BLUEBELLS SCHOOL INTERNATIONAL

Pre-Board-I Examinations 2024-25

CLASS- XII | **16th November, 2024** | **Duration -3 hrs**

SUBJECT-Mathematics | **Max. Marks- 80**

SYLLABUS –

Full Syllabus as per CBSE (excluding Vectors and 3D)

INSTRUCTIONS-

Part A has 20 questions of 1 mark each
Part B has 5 questions of 2 marks each
Part C has 6 questions of 3 marks each
Part D has 4 questions of 5 marks each
Part E has 3 case studies of 4 marks each
Graph sheet required

SECTION – A

Q1 If $A=\begin{bmatrix} x+3 & 8 \\ 3 & 2 \end{bmatrix}$ is non-invertible matrix, then the value of x is **1**

(a) 12 (b) 6 (c) 3 (d) 9

Q2 Let $A=\{1,2,3,....100\}$. **1**

Let a relation R be defined on A, given by $R=\{(x,y): xy \text{ is a perfect square}\}$.

Then the equivalence class [2] is

(a) {2, 8, 18, 32, 50} (b) {2, 8, 18, 32}

(c) {2, 8, 18, 32, 50, 72, 98} (d) None of these

Q3 Let $\sin^{-1}(2x)+\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{2}$. Then the value of 'x' is **1**

(a) $\frac{1}{8}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 8

Q4 If $A=\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ and $A+A'=I$, then the value of α is **1**

(a) $\frac{\pi}{6}$. (b) $\frac{2\pi}{3}$ (c) π (d) $\frac{\pi}{3}$

Q 5 $\int e^x\left(\log\sqrt{x}+\frac{1}{2x}\right)dx =$ **1**

(a) $e^x\times\log x+C$ (b) $e^x\times\log\sqrt{x}+C$ (c) $e^x\times\frac{1}{2x}+C$ (d) $\frac{e^x}{\log\sqrt{x}}+C$

Q 6 If A is a square matrix of order 3 such that $|A|\neq 0$, then which of the following is not true? **1**

(a) $|\text{adj.}A|=|A|^2$ (b) $|A|=|A'|$ (c) $|A|^{-1}=|A^{-1}|$ (d) A is a singular matrix

Q 7 If $y = \sin(2\sin^{-1}x)$, then $(1-x^2)y_2$ is equal to

(a) $-xy_1 + 4y$ (b) $-xy_1 - 4y$ (c) $xy_1 - 4y$ (d) $xy_1 + 4y$

Q 8 If $y = \tan^{-1}(e^{2x})$, then $\frac{dy}{dx}$ is equal to 1

(a) $\frac{2e^{2x}}{1+e^{4x}}$ (b) $\frac{1}{1+e^{4x}}$ (c) $\frac{2}{e^{2x}+e^{-2x}}$ (d) $\frac{1}{e^{2x}-e^{-2x}}$

Q 9 The number of arbitrary constants in the particular solution of a differential equation of second order is (are) 1

(a) 0 (b) 1 (c) 2 (d) 3

Q 10 The objective function of a linear programming problem, is 1

(a) a constant (b) a linear function to be optimized

(c) an inequality (d) a quadratic expression

Q 11 $\int_1^e \frac{\log x}{x} dx$ is equal to 1

(a) $\frac{e^2}{2}$ (b) 1 (c) $\frac{1}{2}$ (d) $-\infty$

Q 12 $\int x\sec^2(5+x^2)dx$ equals 1

(a) $\tan(5+x^2)+C$ (b) $\frac{1}{2}\tan(5+x^2)+C$ (c) $-\frac{1}{2}\tan(5+x^2)+C$ (d) $-\tan(5+x^2)+C$

Q 13 The corner points of the feasible region determined by the system of linear inequalities are $(0,0)$, $(4,0)$, $(2,4)$ and $(0,5)$. If the maximum value of $z = ax + by$, where $a, b > 0$ occurs at both $(2,4)$ and $(4,0)$, then 1

(a) $a = 2b$ (b) $2a = b$ (c) $a = b$ (d) $3a = b$

Q 14 Integrating factor for differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$, is 1

