

Section A

1. $7 \equiv 1 \pmod{3}$
 $\Rightarrow 7^6 \equiv 1^6 \pmod{3}$

$\therefore 7^6 \pmod{3} = 1$

option a) 1

2. When A runs 1000m; B runs 950m
and C runs 931m

$\Rightarrow$ When B runs 950m; C runs 931m

$\therefore$ When B runs 1000m; C runs $\frac{931}{950} \times 1000$
ie 980m

$\therefore$ B can allow C 20m.

option d) 20m

3. Speed of the boat in still water is $x = 15 \text{ km/h}$.
Speed of the current is $y = 3 \text{ km/h}$.
Speed of downstream = $x + y = 15 + 3 = 18 \text{ km/h}$.

$$\begin{aligned}\text{Distance travelled} &= \text{Speed} \times \text{Time} \\ &= 18 \text{ km/h} \times \frac{12}{60} \text{ hours} \\ &= \underline{\underline{3.6 \text{ km}}}\end{aligned}$$

option d) 3.6 km

4. Given $A^2 - A + I = O$

$$\Rightarrow I = A - A^2 \rightarrow \times \bar{A}^1 \text{ on both sides.}$$

$$\Rightarrow I \bar{A}^1 = A \bar{A}^1 - A \cdot A \cdot \bar{A}^1$$

$$\Rightarrow \bar{A}^1 = I - A I$$

$$\therefore \bar{A}^1 = \underline{\underline{I - A}}$$

option c) $I - A$.

5. We have $|kA| = k^n |A|$, where n is the order of the matrix.

$$\therefore |3A| = 3^3 |A| = 27 \times 5 = \underline{\underline{135}}$$

option d) 135

6. $A = \begin{bmatrix} 0 & a & 1 \\ -1 & b & 1 \\ -1 & c & 0 \end{bmatrix}$; given A is a skew

symmetric matrix.

$$\therefore A = -A^1$$

$$\begin{bmatrix} 0 & a & 1 \\ -1 & b & 1 \\ -1 & c & 0 \end{bmatrix} = - \begin{bmatrix} 0 & -1 & -1 \\ a & b & c \\ 1 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow -a = -1 ; \boxed{b = 0} ; c = -1$$

$$\Rightarrow \boxed{a = 1}$$

$$\therefore (a+b+c)^2 = 0$$

option b) 0

7. c) Secular trend.

8. a) a rising trend.

$$9. \int \frac{x^2+1}{x^2-1} dx$$

$$= \int 1 + \frac{2}{x^2-1} dx$$

$$= x + \frac{1}{2} \times 2 \log \left| \frac{x-1}{x+1} \right| + C$$

option a) $x + \log \left| \frac{x-1}{x+1} \right| + C$

$$10. e^{y-x} \frac{dy}{dx} = 1$$

$$\Rightarrow e^y \cdot e^{-x} \frac{dy}{dx} = 1$$

$$\Rightarrow e^y dy = e^x dx$$

$$\Rightarrow e^y = e^x + C$$

option a) $e^y = e^x + C$

11. option b) $E(X^2) - (E(X))^2$

12. option b) $t = \frac{\bar{x} - \bar{y}}{S \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

13. option a) normal distribution

14. Given $a=95$; $b=2$

Trend equation is $Y_t = a + bx$

$$x = 2022 - 2016 = 6.$$

$$\therefore Y_t = 95 + 2(6) = 95 + 12 = 107$$

option c) 107

15. Rate of Return = $\left(\frac{\text{Dividend}}{\text{Investment}} \right) \times 100$

$$\text{Number of Shares} = \frac{\text{Total Investment}}{\text{Price per Share}}$$

$$= \frac{9600}{80} = \underline{120}$$

$$\therefore \text{Dividend} = \text{Number of Shares} \times \text{Dividend Rate} \\ = 120 \times 18 = ₹2160$$

$$\therefore \text{Rate of Return} = \left(\frac{2160}{9600} \right) \times 100 = 22.5\%$$

option b) 22.5%.

16. $EMI = \frac{P + Pni}{n}$

$$= \frac{25,000 + \frac{25,000 \times 11 \times 36}{100}}{36}$$

$$\approx ₹923.61$$

option d) Rs. 923.61

$$17. \quad P = \frac{R}{i} = \frac{1,000}{\left(\frac{5}{100}\right)} = \frac{100,000}{5} = \underline{\underline{20,000}}$$

option d) Rs. 20,000.

$$18. \quad r_{\text{eff}} = \left(1 + \frac{8}{100p}\right)^p - 1$$

$$= \left(1 + \frac{6}{200}\right)^2 - 1$$

$$= (1.03)^2 - 1 = 1.0609 - 1 = 0.0609$$

$$\therefore r_{\text{eff}} = 6.09\%$$

option c) 6.09%.

$$19. \quad \text{Annual Depreciation} = \frac{\text{Cost} - \text{Scrap Value}}{\text{Useful Life}}$$

$$= \frac{50,000 - 10,000}{4} = 10,000$$

$$\therefore \text{Depreciation rate (\%)} = \frac{\text{Annual depreciation}}{\text{Total depreciation}} \times 100$$

$$= \frac{10,000}{40,000} \times 100$$

$$= 25\%$$

(a) Both A and R are true and R is the

correct explanation of the assertion.

