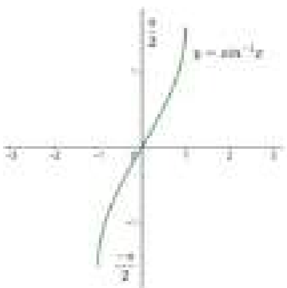
BANGALORE SAHODAYA SCHOOLS COMPLEX ASSOCIATION

PRE-BOARD EXAMINATION (2024-2025)

Grade XII

ANSWER KEY

SUBJECT:MATHEMATICS-041(Set1)

Q. No.	Answers	Marks
1.	**SECTION –A**	
1.	**D**	
2.	**A**	
3.	**A**	
4	**C**	
5.	**C**	
6.	**B**	
7.	**C**	
8.	**A**	
9.	**A**	
10.	**A**	
11.	**B**	
12.	**D**	
13.	**C**	
14.	**D**	
15.	**B**	
16.	**B**	
17.	**C**	
18.	**A**	
19.	**A**	
20	**A**	
	SECTION B	
21	Principal value branch is $[-\frac{\pi}{2},\frac{\pi}{2}]$	1m
		1m
22	$y = sin^{-1}\left(\sqrt{x}\right)$	
	$y' = \frac{1}{\sqrt{1-\left(\sqrt{x}\right)^2}}\frac{d}{dx}\left(\sqrt{x}\right)$	½
		1

$= \frac{1}{\sqrt{1-x}}\frac{1}{2\sqrt{x}}$ ½

$y' = \frac{1}{2\sqrt{x-x^2}}$

OR ½ m

$siny = xsin(a+y)$

$x = \frac{siny}{sin(a+y)}$

Differentiate with respect to y on both sides, we get

$\frac{dx}{dy} = \frac{cos\, y\, sin(a+y) - siny.cos(a+y)}{sin^2(a+y)}$ 1m

$\frac{dx}{dy} = \frac{sin[(a+y)-y]}{sin^2(a+y)}$ ½ m

$\frac{dx}{dy} = \frac{sina}{sin^2(a+y)}$

$\frac{dy}{dx} = \frac{sin^2(a+y)}{sina}$

23 Given $R(x) = 13x^2 + 26x + 15$

Now, $MR = \frac{d}{dx}(R(x)) = 26x + 26$ 1m

MR= $26 \times 17 + 26$

=468 1 m

24 We have

position vector of A = $(\hat{i} + 2\hat{j} + 3\hat{k})$ and position vector of B = $(4\hat{i} + 5\hat{j} + 6\hat{k})$.

$\therefore \overrightarrow{AB} = (p.\mathbf{v}.\text{ of } B) - (p.\mathbf{v}.\text{ of } A)$ ½ m

= $(4\hat{i} + 5\hat{j} + 6\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$ = $(3\hat{i} + 3\hat{j} + 3\hat{k})$, and ½ m

$|\overrightarrow{AB}| = \sqrt{3^2 + 3^2 + 3^2} = \sqrt{27}$

$\therefore$ unit vector in the direction of $\overrightarrow{AB}$

= $\frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \frac{(3\hat{i}+3\hat{j}+3\hat{k})}{\sqrt{27}} = \frac{3(\hat{i}+\hat{j}+\hat{k})}{3\sqrt{3}} = \frac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}}$

= $\left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right)$. 1m

25 Given :$\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$,

$\vec{b} = \widehat{4\imath} - 2\hat{\jmath} + 3\hat{k}$

$\vec{c} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$

$\vec{r} = 2\vec{a} - \vec{b} + 3\vec{c}$=$\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

Since the required vector has magnitude 6units and parallel to vector r. 1m

Required vector is $6\vec{r} = 6\left|\frac{\hat{\imath}-2\hat{\jmath}+2\hat{k}}{\sqrt{1+4+4}}\right|$=$2\hat{\imath} - 4\hat{\jmath} + 4\hat{k}$

OR 1m

½ m

We have, $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$
$\vec{B} = 5\hat{i} + 4\hat{j} - \hat{k}$ ½ m
$\vec{C} = 3\hat{i} + 6\hat{j} + 2\hat{k}$ 1m
$\vec{D} = \hat{i} + 2\hat{j} + 0\hat{k}$
$\overrightarrow{AB}$= position vector of B- position vector of A $\vec{B} - \vec{A} = 5\hat{i} + 4\hat{j} - \hat{k} - (2\hat{i} + 3\hat{j} + 4\hat{k}) = 3\hat{i} + \hat{j} - 5\hat{k}$

