SECOND TERM EXAMINATION—2025-26

CLASS–XII

SUBJECT–MATHEMATICS

Time : 3 Hrs. **M.M. : 80**

General Instructions:

1. *This question paper contains five sections – A, B, C, D and E. Each part is compulsory. However, there are internal choices in some questions.*
2. *Section - A has 18 MCQs and 2 Assertion-Reasoning based questions of 1 mark each.*
3. *Section B has 5 Very Short Answer-type questions of 2 mark each.*
4. *Section C has 6 Short Answer-type questions of 3 mark each.*
5. *Section D has 4 Long Answer-type questions of 5 mark each.*
6. *Section E has 3 source based/case based questions of 4 mark each with subparts.*

SECTION A (1 × 20)

$\frac{y-4}{3} = \cos x$

1. The function $f : R \rightarrow R$ defined by $f(x) = 4 + 3 \cos x$ is

   a) Bijective b) One-one but not onto

   c) Onto but not one-on d) Neither one-one nor onto

2. $\tan^{-1} \sqrt{3} - \sec^{-1}(-2)$ is equal to

   a) π b) $-\frac{\pi}{3}$

   c) $\frac{\pi}{3}$ d) $\frac{2\pi}{3}$

3. The value of the expression $\sin\{\cot^{-1}[\cos(\tan^{-1}(1))]\}$

a) $\sqrt{\frac{2}{3}}$

b) 0

c) $\frac{1}{\sqrt{3}}$

d) 1

4. If $A = \begin{bmatrix} 0 & 1 & c \\ -1 & a & -b \\ 2 & 3 & 0 \end{bmatrix}$ is a skew symmetric matrix then the value of a+b+c is

a) 1

b) 2

c) 3

d) 4

5. If for a square matrix A, A . (adj A) $= \begin{bmatrix} 2025 & 0 & 0 \\ 0 & 2025 & 0 \\ 0 & 0 & 2025 \end{bmatrix}$, then the value of $|A| + |\text{adj } A|$ is equal to

a) 1

b) 2025 + 1

c) $(2025)^2 + 45$

d) $2025 + (2025)^2$

6. If $y^2(2 - x) = x^3$, then $\frac{dy}{dx}$ at (1, 1) is equal to

a) 2

b) -2

c) $\frac{2}{3}$

d) $-\frac{3}{2}$

7. If $\frac{d}{dx}[f(x)] = ax + b$ and $f(0) = 0$, then $f(x)$ is equal to

a) a + b

b) $\frac{ax^2}{2} + bx$

c) $\frac{ax^2}{2} + bx + C$

d) b

8. The degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^2y}{dx^2}$ is

a) 4 b) $\frac{3}{2}$

c) 2 d) not defined

9. The general solution of the differential equation x dy + y dx = 0

a) xy = c b) x + y = c

c) $x^2 + y^2 = c^2$ d) log y = log x + c

10. If $\vec{a}$ is any non zero vector, the value of $(\vec{a}\cdot\hat{i})\hat{i} + (\vec{a}\cdot\hat{j})\hat{j} + (\vec{a}\cdot\hat{k})\hat{k}$ is equal to

a) $\vec{a}$ b) $2\vec{a}$

c) $3\vec{a}$ d) $4\vec{a}$

11. ABCD is a rhombus whose diagonals intersect at E. Then $\overrightarrow{EA} + \overrightarrow{EB} + \overrightarrow{EC} + \overrightarrow{ED}$ equals to

a) $\vec{0}$ b) $\overrightarrow{AD}$

c) $2\overrightarrow{BD}$ d) $2\overrightarrow{AD}$

12. If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of both X-axis and Z-axis, then the angle which it makes with the positive direction of Y-axis is

a) 0 b) $\frac{\pi}{4}$

c) $\frac{\pi}{2}$ d) π

13. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{3}$, then the angle between $2\vec{a}$ and $-\vec{b}$ is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$

c) $\frac{5\pi}{6}$ d) $\frac{11\pi}{6}$

14. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is

a) 0° b) 30°

c) 45° d) 90°

15. The coordinates of the foot of the perpendicular drawn from the point (0, 1, 2) on the X-axis are given by

a) (1, 0, 0) b) (2, 0, 0)

c) $(\sqrt{5}, 0, 0)$ d) (0, 0, 0)

16. The reflection of the point (α, β, γ) in the XY plane is

a) $(\alpha, \beta, 0)$ b) $(0, 0, \gamma)$

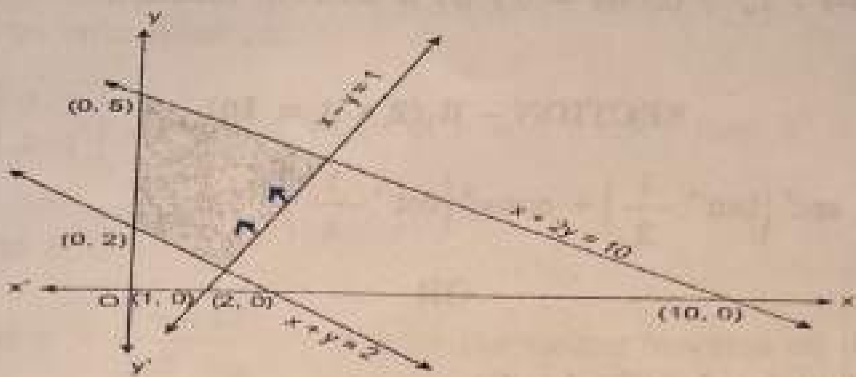
c) $(-\alpha, -\beta, \gamma)$ d) $(\alpha, \beta, -\gamma)$