Q0. $Z = ax - by + 1900$

$$Z = a(5) - b(0) + 1900 = 1950$$

$$\Rightarrow 5a = 50$$

$$\Rightarrow \boxed{a = 10}$$

Also $Z = a(0) - b(5) + 1900 = 1550$

$$-5b = -350$$

$$\Rightarrow \boxed{b = 70}$$

$$\therefore Z = 10(5) - 70(3) + 1900 \text{ at } (5, 3).$$

$$Z = \underline{1740}$$

(a) Both A and R are true and R is the correct explanation of the assertion.

Section B

21.

(a) Given: Cost of one variety of Rice = ₹ 161 per kg.
Cost of another variety = ₹ 179 per kg.

By alligation Mixture,

Cost of Cheaper (c)
₹ 161

Cost of dearer (d)
₹ 179

mean (m)
₹ x

d - m
₹ 179 - x

m - c
x - 161

} $\frac{1}{2}m$

$$\frac{\text{Quantity of cheaper}}{\text{Quantity of dears}} = \frac{d-m}{m-c} = \frac{5}{4} \quad -\frac{1}{2}$$

$$\Rightarrow \frac{179-x}{x-161} = \frac{5}{4}$$

$$\Rightarrow 4(179-x) = 5(x-161)$$

$$\Rightarrow 716-4x = 5x-805$$

$$\Rightarrow 9x = 1521 \quad -\frac{1}{2}$$

$$\therefore x = \underline{\underline{\text{₹}169}}$$

$\therefore$ Mean price = ₹169.

profit = Selling price - Mean price

$$= 202.80 - 169 = 33.80$$

$$\therefore \text{Profit percentage} = \frac{33.8}{169} \times 100 \approx \underline{\underline{20\%}} \quad -\frac{1}{2}$$

OR

$$(b) -2\frac{1}{2} \leq \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}$$

$$\Rightarrow -\frac{5}{2} \leq \frac{x}{2} - \frac{4}{3} < \frac{1}{6}$$

$$\Rightarrow -\frac{5}{2} + \frac{4}{3} \leq \frac{x}{2} < \frac{1}{6} + \frac{4}{3}$$

$$\Rightarrow \frac{-15+8}{6} \leq \frac{x}{2} < \frac{1+8}{6}$$

$$-\frac{7}{6} \leq \frac{x}{2} < \frac{9}{3} \quad \times 2$$

$$\Rightarrow -\frac{7}{3} \leq x < \frac{18}{3}$$

$$\text{ie } \boxed{-\frac{7}{3} \leq x < 6} \Rightarrow -2.33 \leq x < 6 \quad -\frac{1}{2}$$

(i) when x is an integer

$$x = \{-2, -1, 0, 1, 2, 3, 4, 5\} \quad -\frac{1}{2}$$

(ii) when x is a real number.

$$x \in \left[-\frac{7}{3}, 6\right) \quad -\frac{1}{2}$$

22.

To find the Last digit of 17^n .

We find $17^n \pmod{10}$.

$$17 \equiv 17 \pmod{10}$$

$$\Rightarrow 17 \equiv 7 \pmod{10}$$

$$\Rightarrow 17^2 \equiv 7^2 \pmod{10}$$

$$\Rightarrow 17^2 \equiv 9 \pmod{10}$$

$$\Rightarrow (17^2)^2 \equiv 81 \pmod{10}$$

$$\Rightarrow 17^4 \equiv 1 \pmod{10}$$

$$\Rightarrow (17^4)^4 \equiv 1^4 \pmod{10}$$

$$17^{16} \equiv 1 \pmod{10}$$

$-\frac{1}{2}$

$$17^{16} \times 17 \equiv 1 \times 17 \pmod{10}$$

$$\Rightarrow \boxed{17^{17} \equiv 7 \pmod{10}}$$

$\therefore$ Last digit of 17^{17} is 7. $-\frac{1}{2}$

23.

$$A = [a_{ij}]_{3 \times 2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}_{3 \times 2} \text{ where } a_{ij} \text{ is}$$

$$\text{Given as } a_{ij} = \frac{1}{2} |-3i + j|$$

$$a_{11} = \frac{1}{2} |-3 \times 1 + 1| = \frac{1}{2} |-2| = 1$$

$$a_{12} = \frac{1}{2} |-3 \times 1 + 2| = \frac{1}{2}$$

$$a_{21} = \frac{1}{2} |-3 \times 2 + 1| = \frac{5}{2}$$

$$a_{22} = \frac{1}{2} |-3 \times 2 + 2| = 2$$

$$a_{31} = \frac{1}{2} |-3 \times 3 + 1| = 4$$

$$a_{32} = \frac{1}{2} |-3 \times 3 + 2| = \frac{7}{2}$$

$$\therefore A = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{5}{2} & 2 \\ 4 & \frac{7}{2} \end{bmatrix}$$

24.

$$(a) \int \frac{\log(\log x)}{x} dx$$

put $\log x = t$

$$\frac{1}{x} = \frac{dt}{dx}$$

$$\therefore \int \frac{\log(\log x)}{x} dx = \int \log t \, dt \quad \text{ILATE}$$

$$= \int \log t (1) dt \quad -\frac{1}{2}$$

$$= \log t \int 1 dt - \int \frac{d[\log t]}{dt} \int 1 dt \cdot dt$$

$$= t \log t - \int \frac{1}{t} \times t \, dt \quad -\frac{1}{2}$$

$$= t \log t - t + C$$

$$= \log x \cdot \log(\log x) - \log x + C \quad -\frac{1}{2}$$

OR

$$(b) \int \frac{1}{\sqrt{5x^2 - 2x}} dx$$

Consider $5x^2 - 2x = 5 \left(x^2 - \frac{2}{5}x \right)$

$$= 5 \left[\left(x - \frac{1}{5} \right)^2 - \left(\frac{1}{5} \right)^2 \right] \quad -\frac{1}{2}$$

