BANGALORE SAHODAYA SCHOOLS COMPLEX ASSOCIATION
PRE-BOARD EXAMINATION (2024-2025)
Grade XII

Grade 12 Preparatory EXAMINATION-02
MATHEMATICS, SET A MARKING SCHEME

Section - A

1)	d) $\boxed{R-\{-10\}}$	$2(6-0)+1(3\lambda)+1(-2\lambda-2)=0 \Rightarrow 12+\lambda-2=0$ $\Rightarrow \lambda=-10$, Singular. For Non-Singular $R-\{-10\}$	1
2)	c) $\boxed{2025\times 2024}$	$P.(\text{adj } P) = -2025\, I_3 \Rightarrow \lvert P\rvert = -2025$ $\lvert \text{adj } P\rvert = \lvert P\rvert^{n-1} = \lvert P\rvert^2 \Rightarrow \lvert P\rvert + \lvert \text{adj } P\rvert = -2025+(2025)^2$ $\Rightarrow 2025(-1+2025) = 2025\times 2024$	1
3)	d) 2p = q	Z=px + qy, Z(3,0) = 3p, Z (1,2) =p + q 3p = p + q => 2p = q	1
4)	a) $\boxed{m\times n}$	$A \to m\times n$ order. Let B be $x\times y$ (order). B' order $= y\times x$ AB' defined $m\times n \; y\times x \Rightarrow n=y$ $B'A$ defined $y\times x \; m\times n \Rightarrow m=x$ B order is $m\times n$	1
5)	b) $\boxed{\pm 1}$	$\lvert A\rvert \lvert A^T\rvert = 1 \Rightarrow \lvert A\rvert = \pm 1$ $\quad \lvert \text{adj}(\text{adj } A)\rvert = \lvert A\rvert^{(n-1)^2}$ $\Rightarrow (\pm 1)^{(n-1)^2} = \pm 1$ based on n	1
6)	c) $\boxed{3}$	$\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^4\right) = 4\left(\frac{dy}{dx}\right)^3\cdot\left(\frac{d^2y}{dx^2}\right)$ $\quad m=2, n=1, m+n=3$	1
7)	a) $\boxed{e^{2\sqrt{x}}}$	$\frac{dy}{dx}+\frac{y}{\sqrt{x}} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}} \Rightarrow P=\frac{1}{\sqrt{x}}$ $\quad IF = e^{\int \frac{1}{\sqrt{x}}dx} = e^{2\sqrt{x}}$	1
8)	b) $\boxed{e^x\log(x+1)+C}$	$\int e^x\left(\frac{1}{x+1}+\log(x+1)\right)dx = e^x.\log(x+1)+C$ $\int e^x\left(f'(x)+f(x)\right)dx = e^x f(x)+C$	1

9)	$\boxed{d)\ \frac{3}{4}}$	$1-p(\text{problem not solved}) = 1-p(\bar{A}\cap\bar{B}\cap\bar{C})$ $= 1-p(\bar{A})\cdot p(\bar{B})\cdot p(\bar{C}) = 1-\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{\cancel{3}}\cdot\frac{\cancel{3}}{4} = \frac{3}{4}$	1
10)	$\boxed{c)\ 6}$	LHL $\lim_{x\to\pi/2}\frac{k\cos x}{\pi-2x} = \lim_{h\to 0}\frac{k\cos(\pi/2-h)}{\pi-\pi+2h}$ $= \lim_{h\to 0}\frac{k\sin h}{2h} = \frac{k}{2}$. RHL $=3 \Rightarrow k=6$	1
11)	c) 0	$a=0,\ b=-c \Rightarrow b+c=0 \quad 2a-(b+c)=0$	1
12)	b) $\frac{\cos x}{2y-1}$	$y=\sqrt{\sin x+y} \Rightarrow y^2=\sin x+y$ $2y\frac{dy}{dx}=\cos x+\frac{dy}{dx} \Rightarrow \frac{dy}{dx}=\frac{\cos x}{2y-1}$	1
13)	b) Max value at Q	$O(0,0)=\overset{Z}{0}$ | $P=(0,40)=120$ $R(40,0)=160$ | $Q=(30,20)=180$	1
14)	b) $x=\frac{1}{e}$	$y=x^x$, $\log y = x\log x$ | $\frac{1}{y}\frac{dy}{dx}=x\cdot\frac{1}{x}+\log x = 1+\log x$; $\frac{dy}{dx}=x^x(1+\log x)$ $1+\log x=0 \Rightarrow \log x=-1=-\log e \Rightarrow x=\frac{1}{e}$	1
15)	c) $10\sqrt{3}$	$\frac{dA}{dt}=\frac{d}{dt}\left(\frac{\sqrt{3}}{4}a^2\right)=\frac{\sqrt{3}}{4}\cdot 2a\cdot\frac{da}{dt}=\frac{\sqrt{3}}{4}\cdot 2\times 10\times 2 = 10\sqrt{3}$	1
16)	d) $\frac{17}{4}$	$\left\|\int_{-2}^{0}x^3dx\right\| + \int_0^1 x^3dx = \left\|\left[\frac{x^4}{4}\right]_{-2}^{0}\right\| + \left[\frac{x^4}{4}\right]_0^1 = \frac{16}{4}+\frac{1}{4} = \frac{17}{4}$	1
17)	a) $\frac{1}{5}\log\left\|\frac{x^5}{x^5+1}\right\|+C$	$\int\frac{x^4}{x^5(x^5+1)}dx$ $\quad t=x^5$, $dt=5x^4dx$ $\Rightarrow \int\frac{1}{5}\frac{dt}{t(t+1)}$ $\frac{1}{5}\int\left(\frac{t+1-t}{t(t+1)}\right)dt$ $=\frac{1}{5}\left[\int\left(\frac{1}{t}-\frac{1}{t+1}\right)dt\right] = \frac{1}{5}\left[\log\|t\|-\log\|t+1\|\right] = \frac{1}{5}\log\left\|\frac{x^5}{x^5+1}\right\|+C$	1
18)	d)	$\cos\alpha+\cos\beta+\cos\gamma\neq 1 \to$ not true is d)	1
19)	d)	Assertion: $\tan^{-1}(\sqrt{3})-\sec^{-1}(-2)=\frac{\pi}{3}-\frac{2\pi}{3}=-\pi/3$ A → false. Reason – True	1
20)	c)	$\|x-6\|\cdot\cos x$ is not differentiable at $x=6$. So it is differentiable in $R-\{6\}$. Assertion True. Reason is false.	1

