**BANGALORE SAHODAYA SCHOOLS COMPLEX ASSOCIATION**
**PRE-BOARD EXAMINATION (2024-2025)**
**Grade XII**

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| **Class: - XII**<br>**Time: - 3 hours** | **SUBJECT: Applied Mathematics**<br>**Code (241) - SET-2** | **Date: - 20.12.2024**<br>**Marks: - 80** |

General Instructions:
   a) This question paper contains **38** questions. All questions are **compulsory**.
   b) Question paper is divided into **five** sections – **A, B, C, D** and **E**.
   c) In **Section A** – Questions number **1** to **18** are multiple choice questions (MCQs) type and Questions number **19** & **20** are Assertion-Reason based questions of 1 mark each.
   d) In **Section B** – Questions number **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.
   e) In **Section C** – Questions number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.
   f) In **Section D** – Questions number **32** to **35** are long answer (LA) type questions, carrying **5** marks each.
   g) In **Section E** – Questions number **36** to **38** are case study based questions, carrying **4** marks each.
   h) There is no overall choice. However, an internal choice has been provided in 2 questions in Section – B, 3 questions in Section – C, 2 questions in Section – D and 1 question in Section – E.
   i) Use of calculators is **NOT** allowed.

## SECTION – A

This section has 20 multiple choice questions of 1 mark each

1. If $a \equiv 7(mod\ 10)$ and $b \equiv 3(mod\ 10)$, then find x, if $a + b \equiv x(mod\ 10)$
   (A) 0  (B) 1  (C) 3  (D) 10

2. A mixture contains two substances, A and B in the ratio 5:7. If 15 litres of substance A is removed and replaced by 15 litres of substance B, the ratio of A and B becomes 3:5. What was the initial quantity of the mixture?
   (A) 350 litres  (B) 360 litres  (C) 370 litres  (D) 380 litres

3. In a 200 m race, A beats B by 20 metres, and B beats C by 20 metres. How much distance does A beats C by
   (A) 40 m  (B) 38 m  (C) 36 m  (D) 35 m

4. For any square matrix A, if $A = A^T$ is always true, then which of the following is true?
   (A) A is symmetric matrix                    (B) A is skew symmetric matrix
   (C) A is diagonal matrix                      (D) A is identity matrix

5. The value of determinant is unaltered if
   (A) Two rows are interchanged                 (B) two columns are interchanged
   (C) every element in a row (or column) is multiplied by the same number

(D) to each element of a row (or column) is added equi-multiples of the corresponding elements of another row or column.

6.  If $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$ is a non-singular matrix of order 3 with $|A| = -17$. Then find the value of $|adj\, A|$

(A) 17        (B) $-17$        (C) 289        (D) $-289$

7.  If $f(x) = \log(x^2 + 1)$, what is $f''(2)$?

(A) $-\frac{6}{25}$        (B) $\frac{3}{25}$        (C) $\frac{6}{25}$        (D) $-\frac{4}{25}$

8.  $\int e^{3\log x}(x^4 + 1)^{-1}dx$ is equal to

(A) $\frac{1}{4}\log(x^4 + 1) + C$        (B) $-\frac{1}{4}\log(x^4 + 1) + C$

(C) $\log(x^4 + 1) + C$        (D) $-\log(x^4 + 1) + C$

9.  The mean of a probability distribution is 4. If the probability mass function is given by P(X = 2) = 0.3, P(X = 4) = 0.4, P(X = 6) = 0.3. What is the variance of the distribution?

(A) 1.6        (B) 2.0        (C) 1.8        (D) 2.4

10. The heights of a population of adult men follow a normal distribution with a mean of 70 inches and a standard deviation of 3 inches. If a man is randomly selected from this population, what is the probability that his height is between 67 inches and 73 inches?

(A) 0.6826        (B) 0.9544        (C) 0.3413        (D) 0.1359

11. A sample of 64 students is taken to estimate the mean number of hours spent studying each week. The sample mean is 15 hours, and the sample standard deviation is 4 hours. What is the 90% confidence interval for the mean number of hours spent studying?

