin tosses, probabilities for getting k heads and $k + 2$ heads are equal. Find k.
    (a) 3   (b) 2   (c) 4   (d) 1   [1]

17. If $|A| = 6$ and $|B| = 3$, find determinant of $5A^3B^2$ where A, B are 3×3 matrices:
    (a) 243000   (b) 48600   (c) 16200   (d) 27000   [1]

18. Flat rate interest is 10% p.a. for a 3-year loan of ₹ 40000. EMI is:
    (a) ₹ 1,444   (b) ₹ 1,200   (c) ₹ 1,333   (d) ₹ 1,100   [1]

    Instructions: Questions number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R).
    Select the correct answer from the options (A), (B), (C) and (D) as given below:
    (a) Both Assertion (A) and Reason (R) are true and (R) is the correct explanation of Assertion (A).
    (b) Both Assertion (A) and Reason (R) but (R) is not the correct explanation of Assertion (A).
    (c) Assertion (A) is true but Reason (R) is false.
    (d) Assertion (A) is false but Reason (R) is true.

19. Assertion (A): For 2×2 invertible matrix M, adj(adj M) = M.
    Reason (R): $|\text{adj } M| = |M|$.

20. Assertion (A): If binomial random variable has parameters n, p, mean is always less than variance.
    Reason (R): Probability lies between 0 and 1.

SECTION-B

This section comprises of 5 very short answer (VSA) type questions of 2 marks each.

21. (A) Find the last digit of $2^{105} + 105!$.   [2]

    OR

    (B) Without computing exact values, prove $\sqrt{7} + \sqrt{2} > \sqrt{6} + \sqrt{3}$

22. Random variable X has distribution: [2]

    $$P(X = x) = \begin{cases} ax & x = 1, 2, 3 \\ b & x = 4 \end{cases}$$

    with mean 3.1. Find a and b.

23. Population grows from 80,000 to 1,20,000 at 6% CAGR. Find number of years. (log 1.5 = 0.176, log 1.06 = 0.025) [2]

24. (A) In a town, average power outages per month = 3. Find probability of exactly 5 outages given at least one outage. (Use $e^{-3} = 0.05$) [2]

OR

(B) Electronic component fails 3 times per 1000 hours. Find probability of more than 2 failures in 5000 hour operation. (Use $e^{-15} = 3.06 \times 10^{-7}$)

25. A machine initially worth ₹ 40,000 depreciates linearly by ₹ 5,000 per year. When will value be half of initial? [2]

SECTION-C

This section comprises of 6 short answer (SA) type questions of 3 marks each.

26. Boat covers distance upstream in 7 hours 30 min and downstream in 3 hour 45 min. Find ratio of boat speed to stream speed. [3]

27. (A) Curve $y = px^3 + qx^2$ has critical point at $(1, -2)$. Find p, q and the other critical point. [3]

OR

(B) Total revenue $R = 12x + \frac{x^2}{2} - \frac{x^4}{48}$. Show at maximum average revenue, $AR = MR$.

28. (A) Bond face value ₹ 70,000, 8% coupon, redeemable after 5 years, quarterly payment. Purchase price if yield is 9% compounded quarterly? $(1.0225)^{-20} = 0.64$ [3]

OR

(B) A machine currently costs ₹ 60,000. Its price is expected to increase by 25% at the time of replacement. The current machine will have a scrap value of ₹ 7,000 after 7 years. To finance the purchase of the new machine, a sinking fund is established. This fund will earn an annual interest rate of 7%, compounded yearly. Calculate the constant annual payment that must be made into the sinking fund so that the accumulated amount, combined with the scrap value, will exactly cover the future replacement cost. [Use $(1.07)^7 = 1.6$]

29. Pipes A, B, C fill tank in 40, 80 and 160 min respectively. Pipe B, C open 15 min, then B closed, A opened. Pipe C closes 5 min before tank full. Total fill time? [3]

30. Sample of 16 holes punched has mean 1.86 cm, variance 0.005. Test at 0.05 level if average hole diameter is 1.84 cm. $t_{0.05,15} = 2.131$. [3]

31. Company produces A and B with labor limits. A needs 5 h/unit, B needs 7 h/unit; max 350 h labor/day; max 70 units A, 50 units B; profits ₹ 25 per A and ₹ 35 per B. Formulate LPP to maximize profit. [3]

SECTION-D

This section comprises of 4 long answer (LA) type questions of 5 marks each

32. (A) The sum of three numbers is 6. If we multiply the third number by 2 and add the first number to the result, we get 7. By adding second and third numbers to three times of the first number, we get 12. Using matrices find the numbers. [5]

OR

(B) Markets X, Y demand and supply:
$x_d(X) = 85 - 4p_X + p_Y$, $x_s(X) = -3 + 18p_X$
$x_d(Y) = 90 + 3p_X - 5p_Y$, $x_s(Y) = -10 + 30p_Y$
Find equilibrium prices using Cramer's rule.

33. Marginal cost $MC = 70 - 24x + 3x^2$ and marginal revenue $MR = 24x - 3x^2$. Find profit-maximizing output and total profit (fixed cost zero). [3]

34. (A) Annual rice production (millions): [3]

Year	Rice Production (in millions)
2018	7.6
2019	8.3
2020	9.1
2021	9.7
2022	10.2

Find least squares trend line, tabulate values and forecast 2026 production.

(B) Monthly traffic (in thousands)

Month	Traffic (in thousands)
March	28
April	34
May	39
June	36
July	41
August	44
September	38
October	46
November	49
December	48

Compute 3-month moving average and plot trend.

35. Mr. Roy buys a car for ₹ 12,00,000 and pays 15% down. He takes a loan for remaining at 11% annual interest compounded monthly, repaid in 4 years. Compute EMI and total interest. $(1.00917)^{48} = 1.51$. [3]

SECTION-E

This section comprises of 3 case-study/passage-based questions of 4 marks each with sub parts.
The first two case study questions have three sub parts (i), (ii), (iii) of marks 1, 1, 2 respectively.
The third case study question has two sub parts of 2 marks each

36. A factory produces bulbs whose lifetimes are normally distributed with mean 1200 hours and standard deviation 100 hours.
(i) What proportion of bulbs last between 1100 and 1350 hours? [1]
(ii) If 400 bulbs are produced, how many are expected to last less than 1050 hours? [1]
(iii) (A) A company guarantees replacement if less than 5% of bulbs fail before a certain time. Find this guarantee period. [2]

OR

(B) If a sample of 25 bulbs is tested, find the probability that the sample mean lifetime exceeds 1225 hours.

37. A commodity has demand and supply functions:

$Q_d = 500 - 10p - 5p^2$, $Q_s = 50 + 15p$
(i)   Find the equilibrium price and quantity. [1]
(ii) Find the price elasticity of demand at equilibrium. [1]
(iii) (A) Compute the producer surplus at equilibrium. [2]

OR

(B) Compute the consumer surplus at equilibrium using integration.

38. Consider the LPP with objective $Z = px + qy$ where $p, q > 0$, subject to

$$x + 2y \le 80, 2x + y \le 100, x \ge 0, y \ge 0.$$

(a) Solve graphically to find all corner points of the feasible region. [2]
(b) If $Z = px + qy$ attains its maximum at both (50, 0) and (40, 20), find the relation between p and q and state the number of optimal solutions in this case. [2]

ST Francis School
Baraut

ESTD. 2016

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
- **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।**
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।
- Please check that this question paper contains **23** printed pages.
- Please check that this question paper contains **38** questions.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- **Please write down the serial number of the question in the answer-book before attempting it.**
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

व्यावहारिक गणित
APPLIED MATHEMATICS

निर्धारित समय : 3 घण्टे

Time allowed : 3 hours

अधिकतम अंक : 80

Maximum Marks : 80

SECTION-A

This section comprises of 18 multiple choice questions and two assertion and reason type questions of 1 mark each.

1. In what ratio must rice at ₹ 58 per kg be mixed with rice ₹ 110 per kg so that the mixture be worth 70 per kg?

    (a) 2:3   (b) 10:3   (c) 4:5   (d) 5:6   [1]

2. In a 1000 metres race, A can give a start of 100 metres to B and a start of 280 metres to C. In the same race, how much start can B give to C?

    (a) 30 m   (b) 20 m   (c) 80 m   (d) 200 m   [1]

3. For the differential equation: $\dfrac{dy}{dx} = \dfrac{xe^y}{1+x^2}$, the solution is:

    (a) $e^{-y} + \dfrac{1}{2}\ln(1+x^2) = C$ (b) $e^y + \ln(1+x^2) = C$ (c) $e^{-y} - \dfrac{1}{2}\ln(1+x^2) = C$ (d) $e^y - \ln(1+x^2) = C$ [1]

4. Match the following columns to complete the sentence and choose the correct option:

Method	Description	Usage
(I) Straight-line trend	(a) A method that smoothens out fluctuations by averaging data over a set period of time	(i) Used for long-term predictions
(II) Moving average method	(b) A method used to establish a linear relationship between time and the variable	(ii) Applied for short-term forecasting
(III) Least squares method	(c) A technique used for long-term forecasting by using a trend line based on data points	(iii) Used to minimize errors in trend analysis

    (a) (I) – (b) – (iii); (II) – (a) – (ii); (III) – (c) – (i)
    (b) (I) – (b) – (i); (II) – (a) – (ii); (III) – (c) – (iii)
    (c) (I) – (a) – (iii); (II) – (b) – (i); (III) – (c) – (ii)
    (d) (I) – (c) – (ii); (II) – (a) – (i); (III) – (b) – (iii)   [1]

5. Find the present value of a sequence of payments of 60 made at the end of each 6 months and continuing forever, if money is worth 4% compounded semi-annually.