17. The number of corner points of the feasible region determined by constraints $x \geq 0$, $y \geq 0$, $x + y \geq 4$ is

a) 0 b) 1

c) 2 d) 3

18. The feasible region corresponding to the linear constraints of a Linear programming Problem is given below



Which of the following is not a constraint to the given linear programming problem?

a) $x + y \geq 2$ b) $x + 2y \leq 10$

c) $x - y \geq 1$ d) $x - y \leq 1$

In the following questions, a statement of Assertion(A) is followed by a statement of Reason(R), choose the correct answer out of the following choices.

A) Both A and R are true and R is the correct explanation of A.

B) Both A and R are true and R is not the correct explanation of A

C) A is true but R is false.

D) A is false but R is true.

19. **Assertions (A) :** For any square matrix A, A and adj (A) always commute.

Reason (R) : Let A be a 3×3 non singular matrix such that $A^3 = 2A$, then $|A| = \pm 2\sqrt{2}$.

20. **Assertion (A) :** $\int_{-2}^{2} (x^7 + \sin^3 x)\, dx = 0$

Reason (R) : $\int_{-a}^{a} f(x)\, dx = 0$ $f(x)$ is an even function.

SECTION – B (2 × 4 = 10)

21. Evaluate : $\sec^2\left(\tan^{-1}\frac{1}{2}\right) + \operatorname{cosec}^2\left(\cot^{-1}\frac{1}{3}\right)$

OR

Find the domain of $\sin^{-1}(x^2 - 4)$.

22. Prove that the greatest integer function defined by $f(x) = [x]$, $0 < x < 3$ is not differentiable at $x = 2$.

23. Evaluate : $\int \dfrac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x}\, dx$

OR

Evaluate : $\int \dfrac{1}{(1+x)^{1/2} + (1+x)^{1/3}}\, dx$

24. If vectors $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $c = 3\hat{i} + \hat{j}$ are such that $\vec{b} + \mu\,\vec{c}$ is perpendicular to $\vec{a}$, then find the value of μ. $-\frac{5}{4}$

25. Find the coordinates of the point where the line through $(-1, 1, -8)$ and $(5, -2, 10)$ crosses the ZX plane.

SECTION – C (3 × 6 = 18)

26. A relation R is defined on N × N (where N is the set of natural numbers) as : $(a, b)\, R\, (c, d) \Leftrightarrow a - c = b - d$. Show that R is an equivalence relation.

OR

Show that the function $f : R \rightarrow R$ defined by $f(x) = \frac{2x}{1+x^2}$ is neither one-one nor onto. Further, find set A so that the given function $f : R \rightarrow A$ becomes an onto function.

27. If $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the value of a and b such that $A^2 + Aa + bI = 0$.

Hence find A^{-1}.

28. Prove that $y = \frac{4 \sin \theta}{2 + \cos \theta} - \theta$ is an increasing function on $\left[0, \frac{\pi}{2}\right]$

29. Solve the following differential equation :

$$\left[y - x \cos\left(\frac{y}{x}\right)\right] dx + \left[y \cos\left(\frac{y}{x}\right) - 2x \sin\left(\frac{y}{x}\right)\right] dx = 0.$$

dy

OR

Find the general solution of the differential equation :

$$(x^2 + 1)\frac{dy}{dx} + 2xy = \sqrt{x^2 + 4}$$

30. An ant is moving along the vector $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$. Few sugar crystals are kept along the vector $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ which is inclined at an angle θ with the vector $\vec{a}$. Then find the angle θ. Also find the scalar projection of $\vec{a}$ on $\vec{b}$.

31. Solve the following linear programming problem graphically:

Minimise : $Z = x + 2y$, subject to constraints : $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x, y \geq 0$.

SECTION – D (5 × 4 = 20)

32. Find the value of a and b, if the function f defined by

$$f(x) = \begin{cases} x^2 + 3x + a, & x \le 1 \\ bx + 2, & x > 1 \end{cases}$$ is differentiable at x = 1.

OR

If x cos (a + y) = cosy, then prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$. Hence, show that $\sin a \frac{d^2y}{dx^2} + \sin 2(a + y) \frac{dy}{dx} = 0$

33. Show that the right circular cone of least curved surface area and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

OR

A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12m, find the dimensions of the rectangle that will produce the largest area of the window.