$\overrightarrow{CD} = \vec{D} - \vec{C} = \hat{i} + 2\hat{j} + 0\hat{k} - (3\hat{i} + 6\hat{j} + 2\hat{k}) = -2\hat{i} - 4\hat{j} - 2\hat{k}$
$\overrightarrow{AB} \cdot \overrightarrow{CD} = (3\hat{i} + \hat{j} - 5\hat{k}) \cdot (-2\hat{i} - 4\hat{j} - 2\hat{k}) = -6 - 4 + 10 = 0$
Therefore, $\overrightarrow{AB} \perp \overrightarrow{CD}$

SECTION C

26 $\frac{dx}{dt} = 2cm/s$ ½ m
$\frac{dx}{dt} = 0.02m/\sec$
$x^2 + y^2 = 5^2$ ½ m
$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$ ½ m
When x = 4
$y = \sqrt{5^2 - 4^2} = 3$ ½ m
$2 \times 4\,(0.02) + 2 \times 3\frac{dy}{dt} = 0$
$\frac{dy}{dt} = \frac{2\times4\times0.02}{2\times3}$ ½ m

$= -\frac{0.08}{3} \times 100$ ½ m
$= -\frac{8}{3}cm/\sec$

27 Let x be the radius , y be the height, l be the slant height of given cone and θ be the semi vertical angle of cone.
$\therefore l^2 = x^2 + y^2 => x^2 = l^2 - y^2 \ldots\ldots(i)$ ½ m
Volume of cone(V) = $\frac{1}{3}\pi x^2 y = \frac{1}{3}\pi(l^2 - y^2)y = \frac{\pi}{3}(l^2 y - y^3)$
$\frac{dV}{dy} = \frac{\pi}{3}(l^2 - 3y^2)$ and $\frac{d^2V}{dy^2} = \frac{\pi}{3}(-6y) = -2\pi y$ ½ m
$Now\,\frac{dV}{dy} = 0 => \frac{\pi}{3}(\,l^2 - 3y^2) = 0$
$=> 3y^2 = l^2$ ½ m
$y = \frac{1}{\sqrt{3}}l$ ½ m
At $y = \frac{1}{\sqrt{3}}\frac{d^2V}{dx^2} = -2\pi\left(\frac{l}{\sqrt{3}}\right)$
$= \frac{-2\pi l}{\sqrt{3}}$ [Negative]
$\therefore$ V is maximum at $y = \frac{1}{\sqrt{3}}$ ½ m
$\therefore$ From eq. (i), $x^2 = l^2 - \frac{l^2}{3} = \frac{2l^2}{3}$
$\Rightarrow x = \sqrt{2}\frac{1}{\sqrt{3}}$
$\therefore$ Semi-vertical angle, $\tan\theta = \frac{x}{y}$
$= \frac{\sqrt{2}\frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{3}}} = \sqrt{2}$
$\Rightarrow \theta = \tan^{-1}\sqrt{2}$ ½ m

28 $I=\int \frac{2x}{x^2+7x+10}dx = \int \frac{2x+7-7}{x^2+7x+10}dx$

$=\int \frac{2x+7}{x^2+7x+10}dx - \int \frac{7}{x^2+7x+10}dx$ ½ m

$=\int \frac{2x+7}{x^2+7x+10}dx - \int \frac{7}{x^2+7x+\frac{49}{4}-\frac{49}{4}+10}dx$

$=\int \frac{2x+7}{x^2+7x+10}dx - \int \frac{7}{\left(x+\frac{7}{2}\right)^2-\frac{9}{4}}dx$ ½ m

$=log|x^2+7x+10| - \frac{7}{3}log\left|\frac{x+2}{x+5}\right| + C$ 1m+1m

OR

$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{sinx+cosx}{\sqrt{sin2x}}\ dx$ 1m

$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{sinx+cosx}{\sqrt{1-(sinx-cosx)^2}}\ dx$