    (a) ₹ 2000   (b) ₹ 3200   (c) ₹ 2500   (d) ₹ 3000   [1]

6. If $0 < x < 1$, which of the following is greatest?

    (a) x (d) x^2 (c) $\dfrac{1}{x}$ (d) $\dfrac{1}{x^2}$ [1]

7. The travel time matrix between 3 bus stops A_1, A_2 and A_3 is given as:

$$A = \begin{bmatrix} 0 & 2 & 3 \\ 2 & 0 & 4 \\ 3 & 4 & 0 \end{bmatrix}$$

If a new bus stop A_4 is added, with the travel times to the existing stops as [2, 3, 5], what is the new symmetric matrix representing the travel times between all 4 bus stops?

(a) $\begin{bmatrix} 0 & 2 & 3 & 2 \\ 2 & 0 & 4 & 3 \\ 3 & 4 & 0 & 5 \\ 2 & 3 & 5 & 0 \end{bmatrix}$
(b) $\begin{bmatrix} 0 & 3 & 5 & 2 \\ 3 & 0 & 4 & 5 \\ 5 & 4 & 0 & 3 \\ 2 & 5 & 3 & 0 \end{bmatrix}$
(c) $\begin{bmatrix} 0 & 2 & 3 & 3 \\ 2 & 0 & 4 & 2 \\ 3 & 4 & 0 & 5 \\ 3 & 2 & 5 & 0 \end{bmatrix}$
(d) $\begin{bmatrix} 0 & 3 & 4 & 5 \\ 3 & 0 & 5 & 2 \\ 4 & 5 & 0 & 3 \\ 5 & 2 & 3 & 0 \end{bmatrix}$ [1]

8. Evaluate: $\int (x+3)(x+2)dx$

(a) $\frac{x^3}{3} + \frac{5x^2}{2} + 2x + C$
(b) $\frac{2}{5e^{2-5x}} + C$
(c) $-\frac{1}{5e^{3-5x}} + C$
(d) $-\frac{1}{6e^{3-5x}} + C$ [1]

9. Simple random sampling states that:
   (a) Members of sample are chosen at regular interval of population.
   (b) Every individual is chosen entirely by chance and every member of the population has an equal chance of being included in sample.
   (c) Every individual is not chosen entirely by chance and every member of the population has an equal chance of being included in sample.
   (d) Every individual is not chosen entirely by chance and every member of the population does not have an equal chance of being included in sample.   [1]

10. A researcher studying smartphone usage habits divides the population into three states based on age: 18 – 30, 31 – 50 and 51+. They then randomly select a proportional number of participants from each group. This is done to ensure:
   (a) The sample is easy to access.
   (b) Each age group is accurately represented in the sample.
   (c) The cost of sampling is minimized.
   (d) Every possible sample has an equal chance of being selected.   [1]

11. Write the coordinates of triangle ABC with vertices $A(4, -1)$, $B(3, 2)$, $C(2, -4)$ in matrix form.

(a) $\begin{bmatrix} 4 & 3 & 2 \\ -1 & 2 & -4 \end{bmatrix}$
(b) $\begin{bmatrix} 4 & 3 & 2 \\ -1 & 2 & 4 \end{bmatrix}$
(c) $\begin{bmatrix} 4 & 5 & 2 \\ -1 & 2 & -4 \end{bmatrix}$
(d) $\begin{bmatrix} 4 & 3 & 2 \\ -1 & 8 & -4 \end{bmatrix}$

Which of the above matrices correctly represents the coordinates of triangle ABC?

12. At what rate converted semi-annually will the present value of a perpetuity of ₹ 500 payable at the end of each 6 months be ₹ 30,000?
(a) 3.3%
(b) 3.2%
(c) 5.2%
(d) 5.6%   [1]

13. Let $A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$. What is adj (adj (A))?

(a) $\begin{pmatrix} 4 & -3 \\ -1 & 2 \end{pmatrix}$
(b) $\begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}$
(c) $\begin{pmatrix} -2 & -3 \\ -1 & -4 \end{pmatrix}$
(d) $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ [1]

14. Consider the differential equation:

$$\left(\frac{d^4y}{dx^4}\right)^2 + 3\frac{d^2y}{dx^2} - 7y = 0$$

What is the sum of the order and degree of this differential equation?
(a) 5
(b) 6
(c) 7
(d) 8   [1]

15. In a Linear Programming Problem (LPP), which of the following conditions must hold true for the existence of an optimal solution in a bounded feasible region?
   (a) The objective function must be linear and all constraints must be strict inequalities.
   (b) The feasible region must be convex and non-empty and the objective function must be continuous.
   (c) The feasible region must be a closed polygon with at least one corner point.
   (d) The objective function must be continuous and the feasible region must be non-convex.

16. Let $B\left(4, \frac{2}{3}\right)$, then n and p parameters of binomial distribution are:

(a) $n = 4, p = \frac{2}{3}$ (b) $p = 4, n = \frac{2}{3}$ (c) $n = 4, p = \frac{8}{3}$ (d) None of these   [1]

17. Let $A = \begin{bmatrix} 3 & 5 \\ -4 & 6 \end{bmatrix}$ then $2|A|$ is

(a) 23   (b) 32   (c) 76   (d) 56   [1]

18. A borrower is paying an EMI of ₹ 2,500 for a loan with a flat-rate interest of 10% per annum for 3 years. If the total repayment amount (including interest) is ₹ 90,000, what is the principal amount of the loan?

(a) ₹ 70,000   (b) ₹ 75,000   (c) ₹ 80,000   (d) ₹ 85,000

Questions number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below:

(a) Both Assertion (A) and Reason (R) are true and (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) but (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

19. Assertion (A): For a non-zero matrix A, of order $m \times n$, a matrix B of the same order. Then B is called the negative matrix of A if $B = -A$.

Reason (R): If $A + B = O$, where O is the zero matrix of the same order. Then B is called the negative matrix of A. [1]

20. Assertion (A): $E(X)$ of a random variable X, is the theoretical mean of X. Therefore, it is a parameter, not a statistic.

Reason (R): $E(X)$ is based on sample data, not on the distribution of the random variable X. [1]

SECTION-B

This section comprises of 5 very short answer (VSA) type questions of 2 marks each.

21. (A) If $a = 31, b = 21, m = 5$, then check $a \equiv b \pmod{m}$ [2]

OR

(B) Solve the following inequality:

$(-2x - 6) < 10$ [2]

22. A class XII has 20 students whose marks (out of 30) are 14, 17, 25, 14, 21, 17, 17, 19, 18, 26, 18, 17, 17, 26, 19, 21, 21, 25, 14 and 19 years. If random variable X denotes the marks of a selected student given that the probability of each student to be selected is equally likely.

Prepare the probability distribution of the random variable X. [2]

23. An investor puts ₹ 50,000 into a mutual fund, and the value grows to ₹ 1,00,000 over time. If the compound annual growth rate is 8%, calculate the number of complete years required for the investment to double. [Use $\log(2) = 0.301$ and $\log(1.08) = 0.034$] [2]

24. (A) A traffic engineer records the number of bicycle riders that use a particular cycle track. He records that an average of 3.2 bicycle riders use the cycle track every hour. Given that the number of bicycles that use the cycle track follow a Poisson distribution, what is the probability that 2 or less bicycle riders will use the cycle track within an hour? [Given $e^{-3.2} = 0.04076220$] [2]

OR

(B) A computer disk manufacturer tests disk quality on random basis before approving it. The approval is based on the number of errors in a test area on each disk and follows Poisson distribution with $\lambda = 0.2$. What is the percentage of test areas having two or a smaller number of errors? [Given $e^{-0.2} = 0.81873075$] [2]

25. An asset costing ₹ 15,000 is expected to have a useful life of 5 years and a scrap value of ₹ 3000. Find the annual depreciation charge using the straight-line method. [2]

SECTION-C

This section comprises of 6 short answer (SA) type questions of 3 marks each.

26. Find the speed of the boat, if a boat moves downstream at the rate of 16 km/h and upstream at the rate of 10 km/h. [3]

27. (A) Find critical points of the following function:

$f(x) = x^3 - 6x^2 + 9x - 10$

Also, classify the critical points. [3]

OR

(B) A toy manufacturing firm assesses its variable cost to be 'x' times the sum of 30 and 'x', where 'x' is the number of toys produced, also the cost incurred on storage is ₹ 1500. Find the total cost function and the marginal cost when 20 toys are produced. [3]

28. (A) Consider a bond with a coupon rate of 10% charged annually. The par value is ₹ 2,000 and the bond has 5 years to maturity. The yield to maturity is 11%. What is the value of the bond? [Given, $(1.11)^{-5} = 0.593451$] [3]

OR

(B) In 10 years, a machine costing ₹ 40,000 will have a salvage value of ₹ 4,000. A New Machine at that time is expected to sell for ₹ 52,000. In order to provide funds for the difference between the replacement cost and the salvage cost, a sinking fund is set up into which equal payments are placed at the end of each year. If the fund earns interest at the rate 7% compounded annually, how much should each payment be?