Section- B

(21) $\sin^{-1}\left(\frac{1}{\sqrt2}\sin x + \frac{1}{\sqrt2}\cos x\right)$ — (1/2)

$= \sin^{-1}\left(\cos\frac{\pi}{4}\sin x + \sin\frac{\pi}{4}\cos x\right)$ — (1/2)

$= \sin^{-1}\left(\sin(x+\pi/4)\right)$ — (1/2)

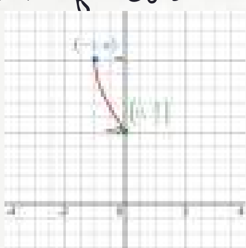
$= x + \pi/4$

as $-\frac{\pi}{4} < x < \frac{\pi}{4} \Rightarrow \frac{\pi}{4}-\frac{\pi}{4} < x+\frac{\pi}{4} < \frac{\pi}{4}+\frac{\pi}{4}$

$\Rightarrow 0 < x+\frac{\pi}{4} < \frac{\pi}{2} \in PVB$ — (1/2)

(OR)

Graph of $\cos^{-1}x$ — (1)



Range of $\cos^{-1}x = [0, \pi]$ — (1/2)

Range of $\tan^{-1}x = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ — (1/2)

(22) $f(1) = 3-1 = 2$ — (1/2)

LHD $\lim_{h\to0}\frac{f(1-h)-f(1)}{-h} = \lim_{h\to0}\frac{(1-h)^2+1-2}{-h} =$ — (1/2)

$= \lim_{h\to0}\frac{h(h-2)}{-h} = 2$

RHD $\lim_{h\to0}\frac{f(1+h)-f(1)}{h} = \lim_{h\to0}\frac{3-(1+h)-2}{h} = -1$ — (1/2)

LHD $\neq$ RHD. So $f(x)$ is not differentiable @ $x=1$ — (1/2)

(23) $f'(x) = x^2(-e^{-x}) + e^{-x}(2x) = e^{-x}(2x-x^2)$ — (1/2)

$f'(x) > 0$ (strictly increasing) as $e^{-x} > 0$ — (1/2)

$(2x-x^2) > 0 \Rightarrow x(x-2) < 0 \Rightarrow 0 < x < 2$ — (1/2)

Hence strictly increasing in $(0, 2)$ — (1/2)

OR

23) $f(x) = \dfrac{4\sin x}{2+\cos x} - x$

$f'(x) = \dfrac{(2+\cos x)\,4\cos x - 4\sin x(-\sin x)}{(2+\cos x)^2} - 1$ — 1/2

$= \dfrac{8\cos x + 4\cos^2 x + 4\sin^2 x}{(2+\cos x)^2} - 1 = \dfrac{8\cos x + 4}{(2+\cos x)^2} - 1$ — 1/2

$= \dfrac{8\cos x + 4 - (2+\cos x)^2}{(2+\cos x)^2} = \dfrac{\cos x\,(4-\cos x)}{(2+\cos x)^2}$

as $x \in [0, \pi/2]$, $\cos x \geq 0$, $4 - \cos x \geq 0$ & $(2+\cos x)^2 \geq 0$ — 1/2

$\Rightarrow f'(x) \geq 0 \Rightarrow f$ is increasing in $[0, \frac{\pi}{2}]$ — 1/2