Substitute $x - \frac{1}{5} = t \Rightarrow dx = dt \quad -\frac{1}{2}$

$$\begin{aligned}
 \therefore \int \frac{dx}{\sqrt{5x^2 - 2x}} &= \int \frac{dt}{\sqrt{5[t^2 - (\frac{1}{5})^2]}} \\
 &= \frac{1}{\sqrt{5}} \int \frac{dt}{\sqrt{t^2 - (\frac{1}{5})^2}} \quad -\frac{1}{2}- \\
 &= \frac{1}{\sqrt{5}} \log \left| t + \sqrt{t^2 - (\frac{1}{5})^2} \right| + C \\
 &= \frac{1}{\sqrt{5}} \log \left| x - \frac{1}{5} + \sqrt{x^2 - \frac{2x}{5}} \right| + C \quad -\frac{1}{2}- \\
 &= \underline{\underline{\quad}}
 \end{aligned}$$

25.

Given: $R = ₹360$

$i = 6\% \text{ p.a.}$

$= \frac{6}{2}\% \text{ per period.}$

$i = 3\% \text{ per period.}$

$$\therefore P = R + \frac{R}{i} \quad -\frac{1}{2}-$$

$$= 360 + \frac{360}{\frac{3}{100}} \quad -\frac{1}{2}-$$

$$= 360 + 120 \times 100$$

$$= 12000 + 360$$

$$P = ₹12,360 \quad -\frac{1}{2}-$$

Section C

26. Given: $f(x) = 2x^3 + 9x^2 + 12x + 20$

Differentiating w.r.t. x ,

$$f'(x) = 6x^2 + 18x + 12$$

For critical points,

$$f'(x) = 0$$

$$\Rightarrow 6x^2 + 18x + 12 = 0$$

$$\Rightarrow x^2 + 3x + 2 = 0$$

$$\Rightarrow x^2 + 2x + x + 2 = 0$$

$$x[x+2] + 1[x+2] = 0$$

$$\Rightarrow (x+1)(x+2) = 0$$

$$\therefore x = -1 \text{ or } -2$$

Using wavy curve method



Intervals	Sign of $f'(x)$	Conclusion
$(-\infty, -2)$	$f'(x) > 0$	f is increasing
$(-2, -1)$	$f'(x) < 0$	f is decreasing
$(-1, \infty)$	$f'(x) > 0$	f is increasing

$\therefore f(x)$ is increasing in $(-\infty, -2) \cup (-1, \infty)$
and decreasing in $(-2, -1)$. -1/2-

27.

Year (t)	No. of Tourists(y)	$x = t - 2007$	x^2	xy
2004	18	-3	9	-54
2005	20	-2	4	-40
2006	23	-1	1	-23
2007	25	0	0	0
2008	24	1	1	24
2009	28	2	4	56
2010	30	3	9	90

$$\Sigma y = 168 \quad \Sigma x = 0 \quad \Sigma x^2 = 28 \quad \Sigma xy = 53$$

$$a = \frac{\Sigma y}{n} = \frac{168}{7} = 24$$

$$b = \frac{\Sigma xy}{\Sigma x^2} = \frac{53}{28} = 1.89$$

$\therefore$ Trend equation $y_t = a + bx$

$$y_t = 24 + 1.89x$$

$\therefore$ For the year 2014 ; $x = 2014 - 2007 = 7$

$$\therefore y_t = 24 + 1.89(7) = 24 + 13.23 = \underline{\underline{37.23 \text{ millions}}}$$

28. Given: $M = 18.5$ thousand kg.

$$n = 14$$

$$\bar{x} = 17.85 \text{ thousand kg}$$

$$s = 1.955$$

} $-\frac{1}{2}$

$H_0: M = 18.5$ (There is no significant difference between the population mean and sample mean)

$-\frac{1}{2}$

$H_1: M < 18.5$ (There is a significant difference between the population mean and sample mean)

Test statistic $t = \frac{\bar{x} - M}{\frac{s}{\sqrt{n}}}$

$-\frac{1}{2}$

$$t = \frac{17.85 - 18.5}{1.955} \times \sqrt{14}$$

$$= \frac{-0.65 \sqrt{14}}{1.955}$$

$$= \frac{-0.65 \times 3.74}{1.955}$$

$-\frac{1}{2}$

$$t \approx -1.245$$

$$|t| = 1.245 ; \text{ given degree of freedom} = n-1 = 13$$

clearly $|t| < t_{13}(0.05)$ } Given $\frac{1}{2}$

$\therefore$ Accept the null hypothesis. $t_{13}(0.05) = 2.16$

The deviation in the mean breaking strength of the steel rods is not statistically significant at the 0.05 level. $\frac{1}{2}$

29.