[Given, $(1.07)^{10} = 1.967151$] [3]

29. Two pipes can fill a cistern in 8 and 12 hours respectively. The pipes are opened simultaneously and it takes 12 minutes more to fill the cistern due to leakage. If the cistern is full, what will be the time taken by the leakage to empty it? [3]

30. Country A has an average farm size of 101 acres, while Country B has an average farm size of 109 acres. Assume the data were attained from two samples with standard deviations of 8 and 12 acres and sample sizes of 8 and 10, respectively. Is it possible to infer that the average size of the farms in the two countries is different at a 0.05? Assume that the populations are normally distributed. [Given, $t_{0.05}(18) = -2.101$] [3]

31. Anya crafts two types of handmade candles: luxurious Serenity Jars and vibrant Energize Tins. To plan her production for the month, she needs to figure out how many of each type to make to maximize her profit. Each Serenity Jar provides an ₹ 18 profit and requires 500 grams of wax, 40 ml of scent oil and 45 minutes of her labor. Each Energize Tin provides a ₹ 12 profit and requires 300 grams of wax, 25 ml of scent oil and 30 minutes of labor. For the upcoming month, she has 60,000 grams of wax, 6,000 ml of scent oil and 7,200 minutes of labor time available. Furthermore, based on her website's demand, she knows she cannot sell more than 100 Serenity Jars. Formulate a linear programming model to determine the optimal number of each candle Anya should produce, defining your decision variables, stating the objective function and writing all constraints as mathematical inequalities. [3]

SECTION-D

This section comprises of 4 long answer (LA) type questions of 5 marks each

32. (A) Three shopkeepers A, B and C are using polythene bags, handmade bags and newspaper bags. A uses 20, 30 and 40 number of bags of respective type. B uses 30, 40 and 20 of each respective kind while C uses 40, 20 and 30 of each type. Each shopkeeper spent ₹ 250, ₹ 220 and ₹ 200 on the bags. Find the cost of each carry bag using matrix method. [5]

OR

(B) A botanist is developing a special nutrient solution to boost the growth of a new hybrid plant. The solution requires precise amounts of three key nutrients: Nitrogen (N), Phosphorus (P) and Potassium (K).

The botanist mixes three commercial fertilizers to get the exact formula:

• Fertilizer A provides units of nutrients in the ratio $(N:P:K) = 2:1:1$
• Fertilizer B provides units of nutrients in the ratio $(N:P:K) = 1:2:1$
• Fertilizer C provides units of nutrients in the ratio $(N:P:K) = 1:1:2$

The desired final nutrient solution must contain a total of:

• 14 units of Nitrogen (N)

- 13 units of Phosphorus (P)
- 12 units of Potassium (K)

Let x, y and z represent the number of scoops of Fertilizer A, B and C used respectively.

(i) Set up the system of three linear equations representing the total amount of each nutrient.

(ii) Using Cramer's Rule only, find out how many scoops of each fertilizer (x, y and z) the botanist must use to achieve the exact nutrient blend.

33. A monopolist has a total cost function $TC = 20q + 10$ and faces a demand curve $q = 50 - 0.5p$.

(a) Find the profit-maximizing price and output before any tax.

(b) If a tax of ₹ 5 per unit is imposed on the monopolist, find the new profit-maximizing output and price.

(c) How much of the tax has been passed on to the consumer? [5]

34. (A) Based on the data available for the sales of an item in a district, by the method of least squares (i) tabulate the trend values (ii) find the best fit for a straight-line trend (iii) compute expected sale trend for year 2002. [5]

Year	Sales (in lakh)
1996	6.5
1997	5.3
1998	4.3
1999	6.1
2000	5.6
2001	7.8

OR

(B) Calculate the 3-year moving averages for the loans (in lakh ₹) issued by co-operative banks for farmers in different states of India based on the values given below. Plot the graph to represent the trend values. [5]

Year	2006	2007	2008	2009	2010	2011	2012	2013	2014
Loan amount (in lakh ₹)	41.85	40.2	38.12	26.5	55.5	23.6	28.36	33.31	41.1

35. Ms. Ananya Desai has been accepted into a prestigious university for a Master's program. The total tuition fee is ₹ 8,00,000. She pays an initial deposit of 25% of the fee and takes an education loan for the remaining amount. The loan has a tenure of 4 years and carries an annual interest rate of 8%, compounded monthly.

Calculate her monthly EMI using the reducing balance method and the total interest paid over the loan period.
[Use $(1.0067)^{48} \approx 1.38$]

SECTION-E

This section comprises of 3 case-study/passage-based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub parts of 2 marks each

36. The mathematics scores of a group of 800 students follow a normal distribution with a mean of 75 and a standard deviation of 10. Based on this data, answer the following questions:

(i) What percentage of students scored below 75 marks? [1]

(ii) Find the number of students who scored more than 90 marks. [1]

(iii) (A) Calculate the number of students scoring between 78 and 85 marks.

Use: $P(Z < 1.5) = 0.9332$, $P(Z < 0.3) = 0.6179$, $P(Z < 1.0) = 0.8413$. [2]

OR

(B) In a nationwide aptitude test, the top 5% of candidates receive a merit certificate. The Z-score for the 95th percentile is 1.645. Find the minimum score a candidate must achieve to earn the certificate.

37. In the smartphone market, the relationship between price and quantity demanded can be modelled using a linear demand function. Suppose the following information is available from market research:

At a price of ₹10,000 per unit, the quantity demanded is 500 units.

At a price of ₹15,000 per unit, the quantity demanded decreases to 300 units.

Based on the above information, answer the following questions:

(i) Formulate the linear demand function based on the given data. [1]

(ii) Suppose the supply function is given by

$$p_s = 2000 + \frac{x}{10}$$

determine the equilibrium price and quantity. [1]

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(iii) (A) Using integration, calculate the consumer surplus at the equilibrium price.

OR

(B) Using integration, calculate the producer surplus at the equilibrium price. [2]

38. Consider a Linear Programming Problem: Maximize $Z = 3x + 5y$

Subject to the constraints:

$$x + 2y \leq 40$$
$$3x + y \leq 45$$
$$x \geq 0, y \geq 0$$

(a) Solve the problem graphically to find the feasible region and its corner points. [2]

(b) If the objective function is changed to $Z = ax + 6y$ and the problem has multiple optimal solutions at (10, 15) and (0, 20), find the value of 'a'. [2]

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- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
- कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।
- Please check that this question paper contains **23** printed pages.
- Please check that this question paper contains **38** questions.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- **Please write down the serial number of the question in the answer-book before attempting it.**
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

व्यावहारिक गणित
APPLIED MATHEMATICS

निर्धारित समय : 3 घण्टे

अधिकतम अंक : 80

Time allowed : 3 hours

Maximum Marks : 80

Prepared By: Amit Mittal, PGT mathematics, St Francis School: Barut

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) This question paper contains **38** questions. **All** questions are **compulsory**.

(ii) This question paper is divided into **five** Sections – **A, B, C, D** and **E**.

(iii) In **Section A**, Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.

(iv) In **Section B**, Questions no. **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.

(v) In **Section C**, Questions no. **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.

(vi) In **Section D**, Questions no. **32** to **35** are Long Answer (LA) type questions carrying **5** marks each.

(vii) In **Section E**, Questions no. **36** to **38** are case study-based questions carrying **4** marks each.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E.

(ix) Use of calculator is **not** allowed.

SECTION-A

This section comprises of 18 multiple choice questions and two assertion and reason type questions of 1 mark each.

1. Consider the inequalities: [1]

$$I_1 : 2x + 3 < 7 \text{ and } I_2 : x - 1 \geq 0$$

Which of the following statements is correct?
(a) The solution set of I_1 is a subset of the solution set of I_2.
(b) The solution set of I_2 is a subset of the solution set of I_1.
(c) The solution sets of I_1 and I_2 are disjoint.
(d) The solution sets of I_1 and I_2 overlap but neither is a subset of the other.

2. What is the main advantage of stratified random sampling over simple random sampling? [1]
(a) It eliminates the need for randomness in selection.
(b) It ensures proportional representation of subgroups within the population.
(c) It reduces the overall cost of conducting the study.
(d) It is only used for populations with uniform characteristics.

3. The cost function for a company is given as $C(x) = x^3 - 3x^2 + 5x + 10$. Based on this function, which of the following is true for the marginal cost? [1]
(a) The marginal cost decreases for $0 < x < 1$
(b) The marginal cost attains its minimum at $x = 1$
(c) The marginal cost increases for $x > 1$
(d) The marginal cost is not constant, but changes quadratically with x.