24) $\vec{AD} = \vec{AB} - \vec{DB} = \hat{i} - \hat{j} + \hat{k}$ — 1/2

Area $= |\vec{AB} \times \vec{AD}| = \sqrt{25+1+16} = \sqrt{42}$ sq units — 1/2

$\vec{AB} \times \vec{AD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix} = 5\hat{i} + \hat{j} - 4\hat{k}$ — 1

$\therefore$ Area $= \sqrt{42}$ sq units

25) Gen Pt on line $\dfrac{x+2}{3} = \dfrac{y+1}{2} = \dfrac{z-3}{2} = \lambda$ is

$P(3\lambda-2, 2\lambda-1, 2\lambda+3)$ — 1/2

Point $Q(1, 3, 3)$

Distance $PQ = \sqrt{(3\lambda-2-1)^2 + (2\lambda-1-3)^2 + (2\lambda+3-3)^2}$

$\Rightarrow PQ^2 = 25 \Rightarrow (3\lambda-3)^2 + (2\lambda-4)^2 + (2\lambda)^2 = 25$ — 1

$9\lambda^2 + 9 - 18\lambda + 4\lambda^2 + 16 - 16\lambda + 4\lambda^2 = 25$

$17\lambda^2 - 34\lambda = 0 \Rightarrow \lambda = 0, 2$

$\therefore$ Points are $(-2, -1, 3)$ and $(4, 3, 7)$ — 1/2

SECTION - C

26) $x = a\cos\theta + b\sin\theta$

$\frac{dx}{d\theta} = a(-\sin\theta) + b\cos\theta$ — ½

$y = a\sin\theta - b\cos\theta$ $\quad \frac{dy}{d\theta} = a\cos\theta - b(-\sin\theta)$ — ½

$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\cos\theta + b\sin\theta}{-a\sin\theta + b\cos\theta} = \frac{x}{-y} = -\frac{x}{y}$ — ½

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{-x}{y}\right) \Rightarrow \frac{y(-1) + x\left(\frac{dy}{dx}\right)}{y^2} = \frac{d^2y}{dx^2}$ — 1

$\Rightarrow y^2\frac{d^2y}{dx^2} = -y + x\frac{dy}{dx} \Rightarrow y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$ — ½

27) $p(\text{getting a six}) = \frac{1}{6} = p(S)$ — ½

$p(\text{not getting a six}) = \frac{5}{6} = p(F)$ — ½

$p(A\text{ wins}) = S, FFS, FFFFS, \ldots$ — ½

$= \frac{1}{6} + \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6} + \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6} = \frac{1}{6} + \left(\frac{5}{6}\right)^2\frac{1}{6} + \left(\frac{5}{6}\right)^4\frac{1}{6} + \cdots$

$\Rightarrow$ Infinite GP with $a = \frac{1}{6}, r = \left(\frac{5}{6}\right)^2$

$S_\infty = \frac{a}{1-r} = \frac{1/6}{1 - \frac{25}{36}} = \frac{1}{6}\times\frac{36}{11} = \frac{6}{11}$ $\quad \therefore p(A\text{ wins}) = \frac{6}{11}$ — 1

$p(B\text{ wins}) = 1 - \frac{6}{11} = \frac{5}{11}$ — ½

(OR)

$S = \{(1,2), (1,3)\,(1,4), (2,1), (2,3), (2,4), (3,1)\,(3,2), (3,4), (4,1), (4,2), (4,3)\}$ $\quad n(S) = 12$ — ½

$p(x=3)$ getting Sum as $3 = \frac{2}{12} = \frac{1}{6}$

$p(x=4)$ " " " $4 = \frac{2}{12} = \frac{1}{6}$

$p(x=5)$ getting Sum as $5 = \frac{4}{12} = \frac{2}{6}$

$p(x=6)$ getting Sum as $6 = \frac{2}{12} = \frac{1}{6}$ — 1

$p(x=7) = \frac{2}{12} = \frac{1}{6}$

prob. distribution table

X	3	4	5	6	7
p_i	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

(½)

Mean $= \sum x_i p_i = \frac{3}{6} + \frac{4}{6} + \frac{10}{6} + \frac{6}{6} + \frac{7}{6} = \frac{30}{6} = 5$ (1)

28) $\int \frac{2\cos x \, dx}{(1-\sin x)(1+\sin^2 x)}$ Let $t = \sin x$, $dt = \cos x \, dx$ (½)

$= \int \frac{2\,dt}{(1-t)(1+t^2)}$ Using partial fractions,

$\frac{2}{(1-t)(1+t^2)} = \frac{A}{1-t} + \frac{Bt+C}{1+t^2} \Rightarrow$

$2 = A(1+t^2) + (Bt+C)(1-t)$

$2 = (A+C) + (A-B)t^2 + (B-C)t$ (½)

