BANGALORE SAHODAYA SCHOOLS COMPLEX ASSOCIATION
PRE-BOARD EXAMINATION (2024-2025)
Grade XII

Class: - XII | **SUBJECT: Applied Mathematics** | **Date: - 20.12.2024**
Time: - 3 hours | **Code - (241) SET-1** | **Marks: - 80**

General Instructions:

1. This question paper contains – **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section **A** has **18 MCQ's** and **02** Assertion Reason based questions of 1 mark each.
3. Section **B** has 5 **Very Short Answers (VSA)** type questions of 2 marks each.
4. Section **C** has 6 **Short Answer (SA)** type questions of 3 marks each
5. Section **D** has 4 **Long Answer (LA)** type questions of 5 marks each.
6. Section **E** has 3 **source based / case based / passage based/integrated units of assessment** (4 marks each) with sub parts.
7. Internal Choice is provided **in 2 questions in Section-B, 2 questions in Section-C, 2 questions in Section-D.** You have to attempt only one alternative in all such questions.

Section A
This section comprises multiple choice questions of 1 mark each.

1. What is the value of $7^6 (mod\ 3)$? 1
   a) 1 b) 2 c) 3 d) 4

2. In a kilometre race, A B and C are three participants. A can give B start of 50 m and C a start of 69 m. In the same race, the start which B can allow for C is _____. 1
   a) 17 m b) 18 m c) 19 m d) 20 m

3. The speed of a boat in still water is 15 km/h and the rate of current is 3 km/h. The distance travelled by boat downstream in 12 minutes is _____ 1
   a) 1.2 km b) 1.8 km c) 2.4 km d) 3.6 km

4. If $A^2 - A + I = O$, then the inverse of matrix A is _____ 1
   a) A^2 b) $A + I$ c) $I - A$ d) $A - I$

5. If A is a square matrix of order 3 and $|A| = 5$, then the value of $|3A|$ is _____ 1
   a) 15 b) 27 c) 45 d) 135

6. If $A = \begin{bmatrix} 0 & a & 1 \\ -1 & b & 1 \\ -1 & c & 0 \end{bmatrix}$ is a skew-symmetric matrix, then the value of $(a+b+c)^2$ is _____ 1
   a) 1 b) 0 c) 4 d) 9

7. The moving averages method is used to find the _____ 1
   a) seasonal variation b) irregular variation

c) secular trend d) cyclical variation

8. If the slope of the trend line is positive, it shows_____ 1

a) a rising trend b) a declining trend

c) stagnation d) irregular trend

9. $\int \frac{x^2+1}{x^2-1}\, dx$ is equal to _____ 1

a) $x + \log|\frac{x-1}{x+1}| + C$ b) $x + \log\log|\frac{x-1}{x+1}| + C$

c) $\log\log|(x-1)(x+1)| + C$ d) $\log|x^2+1| + C$

10. The solution of the given differential equation $e^{y-x}\frac{dy}{dx} = 1$ is _____ 1

a) $e^y = e^x + C$ b) $e^y = e^{-x} + C$

c) $e^y = -e^x + C$ d) $e^{-y} = e^x + C$

11. Let X be a discrete random variable. Then the variance of X is _____ 1

a) $\sqrt{E(X^2) - \big(E(X)\big)^2}$ b) $E(X^2) - \big(E(X)\big)^2$

c) $E(X^2) - E(X)$ d) $E(X^2) + \big(E(X)\big)^2$

12. The test statistic t for testing the significance of difference between the means of two independent samples is given by _____ 1

a) $t = \frac{\bar{x}-\bar{y}}{\sqrt{S}}$ b) $t = \frac{\bar{x}-\bar{y}}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$

c) $t = \frac{\bar{x}-\bar{y}}{\frac{S}{\sqrt{n-1}}}$ d) $t = \frac{\bar{x}+\bar{y}}{S\sqrt{\frac{1}{n_1}-\frac{1}{n_2}}}$

13. If the sample size is large, then the shape of sampling distribution is approximately equal to _____. 1

a) normal distribution b) t- distribution

c) binomial distribution d) Poisson's distribution

14. If $a = 95, b = 2$ and deviation of 2022 from 2016 is 6, then the trend value for 2022 is _____. 1

a) 103 b) 99 c) 107 d) 98

15. A man invests Rs. 9,600 on Rs. 100 shares at Rs. 80. If the company pays him 18% dividend, his rate of return is _____. 1

a) 22% b) 22.5% c) 24% d) 25%

16. Anshul plans a trip to Goa and estimates that he requires Rs. 25,000 for the same. He takes a loan from the credit card with 11% annual interest rate for 3 years. The EMI under the flat rate system is given by _____. 1

a) Rs. 1,001.23 b) Rs. 778.65

c) Rs. 450.5 d) Rs. 923.61

17. The present value of a perpetuity of Rs. 1,000 payable at the end of each year, if money is worth 5% per annum is_____. 1

a) Rs. 18,000 b) Rs. 19,000

c) Rs. 21,000 d) Rs. 20,000

18. The effective rate of interest equivalent to the nominal rate 6% compounded semi-annually is _____. 1

a) 6.05% b) 6.07% c) 6.09% d) 6.1%

Question number 19 and 20 are Assertion and Reason based question. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both A and R are true and R is the correct explanation of the assertion.