4. The absolute minimum value of the function $f(x) = \ln(x) - x^2$ in the interval $[1, 3]$ is: [1]
(a) -8 (b) -6 (c) -4 (d) -2

5. In a one-sample t-test, the null hypothesis is rejected when: [1]
(a) The calculated t-value is smaller than the critical t-value.
(b) The calculated t-value is greater than the critical t-value.
(c) The calculated t-value lies outside the critical t-value range for a given significance level.
(d) The calculated t-value is equal to the sample mean.

6. For the system of equations: [1]

$$2x + 3y = 8$$
$$4x - y = 2$$

What is the value of the variable y obtained using Cramer's Rule?

(a) $y = 1$　　　　(b) $y = 2$　　　　(c) $y = -1$　　　　(d) $y = -2$

7. A random variable X follows a discrete probability distribution given by the following probability mass function (pmf):

$$P(X = x) = \begin{cases} \dfrac{1}{4}, & x = 1 \\ \dfrac{1}{4}, & x = 2 \\ \dfrac{1}{4}, & x = 3 \\ \dfrac{1}{4}, & x = 4 \end{cases}$$

What is the expected value of X? [1]

(a) 2.5　　　　(b) 3　　　　(c) 4　　　　(d) 3.5

8. A company purchases a machine for ₹ 50,000. The machine is expected to have a useful life of 10 years, after which it will have a scrap value of ₹ 5,000. The company uses linear depreciation to calculate the depreciation expense each year. Which of the following is the correct depreciation expense for the 7th year? [1]

(a) ₹ 4,500　　　　(b) ₹ 4,000　　　　(c) ₹ 5,000　　　　(d) ₹ 3,500

9. A milkman mixes water with milk and sells the mixture at the cost price of milk. If he mixes 2 litres of water with 8 litres of milk, what is his gain percentage? [1]

(a) 15%　　　　(b) 20%　　　　(c) 25%　　　　(d) 30%

10. In a large batch of smartphones, 12% are found to be defective. A random sample of 120 smartphones is taken with replacement for inspection. What is the variance of the number of defective smartphones in the sample? [1]

(a) 12.672　　　　(b) 10.56　　　　(c) 11.52　　　　(d) 13.44

11. What is the remainder when 2^{27} is divided by 9? [1]

(a) 1　　　　(b) 2　　　　(c) 8　　　　(d) 5

12. A tank can be filled by a pipe in 6 hours. Another pipe can empty the same tank in 9 hours. If both pipes are opened together, in how much time will the tank be filled? [1]

(a) 12 hours　　　　(b) 18 hours　　　　(c) 9 hours　　　　(d) 36 hours

13. What is the primary goal of time series forecasting? [1]

(a) To identify the trend component of the data　　　(b) To predict future values based on historical data
(c) To estimate the seasonal component of the data　　　(d) To smooth the time series data

14. The present value of a series of payments of ₹ 500 made at the end of every 3 months and continuing forever. If the interest rate is 8% per annum compounded quarterly, what is the present value of the sequence? [1]

(a) ₹ 25,000　　　　(b) ₹ 20,000　　　　(c) ₹ 30,000　　　　(d) ₹ 40,000

15. It is known that $a > b$. Which of the following statements must always be true? [1]

(a) $a^2 > b^2$　　　(b) $a + c > b + c$　　　(c) $ac > bc$　　　(d) $\dfrac{1}{a} < \dfrac{1}{b}$

16 Let A be a 4×4 matrix such that $\det(A) = 5$ and let $B = A^T . A$ (the product of A and its transpose). What is the value of $\det(B)$? [1]

(a) 25　　　　(b) 5　　　　(c) 625　　　　(d) 125

17. Let A be a 3×3 matrix such that $A^2 = 2A - I$, where I is the identity matrix. Then what is A^4? [1]

(a) $4A - 3I$　　　(b) $2A^3 - I$　　　(c) $8A - 7I$　　　(d) $6A - 5I$

18. The order and degree of the differential equation: [1]

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$$

(a) Order = 1, Degree = 1　　(b) Order = 2, Degree = 3　　(c) Order = 2, Degree = 1　　(d) Order = 1, Degree = 3

Questions number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below:

(a) Both Assertion (A) and Reason (R) are true and (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) but (R) is not the correct explanation of Assertion (A).

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(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

19. Assertion (A): A sinking fund is a financial arrangement where a company or individual sets aside money over time to repay a debt or make a large future expenditure. [1]

Reason (R): The sinking fund involves periodic deposits of a fixed amount, which when compounded at a specified rate, accumulate to the required future value to meet the future liability or expenditure.

20. Assertion (A): If A is a symmetric matrix, then $|A| = |A^T|$. [1]

Reason (R): The determinant of the transpose of a matrix is equal to the determinant of the matrix.

SECTION-B

This section comprises of 5 very short answer (VSA) type questions of 2 marks each.

21. (A) A chemist has two solutions of acid: one is 30% acid, and the other is 50% acid. How many litres of each solution should be mixed to obtain 20 litres of a 40% acid solution? [2]

OR

(B) A pipe can fill a tank with water in 4 hours. However, due to a leakage, it took $5\frac{1}{3}$ hours to fill the tank. How much time will the leakage take to empty the tank completely if no other pipe is running?

22. In a race, A can run 100 metres in 10 seconds and B can run 100 metres in 12 seconds. If they start the race at the same time, how much distance will A be ahead of B when A finishes the race? [2]

23. A boat travels 40 km downstream in 4 hours and returns upstream in 5 hours. Find the speed of the boat in still water and the speed of the stream. [2]

24. (A) In a factory, 80% of the products manufactured are defect-free and 20% are defective. If 6 products are selected randomly for quality testing, what is the probability that exactly 4 of them are defect-free? [2]

OR

(B) The height of adult women in a city follows a normal distribution with a mean of 160 cm and a standard deviation of 8 cm. Find the probability that a randomly selected woman has a height:

(i) less than 150 cm

(ii) between 150 cm and 170 cm. (Use $P(Z < -1.25) = 0.1056$ and $P(Z < 1.25) = 0.8944$)

25. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, find $A^2 - 5A + 4I$ and hence find a matrix X such that $A^2 - 5A + 4I + X = 0$. [2]

SECTION-C

This section comprises of 6 short answer (SA) type questions of 3 marks each.

26. A factory produces at least 40 units of a product daily to meet market demand. The cost of producing x units is given by the function $C(x) = 5x + 100$, where x is the number of units produced in a day. If the factory wants to keep the daily production cost below ₹ 600, find the range of units that can be produced. [3]

27. (A) Two machines in a factory are tested for their mean output rate. Assuming that the output rates follow a normal distribution with unknown but equal variances, test the hypothesis that the mean output rates of the two machines are the same. The following data is collected: [3]

Machine	Sample Size (n)	Mean Output Rate (x̄)	Standard Deviation (s)
Machine A	12	260 units/hour	15 units/hour
Machine B	10	260 units/hour	18 units/hour

Use $t_{20}(0.05) = 2.086$ for the critical value of the t-distribution.

OR

(B) A fitness trainer claims that the average weight loss of clients following a specific program is 10 kg in 3 months. To verify this claim, a random sample of 8 clients is tested and the weight loss (in kg) is recorded as: 9, 11, 8, 7, 12, 10, 9 and 6. Test the validity of the trainer's claim at 5% level of significance.

[Given $t_7(0.05) = 2.365$]

28. A café manager records the number of customers arriving during lunchtime. On average, 4 customers arrive at the café every 30 minutes. Assume that the number of customer arrivals follows a Poisson distribution. [3]

(a) What is the probability that exactly 2 customers will arrive within 30 minutes?

(b) What is the probability that 3 or fewer customers will arrive within 30 minutes?
(Given $e^{-4} = 0.0183$)

29. Mr Sameer started a savings account with a deposit of ₹ 20,000 at the beginning of 2020. By the end of 2021, his account balance grew to ₹ 30,000. Due to a financial need, he withdrew ₹ 5,000 at the end of 2021, leaving ₹ 25,000 in the account at the start of 2022. Over the next two years, the account balance grew steadily, reaching ₹ 40,000 by the end of 2023. [3]

Calculate the CAGR (Compounded Annual Growth Rate) of his savings over the four years.

(Use $(2)^{\frac{1}{4}} = 1.1892$)

30. A small bakery produces cookies and cakes. The total number of cookies and cakes it can produce daily cannot exceed 50. It takes 45 minutes to bake a cake and 15 minutes to prepare a batch of cookies. The bakery operates for a maximum of 10 hours a day. The profit per batch of cookies is ₹ 50 and the profit per cake is ₹ 200. [3]
Formulate an LPP to determine how many batches of cookies and cakes the bakery should produce daily to maximise profit, given that at least one batch of cookies and one cake must be produced daily.
(Note: No need to solve for the feasible region or optimal solution.)

31. (A) A cylindrical die with a square base is rolled along its rectangular faces. The die has 4 faces numbered 1, 2, 3 and 4, and it can land with any of these faces on the bottom. Let X denote the number shown on the bottom face after the die is rolled and the following probability distribution of X is given: [3]

X	1	2	3	4
P(X)	p	2p	3p	1p

Given that the total probability is 1, answer the following:

(i) Find the value of p.

(ii) Calculate the mean, E(X), of the distribution.