Comparing co-efficients, $A-B=0$, $B-C=0 \Rightarrow A=B=C$

$A+C = 2 \Rightarrow 2A = 1 \Rightarrow A=B=C=1$ (½)

$\Rightarrow \int \frac{2\,dt}{(1-t)(1+t^2)} = \int\left(\frac{1}{1-t} + \frac{t+1}{1+t^2}\right)dt = \int \frac{1}{1-t}dt + \int \frac{t}{1+t^2}dt + \int \frac{1}{1+t^2}dt$

$= \frac{\log|1-t|}{-1} + \frac{1}{2}\int \frac{2t}{1+t^2}dt + \tan^{-1}t + C$ $u = 1+t^2$, $du = 2t\,dt$

$= -\log|1-t| + \frac{1}{2}\int \frac{du}{u} + \tan^{-1}t + C$ (1½)

$= -\log|1-\sin x| + \frac{1}{2}\log|1+\sin^2 x| + \tan^{-1}(\sin x) + C$

29) DE: $(x^2y + yx\sqrt{y^2-x^2})dx - x^3 dy = 0$ $\div$ by x^3/transpose

$\frac{dy}{dx} = \frac{y}{x} + \frac{y}{x^2}\sqrt{y^2-x^2} = \frac{y}{x} + \frac{y}{x}\sqrt{\left(\frac{y}{x}\right)^2 - 1}$ ∴ Homogenous DE (1)

Let $y = vx$

$\frac{dy}{dx} = v(1) + x\frac{dv}{dx}$ (½)

29) continued...

$$\cancel{v} + x\frac{dv}{dx} = \cancel{v} + v\sqrt{v^2-1} \Rightarrow x\frac{dv}{dx} = v\sqrt{v^2-1} \quad — \frac{1}{2}$$

$$\Rightarrow \int \frac{dv}{v\sqrt{v^2-1}} = \int \frac{dx}{x}$$

$$\sec^{-1} v = \log|x| + \log c = \log(cx) \quad — \frac{1}{2}$$

$$\sec^{-1}\left(\frac{y}{x}\right) = \log cx \text{ (or) } \log|x| + C \quad — \frac{1}{2}$$

(OR)

$$(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2 \quad \text{Divide by } (x+1)$$

$$\frac{dy}{dx} - \frac{1}{(x+1)}y = e^{3x}(x+1). \text{ This is LDE } \frac{dy}{dx} + Py = Q \quad — \frac{1}{2}$$

$$P = \frac{-1}{x+1}, \quad Q = e^{3x}(x+1) \quad — \frac{1}{2}$$

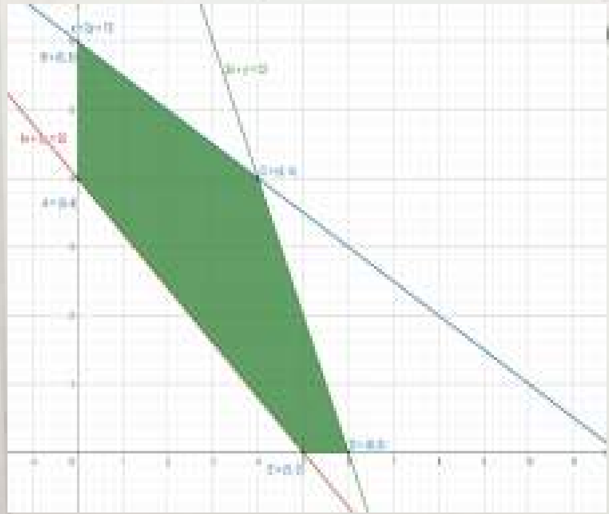
$$IF = e^{\int P dx} = e^{\int \frac{-1}{x+1} dx} = e^{-\log|x+1|} = \frac{1}{x+1} \quad — \frac{1}{2}$$

$$\text{Solution: } y(IF) = \int Q(IF)\,dx \quad — \frac{1}{2}$$

$$y\left(\frac{1}{x+1}\right) = \int e^{3x}\cdot(x+1)\cdot\frac{1}{(x+1)}\,dx = \int e^{3x}dx = \frac{e^{3x}}{3} + C \quad 1$$

$$y = \frac{e^{3x}(x+1)}{3} + C(x+1)$$

30)



Correct Graph $1\frac{1}{2}$

1. $x + 2y \leq 12$
   $\frac{x}{12} + \frac{y}{6} \leq 1$
   origin included
2. $2x + y \leq 12$
   $\frac{x}{6} + \frac{y}{12} \leq 1$
   origin included
3. $4x + 5y \geq 20$
   $\frac{x}{5} + \frac{y}{4} \geq 1$
   origin not included

Bounded region ABCDEA.

Maximum occurs at corner point.