$\{\ Let\ (sinx-cosx)^2 = sin^2x + cos^2x - 2sinx\ cosx$

$(sinx-cosx)^2 = 1 - sin2x$

$sin2x = 1 - (sinx-cosx)^2\}$

Let $t = sinx - cosx$

$dt = (cosx + sinx)dx$

$\int \frac{1}{\sqrt{1-t^2}}\ dt = \sin^{-1} t = \sin^{-1}(sinx - cosx)$

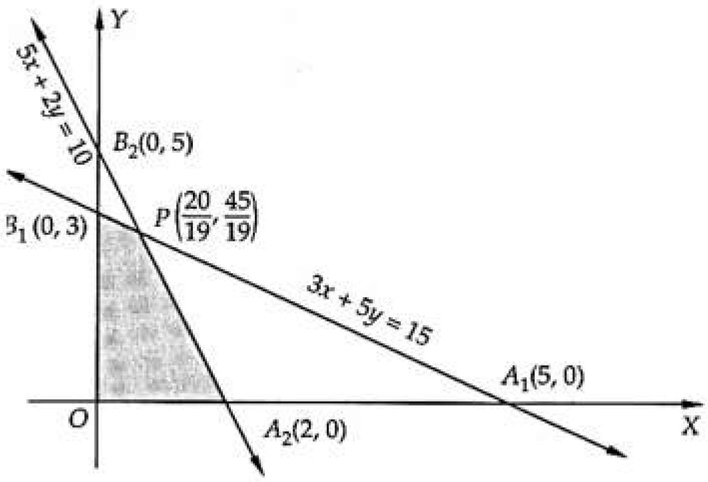
$= \sin^{-1}[(sinx-cosx)]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ 1m

$= \sin^{-1}\left[\left(sin\frac{\pi}{3} - cos\frac{\pi}{3}\right)\right] - \left[\left(sin\frac{\pi}{6} - cos\frac{\pi}{6}\right)\right]$

$= \sin^{-1}[\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) - \sin^{-1}\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)$

$= \sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right) + \sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$

$= 2\sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$ 1m

29 

2m

These points have been obtained by solving the equations of the corresponding intersecting lines, simultaneously.

The values of the objective function at these points are given in the following table:

Point (x, y)	Value of the objective function Z = 5x + 3 y
O (0, 0)	$Z = 5 \times 0 + 3 \times 0 = 0$
A_2 (2, 0)	$Z = 5 \times 2 + 3 \times 0 = 10$
$P\left(\frac{20}{19}, \frac{45}{19}\right)$	$Z = 5 \times \frac{20}{19} + 3 \times \frac{45}{19} = \frac{235}{19}$
B_1 (0, 3)	$Z = 5 \times 0 + 3 \times 3 = 9$

Clearly, the objective function Z has maximum at $P\left(\frac{20}{19}, \frac{45}{19}\right)$. Hence, $x = \frac{20}{19}$, $y = \frac{45}{19}$ is the optimal solution of the given LPP and the optimal value of Z is $\frac{235}{19}$. 1m

30 $\vec{c} = \vec{a} + \vec{b} = 5\,\hat{\imath} + 0\,\hat{\jmath} + \hat{k}$] ½ m

$\vec{d} = \vec{a} - \vec{b} = -\hat{\imath} - 2\,\hat{\jmath} + 5\,\hat{k}$ ½ m

$|\vec{c}| = \sqrt{26}$, $|\vec{d}| = \sqrt{30}$, $\vec{c} \times \vec{d} = 2\,\hat{\imath} - 26\,\hat{\jmath} - 10\,\hat{k}$ 1m

$\text{Sin}\,\theta = 1 \therefore \theta = \frac{\pi}{2}$ 1m

OR

$$\vec{d} = \lambda(\vec{c} \times \vec{b}) = \lambda \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 1 & -1 \\ 1 & -4 & 5 \end{vmatrix} = \lambda\hat{\imath} - 16\lambda\hat{\jmath} - 13\lambda\hat{k}$$ 1 ½ m

$\vec{d}.\vec{a} = 21$

$\Rightarrow 4\,\lambda - 80\lambda + 13\lambda = 21$

$\Rightarrow \lambda = \frac{-1}{3}$ 1m

$\vec{d} = \frac{-1}{3}\hat{\imath} + \frac{16}{3}\hat{\jmath} + \frac{13}{3}\hat{k}$ ½ m

31 Let E_1, E_2 and E_3 denote the events that the vehicle is a scooter, a car and a truck, respectively.

Let A be the event that the vehicle meets with an accident.