OR

(B) A person is taking part in a quiz competition. The probability of answering a question correctly on any given trial is 0.04. How many minimum questions must the person attempt to have a probability of at least one correct answer greater than or equal to 0.75?

(Use $\log_{10} 0.96 = -0.0177$ and $\log_{10} 0.25 = -0.6021$)

SECTION-D

This section comprises of 4 long answer (LA) type questions of 5 marks each

32. (A) The monthly sales figures (in ₹ thousands) for a grocery store over 12 months are given below:

Month	Sales
Jan	120
Feb	150
Mar	170
Apr	140
May	180
Jun	200
Jul	210
Aug	190
Sep	220
Oct	230
Nov	240
Dec	250

The store manager wants to analyse the sales trends to improve stock management and promotional strategies. Calculate the 3-month moving averages for the given data. [3]

Identify the trend in sales over the year based on the moving averages. [2]

OR

(B) A small-scale electronics manufacturing company tracks the number of units sold per year and their respective prices over the past 7 years. The data is as follows:

Year	Number of Units Sold (in thousands)	Price per Unit (₹)
2016	42	45
2017	45	50
2018	50	55
2019	55	60
2020	60	65
2021	65	70
2022	70	75

Using the least squares method, find the straight-line trend equation for the number of units sold in relation to the price per unit. Then, calculate the trend values.

33. Ravi owns a small-scale bakery that specializes in two types of cakes: Chocolate Delight and Vanilla Supreme. Each cake requires three key resources: flour, sugar and labour hours. Ravi wants to maximise his profits while ensuring the production constraints are met.

Details:

(1) Profit per cake:
   - Chocolate Delight: ₹ 40 per cake
   - Vanilla Supreme: ₹ 50 per cake

(2) Resources required per cake:

Resource	Chocolate Delight	Vanilla Supreme	Available per day
Flour (grams)	200	300	15,000
Sugar (grams)	100	200	10,000
Labour (hours)	1.5	2	90

Based on above information, answer the following questions:

(a) Considering the number of Chocolate Delight (x) and Vanilla Supreme (y) cakes and profit should be maximised, formulate the linear inequalities for the given scenario and explain their significance in Ravi's production constraints. [2]

(b) Ravi decides to test a solution where he produces 40 Chocolate Delight cakes and 20 Vanilla Supreme cakes. Verify if this solution satisfies all the constraints. If it does, calculate the total profit. If not, identify which constraint is violated. [3]

34. A company, XYZ Electronics, manufactures and sells smartphones. The company has observed the following data points for its smartphone sales:
   - When the price of a smartphone was ₹ 25,000, they sold 30 units per month.
   - When the price dropped to ₹ 20,000, they sold 80 units per month.
   - When the price was ₹ 35,000, there were 150 smartphones available for sale each month.
   - When the price was reduced to ₹ 25,000, there were only 50 smartphones remaining for sale per month.

(i) Find the demand function for smartphones in the form of $D(x) = ax + b$, where x is the number of smartphones sold and $D(x)$ is the price of the smartphone. [2]

(ii) Calculate the consumer surplus if the supply function is given as $S(x) = 120x + 4000$, where $S(x)$ represents the price at which x smartphones. [3]

35. A new car is expected to cost ₹ 75,000 at the time of purchase. After 5 years, the car's estimated salvage value is ₹ 30,000. To prepare for the difference between the purchase cost and the salvage value, a sinking fund is established. Equal payments are deposited at the end of each year into this fund, which earns interest at an annual rate of 6% compounded annually. [5]

You are required to:

(i) Determine how much should be paid into the sinking fund each year to accumulate the difference between the purchase price and the salvage value.

(ii) Find the annual depreciation of the car's value over 5 years using the straight-line method.

(iii) Calculate the rate of depreciation under the straight-line method.

[Use $(1.06)^5 = 1.3382$ for the calculation.]

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This section comprises of 3 case-study/passage-based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub parts of 2 marks each

36. A company manufactures a product, and its cost and revenue functions depend on the number of units produced and sold. The company wants to maximise its profit and minimise its cost. The cost and revenue functions are given below:

- The cost function $C(x)$ represents the total cost of producing x units of the product. It is given by:

$$C(x) = 50x + 1000$$

Where

x = number of units produced.

$50x$ is the variable cost per unit and 1000 is the fixed cost.

- The revenue function $R(x)$ represents the total revenue earned by selling x units of the product. It is given by:

$$R(x) = 200x - 0.5x^2$$

Where,

x = number of units sold.

$200x$ is the revenue per unit and $-0.5x^2$ accounts for diminishing returns as the quantity sold increases.

- The profit function $P(x)$ is given by:

$$P(x) = R(x) - C(x)$$

Where,

$P(x)$ = profit for x units produced and sold.

Based on above information, answer the following questions:

(i) Find the number of units x that maximises the revenue function $R(x)$. [1]

(ii) Determine the total revenue when 30 units are sold. [1]

(iii) (A) Find the number of units x that maximises the profit $P(x) = R(x) - C(x)$. Also, calculate the maximum profit.

[2]

OR

(B) The company wants to minimise its cost. Find the number of units x that minimises the cost function $C(x)$ and calculate the minimum cost. [2]

37. Riya is a young professional who recently started her first job. She plans to purchase a high-performance laptop costing ₹ 60,000 to support her work and personal projects. Since her savings are limited, she decides to take a loan from a bank. The bank offers her an EMI scheme with an annual interest rate of 12% for a tenure of 2 years. Riya is curious about how much she will need to pay as EMI each month. The bank uses the following formula to calculate the EMI:

$$EMI = \frac{P.r.(1+r)^n}{(1+r)^n - 1}$$

Where

- P = Loan amount (Principal)
- r = Monthly interest rate
- n = Loan tenure in months

Help Riya calculate her EMI and answer some additional questions about her loan.

(i) What is the value of r (the monthly interest rate) for this case? [1]

(ii) How many total EMIs will Riya need to pay over the loan period? [1]

(iii) (A) Using the formula provided, calculate the EMI amount (up to two decimal places). Show all necessary steps in your calculations.

[Use $(1.01)^{24} = 1.26793$] [2]

OR

(B) If Riya increases her loan tenure to 3 years while keeping the same interest rate, will her EMI increase or decrease? Explain why, without performing detailed calculations. [2]

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38. One of the prominent applications of matrices is in cryptography. Cryptography is a type of secure communication where a message is transmitted from a sender to a receiver.
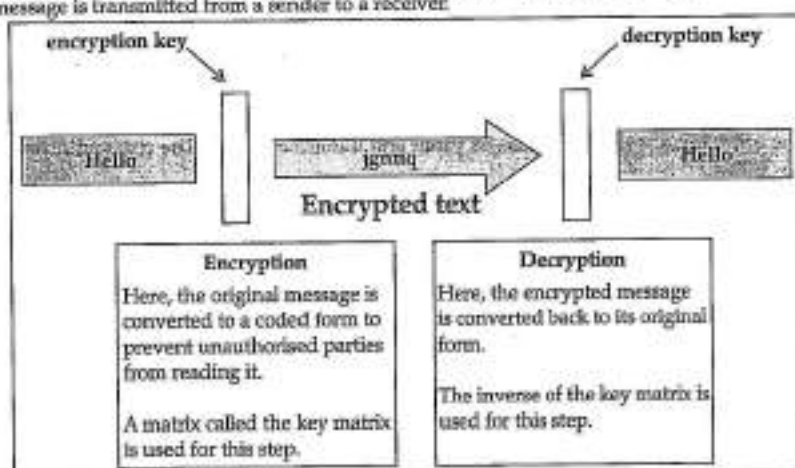
An encryption process with a key matrix, $K = \begin{bmatrix} 3 & 7 \\ 4 & 1 \end{bmatrix}$ is shown below.

Assume the coordinates of a location, $P = (4, 18)$, is to be encrypted. Matrix multiplication is performed between the key matrix and the coordinates to obtain the encrypted form of P as:

$$K \times P = \begin{bmatrix} 3 & 7 \\ 4 & 1 \end{bmatrix} \times \begin{bmatrix} 4 \\ 18 \end{bmatrix} = \begin{bmatrix} 138 \\ 34 \end{bmatrix}$$

Decryption is carried out by using the inverse of the key matrix as:

$K^{-1} \times$ encrypted matrix $=$ original matrix

(i) Using the same key matrix, K, find the encrypted form of $Q = (19, -20)$.   [2]

A new key matrix, $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, is proposed for the encryption system. For the process to work, this matrix must be invertible. State the necessary and sufficient mathematical condition that the elements of matrix M must satisfy for it to be invertible.

(ii) Find the matrix used for decryption corresponding to K. Show your work.   [2]

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- इस प्रश्न–पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न–पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न–पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर–पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।
- Please check that this question paper contains **23** printed pages.
- Please check that this question paper contains **38** questions.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- **Please write down the serial number of the question in the answer-book before attempting it.**
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

व्यावहारिक गणित
APPLIED MATHEMATICS

निर्धारित समय : 3 घण्टे

Time allowed : 3 hours

अधिकतम अंक : 80

Maximum Marks : 80

Prepared By: Amit Mittal, PGT mathematics, St Francis School: Barut

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i)   This question paper contains **38** questions. **All** questions are **compulsory**.