Corner point table

	$Z = 50x + 30y$
A(0,4)	120
B(0,6)	180
C(4,4)	320
D(6,0)	300
E(5,0)	250

— (1)

Max value at C(4,4)

Value is 320 — (1/2)

31) $\int_{-1}^{2} |x^3 - x|\, dx = \int_{-1}^{2} |x(x-1)(x-2)|\, dx \Rightarrow x = 0, 1, 2.$

$x^3 - x = 0$ — (1/2)

$\Rightarrow x = 0, 1, 2$

$x^3 - x$ +ve $-1 < x < 0,\ 1 < x < 2$

$x^3 - x$ -ve $0 < x < 1$ — (1/2)

$\int_{-1}^{0} (x^3 - x)\,dx + \int_{0}^{1} -(x^3 - x)\,dx + \int_{1}^{2} (x^3 - x)\,dx$

$= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} + \left[-\frac{x^4}{4} + \frac{x^2}{2}\right]_{0}^{1} + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{1}^{2}$ — (1/2)

$= 0 - \left(\frac{1}{4} - \frac{1}{2}\right) + \left(\frac{-1}{4} + \frac{1}{2}\right) + \left(\frac{16}{4} - \frac{4}{2} - \left(\frac{1}{4} - \frac{1}{2}\right)\right)$

$= \frac{1}{4} + \frac{1}{4} + 2 + \frac{1}{4} = 2 + \frac{3}{4} = \frac{11}{4}$ — (1/2)

$\boxed{\text{Section - D}}$

32) Given $(a,b)\, R\, (c,d) \Leftrightarrow ad(b+c) = bc(a+d)$

(i) Reflexive: $ab(a+b) = ab(a+b)$

$ab(b+a) = ba(a+b)$

commutative property of addition/multiplication on N — (1)

$\Rightarrow (a,b)\, R\, (a,b),\ \forall (a,b) \in$

$\therefore$ Reflexive.

32) continued...

Symmetric : Let $(a,b), (c,d)$ be an arbitrary element of $N \times N$ such that $(a,b) R (c,d)$

$ad(b+c) = bc(a+d)$

$bc(a+d) = ad(b+c) \Rightarrow cb(d+a) = da(c+b)$ — 1

$\Rightarrow cb(d+a) = da(c+b)$ {Commutative property of multiplication/ addition on N}

$\Rightarrow (c,d) R (a,b)$

$\therefore (a,b) R (c,d) \Rightarrow (c,d) R (a,b)$ for all $(a,b), (c,d) \in N \times N$

Transitive: $(a,b), (c,d), (e,f)$ be arbitrary elements of $N \times N$

Such that $(a,b) R (c,d)$, $(c,d) R (e,f)$

$ad(b+c) = bc(a+d)$ — 1/2

$\frac{b+c}{bc} = \frac{a+d}{ad}$

$\frac{1}{b}+\frac{1}{c} = \frac{1}{a}+\frac{1}{d}$ — ①

$cf(d+e) = de(c+f)$ — 1/2

$\frac{d+e}{de} = \frac{c+f}{cf}$

$\frac{1}{d}+\frac{1}{e} = \frac{1}{f}+\frac{1}{c}$ — ②

Adding ① & ② $\cancel{\frac{1}{c}}+\frac{1}{b}+\cancel{\frac{1}{d}}+\frac{1}{e} = \frac{1}{a}+\cancel{\frac{1}{d}}+\cancel{\frac{1}{c}}+\frac{1}{f}$

$\frac{1}{b}+\frac{1}{e} = \frac{1}{a}+\frac{1}{f} \Rightarrow \frac{b+e}{be} = \frac{a+f}{af}$ — 1

$\Rightarrow af(b+e) = be(a+f)$

$\Rightarrow (a,b) R (e,f)$ — 1/2

$\Rightarrow (a,b) R (c,d)$ & $(c,d) R (e,f) \Rightarrow (a,b) R (e,f)$

for all $(a,b), (c,d), (e,f) \in N \times N$. $\therefore$ R is transitive

As it is reflexive, symmetric & transitive

$\therefore$ It is an equivalence relation. — 1/2

(OR)

32) One-one:

Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in R$ — (1/2)

$\Rightarrow \frac{\cancel{2}x_1}{1+x_1^2} = \frac{\cancel{2}x_2}{1+x_2^2} \Rightarrow x_1 + x_1x_2^2 = x_2 + x_2x_1^2$

$\Rightarrow (x_1 - x_2) - (x_1x_2^2 - x_2x_1^2) = 0$ — (1)

$\Rightarrow (x_1-x_2) - x_1x_2(x_1-x_2) = 0$

$\Rightarrow (x_1-x_2)(1-x_1x_2) = 0 \Rightarrow x_1 = x_2$ or $x_1x_2 = 1$

So if $x_1x_2 = 1$, $x_1 \neq x_2$. Hence f is not one-one — (1/2)

onto

Let $y = f(x)$ where $x \in R$, $y = 0$ $x = 0$ — (1/2)