(a) $2\sqrt{x}$ (b) $e^{2\sqrt{x}}$ (c) $e^{\sqrt{x}}$ (d) $e^{-2\sqrt{x}}$

Q 15 If A and B are two independent events with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then $P(B'|A)$ is equal to 1

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d)

Q 16 The function $f(x) = \frac{x-1}{x(x^2-1)}$ is discontinuous at 1

(a) exactly one point (b) exactly two points

(c) exactly three points (d) no point

Q 17 The value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cos x - \sin x}{1+\sin 2x} dx$ is 1

(a) $\frac{\pi}{12}$ (b) π (c) $\frac{\pi}{2}$ (d) 0

Q 18 Sum of order and degree of the differential equation $\left(\frac{dy}{dx}\right)^3 + y\left(\frac{d^2y}{dx^2}\right) = 0$. is **1**

(a) 1 (b) 2 (c) 3 (d) 4

Assertion -Reason based questions

In the following questions a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true

Q 19 **Assertion (A):** The value of $\tan\left(\frac{\sec^{-1}x + \operatorname{cosec}^{-1}x}{2}\right) = 1$, if $x \in \mathbb{R} - (-1, 1)$. **1**

Reason (R): For $y = \tan^{-1} x$, we always have $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Q 20 **Assertion (A):** Let $X = \{1, 2, 3\}$ and $S: X \to X$ such that $S = \{(1,1), (2,2), (3,3), (1,2)\}$. Then, the relation S is a reflexive relation on X. **1**

Reason (R): A relation S defined in a set X is called symmetric relation, if $(a, b) \in S$ implies $(b, a) \in S$ for all $a, b \in X$.

SECTION- B

Q21 Find the value of $\sin^{-1}\left[\sin\left(-\frac{17\pi}{8}\right)\right]$. **2**

OR

Find range of the function $f(x) = \tan^{-1} x + \frac{1}{2}\sin^{-1} x$.

Q22 The side of an equilateral triangle is increasing at the rate of 2 cm/s. At what rate is its area increasing when the side of the triangle is 20 cm? **2**

Q23 If $y = e^x + e^{-x}$, then show that $\frac{dy}{dx} = \sqrt{y^2 - 4}$. **2**

Q24 Find the inverse of $A = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$, also find the value of AA^{-1}. **2**

OR

Find the value of $(x - y)$ from the matrix equation

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} -3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}.$$

Q25 If $3\tan^{-1} x = \pi$ and $\cot^{-1} y = \frac{\pi}{4}$, then write the value of $(x + y)^2$. **2**

SECTION- C

Q26 Let L be the set of all lines in a plane and R be the relation in L defined as

$R = \{(L_1, L_2) : L_1 \text{ is perpendicular to } L_2\}$.

Show that R is symmetric but neither reflexive nor transitive.

OR

Prove that the function $f : N \to N$ defined by $f(x) = x^2 + x + 1$ is one-one but not onto. 3

Q27 Find : $\int \frac{x^3+1}{x^3-x}\,dx$. 3

Q28 A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn randomly one-by-one without replacement and are found to be both kings. Find the probability of the lost card being a king. 3

Q29 Find the particular solution of the D.E. : $\frac{dy}{dx} - 3y\cot x = \sin 2x$, given that $y = 2$ when $x = \frac{\pi}{2}$. 3

Q30 Find the value of $\int_0^1 \tan^{-1}\left(\frac{1-2x}{1+x-x^2}\right)dx$ 3

Q31 An urn contains 5 red, 2 white and 3 black balls. Three balls are drawn, one-by-one, at random without replacement. Find the probability distribution of the number of white balls. Also, find the mean of the number of white balls drawn. 3

SECTION- D

Q32 If $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$, then find A^{-1} and use it to solve the following system of the equations: 5

$x + 2y - 3z = 6,\ 3x + 2y - 2z = 3,\ 2x - y + z = 2.$

Q33 A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at a constant rate of 5 cubic metres per hour. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4 m. 5