(ii) This question paper is divided into **five** Sections – **A, B, C, D** and **E.**

(iii) In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of 1 mark each.

(iv) In **Section B,** Questions no. **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.

(v)   In **Section C,** Questions no. **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.

(vi) In **Section D,** Questions no. **32** to **35** are Long Answer (LA) type questions carrying **5** marks each.

(vii) In **Section E,** Questions no. **36** to **38** are case study-based questions carrying **4** marks each.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E.

(ix) Use of calculator is **not** allowed.

SECTION-A

This section comprises of 18 multiple choice questions and two assertion and reason type questions of 1 mark each.

1. Consider the inequalities: [1]
$$I_1 : 2x + 3 < 7 \text{ and } I_2 : x - 1 \geq 0$$
Which of the following statements is correct?
   (a) The solution set of I_1 is a subset of the solution set of I_2.
   (b) The solution set of I_2 is a subset of the solution set of I_1.
   (c) The solution sets of I_1 and I_2 are disjoint.
   (d) The solution sets of I_1 and I_2 overlap but neither is a subset of the other.

2. What is the main advantage of stratified random sampling over simple random sampling? [1]
   (a) It eliminates the need for randomness in selection.
   (b) It ensures proportional representation of subgroups within the population.
   (c) It reduces the overall cost of conducting the study.
   (d) It is only used for populations with uniform characteristics.

3. The cost function for a company is given as $C(x) = x^3 - 3x^2 + 5x + 10$. Based on this function, which of the following is true for the marginal cost? [1]
   (a) The marginal cost decreases for $0 < x < 1$
   (b) The marginal cost attains its minimum at $x = 1$
   (c) The marginal cost increases for $x > 1$
   (d) The marginal cost is not constant, but changes quadratically with x.

4. The absolute minimum value of the function $f(x) = \ln(x) - x^2$ in the interval $[1, 3]$ is: [1]
   (a) -8 (b) -6 (c) -4 (d) -2

5. In a one-sample t-test, the null hypothesis is rejected when: [1]
   (a) The calculated t-value is smaller than the critical t-value.
   (b) The calculated t-value is greater than the critical t-value.
   (c) The calculated t-value lies outside the critical t-value range for a given significance level.
   (d) The calculated t-value is equal to the sample mean.

6. For the system of equations: [1]
$$2x + 3y = 8$$
$$4x - y = 2$$

(c) The time series is completely random

(d) The time series does not have any trend

14. The present value of a perpetuity is ₹ 40,000, and the payment made at the end of each year is ₹ 2,000. What is the rate of interest per annum? [1]

(a) 5% (b) 6% (c) 7% (d) 8%

15. Given the equation $y = Ce^{kx}$, where C and k are constants, what is the differential equation obtained by eliminating the constants C and k? [1]

(a) $\dfrac{d^2y}{dx^2} = \dfrac{\left(\dfrac{dy}{dx}\right)^2}{y}$ (b) $\dfrac{d^2y}{dx^2} = y$ (c) $\dfrac{dy}{dx} = \dfrac{1}{y}$ (d) $\dfrac{d^2y}{dx^2} = \dfrac{dy}{dx}$

16. Let A be an invertible $n \times n$ matrix. Which of the following is equal to $(A^{-1})^T$? [1]

1. $(A^T)^{-1}$ 2. $(A^{-1})^2$ 3. $\dfrac{1}{\det(A)}\text{adj}(A^T)$ 4. $A^T A^{-1}$

(a) Only 1 (b) Only 1 and 3 (c) Only 2 and 4 (d) Only 3

17. Let A be a 2×2 matrix such that $A^2 - 5A + 6I = 0$, where I is the identity matrix. Find A^{-1}. [1]

(a) $\dfrac{1}{6}(5I - A)$ (b) $\dfrac{1}{6}(6I - A)$ (c) $\dfrac{1}{5}(5I + A)$ (d) $\dfrac{1}{5}(6I + A)$

18. For the equation: [1]

$$\dfrac{d^2y}{dx^2} + \sin(x)\dfrac{dy}{dx} + y = 0$$

What is the order and degree of the differential equation?

(a) Order = 2, Degree = 1

(b) Order = 1, Degree = 1

(b) Order = 2, Degree = not defined

(d) Order = 1, Degree = 2

Questions number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below:

(a) Both Assertion (A) and Reason (R) are true and (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) but (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

19. Assertion (A): The EMI is used in loans where the principal and interest are repaid through fixed monthly payments, while a sinking fund is used for accumulating funds for future large expenses. [1]

Reason (R): The difference between an EMI and a sinking fund is that in an EMI, each payment contributes towards both the principal and the interest, whereas in a sinking fund, the amount deposited grows at a specified interest rate to meet future liabilities.

20. Assertion (A): If A is a 3×3 matrix such that $adj(A) = kA$, where k is a scalar, then A is singular if $k = 0$. [1]

Reason (R): A matrix A is singular if and only if its determinant is zero.

SECTION-B

This section comprises of 5 very short answer (VSA) type questions of 2 marks each.

21. (A) In what ratio must water be added to fruit juice costing ₹ 80 per litre so that the resulting mixture is worth ₹ 64 per litre? [2]

OR

(B) A tank is being filled by two pipes, A and B. Pipe A can fill the tank in 8 hours, and pipe B can fill the tank in 12 hours. If both pipes are open together, a leakage at the bottom of the tank causes it to take 6 hours longer to fill than it would have without the leakage. In how much time the tank will fill with leakage?

22. In a 500 metre race, A can finish the race in 50 seconds, while B can finish the race in 60 seconds. If they both start the race together and run at their constant speeds, how much distance will B be behind A when A finishes the race? [2]

23. A boat can go 30 km upstream and 40 km downstream in 5 hours. The speed of the stream is 2 km/h. Find the speed of the boat in still water. [2]

24. (A) In a survey, 60% of people prefer tea over coffee. If 8 people are randomly selected, what is the probability that exactly 5 of them prefer tea? [2]

OR

(B) The test scores of a group of students are normally distributed with a mean of 75 and a standard deviation of 10. Find the probability that a randomly selected student will have:

(i) a score greater than 85

(ii) a score between 70 and 80.

(Use $P(Z > 1) = 0.8413$ and $P(Z < 0.5) = 0.6915$)

25. A company produces two types of products, A and B, each requiring time on two machines, M_1 and M_2, for production. The time required (in hours) for each product on each machine is given by the matrix T [2]

$$T = \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}$$

where:

The first row represents the time required for product A on machines M_1 and M_2 (2 hours and 3 hours respectively).
The second row represents the time required for product B on machines M_1 and M_2 (3 hours and 2 hours, respectively).

If the company operates the machines for a total of 100 hours each, determine how many units of product A and product B can be produced.

SECTION-C

This section comprises of 6 short answer (SA) type questions of 3 marks each.

26. A digital lock system accepts a 4-digit PIN. The system only checks whether the entered PIN is congruent modulo 9 with the correct PIN, instead of verifying the exact number (to reduce computation). [3]

If the correct PIN is 7531 ,

(a) What remainder must a valid entered PIN give when divided by 9?

(b) Instead of listing specific examples, determine in general how many distinct 4-digit PINs will unlock the system.

(c) Discuss one practical risk of using such a congruence-based security system.

27. (A) Two branches of a retail chain test the average time it takes to process customer orders. Assuming the processing times follow a normal distribution with unknown but equal variances, test the hypothesis that the mean processing times are the same for both branches. The data is as follows: [3]

Branch	Sample Size (n)	Mean Processing Time (minutes, $\bar{x}$)	Standard Deviation (s)
Branch A	16	12	3
Branch B	14	10	2.5

Use $t_{28}(0.05) = 2.048$ for the critical value of the t-distribution.

OR

(B) A car manufacturer claims that their electric vehicle has an average range of 350 km on a single charge. A consumer group tests this claim by measuring the range (in km) of a random sample of 7 vehicles and the results are as follows: 340, 360, 355, 345, 330, 325 and 350. Test the validity of the manufacturer's claim at the 5% level of significance.
[Given, $t_6(0.05) = \pm 2.447$]

28. A security system at a museum records the number of visitors entering an exhibit every hour. On average, 5 visitors enter the exhibit per hour. Assuming the number of visitors follows a Poisson distribution: [3]

(I) What is the probability that no more than 2 visitors enter the exhibit in an hour?

(ii) Determine the mean and variance for the number of visitors.

(Given, $e^{-5} = 0.0067$)

29. Mrs Kavya invests ₹ 50,000 in a diversified equity mutual fund at the beginning of 2018. Over the years, the value of her investment changes as follows: [3]

End of 2018: ₹ 45,000 (due to a market correction)
End of 2019: ₹ 60,000 (market recovery and growth)
End of 2020: ₹ 72,000 (continued growth)
End of 2021: ₹ 90,000 (strong returns due to market rally)

Calculate the Compounded Annual Growth Rate (CAGR) of her investment over the 4 years.