(b) Both A and R are true but R is not the correct explanation of the assertion.

(c) A is true, but R is false.

(d) A is false, but R is true.

19. **Assertion(A):** If a machine costs Rs. 50,000 with an estimated useful life of 4 years and its scrap value is Rs. 10,000, then the depreciation rate is 25%. 1

Reason(R): Depreciation rate (%) $= \frac{\text{Annual depreciation}}{\text{Total depreciation}} \times 100$.

20. The corner points of the feasible region for L.P.P. are $(4, 0)$, $(5, 0)$, $(5, 3)$, $(3, 5)$, $(0, 5)$ and $(0, 4)$.The objective function $Z = ax - by + 1900, a, b > 0$ has maximum value 1950 at $(5, 0)$ and minimum value 1550 at $(0, 5)$. 1

Assertion (A): The value of Z at point $(5, 3) = 1740$.

Reason (R): $a = 10, b = 70$.

Section B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. (a) A shopkeeper mixes two varieties of rice, one costs Rs. 161 per kg and other costs Rs. 179 per kg, in the ratio 5:4 respectively. He sells the mixed variety at 2

Rs. 202.80 per kg. Find his profit percentage.

OR

(b) If $-2\frac{1}{2} \leq \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}$, then find the values of x when

(i) is an integer (ii) x is a real number

22. Find the last digit of 17^{17}. 2

23. Construct a matrix $A = \left[a_{ij} \right]_{3\times 2}$ whose elements a_{ij} are given by 2

$a_{ij} = \frac{1}{2} | -3i + j |$.

24. (a) Evaluate: $\int \frac{\log(\log x)}{x}\, dx$. 2

OR

(b) Evaluate: $\int \frac{1}{\sqrt{5x^2 - 2x}}\, dx$.

25. Find the present value of a sequence of payments of Rs. 360 made at the beginning of every six months continuing forever, if money is worth 6% p.a. compounded half yearly. 2

Section C

This section comprises short answer (SA) type questions of 3 marks each.

26. Find the intervals in which the function $f(x) = 2x^3 + 9x^2 + 12x + 20$ is 3

(i) increasing

(ii) decreasing

27. Fit a straight-line trend by the method of least square to estimate the number of tourists (in millions) that had arrived in the year 2014 for the following data: 3

Years	2004	2005	2006	2007	2008	2009	2010
Number of Tourists (in millions)	18	20	23	25	24	28	30

28. A company manufacturers steel rods with the average breaking strength of steel rods specified to be 18.5 thousand kg. For this, a sample of 14 rods were tested. The mean and standard deviation obtained were 17.85 and 1.955, respectively. Test the significance of the deviation. (Given $t_{13}(0.05) = 2.16$) 3

29. A person borrowed Rs. 20,00,000 from a bank to purchase a flat and decided to repay the loan by equal monthly installments in 15 years. The bank charges interest 12% per annum compounded annually. Determine the EMI by using reducing balance method. (Given $(1.01)^{-180} = 0.1668$) 3

30. (a) The rate of growth of a population is proportional to the number present. If the population of a city doubled in the past 25 years and the present population is 1,00,000, when will the city have a population of 5,00,000? 3

(Use $\log 5 = 1.609, \log 2 = 0.6931$)

OR

(b) In a bank, principal increases continuously at the rate of 5% per year. An amount of Rs. 1,000 is deposited with this bank, how much will it worth after 10 years? (Use $e^{0.5} = 1.648$)

31. (a) Solve the following LPP graphically using corner point method: 3

Maximize $Z = x + y$

Subject to the constraints,

$x + y \leq 3; y - 2x \leq 1 ; x \leq 2$

$x \geq 0, y \geq 0$

OR

(b)A medical company has factories at two cities C_1 and C_2. From these cities, supply is made to each of its three agencies A, B and C using CNG vehicles. The monthly requirements of agencies are respectively 40, 40 and 50 packets of medicines, while the production capacities at two factories are 60 and 70 packets respectively. The transportation cost per packet from the factories to the agencies are given below:

Transportation cost per packets (in Rs.)		
	C_1	C_2
A	5	4
B	4	2
C	3	5

Express the given problem as an LPP to minimize the cost. (No need to solve)

Section D

This section comprises long answer (LA) type questions of 5 marks each.