It is given that there are 3000 scooters, 4000 cars and 5000 trucks.

Total number of vehicles = 3000 + 4000 + 5000 = 12000

Therefore, we have,

$P(E_1) = \frac{3000}{12000} = \frac{1}{4}$

$P(E_2) = \frac{4000}{12000} = \frac{1}{3}$

$P(E_3) = \frac{5000}{12000} = \frac{5}{12}$ 1m

The probability that the vehicle, which meets with an accident is a scooter is given by $P(\frac{E_1}{A})$.

Now, we have,

$P(\frac{A}{E_1}) = 0.02 = \frac{2}{100}$

$P(\frac{A}{E_2}) = 0.03 = \frac{3}{100}$

$P(\frac{A}{E_3}) = 0.04 = \frac{4}{100}$

Using Bayes' theorem, we have, 1m

i. Required probability = $P(\frac{E_1}{A}) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)++P(E_2)P(A/E_2)}$

$$= \frac{\frac{1}{4}\times\frac{2}{100}}{\frac{1}{4}\times\frac{2}{100}+\frac{1}{3}\times\frac{3}{100}+\frac{5}{12}\times\frac{4}{100}}$$

$$= \frac{\frac{1}{2}}{\frac{1}{2}+1+\frac{5}{3}} = \frac{\frac{1}{2}}{\frac{3+6+10}{6}} = \frac{3}{19}$$ 1.5m

OR

We have p=probability of getting spade in a draw $=\frac{1}{4}$

q= $1-\frac{1}{4} = \frac{3}{4}$ 1.5m

let X denote a success of getting a spade in a throw then X follows binomial distribution with parameters n=3

P(X=r)= where r=0,1,2,3

So the probability distribution of X is given by ½ + ½ m

R	0	1	2	3
P(X=r)	$\left(\frac{3}{4}\right)^3$	$3\times\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^1$	$3\times\left(\frac{3}{4}\right)^1\times\left(\frac{1}{4}\right)^2$	$\left(\frac{1}{4}\right)^3$

1

Mean E(x) $=0\times\left(\frac{3}{4}\right)^3+1\times3\times\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^1+2\times3\times\left(\frac{3}{4}\right)^1\times\left(\frac{1}{4}\right)^2+3\times\left(\frac{1}{4}\right)^3$

$= \frac{48}{64} = \frac{3}{4}$

2m

1m

SECTION D

32 According to the question ,

Given equation of circle is $x^2 + y^2 = 16$ ...(i)

Equation of line given is ,

$\sqrt{3}y = x$ ...(ii)

$\Rightarrow y = \frac{1}{\sqrt{3}}x$ represents a line passing through the origin.

To find the point of intersection of circle and line , substitute eq. (ii) in eq.(i) , we get

$x^2 + \frac{x^2}{3} = 16$

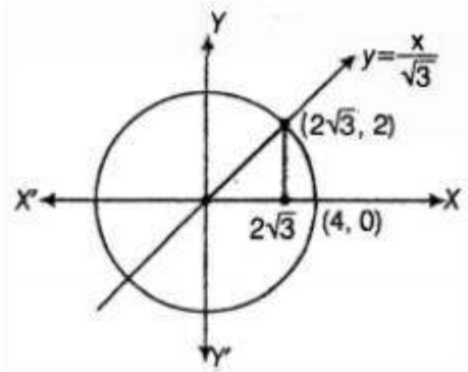
$\frac{3x^2+x^2}{3} = 16$

$\Rightarrow 4x^2 = 48$

$\Rightarrow x^2 = 12$ 1m

$\Rightarrow$ x= $\pm 2\sqrt{3}$

When x= $2\sqrt{3}$, then $y = \frac{2\sqrt{3}}{\sqrt{3}} = 2$ 1m



Required area = Area under the line + Area under the circle

$= \int_0^{2\sqrt{3}} \frac{1}{\sqrt{3}} x\,dx + \int_{2\sqrt{3}}^{4} \sqrt{16-x^2}\,dx$

$= \frac{1}{\sqrt{3}}\left[\frac{x^2}{2}\right]_0^{2\sqrt{3}} + \left[\frac{x}{2}\sqrt{16-x^2} + \frac{(4)^2}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_{2\sqrt{3}}^{4}$ 1m