(Use $(1.8)^{\frac{1}{4}} = 1.1583$)

30. A workshop produces two types of furniture: chairs and tables. The workshop can produce at most 30 pieces of furniture per day due to space constraints. Producing a chair requires 2 hours of labour and producing a table requires 5 hours of labour. The workshop has a maximum of 80 labour hours available daily. The profit per chair is ₹ 150 and the profit per table is ₹ 500.

Formulate a Linear Programming Problem (LPP) to determine how many chairs and tables should be produced daily to maximise profit, given that at least 5 chairs must be produced daily. [3]

(Note: No need to solve for the feasible region or optimal solution.)

31. (A) A factory produces two types of bulbs: Type A and Type B. The probability distribution of the number of defective bulbs in a sample of 10 bulbs is given in the table below, where X represents the number of defective bulbs: [3]

X	0	1	2	3	4
P(X)	p	2p	3p	4p	5p

Given that the total probability sums to 1, answer the following:
(i) Find the value of p.
(ii) Find the mean, $E(X)$, of the distribution.

OR

(B) A scientist is conducting an experiment where the probability of a faulty sensor giving an incorrect reading in a single test is 0.03. How many minimum tests must be performed so that the probability of having at least one faulty sensor reading is 0.8 or more?

(Use $\log_{10} 0.2 = -0.69897$ and $\log_{10} 0.97 = -0.01322$)

SECTION-D

This section comprises of 4 long answer (LA) type questions of 5 marks each

32. (A) The yearly revenues (in ₹ crores) of a renewable energy company over the past 8 years are as follows:

Year	2016	2017	2018	2019	2020	2021	2022	2023
Revenue (₹ cr)	25	28	32	30	35	40	38	45

The company is analysing its growth trend over these years to plan for future expansions.
(i) Calculate the 4-year moving averages for the given revenue data. [2]
(ii) Plot a trend line graph based on the calculated moving averages and interpret the growth trend. [3]

OR

(B) The annual revenue (in ₹ crores) of a food processing company for the past six years is as follows:

Year	2018	2019	2020	2021	2022	2023
Revenue (₹ crores)	50	55	60	62	68	72

The company wants to identify a trend in its revenue growth over the years using the method of least squares.
(i) Fit a straight-line trend in the form $Y = a + bX$, where Y is the revenue and X represents the year. [2]
(ii) Use the fitted trend equation to calculate the trend values for each year. [3]

33. (A) The equation of the path traversed by the ball headed by the footballer is $y = ax^2 + bx + c$ (where $0 \le x \le 14$ and $a, b, c \in R$ and $a \ne 0$) with respect to a XY-coordinate system in the vertical plane. The ball passes through the points (2, 15), (4, 25) and (14, 15). Determine the values of a, b and c by solving the system of linear equations in a, b and c, using matrix method. Also find the equation of the path traversed by the ball. [5]

OR

(B) A farmer grows two types of crops, Crop A and Crop B, on his farm. Each crop requires a certain amount of water and fertiliser, and the farmer has a limited amount of both resources available. The farmer has 400 litres of water and 600 kg of fertiliser available for the season. The following constraints describe the water and fertiliser requirements for both crops:

Crop A requires 10 litres of water and 20 kg of fertilizer per hectare.

Crop B requires 15 litres of water and 25 kg of fertiliser per hectare.

The farmer wants to plant both crops while maximising the use of the available resources. How many hectares of Crop A and Crop B should the farmer plant? Use Cramer's rule to solve the question.

34. A company, ABC Furniture Ltd., manufactures wooden tables. The company has the following data:

When the price of a wooden table is ₹ 8,000 , the company supplies 50 tables per month.

When the price increases to ₹ 10,000, the company supplies 70 tables per month.

The supply function is assumed to be linear, i.e., $S(x) = ax + b$, where x is the number of tables supplied and $S(x)$ is the price at which these tables are sold.

The demand function is given by $D(x) = 15000 - 100x$, where x is the number of tables demanded and $D(x)$ is the price consumers are willing to pay.

You are required to:

(i) Find the supply function $S(x)$ using the given data points. [2]

(ii) Calculate the producer surplus. [3]

35. A furniture manufacturing company produces office desks and anticipates that a desk will cost ₹ 20,000 at the time of purchase. After a period of 6 years, the desk's residual value is expected to be ₹ 6,000. To prepare for the difference between the initial price and the residual value, the company creates a sinking fund where equal annual contributions are made at the end of each year. The fund earns an interest rate of 5% compounded annually.

(i) Calculate the annual contribution required to the sinking fund to cover the difference between the original cost and the residual value. [2]

(ii) Compute the annual depreciation of the desk's value using the straight-line depreciation method over the 6-year period. [2]

(iii) Determine the annual depreciation rate using the straight-line method. [1]

You may use the value $(1.05)^6 = 1.3401$ in your calculations.

SECTION-E

This section comprises of 3 case-study/passage-based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub parts of 2 marks each

36. A particle is moving along a straight line, and its position at any time t seconds is given by the equation $s(t) = t^3 - 6t^2 + 9t + 1$ where $s(t)$ is the position in metres. Scientists are studying the particle's motion to understand its behaviour, such as when it stops, changes direction, and its rate of acceleration. The particle's velocity and acceleration at different times provide key insights into its dynamics and can help predict its future movement.

Based on above information, answer the following questions:

(i) Find the velocity function $v(t)$ of the particle. [1]

(ii) Determine the time(s) t when the particle is at rest. [1]

(iii) (A) Determine the time interval(s) during which the particle is moving in the positive direction (i.e., when $v(t) > 0$). Clearly justify your answer by solving the inequality. [2]

OR

(B) Derive the acceleration function $a(t)$, which represents the rate of change of velocity, and calculate the acceleration of the particle at $t = 2$. [2]

37. Amit wants to renovate his home and takes a loan of ₹ 8,00,000 from a bank. The bank offers two loan repayment options:

Option 1: Repayment over 5 years at an annual interest rate of 9%.

Option 2: Repayment over 7 years at an annual interest rate of 10%.

The bank calculates the EMI for both options using the formula:

$$EMI = \frac{P \cdot r \cdot (1+r)^n}{(1+r)^n - 1}$$

Where:

P = Loan amount (Principal)

r = Monthly interest rate

n = Loan tenure in months

Amit is confused about which option to choose and approaches you for help.

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Based on above information, answer the following questions:

(i) For Option 1, calculate the monthly interest rate (r) and the number of EMIs Amit needs to pay. [1]

(ii) For Option 1, calculate the monthly interest rate (r) and the number of EMIs Amit needs to pay. [1]

(iii) (A) Compare the two options by calculating the EMI for each. Based on the monthly payments, which option is better for Amit if he prefers lower monthly payments? Show all calculations.
(Use $1.0075^{60} \approx 1.56568$, $1.00833^{84} \approx 2.00736$) [2]

OR

(B) Without calculating the EMI, explain which option might result in a higher total interest payment over the loan tenure. Support your reasoning conceptually. [2]

38. A logistics company operates in a city and delivers two types of parcels: Standard Packages and Express Packages. To maximise efficiency and profits, the company must allocate its delivery fleet, time and fuel resources effectively.

Details:

(1) Revenue per delivery:
Standard Package: ₹ 100
Express Package: ₹ 150

(2) Resources required per delivery:

Resource	Standard Package	Express Package	Maximum Available Daily
Fleet Capacity (vans)	1	1	20
Delivery Time (hours)	2	3	50
Fuel Consumption (litres)	4	5	80

Assume that the company should deliver Standard Packages (x) and Express Packages (y) daily in order to maximise revenue while meeting all resource constraints.

Based on above information, answer the following questions:

(i) Formulate the Linear Programming Problem (LPP) for the company, including the objective function and all constraints. Explain how the constraints reflect the resource limitations of the company. [2]

(ii) Discuss a scenario where the company delivers 10 Standard Packages and 8 Express Packages. Without solving, analyse which resource (if any) would be exhausted first and why. Support your answer with calculations and reasoning. [2]

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SET – 5

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Candidates must write the Q.P Code on the title page of the Answer book.

व्यावहारिक गणित
APPLIED MATHEMATICS

निर्धारित समय : 3 घण्टे

अधिकतम अंक : 80

Time allowed : 3 hours

Maximum Marks : 80

Prepared By: Amit Mittal, PGT mathematics, St Francis School: Barut

1. This question paper contains 38 questions. All questions are compulsory.
2. Question paper is divided into FIVE sections-Section A, B, C, D and E.
3. In Section A, Question number 1 to 18 are Multiple Choice Questions (MCQs) and Question number 19 and 20 are Assertion-Reason based questions carrying 1 mark each.
4. In Section B, Question number 21 to 25 are Very Short Answer (VSA) type questions carrying 2 marks each.
5. In Section C, Question number 26 to 31 are Short Answer (SA) type questions carrying 3 marks each.
6. In Section D, Question number 32 to 35 are Long Answer (LA) type questions carrying 5 marks each.
7. In Section E, Question number 36 to 38 are Case Study Based question carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 2 questions in Section E.
9. Use of calculator is NOT allowed.