If $y \neq 0$, $y = \frac{2x}{1+x^2} \Rightarrow yx^2 + y - 2x = 0$

$\Rightarrow yx^2 - 2x + y = 0$. Solve for $x = \frac{2 \pm \sqrt{4-4y^2}}{2y}$ — (1)

$\Rightarrow x = \frac{1 \pm \sqrt{1-y^2}}{y}$

For x to be real, $1-y^2 \geq 0 \Rightarrow y^2 \leq 1 \Rightarrow -1 \leq y \leq 1$ — (1/2)

Range = $[-1, 1] \neq$ codomain. $\therefore$ f is not onto — (1/2)

For function to become onto, $A = [-1, 1]$ — (1/2)

33) $|A| = 3(3-6) - 2(-12-14) + 1(12+7) = 62 \neq 0$, A^{-1} exists. — (1/2)

Cofactors — (2)

$C_{11} = -3$	$C_{21} = 9$	$C_{31} = 5$
$C_{12} = 26$	$C_{22} = -16$	$C_{32} = -2$
$C_{13} = 19$	$C_{23} = -5$	$C_{33} = -11$

$$\text{adj}\, A = \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{62}\begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix}$$ — (1/2)

$3x+4y+7z=14$

$2x-y+3z=4$

$x+2y-3z=0$

$$\underset{A^T}{\begin{bmatrix}3&4&7\\2&-1&3\\1&2&-3\end{bmatrix}}\underset{X}{\begin{bmatrix}x\\y\\z\end{bmatrix}}=\underset{B}{\begin{bmatrix}14\\4\\0\end{bmatrix}}$$

$\Rightarrow (A^T)X=B$.

Premultiply by $(A^T)^{-1}$

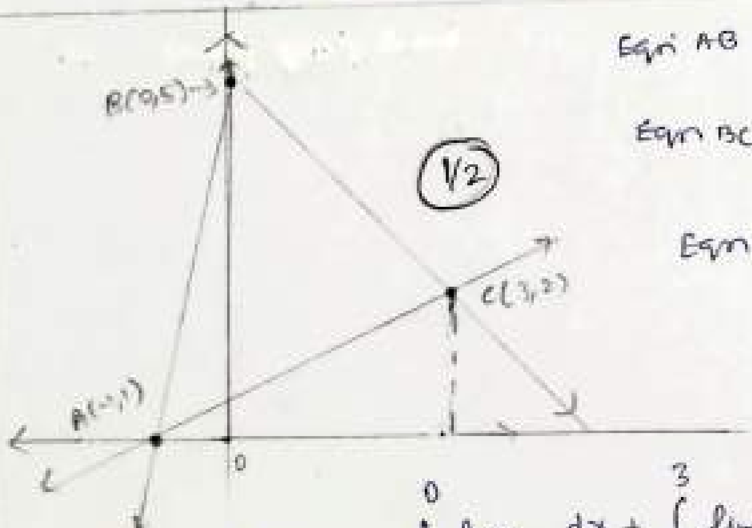
$(A^T)^{-1}\cdot A^T\cdot X=(A^T)^{-1}\cdot B \Rightarrow IX=(A^T)^{-1}\cdot B \Rightarrow X=(A^T)^{-1}B$ — (1)

$X=(A^{-1})^T B \quad \{(A^{-1})^T=(A^T)^{-1}\}$

$$X=\frac{1}{62}\begin{bmatrix}-3&9&5\\26&-16&-2\\19&5&-11\end{bmatrix}^T\begin{bmatrix}14\\4\\0\end{bmatrix}=\frac{1}{62}\begin{bmatrix}-3&26&19\\9&-16&5\\5&-2&-11\end{bmatrix}\begin{bmatrix}14\\4\\0\end{bmatrix}$$ — (1)

$$X=\frac{1}{62}\begin{bmatrix}-42+104\\126-64\\70-8\end{bmatrix}=\frac{1}{62}\begin{bmatrix}62\\62\\62\end{bmatrix}=\begin{bmatrix}1\\1\\1\end{bmatrix}\Rightarrow x=1,\ y=1,\ z=1$$

34)



Eqn AB $\Rightarrow \frac{x-0}{+1}=\frac{y-5}{+4} \Rightarrow y=4x+5$ — (1/2)

Eqn BC $\Rightarrow \frac{x-0}{3}=\frac{y-5}{-3} \Rightarrow y=-x+5$ — (1/2)

Eqn AC $\Rightarrow \frac{x+1}{4}=\frac{y-1}{1} \Rightarrow y=\frac{x+5}{4}$ — (1/2)

Required Area $=\int_{-1}^{0} line_{AB}\,dx+\int_{0}^{3} line_{BC}\,dx-\int_{-1}^{3} line_{AC}\,dx$ — (1)

$=\int_{-1}^{0} 4x+5\,dx+\int_{0}^{3}(-x+5)\,dx-\frac{1}{4}\int_{-1}^{3}(x+5)\,dx$