$= \frac{1}{2\sqrt{3}}\left[(2\sqrt{3})^2 - 0\right] + \left[0 + 8\sin^{-1}(1) - \frac{2\sqrt{3}}{2}\sqrt{16-12} - 8\sin^{-1}\left(\frac{2\sqrt{3}}{4}\right)\right]$ 1m

$= 2\sqrt{3} + 8\left(\frac{\pi}{2}\right) - \frac{2\sqrt{3}}{2} \times 2 - 8\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= 2\sqrt{3} + 4\pi - 2\sqrt{3} - 8\left(\frac{\pi}{3}\right)$

$= 4\pi - \frac{8\pi}{3}$

$= \frac{12\pi - 8\pi}{3}$ 1m

$= \frac{4\pi}{3}$ sq units.

33 Let $\frac{1}{x} = p, \frac{1}{y} = q,$ and $\frac{1}{z} = r$ ½ m

Then, the given system of equations is as follows:

2p + 3q +10r = 4

4p - 6q + 5r = 1

6p + 9q - 20r = 2

This system can be written in the form of AX = B , where

A = $\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$, X= $\begin{bmatrix} p \\ q \\ r \end{bmatrix}$ and B = $\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$ 1m

Now, |A| = 2(120 - 45) - 3(- 80 -30) +10(36 +36)

= 150 + 330 + 720 1m

= 1200

Thus, A is non- singular. Therefore, its inverse exists.

Now,

$A_{11} = 75, A_{12} = 110, A_{13} = 72$

$A_{21} = 150, A_{22} = -100, A_{23} = 0$

$A_{31} = 75, A_{32} = 30, A_{33} = -24$

$\therefore A^{-1} = \frac{1}{|A|} adjA$ 1m

$$= \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

Now, ½ m

$X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{1200}\begin{bmatrix} 300+150+150 \\ 440-100+60 \\ 288+0-48 \end{bmatrix}$$

$$= \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$ 1m

Therefore, $p = \frac{1}{2}$, $q = \frac{1}{3}$ and $r = \frac{1}{5}$

Hence, x = 2, y = 3 and z = 5.

34 Consider equation

$tanx.\frac{dy}{dx} = 2xtanx + x^2 - y;$

$tanx.\frac{dy}{dx} - y = 2xtanx + x^2$

$\frac{dy}{dx} - cotx\,.y = (2xtanx + x^2)cotx$ 1m

Here $P(x) = cotx, Q(x) = (2xtanx + x^2)cotx$

Integrating factor (I.F)=$e^{\int cotx\,dx} = e^{log|sinx|} = sinx$ 1m

Solution is $(I.F)y = \int (I.F).Q(x)dx$

$$sinx.y = \int sinx(2xtanx + x^2)cotxdx = \int 2xsinx\,dx + \int x^2cosxdx$$ 1m

$$= \int 2xsinxdx + x^2\,.sinx - \int 2xsinxdx$$

$$sinx.y = x^2sinx + C$$ 1m

$Given\ that\ y = 0\ when\ x = \frac{\pi}{2}$

$0 = \frac{\pi^2}{4}.sin\frac{\pi}{2} + C$ ½ m

$C = -\frac{\pi^2}{4}$

Substituting value of C

$sinx.y = x^2sinx - \frac{\pi^2}{4}$ ½ m

OR

$$(x-y)\frac{dy}{dx}=x+2y$$
$$\frac{dy}{dx}=\frac{x+2y}{x-y}$$ ½
$$let\ y=vx\ \Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$$ ½ m
$$v+x\frac{dv}{dx}=\frac{1+2v}{1-v}$$
$$x\frac{dv}{dx}=\frac{1+2v}{1-v}-v$$ ½ m
$$x\frac{dv}{dx}=\frac{1+v+v^2}{1-v}$$
$$\int\frac{1-v}{1+v+v^2}dv=\int\frac{dx}{x}$$ ½ m
$$\frac{1}{2}\int\frac{2-2v}{1+v+v^2}dv=\int\frac{dx}{x}$$
$$\frac{1}{2}\int\frac{3-(1+2v)}{1+v+v^2}dv=\int\frac{dx}{x}$$
$$\frac{1}{2}\left[3\int\frac{1}{1+v+v^2}dv-\int\frac{1+2v}{1+v+v^2}dv\right]=\int\frac{dx}{x}$$
$$\frac{1}{2}\left[\int\frac{1}{1+v+v^2}dv-\int\frac{1+2v}{1+v+v^2}dv\right]=\int\frac{dx}{x}$$
$$\frac{3}{2}\int\frac{1}{\left(v+\frac{1}{2}\right)^2+\frac{3}{4}}dv-\frac{1}{2}\int\frac{1+2v}{1+v+v^2}dv=\int\frac{dx}{x}$$
$$\Longrightarrow\frac{3}{2}\times\frac{2}{\sqrt{3}}tan^{-1}\left(\frac{v+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)-\frac{1}{2}\ log|1+v+v^2|=log|x|+C$$ 1+1
$$\Rightarrow\sqrt{3}\,tan^{-1}\left(\frac{\frac{2y+x}{x}}{\sqrt{3}}\right)-\frac{1}{2}log\left|1+\frac{y}{x}+\frac{y^2}{x^2}\right|=\log|x|+C$$