Given: $P = \text{Rs. } 20,00,000$

$n = 15 \text{ years} = 15 \times 12 = 180$ } $\frac{1}{2}$

$i = 12\% \text{ per annum}$

$= \frac{12}{1200} = 0.01 \text{ per period.}$

$$EMI = \frac{Pi}{1 - (1+i)^{-n}} \quad \frac{1}{2}$$

$$= \frac{20,00,000 \times 0.01}{1 - (1+0.01)^{-180}} \quad \frac{1}{2}$$

$$= \frac{20,000}{1 - 0.1668} = \frac{20,000}{0.8332} \quad 1$$

$$= \underline{\underline{\text{₹ } 24,003.84}} \quad \frac{1}{2}$$

30.

(a) Given: Current population = 1,00,000

Let P is the population at time t .

The population doubled in 25 years.

$$\frac{dP}{dt} \propto P$$

$-\frac{1}{2}-$

$$\Rightarrow \frac{dP}{dt} = kP$$

$$\Rightarrow \frac{dP}{P} = k dt$$

$-\frac{1}{2}-$

$$\log P = kt + C$$

When $t=0$; Let $P = P_0$

$$\therefore \log P_0 = C$$

$-\frac{1}{2}-$

$$\therefore \log\left(\frac{P}{P_0}\right) = kt$$

When $t = 25$ yrs; $P = 2P_0$

$-\frac{1}{2}-$

$$\therefore \log\left(\frac{2P_0}{P_0}\right) = 25k$$

$$\Rightarrow k = \frac{1}{25} \log 2 = \frac{0.6931}{25} \approx 0.0277$$

$\therefore$ Now When $P = 5,00,000$

$-\frac{1}{2}-$

$$\log\left(\frac{5,00,000}{1,00,000}\right) = (0.0277)t$$

$$\log 5 = 0.0277 t$$

$$\therefore t = \frac{1.609}{0.0277} = \frac{16090}{277}$$

$$\therefore t \approx 58 \text{ years}$$

-1/2-

OR

(b) Let P be the principal.

rate $r = 5\%$.

When $t=0$; $P = ₹1,000$.

-1/2-

Given: $\frac{dp}{dt} = 5\% \text{ of } P$

$$\Rightarrow \frac{dp}{dt} = \frac{5}{100} P$$

-1/2-

$$\Rightarrow \frac{dp}{p} = \frac{1}{20} dt$$

-1/2-

$$\log P = \frac{1}{20} t + C$$

When $t=0$; $C = \log 20,000$

$$\therefore \log P = \frac{1}{20} t + \log 20,000$$

When $t=10$; what is P ?

$$\therefore \log P = \frac{10}{20} + \log 20,000$$

$$\Rightarrow \log\left(\frac{P}{20,000}\right) = \frac{1}{2} = 0.5$$

$$\frac{P}{20,000} = e^{0.5}$$

$$\Rightarrow P = 20,000 e^{0.5}$$

$$P = 20,000 \times 1.648$$

$$\therefore P = \underline{\underline{32,960.000}}$$

31. Given: Maximize $Z = x + y$
Subject to the constraints,

$$x + y \leq 3$$

$$y - 2x \leq 1$$

$$x \leq 2$$

$$x \geq 0, y \geq 0$$

Consider $x + y = 3$; $y - 2x = 1$

x	0	3
y	3	0

x	0	$-\frac{1}{2}$
y	1	0

checking $(0,0)$ condition,

$0 \leq 3$ is true, $0 \leq 1$ is true, $0 \leq 2$ is true.

$\therefore (0,0)$ is part of the solution region.
for all the constraints. $\frac{1}{2}$

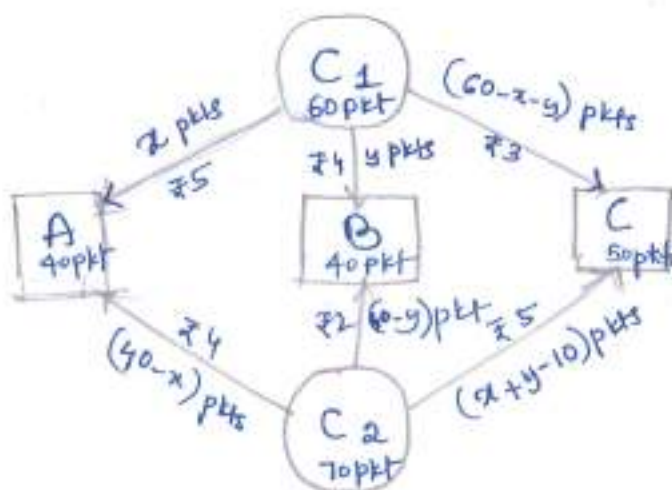
Corner Points	Value of $Z = x + y$
$O(0,0)$	0
$A(0,1)$	1
$B(\frac{2}{3}, \frac{7}{3})$	3
$C(1,3)$	4 $\rightarrow M$
$D(2,0)$	2

-/-

$\therefore$ Value of Z is maximum at $x=1$ and $y=3$. ~~and~~ and the maximum value is 4.