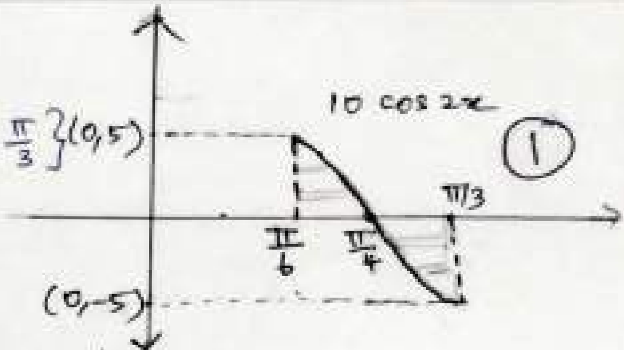
$=\left.\frac{4x^2}{2}+5x\right]_{-1}^{0}+\left[-\frac{x^2}{2}+5x\right]_{0}^{3}-\frac{1}{4}\left[\frac{x^2}{2}+5x\right]_{-1}^{3}$ — (1 1/2)

$=-(2-5)+\left(-\frac{9}{2}+15\right)-\frac{1}{4}\left[\frac{9}{2}+15-\left(\frac{1}{2}-5\right)\right]$

$=3+\frac{21}{2}-\frac{1}{\cancel{4}}\cdot\cancel{24}^{6}=\frac{15}{2}$ sq units — (1/2)

(OR)

34) $y = 10\cos 2x \quad \left\{ \frac{\pi}{6} \le x \le \frac{\pi}{3} \right\}$ (1)



Required area =

$$= \int_{\pi/6}^{\pi/4} 10\cos 2x \, dx + \left| \int_{\pi/4}^{\pi/3} 10\cos 2x \, dx \right| \quad \text{— (1)}$$

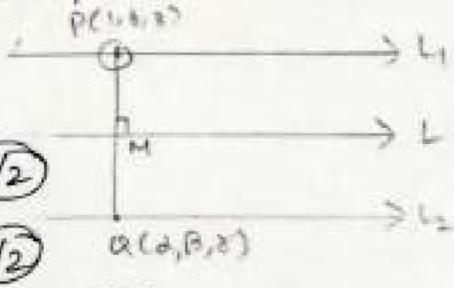
$$= 10 \frac{\sin 2x}{2}\Big]_{\pi/6}^{\pi/4} + \left| 10 \frac{\sin 2x}{2}\Big]_{\pi/4}^{\pi/3} \right| = 5\sin 2x\Big]_{\pi/6}^{\pi/4} + \left| 5\sin 2x\Big]_{\pi/4}^{\pi/3} \right| \quad \text{— (1)}$$

$$= 5\left(\sin \pi/2 - \sin \pi/3\right) + \left| 5\left(\sin 2\pi/3 - \sin \pi/2\right) \right| \quad \text{— (1)}$$

$$= 5\left(1 - \frac{\sqrt{3}}{2}\right) + 5\left| \frac{\sqrt{3}}{2} - 1 \right| = 5\left(1 - \frac{\sqrt{3}}{2}\right) + 5\left(1 - \frac{\sqrt{3}}{2}\right) \quad \text{— (1/2)}$$

$$= 10\left(1 - \frac{\sqrt{3}}{2}\right) \text{ sq units} \quad \text{— (1/2)}$$

35) Let M be the foot of perpendicular from P(1, 6, 3) and Q be the image of point P with respect to line L



Line L : $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$

Gen Point on L : $\lambda,\ 2\lambda+1,\ 3\lambda+2$ — (1/2)

DR (Direction Ratios) of L $\langle 1, 2, 3 \rangle$ — (1/2)

DR PM are $\langle \lambda-1,\ 2\lambda-5,\ 3\lambda-1 \rangle$ — (1/2)

PM ⊥ L ⇒ dot product = 0 ⇒ $(\lambda-1)1 + (2\lambda-5)2 + (3\lambda-1)3 = 0$

$\lambda - 1 + 4\lambda - 10 + 9\lambda - 3 = 0 \Rightarrow 14\lambda - 14 = 0 \Rightarrow \lambda = 1$ (1)

Coordinates of M (1, 3, 5). M is the midpoint of PQ. (1/2)

Let Q (α, β, γ) ⇒ $\frac{\alpha+1}{2} = 1,\ \frac{\beta+6}{2} = 3,\ \frac{\gamma+3}{2} = 5 \Rightarrow \alpha = 1,\ \beta = 0,\ \gamma = 7$ (1)

∴ Q ∴ (1, 0, 7)

Equation of L2; L2 passes through Q(1, 0, 7). DR $\langle 1, 2, 3 \rangle$ (parallel to L) (1/2)

$$\Rightarrow \frac{x-1}{1} = \frac{y-0}{2} = \frac{z-7}{3} \quad (1/2)$$

Case-study - Section-E

36) (i) First square length = 'x' m

Side = $\frac{x}{4}$ m — (1/2)