Sr. No.	SECTION–A	Marks
1	The value of $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} dx$ is (a) 0 (b) $\frac{a}{4}$ (c) $\frac{a}{2}$ (d) a	1
2	A specific characteristic of a sample is known as (a) Population (b) parameter (c) statistic (d) variance	1
3	The demand for a certain product is represented by the function $P=200+20x-x^2$ where x is the number of units demanded and p is the price in rupee per unit. than the marginal revenue when 10 units are sold is (a) 0 (b) 100 (c) 200 (d) 300	1
4	If an edge of a variable cube is increasing at the rate of 0.5 cm/sec, at what rate is its surface area increasing when its edge is 12 cm ? (a) 12 cm²/s (b) 60 cm²/s (c) 70 cm²/s (d) 72 cm²/s	1

5	Consider the following hypothesis test : $H_0 : \mu = 16$ $H_a : \mu \neq 16$ A sample of 48 provided a sample mean $\bar{x} = 17$ and a sample standard deviation S=4.5 then the value of the test statistic is (a) 1 (b) 1.54 (c) 2 (d) 2.45	1
6	If the objective functions for an LPP is Z= 3x-4y and the corner points for the bounded feasible region are (0,0) , (5,0), (6,5) , (6, 8) , (4, 10) and (0,8) then the minimum value of Z occurs at (a) (0,0) (b) (0,8) (c) (5, 0) (d) (4, 10)	1
7	The mean of the Binomial distribution B $(4, \frac{1}{3})$ is (a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) 4 (d) 3	1
8	At what rate of interest will the present value of a perpetuity of ₹ 500 payable at the end of every 6 months be ₹ 10000? (a) 5 % (b) 7.5% (c) 10% (d) 12.5%	1
9	Out of the following matrices , choose that matrix which is a scalar matrix : (a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	1
10	How many times must a man toss a fair coin so that the probability of having at least one head is more than 80%? (a) 2 (b) 3 (c) 4 (d) 5	1
11	A man invests ₹11200 in a company paying 6% dividend when its ₹ 100 shares can be bought for ₹ 140 ,then his annual income is (a) ₹ 240 (b) ₹ 360 (c) ₹480 (d) ₹ 600	1
12	The solution of the given inequality $3(2 - x) \geq 2 (1 - x)$ is (a) $(-\infty, -4]$ (b) $(-\infty, 4]$ (c) $(4, \infty)$ (d) $[-4, 4]$	1
13	Seasonal variations are (a) Short term (b) Long term (c) Sudden (d) None of these.	1
14	The present value of a sequence of payments of ₹ 1000 made at the end of every 6 months and continuing forever, if money is worth 8% per annum compounded semi-annually is (a) ₹ 20000 (b) ₹ 24000 (c) ₹ 25000 (d) ₹ 35000	1
15	The interval in which the function f(x) = $2x^3 + 9x^2 + 12x + 20$ is increasing (a) $(-2, -1)$ (b) $(2, 4)$ (c) $(-\infty, -2) \cup (-1, \infty)$ (d) $(-\infty, -4) \cup (-2, \infty)$	1
16	If A= $\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then the value of $|adj\ A|$ is (a) a^{27} (b) a^9 (c) a^6 (d) a^2	1
17	The value of x for which the matrix A = $\begin{bmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{bmatrix}$ is singular (a) -1 (b) 0 (c) 1 (d) 2	1
18	The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 - \left(\frac{dy}{dx}\right) = y^3$ is (a) $\frac{1}{2}$ (b) 2 (c) 3 (d) 4	1

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	Assertion – Reason Based Questions	
	In the question number 19 and 20, a statement of Assertion (A) us followed by a statement of reason (R). Choose the correct option. (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.	
19	Assertion (A) Rohan can row with a speed of 16 km/h in still water and the speed of stream is 12 km/h. Then, speed of Rohan downstream will be 26 km/h. Reason (R) If the speed of of the boat in still water x km/h and speed of stream is y km/h, then speed of the downstream is (x+y) km/h.	1
20	Assertion (A) If x is real, then the minimum value of $x^2-8x+17$ is 1. Reason (R) If f"(x) > 0 at critical point, then the value of the function at critical point will be the minimum value of the function.	1
	SECTION–B	
21	Pipe A can fill the tank 2 times faster than pipe B. If both pipes A and B running together can fill the tank in 24 minutes, find how much time will pipe B alone take to fill the tank? OR In a 600 m race. A beats B by 37.5 m or by 10 seconds, find the time taken by B to finish the race.	2
22	The speed of a boat in still water is 12 km/hr. It takes twice as long as to go upstream to a point as to return downstream to the starting point. What is the speed of the stream?	2
23	Find the remainder when 783 x 657 x 594 x 432 x 346 x 251 is divided by 5	2
24	If X is normally distributed with mean 10 and standard deviation 4, find x such that the probability of X between 10 and x is 0.4772. (Given F (2) = 0.9772) OR Two dice are rolled simultaneously. Find the probability distribution of sum of digits on the two dice.	2
25	Find the value of x such that : $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$	2
	SECTION–C	
26	If X has a Poisson distribution such that P (X=1) = P (X=2) , find P (X=4). (Given $e^{-2} = 0.1353$) OR A pair of dice is thrown 7 times. If getting "a doublet" is considered as success. what is the probability of getting atleast 6 successes?	3
27	Ten students are selected at random from a college and their heights are found to be 100, 104,108,110,118,120,122,124,126 and 128 cms. In the light of these data, discuss the suggestion that the mean height of the students of the college is 110 cms. (Given t_9 (0.05)–2.262)	3
28	A man invested a certain sum in 7.5% stock at 103 and sold when the price rose to 107, gaining thereby a sum of ₹ 36. What sum did he invest? How much stock did he buy? OR Which is the better investment 10.5% stock at ₹ 90 or, 11% stock at par?	3
29	A pump can fill a tank with water in 20 minutes and another pump can fill the same tank in 30 minutes. If both the pumps are opened together, then how much time will be taken to fill the tank completely?	3

Prepared By: Amit Mittal, PGT mathematics, St Francis School: Barut

30	From a lot of 30 bulbs which includes 6 defectives, a sample of 4 bulbs is drawn at random one by one with replacement. Find the probability distribution of the number of defective bulbs. Find the mean of the distributions.	3
31	A dietician wishes to mix two types of food in such a way that the vitamin contents of the mixture contain at least 8 units of Vitamin A and 10 units of vitamin C. Food 'I' contains 2 units per kg of vitamin A and 1 unit per kg of Vitamin C while food 'II' contains 1 unit per kg of vitamin A and 2 units per kg of vitamin C. It costs ₹ 50 per kg to purchase food 'I' and ₹ 70 per kg to produce food ' II' Formulate the above linear programming problem to minimize the cost of such a mixture. (**Note** : No need to find the feasible region and optimal solution.)	3

SECTION–D

32	Fit a straight line trend by the method of least square for the following data. Also, tabulate the trend values.	5

year	2004	2005	2006	2007	2008	2009	2010
Profit (₹'000)	114	130	126	144	138	156	164

33	Evaluate $\int \frac{2x}{(x^2+1)(x^2+2)^2} dx$ OR The marginal revenue function of a commodity is given by MR= 11-3x +4x^2 , find the revenue function. Also, find the demand function.	5
34	If A=$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and B=$\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$,find AB. Hence, solve the system of equations: x-y=3 , 2x+3y+4z=17 , y+2z=7.	5
35	Mr. Parekh invested ₹ 52000 on ₹ 100 shares at a discount of ₹20 paying 8% dividend. At the end of the year he sells the shares at premium of ₹20. Find his rate of return. OR A dining table costing ₹ 36000 has a useful life of 15 years. If annual depreciation is ₹2000, find its scrap value using linear method.	5

	SECTION E	
36	**CASE STUDY –I** P(x) = -5x^2+125x +37500, x>0 is the total profit function of a company where x is the production of the company. Based on the above information, answer the following questions. (i) What will be the production when the profit is maximum? (ii) What will be the maximum profit? (iii)When the production is 2 units what will be the profit of the company? OR (iii) What will be production of the company when the profit is 38250?	1 1 2 OR 2

37	**CASE STUDY –II** A train can carry a maximum of 200 passengers. A profit of ₹ 1000 is made on each executive class ticket and a profit of ₹ 600 is made on each economy class ticket. The IRCTC reserves atleast 20 seats for executive class. However atleast 4 times as many passengers prefer to travel by economy class than by the executive class. It is given that the number of executive class ticket is ₹. x and that of economy class ticket is ₹ .Based on the given information answer the following questions (i) Formulate objective function for the given LPP. (ii) Determine how many tickets of each must be sold in order to maximize the profit for the IRCTC. (iii) Write the corner points of the feasible region. OR (iii) What is the maximum profit?	1 1 2 OR 2
38	**CASESTUDY –III** A loan is the leading of money by one or more individual, organisations, or other entities to other individuals, organisations etc. The recipient incurs a debt and is usually liable to pay interest on that debt until it is repaid as well as to repay the principal amount borrowed. A loan of ₹ 4000000 at the interest rate of 6.75% per annum compounded monthly is to be amortized by equal payment at the end of each month for 10 year. Based on the above information, answer the following questions. (i) Find the size of each monthly payment. (Given $1.005625^{120} = 1.9603$ (ii) Find the Principal outstanding at the beginning of 61st month. (Given $1.005625^{60} = 1.4001$}	2 2

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