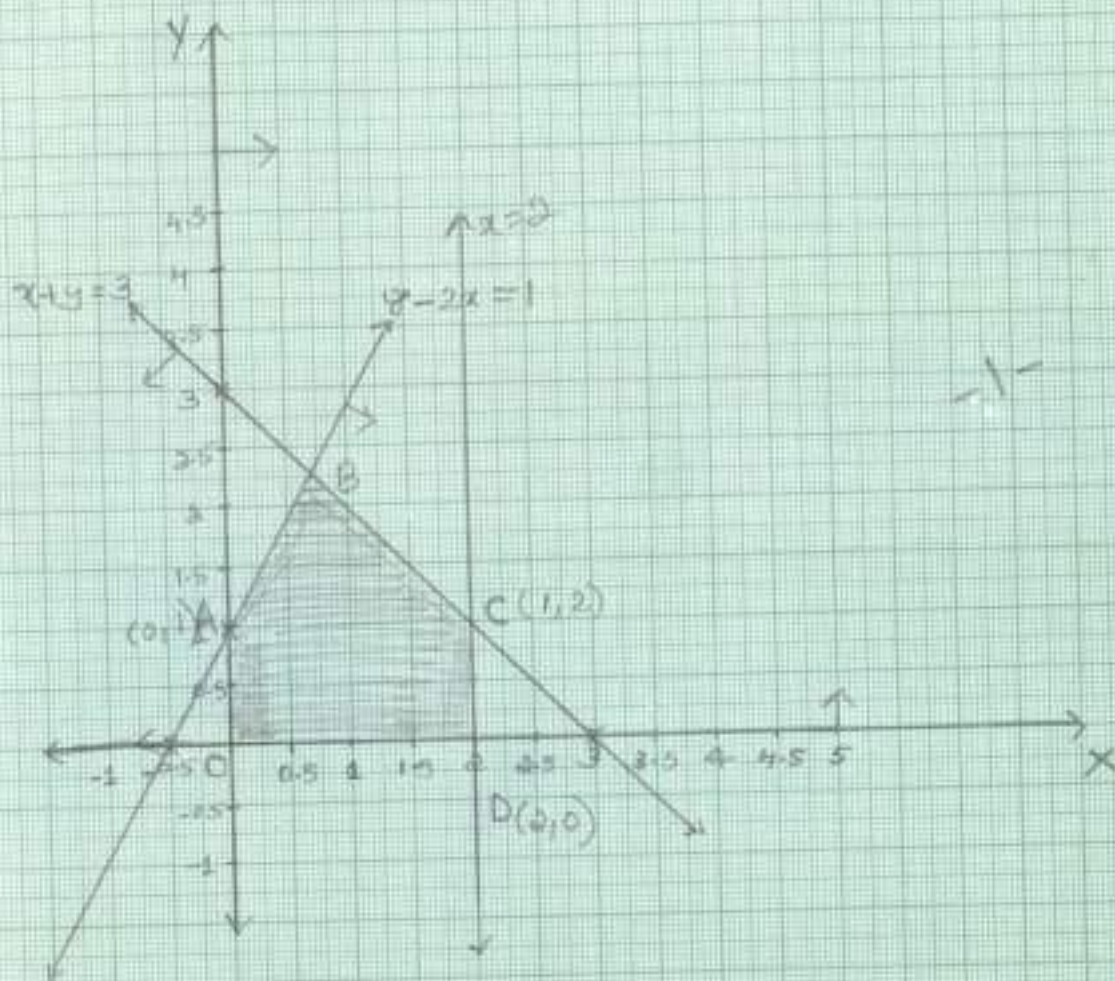
OR

(b)



-/-

31. 3)



Pg-20

Mathematical formulation of the given problem as an LPP is

$$\text{Minimize } Z = 5x + 4y + 3(60 - x - y) + 4(40 - x) + 2(40 - y) + 5(x + y - 10)$$

$$Z = 5x + 4y + 180 - 3x - 3y + 160 - 4x + 80 - 2y + 5x + 5y - 50$$

i.e. Minimize $Z = 3x + 4y + 370$

Subject to the constraints,

$$x + y \leq 60$$

$$x \leq 40, y \leq 40$$

$$x + y \geq 10$$

$$x, y \geq 0$$

Section D

32. (a)

$$AB = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 6-1-1 & 2-3+1 & -2-1+3 \\ -3+2+1 & -1+6-1 & 1+2-3 \\ 3-1-2 & 1-3+2 & -1-1+6 \end{bmatrix}$$

$$AB = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= 4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 4I$$

$$\therefore AB = 4I \Rightarrow A \left[\frac{B}{4} \right] = I \therefore A^{-1} = \frac{B}{4}$$

The given system of equations are

$$2x - y + z = -1$$

$$-x + 2y - z = 4$$

$$x - y + 2z = -3$$

can be written in the matrix form as

$$AX = B \text{ where}$$

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{4} \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix} = \begin{bmatrix} 4/4 \\ 8/4 \\ -4/4 \end{bmatrix}$$

$$\therefore x = 1; y = 2; z = -1$$

OR

$$(b) C = xI + yO + z$$

Given:

$$12 = 2x + 3y + z$$

$$13 = 6x + 2y + z$$

$$15 = 5x + 3y + z$$

The above system of equations can be written in the matrix form as $AX = B$

Where $A = \begin{bmatrix} 2 & 3 & 1 \\ 6 & 2 & 1 \\ 5 & 3 & 1 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} 12 \\ 13 \\ 15 \end{bmatrix}$