Q34 Solve the following linear programming problem graphically. 5

Minimise $z = 3x + 2y$

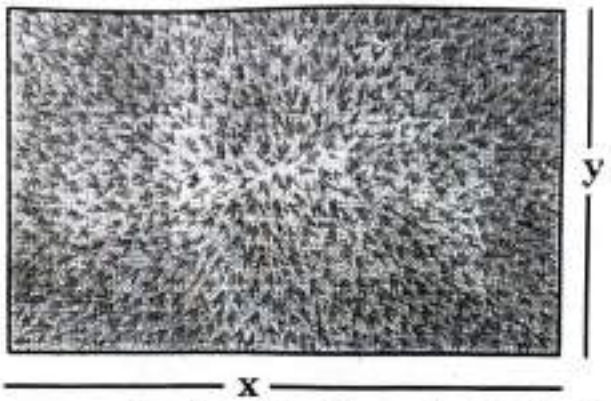
Subject to the constraints : $x \geq 0, y \geq 0, 3x + 4y \leq 60, y \geq 3, x \geq 2y$.

Also, write the maximum value of z.

Q35 Find the area enclosed between by the circle $x^2 + y^2 = 16$ and the lines $\sqrt{3}y = x$, $x = 0$ in the first quadrant using integration. 5

SECTION- E

CASE STUDY I : Read the following passage and the answer the questions given below.



Manjit wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that :

- If its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain same,
- If length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2.

(i) Assume that the length and breadth of the land be x and y (in metres) respectively. Find the equations in terms of x and y. **1**

(ii) Using matrices represent the linear equations obtained above in **(i)**. **1**

(iii) Using matrices, determine the dimensions of the land (in metres). Also write the area of the rectangular plot of land (in square metres). **2**

OR

(iii) Suppose that, Manjit gave the information about his plot in the following manner :
If its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain the same, but if length is decreased by 20 m and breadth is decreased by 10 m, then its area will be decreased by 4800 m^2. In this situation, what will be dimensions of the plot? Assume that the length and breadth of the land be x and y (in metres) respectively. Use matrices.

Q37 **CASE STUDY II :** Read the following passage and answer the questions given below.
A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank.

A tap is connected to such a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uniform rate of 2 cm^3/s.

The semi-vertical angle of the conical tank is 45°.

(i) Find the volume of water in the tank in terms of its radius r. **1**

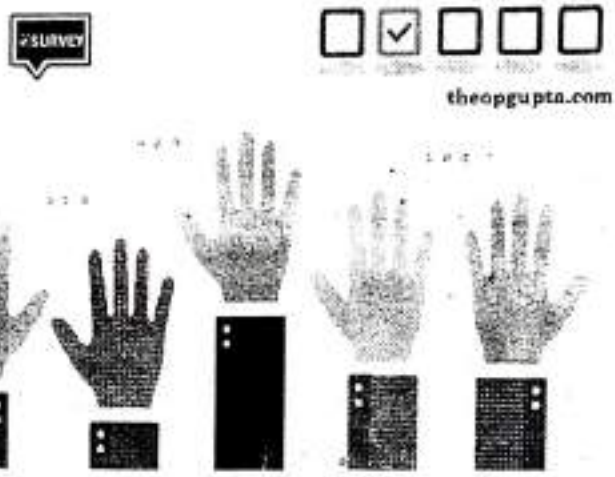
(ii) Find rate of change of radius at an instant when $r = 2\sqrt{2}$ cm. **1**

(iii) Find the rate at which the wet surface of the conical tank is decreasing at an instant when radius $r = 2\sqrt{2}$ cm. **2**

OR

(iii) Find the rate of change of height 'h' at an instant when slant height is 4 cm.

Q38 **CASE STUDY III:** Read the following passage and answer the questions given below.



Utkarsh was doing a survey on a school. Theme of the survey was 'the average number of hours spent on study' by students selected at random. At the end of survey, he prepared the following report related to the data.

Let X denotes the average number of hours spent on study by students. The probability that X can take the values x, has the following form, where k is some constant.

$$P(X=x)=\begin{cases}0.2, & \text{if } x=0\\ kx, & \text{if } x=1 \text{ or } 2\\ k(6-x), & \text{if } x=3 \text{ or } 4\\ 0, & \text{otherwise}\end{cases}$$

(i) What is the value of k? **1**

(ii) What is the probability that the average study time of students is not more than 1 hour? **1**

(iii) What is the probability that the average study time to students is at least 3 hours? **1**

What is the probability that the average study time of students is exactly 2 hours? **2**

OR

(iii) What is the probability that the average study time of students is at least 1 hour?