is the required solution

1

35 $\vec{a}_1=\hat{i}+\hat{j},\vec{b}_1=2\hat{i}-\hat{j}+\hat{k}$ ½ m

$\vec{a}_2=2\hat{i}+\hat{j}-\hat{k},\vec{b}_2=3\hat{i}-5\hat{j}+2\hat{k}$ ½ m

$\vec{a}_2-\vec{a}_1=\hat{i}-\hat{k}$ ½ m

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&-1&1\\3&-5&2\end{vmatrix}$$

$=\hat{i}(-2+5)-\hat{j}(4-3)+\hat{k}(-10+3)$ 1m

$=3\hat{i}-\hat{j}-7\hat{k}$

$\left|\vec{b}_1\times\vec{b}_2\right|=\sqrt{9+1+49}=\sqrt{59}$ ½ m

Also, $(\vec{b}_1\times\vec{b}_2).(\vec{a}_2-\vec{a}_1)=(3\hat{i}-\hat{j}-7\hat{k})(\hat{i}-\hat{k})=3+7+0=10$ 1m

$d=\left|\frac{(\vec{b}_1\times\vec{b}_2).(\vec{a}_2-\vec{a}_1)}{|\vec{b}_1\times\vec{b}_2|}\right|=\frac{10}{\sqrt{59}}$ 1m

OR

Suppose the point (1, 0, 0) be P and the point through which the line passes be Q(1,-1,-10). The line is parallel to the vector ½ m

$\vec{b} = 2\hat{i} - 3\hat{j} + 8\hat{k}$ ½ m

Now,

$\overrightarrow{PQ} = 0\hat{i} - \hat{j} - 10\hat{k}$

$\therefore \vec{b} \times \overrightarrow{PQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 8 \\ 0 & -1 & -10 \end{vmatrix}$ 1

$= 38\hat{i} + 20\hat{j} - 2\hat{k}$

$\Rightarrow |\vec{b} \times \overrightarrow{PQ}| = \sqrt{38^2 + 20^2 + 2^2}$ ½ m

$= \sqrt{1444 + 400 + 4}$

$= \sqrt{1848}$

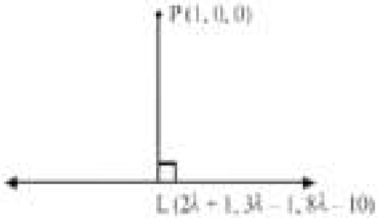
$d = \frac{|\vec{b} \times \overrightarrow{PQ}|}{|\vec{b}|}$

$= \frac{\sqrt{1848}}{\sqrt{77}}$ ½ m

$= \sqrt{24}$

$= 2\sqrt{6}$

Suppose L be the foot of the perpendicular drawn from the point P(1,0,0) to the given line-



The coordinates of a general point on the line

$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ are given by

$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} = \lambda$

$\Rightarrow x = 2\lambda + 1$

$y = -3\lambda - 1$

$z = 8\lambda - 10$

Suppose the coordinates of L be

$(2\lambda + 1, -3\lambda - 1, 8\lambda - 10)$

Since, The direction ratios of PL are proportional to, 1m

$2\lambda + 1 - 1, -3\lambda - 1 - 0, 8\lambda - 10 - 0$, i.e., $2\lambda, -3\lambda - 1, 8\lambda - 10$

Since, The direction ratios of the given line are proportional to 2, -3, 8, but PL is perpendicular to the given line.