(A) (14.02, 15.98)        (B) (14.1775, 15.8225)

(C) (14.897, 15.103)        (D) (14.8775, 15.1225)

12. A research study claims that the average height of students in a school is 150 cm. A random sample of 100 students shows a sample mean of 152 cm and a sample standard deviation of 10 cm. At a 5% significance level, which of the following is the correct hypothesis test to test if the average height is significantly different from 150 cm?

(A) Null hypothesis: H0: $\mu = 150$, Alternative hypothesis: H1: $\mu \neq 150$

(B) Null hypothesis: H0: $\mu = 152$, Alternative hypothesis: H1: $\mu \neq 152$

(C) Null hypothesis: H0: $\mu \neq 150$, Alternative hypothesis: H1: $\mu = 150$

(D) Null hypothesis: H0: $\mu = 150$, Alternative hypothesis: H1: $\mu > 150$

13. A government agency wants to analyse and predict the unemployment rate over the next few years based on past quarterly data. Which time series component should the agency primarily focus on?

(A) Trend        (B) Seasonal Variations

(C) Cyclical Fluctuations        (D) Irregular Variations

14. A company is analysing the historical data of its stock prices to predict future prices. The stock prices exhibit sudden changes due to unexpected events (e.g., market crashes or corporate announcements). Which time series component is most affected by such events?

(A) Trend        (B) Seasonal Variations

(C) Irregular Variations        (D) Cyclical Variations

15. If the interest rate decreases from 6% to 4%, how does the value of a perpetuity change, assuming the annual payment remains the same?

(A) The value of the perpetuity will decrease.

(B) The value of the perpetuity will remain the same

(C) The value of the perpetuity will increase

(D) The perpetuity will no longer be valid

16. An individual invests Rs. 5,000 in a mutual fund. After 2 years, the investment grows to Rs. 6,500. What is the ROI?

(A) 25%  (B) 30%  (C) 35%  (D) 40%

17. A company purchases an asset for Rs. 10,000 with a 10% depreciation rate using the declining balance method. What will be the book value at the end of the second year?

(A) Rs. 8100  (B) Rs. 8500  (C) Rs. 8900  (D) Rs. 7800

18. For the following system of inequalities:

$$x + y \geq 6; \; 3x - 2y \leq 8 \; x \geq 0, y \geq 0$$

Which of the following is not a valid corner point of the feasible region?

(A) (4,2)  (B) (5,1)  (C) (2,4)  (D) (3,3)

### Assertion-Reason Based Questions

**Direction:** In questions numbers 19 and 20, two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the following options:

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true and the Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A):** The one-sample t-test can produce biased results if the underlying population distribution is highly skewed and the sample size is small.

**Reason (R):** When the sample size is small, the t-distribution does not provide an accurate approximation of the true distribution of the test statistic under skewed conditions.

20. **Assertion (A):** In the method of least squares, the sum of the residuals (the differences between observed and predicted values) may or may not be zero.

**Reason (R):** The least squares method minimizes the squared residuals, and in the process, the sum of the residuals is forced to be zero to ensure that the regression line is balanced around the data points.

## SECTION – B

This section has 5 very short answer type questions of 2 marks each

21. If $x = e^{x/y}$, prove that $\frac{dy}{dx} = \frac{x-y}{x \log x}$.

22. Evaluate the definite integral: $\int_1^e \left( \frac{e + \log x}{3x} \right) dx$

(OR)

Evaluate the indefinite integral: $\int x^2 . \log x \, dx$

23. A factory produces light bulbs, and a random sample of 64 bulbs has a mean lifetime of 2000 hours with a standard deviation of 100 hours. Find the 90% confidence interval for the population mean lifetime. (Given: $Z_{0.05} = 1.669$)

24. A Lamborghini car costing Rs. 1,20,00,000 has a scrap value of Rs. 24,00,000. If annual depreciation charge is Rs. 8,00,000, find its useful life in years.