32. (a) Given two matrices A and B, $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$, find AB. Using this product AB, solve the following system of equations: 5

$2x - y + z = -1, -x + 2y - z = 4, x - y + 2z = -3$

OR

(b) The daily cost C of operating a hospital is a linear function of the number of indoor patients (I) and outdoor patients (O) plus fixed cost (z) i.e

$C = xI + yO + z$. Find the values of costs x, y and z through a system of linear equations, using inverse coefficient matrix method:

Day	C(in Rs. 1000)	Patients (in multiple of 10')	
		I	**O**
1	12	2	3
2	13	6	2
3	15	5	3

33. Mr. X wants to construct a rectangular garden using a brick wall on the one side of the garden and wire fencing for other three sides. He has 200 ft. of wire fencing. If x denotes the length of the side of garden perpendicular to the brick wall and y 5

denotes the length of side parallel to the brick wall then find:

(i) the relation representing total length of fencing wire.

(ii) area of the garden as a function of x.

(iii) maximum value of the area of the garden.

34. (a) If 5% students appearing in an examination fail, find the probability that out 100 students appearing in the examination: 5

(i) none failed

(ii) 5 students failed

(iii) at most 3 failed

[Use $e^{-5} = 0.006738$]

OR

In a sample of 1000 cases, the mean of a certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal, find how many students score

(i) between 12 and 15?

(ii) above 18?

(iii) below 8?

$(F(0.4) = 0.6554, F(0.8) = 0.7881, F(1.6) = 0.9452, F(2.4) = 0.9918)$

35. A man borrows Rs. 10,00,000 and agrees to pay interest quarterly at an annual rate of interest of 8%. At the same time, he sets up a sinking fund in order to repay the loan at the end of 5 years. If the sinking fund earns interest at 6% per annum, compounded quarterly, find the quarterly cost of the debt. 5

[Given $(1.015)^{20} = 1.346852$]

Section E

This section comprises 3 case study-based questions of 4 marks each.

36. **Case Study-I**

At usual rowing rate Mr. X can travel 12 km downstream in a certain river, 6 hours less than it takes to travel same distance upstream. But if he doubles his usual rowing speed for his 24 km round trip, the downstream 12 km would then take only 1 hour less than the upstream 12 km. Consider the rowing speed in still water as x km/hr and speed of water current as y km/hr.

Based on the above information, answer the following questions:

(i) Form the equation in terms of x and y for first condition. 1

(ii) What will be the new speed upstream and downstream in second condition? 1

(iii) (a) Find the speed of the water current. 2

OR

(b) A man rows a boat at 5 km/hr in still water. If the speed of water current in the river is 1 km/hr and it takes him 1 hour to row to a place and come back, how far off is the place.

37. **Case Study-II**

In a military operation, the effort is to inflict maximum damage to enemy at minimum cost and loss. In an industry, the management always tries to utilize its resources in the best possible manner. A salaried person tries to make investments in such a manner that the returns on investment are high but at the same time income tax liability is kept low. In all the above cases, if constraints are represented by linear equations/inequations and a particular plan of action with several alternatives, we use linear programming. The linear programming is a method for determining optimum values of a linear function subject to constraints expressed as linear equations or inequations.

A company uses three machines to manufacture two types of shirts, half sleeves and full sleeves. The number of hours required per week on machine M_1, M_2 *and* M_3 for one shirt of each type is given in the following table:

	M_1	M_2	M_3
Half sleeves	1	2	$\frac{8}{5}$
Full sleeves	2	1	$\frac{8}{5}$

None of the machines can be in operation for more than 40 hours per week. The profit on each half sleeves shirt is Rs. 1 and profit on each full sleeve shirt is Rs. 1.50.

Based on the above information, answer the following questions:

(i) Formulate the objective function and the constraint of the above linear programming problem. 2

(ii) How many of each type shirts should be made per week to maximize the company's profit? 2

38. **Case study- III**

A photocopier (also called copier or copy machine) is a machine that makes copies of documents and other visual images onto paper or plastic film quickly and cheaply. Most modern photocopiers use a technology called xerography, a dry process that uses electrostatic charges on a light-sensitive photoreceptor to first attract and then transfer toner particles (a powder) onto paper in the form of an image. The toner is then fused onto the paper using heat, pressure, or a combination of both. Copiers can also use other technologies, such as inkjet, but xerography is standard for office copying.

An office has four copying machines, and the random variable X measures how many of them are in use at a particular moment in time. Suppose that

$P(X = 0) = 0.08, P(X = 1) = 0.11, P(X = 2) = 0.27$ and $P(X = 3) = 0.33$.

Based on the above information, answer the following questions:

(i) What is $P(X = 4)$? 1

(ii) What is $P(X \geq 2)$? 1

(iii) (a) What is expected number of copying machines in use at a particular moment in time? 2

OR

(b) Calculate the variance and standard deviation of the number of copying machines in use at a particular moment.