To find $x, y, z;$ $AX = B$

$$\Rightarrow X = A^{-1}B$$

$$A^{-1} = \frac{1}{|A|} \text{adj} A$$

$$\therefore |A| = \begin{vmatrix} 2 & 3 & 1 \\ 6 & 2 & 1 \\ 5 & 3 & 1 \end{vmatrix}$$

$$= 2(2-3) - 3(6-5) + 1(18-10)$$

$$= -2 - 3 + 8 = 3 \neq 0$$

$$\therefore A^{-1} \text{ exists.}$$

$$A_{11} = -1 \quad ; \quad A_{12} = -(1) \quad ; \quad A_{13} = 8$$

$$A_{21} = -(0) \quad ; \quad A_{22} = -3 \quad ; \quad A_{23} = -(-9)$$

$$A_{31} = 1 \quad ; \quad A_{32} = -(-4) \quad ; \quad A_{33} = -14$$

$$\therefore \text{adj } A = \begin{bmatrix} -1 & -1 & 8 \\ 0 & -3 & 9 \\ 1 & 4 & -14 \end{bmatrix}^T$$

$$= \begin{bmatrix} -1 & 0 & 1 \\ -1 & -3 & 4 \\ 8 & 9 & -14 \end{bmatrix}$$

$$\therefore X = \frac{1}{3} \begin{bmatrix} -1 & 0 & 1 \\ -1 & -3 & 4 \\ 8 & 9 & -14 \end{bmatrix} \begin{bmatrix} 12 \\ 13 \\ 15 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} -12+0+15 \\ -12-39+60 \\ 96+117-210 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 3 \\ -9 \\ 3 \end{bmatrix} = \begin{bmatrix} 3/3 \\ -9/3 \\ 3/3 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix}$$

$$\therefore x = 1 \quad ; \quad y = -3 \quad ; \quad z = 1$$

33.

(i) the relation representing the total length of fencing wire is

$$x + x + y = 200$$

$$\Rightarrow \boxed{2x + y = 200}$$

-1-

(ii) Area of the garden as a function of x :

$$A = xy$$

$$\Rightarrow A = x(200 - 2x)$$

$$A = 200x - 2x^2$$

-1-

(iii) To find the maximum area, we use second derivative test.

$$A = 200x - 2x^2$$

Diff w.r.t. x ,

$$\frac{dA}{dx} = 200 - 4x$$

$$\text{put } \frac{dA}{dx} = 0 \Rightarrow 200 = 4x$$

$$\boxed{x = 50}$$

-1-

$$\therefore y = 200 - 2(50) = 100$$

Differentiate $\frac{dA}{dx}$ w.r.t. x again, ✓

$$\frac{d^2A}{dx^2} = -4 < 0$$

∴ Area is maximum at $x=50$ -1-

$$\therefore \text{Area} = 50 \times 100 = \underline{\underline{5000 \text{ sq. ft.}}}$$

34. (a)

$$\text{Given: } p = 5\% = \frac{5}{100} ; P(X=r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!}$$

$$n = 100$$

$$\lambda = np = 100 \times \frac{5}{100} = 5 \quad \frac{1}{2}-$$

$$e^{-5} = 0.006738.$$

$$(i) P(X=0) = \frac{e^{-5} (5)^0}{0!} = \underline{\underline{e^{-5} = 0.006738}} \quad -1-$$

$$(ii) P(X=5) = \frac{e^{-5} \times 5^5}{5!} = \frac{0.006738 \times 5^5}{120} \approx \underline{\underline{0.1754}} \quad -1-$$

$$(iii) P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3) \\ = \frac{e^{-5} \times 5^0}{0!} + \frac{e^{-5} \times 5^1}{1!} + \frac{e^{-5} \times 5^2}{2!} + \frac{e^{-5} \times 5^3}{3!} \quad -1-$$

$$P(X \leq 3) = e^{-5} \left(1 + 5 + \frac{25}{2} + \frac{125}{6} \right) \quad \frac{1}{2}$$

$$= 0.006738 \left(\frac{36 + 75 + 125}{6} \right)$$

$$= 0.001123 (236)$$

$$\approx \underline{\underline{0.265028}} \quad \frac{1}{2}$$

OR

(b) Given: $\mu = 14$

$$SD(\sigma) = 2.5 \quad \frac{1}{2}$$

$$\text{We have, } Z = \frac{X - \mu}{\sigma}$$

$$(i) P(12 < X < 15)$$

$$= P\left(\frac{12-14}{2.5} < Z < \frac{15-14}{2.5}\right) \quad \frac{1}{2}$$

$$= P\left(\frac{-2}{2.5} < Z < \frac{1}{2.5}\right)$$

$$= P(-0.8 < Z < 0.4) \quad \frac{1}{2}$$

$$= F(0.4) - F(-0.8)$$

$$= 0.1554 - [1 - F(0.8)] = 0.1554 - 1 + 0.7881$$

$$= \underline{\underline{0.4435}}$$

$$\therefore \text{Number of students} = 1000 \times 0.4435 \quad \frac{1}{2}-$$

$$\approx \underline{\underline{444}} \text{ students.}$$

(ii) Above 18

$$P(X > 18) = P\left(Z > \frac{18-14}{2.5}\right)$$

$$= P\left(Z > \frac{4}{2.5}\right) \quad \frac{1}{2}-$$

$$= P(Z > 1.6)$$

$$= 1 - P(Z < 1.6) \quad \frac{1}{2}-$$

$$= 1 - F(1.6)$$

$$= 1 - 0.9452$$

$$= \underline{\underline{0.0548}} \quad \frac{1}{2}-$$

$$\therefore \text{Number of students} = 1000 \times 0.0548 \quad \frac{1}{2}-$$

$$\approx \underline{\underline{55}}$$

(iii) below 8.

$$P(X < 8) = P\left(Z < \frac{8-14}{2.5}\right) = P\left(Z < \frac{-6}{2.5}\right)$$

$$= P(Z < -2.4)$$

$$= F(-2.4) = 1 - F(2.4) \quad \frac{1}{2}$$

$$= 1 - 0.9918$$

$$= 0.0082$$

$$\therefore \text{Number of Students} = 1000 \times 0.0082$$

$$\approx \underline{\underline{8}} \quad \frac{1}{2}$$

35.