(OR)

Find the effective rate which is equivalent to a stated rate of 9% compounded quarterly. (Given: $(1.0225)^4 = 1.093$)

25. A factory produces two types of products, A and B. Each product A requires 2 hours of labour and 3 kg of raw material. Each product B requires 3 hours of labour and 2 kg of raw material. The factory has 120 hours of labour and 100 kg of raw material available. If the profit per unit of product A is Rs. 30 and for product B is Rs. 40, formulate a Linear Programming Problem to maximize the profit.

## SECTION – C

This section has 6 short answer type questions of 3 marks each

26. How many litres of water will have to be added to 1725 litres of the 50% solution of acid so that the resulting mixture will contain more than 30% but not less than 40% acid solution?

27. If $A = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$, find matrix C such that $5A + 3B + 2C$ is a null matrix.

(OR)

If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ verify that $A^2 - 4A - 5I = 0$.

28. In a binomial distribution, the sum of the mean and variance is 25, and the product of the mean and variance is 150. Find the values of the number of trials and the probability of success.

(OR)

The marks of students of class XII follow normal distribution with mean 75 and standard deviation 4. Find the probability that a student selected at random will get marks

(i)     Less than 65

(ii)    More than 80

(iii)   Between 70 and 85

(Given: $P(Z < -2.5) = 0.0062$;   $P(Z < 1.25) = 0.8944$;

$P(Z < 2.5) = 0.998$);     $P(Z < -1.25) = 0.1056$)

29. The following table shows the monthly average temperatures (in Celsius) for a year:

| January | -5 | April | 5 |
|---|---|---|---|
| February | -2 | May | 10 |
| March | 0 | June | 15 |
| July | 20 | October | 8 |
| August | 18 | November | 2 |
| September | 13 | December | -3 |

Calculate the four-month centred moving average for each month starting from January and ending in December.

30. A machine costing Rs. 2,00,000 has effective life of 7 years and the scrap value is Rs. 20,000. What amount should the company put into a sinking fund earning 5% per annum, so that it can replace the machine after its useful life? Assume that a new machine will cost Rs. 3,30,000 after 7 years. (Given: $(1.05)^7 = 1.407$)

31. The population of a city increases from 1,00,000 to 2,50,000 over a period of time. If the compound annual growth rate is 8%, calculate the number of years it will take for the population to grow from 1,00,000 to 2,50,000.
(Given: $log(2.5) = 0.39794, log(1.08) = 0.03390$)

(OR)

The cost of a spice jet aeroplane depreciates by Rs. 7,20,000 during second year and by Rs. 6,48,000 during the third year. Calculate

(i)     Rate of depreciation per annum.

(ii)    The original cost of the spice jet aeroplane.

## SECTION – D

This section has 4 long answer type questions of 5 marks each

32. A tank has three pipes connected to it: Pipe A can fill the tank in 12 hours, Pipe B can fill the tank in 18 hours and Pipe C can empty the tank in 24 hours. Initially, Pipe A and Pipe B are both opened, and after 3 hours, Pipe C is also opened. After 2 more hours, Pipe A is closed. After 1 hour, Pipe B is also closed, leaving only Pipe C open. The tank was completely empty at the beginning. How long will it take to fill the tank completely?

33. In a murder case, the body of the victim was found in a room at 8:00 AM. The temperature of the body at that time was 85°F. The temperature of the room was 68°F. At 10:00 AM, the body temperature was measured to be 78°F. Then, determine the time of death of the victim assuming the body was initially at a normal human body temperature of 98.6°F. (Given: $\log\left(\frac{10}{17}\right) = 0.454$ ; $\log\left(\frac{30.6}{17}\right) = 0.5878$)

(OR)

If $y(t)$ is a solution of $(1 + t^2)\frac{dy}{dt} - ty = 2t$ and $y(0) = -1$, then show that $y(1) = -2 + \sqrt{2}$.

