CAMBRIDGE SCHOOL

PRE-BOARD 2024-25

Series	S	I	N	GN

SET	1

Candidate must write the Set No. on the title page of the answer book.

- Please check that this question paper contains 7 printed pages.
- Set number given on the right-hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains __38_ questions.
- 15-minute time has been allotted to read this question paper. During this duration, students will read the question paper only and will not write any answer on the answer sheet.

MATHEMATICS (041)

CLASS XII

Time allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

(i) **This question paper contains five sections-A, B, C, D and E. Each section is compulsory. There are internal choices in some questions.**

(ii) **Section A has 18 MCQs and 02 Assertion-Reasoning based questions of 1 mark each.**

(iii) **Section B has 05 questions of 02 marks each.**

(iv) **Section C has 06 questions of 3 marks each.**

(v) **Section D has 04 questions of 5 marks each.**

(vi) **Section E has 03 questions of case study questions with sub parts (4 marks each).**

SECTION A **1×20=20**

1. Evaluate: $\int \frac{3x^2}{x^6+1}dx =$

(a) $\log(x^6+1)+c$ (b) $\tan^{-1}x^3+c$ (c) $3\tan^{-1}x^3+c$ (d) $\log x^2+c$

2. $Evaluate: \int_{-\pi}^{\pi} x^3\cos^3 x\,dx =$

(a) π (b) $\frac{\pi}{4}$ (c) 2π (d) 0

3. If A is a square matrix and $A^2 = A$, then $(I+A)^2 - 3A$ is equal to:

(a) I (b) A (c) 2A (d) 3I

4. A square matrix $A = [a_{ij}]_{n \times n}$ is called a diagonal matrix if $a_{ij} = 0$ for

(a) i = j (b) i < j (c) i > j (d) i ≠ j

5. If $|A| = |kA|$, where A is a square matrix of order 2, then the sum of all possible values of k is :

(a) 1 (b) -1 (c) 2 (d) 0

6. The domain of the function $\sin^{-1}(3x-1)$ is:

(a)[0,1] (b) [-1,1] (c) $[0,\frac{2}{3}]$ (d) none of these

7. If A is a square matrix such that $|A|= 5$, the value of $|AA^t|$ is

(a) 5 (b) 25 (c) 125 (d) none of these

8. If $xe^y = 1$, then the value of $\frac{dy}{dx}$ at x = 1 is:

(a) -1 (b) 1 (c) - e (d) $-\frac{1}{e}$

9. Order and degree of differential equation $\frac{d^2y}{dx^2} = \left[y + \left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{4}}$

(a) 4 and 2 (b) 1 and 2 (c) 1 and 4 (d) 2 and 4

10. Area of the region bounded by curve $y^2 = 4x$ and the x-axis between x=0 and x =1 is :

(a) $\frac{2}{3}$ (b) $\frac{8}{3}$ (c) 3 (d) $\frac{4}{3}$

11. If A and B are two events such that P(A) = 0.2, P(B)= 0.4 and P(AUB)=0.5, then the value of P(A/B) is:

(a)0.1 (b) 0.25 (c) 0.5 (d) 0.08

12. Integrating factor of differential equation $x\frac{dy}{dx} + 2y = x^2$

(a) $\frac{1}{x^2}$ (b) x^2 (c) x (d) $\frac{1}{x}$

13. The function $f(x) = \frac{2}{x} + \frac{x}{2}$ has a local maxima at x equal to :

(a) 2 (b) 1 (c) 0 (d) -2

14. The value of p for which the vectors: $2\hat{i} + p\hat{j} + \hat{k}$ and $-4\hat{i} - 6\hat{j} + 26\hat{k}$ are perpendicular to each other is:

(a)3 (b) -3 (c) $-\frac{17}{3}$ (d) $\frac{17}{3}$

15. If $|\vec{a}| = 10$ and $|\vec{b}| = 2$ and $\vec{a}.\vec{b} = 12$, find $|\vec{a} \times \vec{b}|$ is :

(a) 5 (b) 10 (c) 14 (d) 16

16. The number of all possible matrices of order 3x3 with each entry -1 or 1 is

(a) 512 (b) 81 (c) 27 (d) 18

17. The objective function $Z = ax + by$ of an LPP has maximum value 42 at (4,6) and minimum value 19 at (3,2). Which of the following is true?
(a) a =9, b =1 (b) a=9 , b =2
(c) a=3 , b= 5 (d) a=5, b=3

18. The corner points of a feasible region for a L.P.P are (0, 0), (0, 8), (4, 10), (6, 8), (6, 5) and (5, 0). Let $Z = 3x - 4y$ be the objective function. The point at which maximum of Z occurs is:
(a) (0, 8) (b) (6, 8) (c) (6, 5) (d) (5, 0)

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason(R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion**: Let R be the relation on set of integers Z given by R = {(a, b) : 2 divides (a-b)} is reflexive relation

Reason: A relation R is said to be equivalence relation if R is reflexive, symmetric and transitive.

20. **Assertion**: If $x = at^2$ and $y = 2at$, then $\frac{d^2y}{dx^2}$ at t =2 is equal to $(-\frac{1}{16a})$

Reason: $\frac{d^2y}{dx^2} = (\frac{dy}{dt} \times \frac{dt}{dx})^2$

SECTION B **2×5=10**

21. Find the value of $\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2}$

OR

Evaluate: $\sin^{-1}(sin\frac{3\pi}{4}) + \cos^{-1}(cos\pi) + \tan^{-1}(1)$

22. The volume of the cube is increasing at the rate of 9 cubic centimetres per second. How fast is the surface area increasing when the length of an edge is 10 centimetres?