Loan amount = Rs. 10,00,000.

Interest rate = 8% p.a. = $\frac{8}{4}\%$ per quarter. $\frac{1}{2}$

$$\therefore \text{Interest} = 10,00,000 \times \frac{2}{100} = \underline{\underline{20,000}}$$

$$\text{Sinking Fund } A = R \left[\frac{(1+i)^n - 1}{i} \right] \quad \frac{1}{2}$$

$$R = ? ; n = 5 \times 4 = 20 ; i = \frac{6\%}{4} = \frac{3}{2}\%$$

$$i = \underline{\underline{0.015}} \quad \frac{1}{2}$$

$$\therefore R = \frac{A \cdot i}{(1+i)^n - 1} = \frac{10,00,000 \times 0.015}{(1.015)^{20} - 1}$$

$$R = \frac{15,000}{1.346852 - 1}$$

$$= \frac{15,000}{0.346852} \approx \frac{15,000}{0.3469}$$

$$R \approx ₹43,240.1$$

$$\therefore \text{Total quarterly cost} = 20,000 + 43,240 \\ = \underline{\underline{₹63,240}} \quad -1-$$

Section E

36.

(i) We have downstream Speed = $(x+y)$ km/h
and upstream speed = $(x-y)$ km/h.

from the question,

$$t_{\text{down}} = \frac{12}{x+y} \quad ; \quad t_{\text{up}} = \frac{12}{x-y} \quad -\frac{1}{2}-$$

$$\therefore t_{\text{up}} - t_{\text{down}} = 6$$

$$\frac{12}{x-y} - \frac{12}{x+y} = 6$$

$$2(x+y) - 2(x-y) = x^2 - y^2 \quad -\frac{1}{2}-$$

$$\Rightarrow \boxed{4y = x^2 - y^2} \Rightarrow y^2 = x^2 - 4y$$

(ii) For the downstream journey, the new speed = $2x+y$ km/h.

and upstream speed = $2x-y$ km/h. $\frac{1}{2}$

According to the question,

$$\frac{12}{2x-y} - \frac{12}{2x+y} = 1$$

$$12(2x+y) - 12(2x-y) = 4x^2 - y^2 \quad \frac{1}{2}$$

$$\Rightarrow \boxed{24y = 4x^2 - y^2} \rightarrow (2)$$

(iii) From (1) and (2)

(a)

$$24y = 4x^2 - (x^2 - 4y)$$

$$\Rightarrow 24y = 3x^2 + 4y \quad \frac{1}{2}$$

$$\Rightarrow 20y = 3x^2 \Rightarrow \boxed{x^2 = \frac{20}{3}y} \rightarrow (3)$$

Substituting (3) in (2)

$$24y = 4 \left[\frac{20}{3}y \right] - y^2$$

$$\Rightarrow y^2 + y \left[24 - \frac{80}{3} \right] = 0 \quad \frac{1}{2}$$

$$\Rightarrow y^2 + y \left[-\frac{8}{3} \right] = 0$$

$$y \left[y - \frac{8}{3} \right] = 0 \Rightarrow y = 0 \text{ or } \boxed{y = \frac{8}{3} \text{ km/h}} \quad -1-$$

OR

(b) Speed of boat in still water = 5 km/h.

Speed of water current = 1 km/h $\frac{1}{2}$

Speed of downstream = $(x+y) = 5+1 = 6 \text{ km/h}$.

Speed of upstream = $x-y = 5-1 = 4 \text{ km/h}$. $\frac{1}{2}$

$$T_{\text{down}} + T_{\text{up}} = 1$$

$$\Rightarrow \frac{x}{6} + \frac{x}{4} = 1 \Rightarrow \frac{2x+3x}{12} = 1 \quad \frac{1}{2}$$

$$\Rightarrow 5x = 12$$

$$x = \frac{12}{5} = 2.4 \text{ km/h} \quad \frac{1}{2}$$

37.