34. In a hospital emergency room, it is observed that on average, 3 patients arrive every hour for emergency care. The number of patient arrivals per hour follows a Poisson distribution. Then find the probability that

(i)     exactly 5 patients will arrive in a given hour

(ii)    no more than 2 patients will arrive in a given hour

(iii)   more than 4 patients will arrive in a given hour.

(Given: $e^{-3} = 0.0498$)

35. Solve the linear programming problem graphically,
$$\text{Minimise } Z = 24x + 36y$$

Subject to the constraints,
$$x + 2y \geq 10; \quad 2x + 2y \geq 12; \quad 3x + y \geq 8;$$
$$x \geq 0, y \geq 0$$

.

(OR)

A factory produces two types of products: Product 1 ($P_1$) and Product 2 ($P_2$). The factory has three departments: Assembly, Packaging, and Quality Control. The following table shows the time required by each product in each department:

| Product | Assembly (hrs) | Packaging (hrs) | Quality Control (hrs) | Profit per unit |
|---------|----------------|-----------------|-----------------------|-----------------|
| $P_1$ | 4 | 3 | 2 | 120 |
| $P_2$ | 3 | 4 | 3 | 100 |

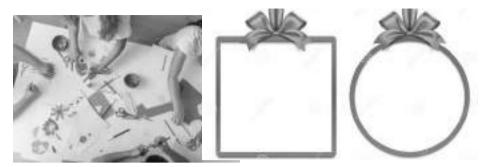The available department times are: 200 hours in Assembly, 180 hours in Packaging and 150 hours in Quality Control. The factory can produce a maximum of 30 units of Product 1 and 40 units of Product 2. Formulate the Linear Programming Problem (LPP) and find the optimal number of units of Product 1 and Product 2 that should be produced to maximize the profit.

## SECTION – E

This section has 3 case study questions of 4 marks each

36. Read the following passage and answer the questions given below.

An art and craft teacher wants to demonstrate new craft works to the students of the class VI. So, the teacher started to distribute a decorative colour ribbon to each student, which is 28m in length. Now, students are instructed to cut the ribbon into two pieces such that one piece is to make a circle and another is to make a square. This idea is to keep the sum of the areas enclosed by a circle and square to be minimum.



Based on the above information, answer the following questions:
(i)      If the ribbon is cut at 'x' metre from one end and made into circle and 'r' represents radius of a circle, then find the circumference of the circle.          (1)
(ii)     Find the area enclosed by the circular ribbon                                        (1)
                                        (OR)
         Find the area enclosed by the square ribbon                                        (1)
(iii)    Find the value of 'x' when combined area is minimum.                               (2)

37. Read the following passage and answer the following questions given below.
Narendar wants to buy a new plot to build a house for their children in his village. When he asked about the dimensions of the plot to the broker, he replied that if its length is decreased by 45 m and breadth is increased by 45 m, then its area will remain same. But if length is decreased by 20 m and breadth is decreased by 10 m, then its area will decrease by 4600 $m^2$.



Based on the above information, answer the following questions:
(i)      Find two relations between x and y.                                                (2)
(ii)     Express above system of equations in matrix form.                                 (1)
(iii)    Find the original area of the plot.                                               (1)

38. A student, Anjali, decides to buy a new laptop for her studies. The cost of the laptop is ₹50,000. Since Anjali doesn't have enough funds, she opts for a loan from a bank that offers an Equated Monthly Installment (EMI) option. The bank gives her a loan of ₹40,000 at an interest rate of 12% per annum, compounded monthly. The EMI is to be paid over 2 years. (Given: $(1.01)^{24} = 1.26824$)



Based on the above information, answer the following questions:

(i)     Calculate the EMI Anjali needs to pay every month for the loan.       (2)

(ii)     Calculate the total amount paid at the end of 2 years.       (1)

(iii)     Calculate the interest paid at the end of 2 years.       (1)

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