34. Evaluate : $\int_{-1}^{3/2} |x \sin \pi x| \, dx$

35. Make a rough sketch of the region $\{(x, y) : 0 \le y \le x^2 + 1, 0 \le y \le x + 1, 0 \le x \le 2\}$ and find the area of the region, using method of integration.

SECTION - ~~D~~ E $(4 \times 3 = 12)$

36. Utkarsh buys 5 pens, 3 bags and 1 instrument box and pays a sum of Rs.160. From the same shop, Daksh buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of Rs.190. Also, Tejasvi buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of Rs.250.

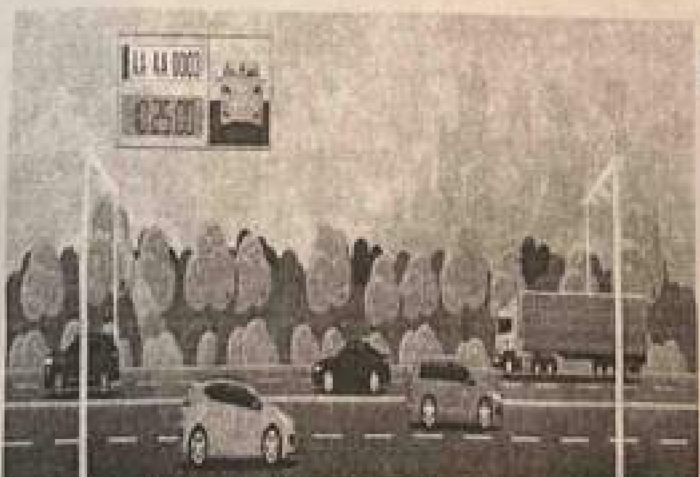
Based on the above information, answer the following questions:

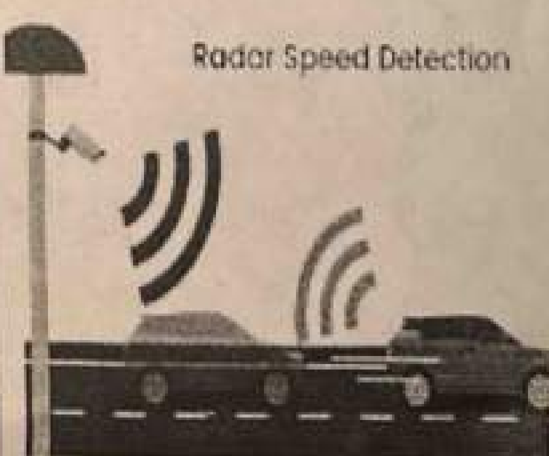
(i) Convert the above situation into a matrix equation of the form $AX = B$. (1)

(ii) Find $|A|$. (1)

(iii) Find A^{-1}. (2)

37. The traffic police has installed Over Speed Violation Detection (OSVD) system at various locations in a city. These cameras can capture a speeding vehicle from a distance of 300 m and even function in the dark. A camera is installed on a ploe at the height of 5m. It detects a car travelling away from the pole at the speed of 20 m per second. At any point, x m away from the base of the pole, the angle of elevation of the speed camera from the car C is θ.





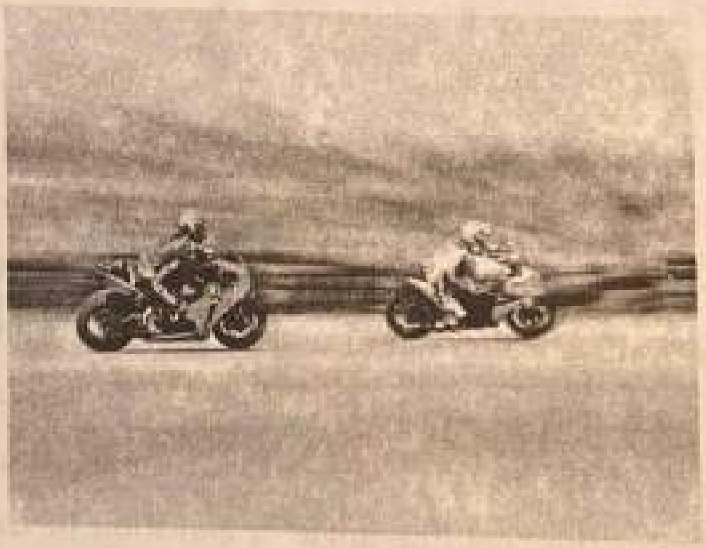
Using the above information answer the following questions:

(a) Express θ in terms of height of the camera installed on the pole and x. (1)

(b) Find $\frac{d\theta}{dx}$ (1)

(c) Find the rate of change of angle of elevation with respect to time at an instant when the car is 50m away from the pole. (2)

38. Two motercycles A and B are running at the speed more than the allowed speed on the roads represented by the lines $\vec{r} = \alpha(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k})$ respectively.



Using the above information answer the following questions:

(a) Find the shortest distance between the given lines. (2)

(b) Find the point at which motorcycles may collide. (2)

–x–x–