Given

	M_1	M_2	M_3	Profit
Half sleeves	1	2	8/5	₹1
Full sleeves	2	1	8/5	₹1.50
Time :	≤ 40	≤ 40	≤ 40	

(i) objective function: Let x and y represent the number of half sleeves and full sleeves shirt resp

Maximize $Z = x + 1.5y$

Subject to the constraints,

$$x + 2y \leq 40$$

$$2x + y \leq 40$$

$$\frac{8}{5}x + \frac{8}{5}y \leq 40 \Rightarrow x + y \leq 25$$

$$x, y, \geq 0$$

(ii)

Consider

$$x + 2y = 40$$

x	0	40
y	20	0

$$0 \leq 40$$

$$2x + y = 40$$

x	0	20
y	40	0

$$0 \leq 40$$

$$x + y = 25$$

x	0	25
y	25	0

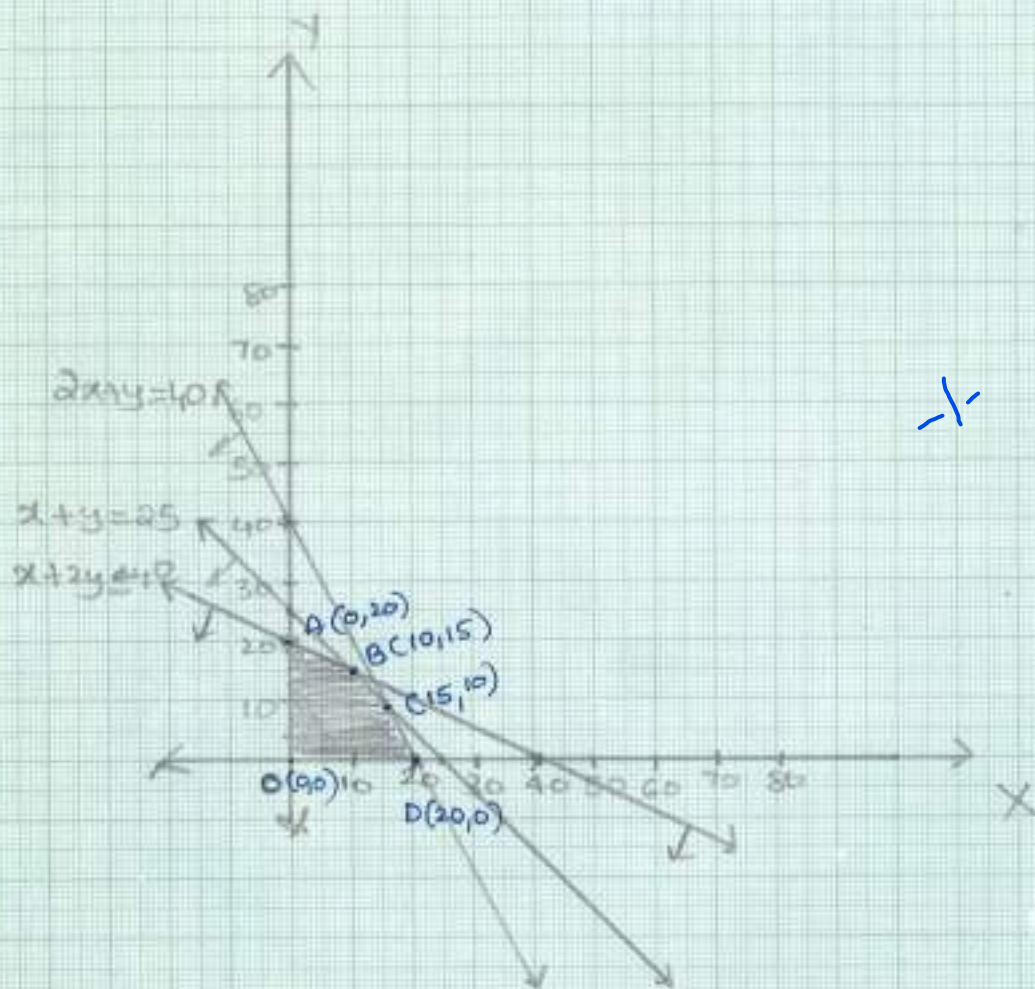
$$0 \leq 25$$

is true for all the 3 constraints.

$(0,0)$ is part of solution region.

Corner Points	Values of $Z = x + 1.5y$
A(0,20)	30
B(10,15)	32.5 — M
C(15,10)	30
D(20,0)	20

37.(b)



∴ Profit is maximum at $x=10$ and $y=15$
with $Z = 32.5$. 1- mark for graph.

38.

Given: $P(X=0) = 0.08$

$$P(X=1) = 0.11$$

$$P(X=2) = 0.27$$

$$P(X=3) = 0.33$$

(i) $P(X=4) = ?$

We have $\sum P = 1$

$$\Rightarrow P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) = 1$$

$$\Rightarrow 0.08 + 0.11 + 0.27 + 0.33 + P(X=4) = 1$$

$$\Rightarrow P(X=4) = 1 - 0.79$$

$$\Rightarrow P(X=4) = \underline{\underline{0.21}}$$

(ii) $P(X \geq 2) = P(X=2) + P(X=3) + P(X=4)$

$$= 0.27 + 0.33 + 0.21$$

$$= \underline{\underline{0.81}}$$

(iii) $E(X) = \sum x P(x)$

(iv)

X	0	1	2	3	4
P(X)	0.08	0.11	0.27	0.33	0.21
X P(X)	0	0.11	0.54	0.99	0.84

$$E(X) = \sum x P(x) = 0.11 + 0.54 + 0.99 + 0.84$$

$$E(X) = \underline{\underline{2.48}}$$

OR

$$\text{Variance} = E(X^2) - (E(X))^2$$

$$= [0.11 + 1.08 + 2.97 + 3.36] - [2.48]^2$$

$$= 7.52 - 6.1504$$

$$= \underline{\underline{1.3696}}$$

$$\text{Standard deviation } (\sigma) = \sqrt{\text{Variance}}$$

$$= \sqrt{1.3696}$$

$$= \underline{\underline{1.1702}}$$