$\therefore 2(2\lambda) - 3(-3\lambda - 1) + 8(8\lambda - 10) = 0$

$\Rightarrow \lambda = 1$ Substituting $\lambda = 1$ in $(2\lambda + 1, -3\lambda - 1, 8\lambda - 10)$ we get the coordinates of L as (3, -4, -2). Equation of the line PL is given by

$\frac{x-1}{3-1} = \frac{y-0}{-4-0} = \frac{z-0}{-2-0}$ 1

$= \frac{x-1}{1} = \frac{y}{-2} = \frac{z}{-1}$

$\Rightarrow \vec{r} = \hat{i} + \lambda(\hat{i} - 2\hat{j} - \hat{k})$

Section E

Question No. 36 to 38 are based on the given text. Read the text carefully and

answer the questions:

36 (i) i. $B = \{1, 2, 3, 4, 5, 6\}$

$R = \{(x, y) : y \text{ is divisible by } x\}$

Now, since x is divisible by x

$\Rightarrow (x, x) \in R$

So R is reflexive

Also, here $(2, 4) \in R$, as 4 is divisible by 2

But $(4, 2) \notin R$ as 2 is not divisible by 4

So R is not symmetric

Now, if $(x, y) \in R$ & $(y, z) \in R$

Then $(x,z) \in R$ 1m

So R is transitive

Hence R is reflexive and transitive

(ii) As A has 2 elements and B has 6 elements 1m

So, number of functions from A to B = 6^2.

iii)

$\because (1, 1) \notin R$

So R is not reflexive

Now, here $(1, 2) \in$ but $(2, 1) \notin R$

So R is not symmetric 1m

Also $(1, 3) \in R$ and $(3, 4) \in R$

But $(1, 4) \notin R$

So R is not transitive. 1m

OR

Number of relations

$= 2^{\text{number of element in A} \times \text{number of element in B}}$ 1m

$= 2^{2 \times 6}$

$= 2^{12}$ 1m

37

Let the side of square to be cut off be 'x' cm. then, the length and the breadth of the box will be (18 - 2x) cm each and the height of the box is 'x' cm.

The volume V(x) of the box is given by $V(x) = x(18 - x)^2$ 1m

. $V(x) = x(18 - 2x)^2$

$\frac{dV(x)}{dx} = (18 - 2x)^2 - 4x(18 - 2x)$

For maxima or minima = $\frac{dV(x)}{dx} = 0$

$\Rightarrow (18 - 2x)[18 - 2x - 4x] = 0$ 1m

$\Rightarrow x = 9$ or $x = 3$

$\Rightarrow x =$ not possible

$\Rightarrow x = 3$ cm

The side of the square to be cut off so that the volume of the box is maximum is x = 3 cm

$\frac{dV(x)}{dx} = (18 - 2x)(18 - 6x)$ 1m

$\frac{d^2V(x)}{dx^2} = (18 - 6x)(-2) + (18 - 2x)(-6)$

$\Rightarrow \frac{d^2V(x)}{dx^2} = -12[3 - x + 9 - x] = -24(6 - x)$

$\Rightarrow \left.\frac{d^2V(x)}{dx^2}\right]_{x=3} = -72 < 0$ 1m

$\Rightarrow$ volume is maximum at x = 3

OR 1m

$V(x) = x(18 - 2x)^2$

When x = 3

$V(3) = 3(18 - 2 \times 3)^2$ 1m

$\Rightarrow$ Volume $= 3 \times 12 \times 12 = 432$ cm^3

38

Let A represents obtaining a sum 10 and B represents black die resulted in even number. 1m

$n(S) = 36$

$n(A) = \{(4, 6), (6, 4), (5, 5)\} = 3$

$n(B) = \{(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\} = 18$ 1m

$n(A \cap B) = \{(4, 6), (6, 4)\} = 2$

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{\frac{2}{36}}{\frac{18}{36}} = \frac{1}{9}$

OR

Let A represents getting doublet and B represents red die resulted in number greater than 4.

$n(S) = 36$ 1m

$n(A) = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\} = 6$

$n(B) = \{(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6), (5, 5), (5, 6), (6, 5), (6, 6)\} = 12$

$n(A \cap B) = \{(4, 4), (5, 5), (6, 6)\} = 3$

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{\frac{3}{36}}{\frac{12}{36}} = \frac{1}{4}$ 1m