Second Square length = $(a-x)$ m, Side = $\left(\frac{a-x}{4}\right)$m. — (1/2)

Combined area $A = \frac{x^2}{16} + \frac{(a-x)^2}{16} = \frac{x^2}{16} + \frac{a^2+x^2-2ax}{16}$
$= \frac{2x^2+a^2-2ax}{16}$ — (1)

(ii) $\frac{dA}{dx} = \frac{4x+0-2a}{16} = \frac{4x-2a}{16}$ — (1/2)

$\frac{dA}{dx} = 0 \Rightarrow 4x = 2a \Rightarrow x = a/2$ (critical point) — (1/2)

$\left.\frac{d^2A}{dx^2}\right|_{x=a/2} = \frac{1}{4} > 0 \Rightarrow$ point of local minima — (1/2)

Side length square 1 $= \frac{a/2}{4} = \frac{a}{8}$ m
Side length Square 2 $= \frac{a-a/2}{4} = \frac{a}{8}$ m — (1/2)

(OR)

$\frac{dA}{dx} = \frac{4x-2a}{16}$. $\frac{dA}{dx} = 0 \Rightarrow 4x = 2a \Rightarrow x = a/2$ — (1)

$\left.\frac{d^2A}{dx^2}\right|_{x=a/2} = \frac{1}{4} > 0$. Minima — (1/2)

Area $= \frac{x^2}{16} + \frac{(a-x)^2}{16} = \frac{1}{16}\left(\frac{a^2}{4}+\frac{a^2}{4}\right) = \frac{1}{16}\cdot\frac{a^2}{2} = \frac{a^2}{32}$ sqm — (1/2)

37) $\vec{PQ} = (3\hat{i}+4\hat{j}-\hat{k}) - (-2\hat{i}+\hat{j}+3\hat{k}) = 5\hat{i}+3\hat{j}-4\hat{k}$ — (1/2)

Angle between $\vec{PQ}$ and $\vec{PR}$. $\cos\theta = \frac{\vec{PQ}\cdot\vec{PR}}{|\vec{PQ}||\vec{PR}|}$ — (1/2)

$|\vec{PQ}| = \sqrt{25+9+16} = \sqrt{50}$
$|\vec{PR}| = \sqrt{25+1+4} = \sqrt{30}$
$\vec{PR} = 5\hat{i}+\hat{j}-2\hat{k}$

$\cos\theta = \frac{25+3+8}{\sqrt{50}\sqrt{30}} = \frac{36}{10\sqrt{15}}$

$\theta = \cos^{-1}\left(\frac{18}{5\sqrt{15}}\right)$ — (1)

37) (ii) $\vec{RQ} = \vec{PQ} - \vec{PR} = 2\hat{j} - 2\hat{k}$ — (1/2)

Let S be the point on PQ (one third fuel is used)

1 : 2

P S Q — (1/2)

$$\vec{OS} = \frac{1(\vec{OQ}) + 2(\vec{OP})}{3}$$

$$\vec{OS} = \frac{(3\hat{i} + 4\hat{j} - \hat{k}) + 2(-2\hat{i} + \hat{j} + 3\hat{k})}{3} = \frac{-\hat{i} + 6\hat{j} + 5\hat{k}}{3}$$ — (1/2)

p.v of S $= -\frac{\hat{i}}{3} + 2\hat{j} + \frac{5}{3}\hat{k}$ — (1/2)

38) Let I+ be the event when a person is infected and I− be the event when person is not infected.

Let T+ be the event a person is tested positive and T− denote where test is negative.

(i) given $p(I+) = \frac{5}{100}$, $p(I-) = \frac{95}{100}$, $p(T+/I-) = \frac{4}{100}$ (false +ve)

$p(T-/I+) = \frac{3}{100}$ (false negative)

$p(T+/I+) = 1 - p(T-/I+) = 1 - \frac{3}{100} = \frac{97}{100}$ — (1/2)

$p(T+) = p(I+)\cdot p(T+/I+) + p(I-)\cdot p(T+/I-)$ — (1/2)

$= \frac{5}{100}\cdot\frac{97}{100} + \frac{95}{100}\cdot\frac{4}{100} = \frac{865}{10000} = 0.0865$ — (1)

(ii) $p(I+/T+) = \frac{p(I+)\cdot p(T+/I+)}{p(I+)} = \frac{(5\times 97)/10^4}{865/10^4} = \frac{97}{173}$ — (1 1/2)

$p(I-/T+) = 1 - \frac{97}{173} = \frac{76}{173}$ — (1/2)

(OR)

$p(T-) = 1 - p(T+) = 1 - \frac{865}{10000} = 0.9135$ — (1/2)

$p(I+/T-) = \frac{p(I+)\cdot p(T-/I+)}{p(T-)} = \frac{15/10^4}{9135/10^4} = \frac{1}{609}$ — (1 1/2)