23. Find the area of the parallelogram whose adjacent sides are represented by the vectors $2\hat{i} - 4\hat{j} - 5\hat{k}$ and $2\hat{i} + 2\hat{j} + 3\hat{k}$.

24. If $f(x) = \sqrt{tan\sqrt{x}}$, then find $f'(\frac{\pi^2}{16})$.

OR

Prove that the greatest integer function defined by $f(x) = [x], 0 < x < 3$ is not differentiable at $x = 2$.

25. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot \vec{d} = 27$

SECTION C

$3 \times 6 = 18$

26. Evaluate: $\int_2^4 [|x-2| + |x-3| + |x-4|]\, dx$

OR

Evaluate: $\int_0^{\frac{\pi}{4}} (\frac{sinx + cosx}{3 + sin2x})\, dx$.

27. Find the vector equation of the line that passes through $(-1, 2, 7)$ and is perpendicular to the lines $\vec{r} = 2\hat{i} + \hat{j} - 3\hat{k} + \lambda(\hat{i} + 2\hat{j} + 5\hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} - 7\hat{k} + \mu(3\hat{i} - 2\hat{j} + 5\hat{k})$.

28. Find the general solution of the differential equation: $(xy - x^2)\, dy = y^2\, dx$

OR

Find the particular solution of the differential equation:

$\frac{dy}{dx} + y\, cotx = 4x\, cosecx$, $x \neq 0$, given that $y = 0$ and $x = \frac{\pi}{2}$.

29. Let a pair of dice be thrown and the random variable X be the sum of the numbers that appear on the two dice. Find the expectation of X.

OR

A and B throw a die alternately till one of them gets a '6' and wins the game. Find their respective probabilities of winning, if A starts the game first.

30. Show that $f(x) = 2x + \cot^{-1} x + \log(\sqrt{1+x^2} - x)$ is increasing in R.

31. Solve the Linear Programming problem graphically:

Minimize: $Z = 5x + 10y$, subject to the constraints

$x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x \geq 0$, $y \geq 0$

SECTION D

$5 \times 4 = 20$

32. Find the area of the region lying in the first quadrant and enclosed by the x-axis, the line y = x and the circle $x^2 + y^2 = 32$.

OR

Using Integration, find the area of the region in the first quadrant enclosed by the line $x + y = 2$ and curve $y^2 = x$ and the x-axis.

33. Two factories decided to award their employee for three values of (a) adaptable to new situation, (b) careful and alert in difficult situations and (c) keeping calm in tense situations, at the rate of ₹ x, ₹ y and ₹ z per person respectively. The first factory decided to honour respectively 2, 4 and 3 employees with total prize money of ₹ 29000. The second factory decided to honour respectively 5, 2 and 3 employees with a total prize money of ₹ 30500. If three prizes per person together cost ₹ 9500, then

(i) Represents the above situation by a matrix equation.
(ii) Solve this equation using matrix method.

34. If $(ax+b)e^{\frac{y}{x}} = x$, then show that:

$x^3\left(\frac{d^2y}{dx^2}\right) = \left(x\frac{dy}{dx} - y\right)^2$

35. Find the shortest distance between the lines $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ and $(2\hat{i} + 4\hat{j} + 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 8\hat{k})$

OR

Find the equations of the diagonals of the parallelogram PQRS whose vertices are P(4,2,-6), Q(5,-3,1), R(12,4,5) and S(11,9,-2). Use these equations to find the point of intersection of diagonals.

SECTION E

(Question numbers 36 to 38 carry 4 marks each.) $4 \times 3 = 12$

This section contains three Case-study / Passage based questions. First two case study questions have three sub-parts (i), (ii) and (iii) of marks 1, 1 and 2 respectively. The third case study question has two sub parts (i) and (ii) of marks 2 each.

36. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by cab, metro, bike or by other means of transport are respectively 0.3, 0.2, 0.1 and 0.4. The probabilities that he will be late are 0.25, 0.3, 0.35 and 0.1 if he comes by cab, metro, bike and other means of transport respectively.

(i)What is the probability that the doctor is late by other means?

(ii)When the doctor arrives late, what is the probability that he comes by metro?

(iii) When the doctor arrives late, what is the probability that he comes by bike or other means?

OR

When the doctor arrives late, what is the probability that he comes by cab or metro?

37. Rajat visited the Exhibition along with his family. The Exhibition had a huge swing, which attracted many children. Rajat found that the swing traced the path of a parabola given by $y = x^2$.

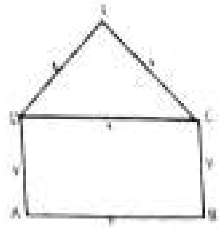
Based on the given information, answer the following questions

(i) If f: R→ R be defined by $f(x) = x^2$, then show that f is neither injective nor surjective.

(ii) If f: N→ N be defined by $f(x) = x^2$, then show that f is injective but not surjective.

(iii) If f: {1,2,3....} → {1,4,9, ...} is defined by $f(x) = x^2$ Then show f is bijective.

38 The windows of a newly constructed building are in the form of rectangle surmounted by an equilateral triangle. The perimeter of each window is 12 m as shown in figure:



Based on the above information answer the following:

(i) For what value of x area of the window is maximum?

(ii) Find the value of y ?