CHENNAI SAHODAYA SCHOOLS COMPLEX
COMMON EXAMINATION
CLASS 12
APPLIED MATHEMATICS (241)
ANSWER KEY
SET – 1

Q.NO	ANSWER	MARKS
1.	Speed of Samarth $= \frac{35}{7} = 5$ m/s Time taken by Samarth to cover 200m $= \frac{200}{5} = 40$s Time taken by Prateek $= 40 - 7 = 33$s (c)	1
2.	$\frac{x+1}{x+2} - 1 \geq 0$ $\Rightarrow \frac{-1}{x+2} \geq 0$ (b) $\Rightarrow x+2 < 0 \Rightarrow x < -2 \quad x \in (-\infty, -2)$	1
3	Since matrix A is symmetric (a) $2x - 3 = x + 7$ $\Rightarrow x = 10$	1
4.	$\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$ $C_3 \to C_3 - 9C_2$ (c) $\begin{vmatrix} 2 & 7 & 2 \\ 3 & 8 & 3 \\ 5 & 9 & 5 \end{vmatrix}$ $C_1 = C_3$ $= 0$	1
5.	$f(x) = \log_a x = \frac{\log x}{\log a}$ (a) $f'(x) = \frac{1}{(\log a)x}$ $f'(e) = \frac{1}{e(\log a)}$	1
6.	$\frac{dy}{dx} = \frac{-2x}{(1+x^2)^2} \leq 0 \quad \because x \geq 0$ (d) f_n is decreasing for $x \geq 0$	1
7	order = 2 degree = 3 (a)	1

Q.NO	ANSWER	MARKS
8.	X : No of black balls = 0, 1, 2 (a) $P(X=0) = \frac{5C_2}{7C_2} = \frac{10}{21}$ $P(X=1) = \frac{5C_1 \times 2C_1}{7C_2} = \frac{5\times2\times2}{7\times6} = \frac{10}{21}$ $P(X=2) = \frac{2C_2}{7C_2} = \frac{1}{21}$ X 0 1 2 P 10/21 10/21 1/21 $E(X) = \frac{12}{21} = 4/7$ PiXi 0 10/21 2/21	1
9	$6 \equiv (-1) \bmod 7 \Rightarrow 6^{12} \equiv 1 \pmod 7$ (a)	1
10.	Irregular Trend (b)	1
11.	Let Sample mean = $\bar{x}$ margin of Error = E (b) $\bar{X}+E = 160$ $\bar{X}-E = 152$ $2E = 8$ $E = 4$	1
12.	40, 42, 44 b	1
13.	$\int_0^{40} \frac{dx}{2x+1} = \frac{1}{2}\log\lvert 2x+1\rvert\Big]_0^{40} = \log K.$ (c) $\frac{1}{2}[\log 81 - \log 1] = \log K.$ $\log 9 = \log K$ $K = 9$	1
14	Glass is filled with 10 parts of I liquid + 4 parts of Second liq Total volume of glass = 14 Water in new mixture = $\frac{20}{100}\times10 + \frac{35}{100}\times4$ (b) $= 2 + 1.4 = 3.14$ req % = $\frac{3.14}{14}\times100 = 24\frac{2}{7}\%$	(1)
15	$\frac{125000-25000}{n} = 12500$ (c) $n = \frac{100000}{12500} = 8$	1
16	$50000 = R\left[\frac{(1.03)^{16}-1}{.03}\right] \Rightarrow \frac{1500}{.6047} = R.$ (c) $R = 2480.57$	1

Q.NO	ANSWER	MARKS
17.	$Z(0,0)=0 \quad Z(7,0)=35 \quad Z(3,4)=43 \quad Z(0,2)=14$ max Z at (3,4) (c)	1
18.	$\int_0^{1/3} 0\,dx + \int_{1/3}^{2/3} dx + 2\int_{2/3}^{1} dx =$ (b) $x\Big]_{1/3}^{2/3} + 2[x]_{1/3}^{1} = \frac{2}{3}-\frac{1}{3}+2\left[1-\frac{2}{3}\right]$ $\frac{1}{3}+\frac{2}{3}=\frac{3}{3}=1$	1
19.	If $x<\lvert x\rvert \Rightarrow x<0$ $\because 2x+1<0 \quad x<-\frac{1}{2} \quad x\in(-\infty,-1/2)$ Assertion True. (b) Reason: True. Hence Assertion True but Reason is not correct Explation of Assertion	1
20.	$2x^2-40=18+14 \Rightarrow 2x^2=72 \quad x=\pm 6$. (c) Assertion: True. $\lvert 2AB\rvert = 2^3\lvert A\rvert\lvert B\rvert = 8\times4\times6=192$ Reason: False / True	1
	Section B (2 Marks	
21)	$A(\text{adj}A) = \lvert A\rvert I$	½
	$\lvert A\rvert = \begin{vmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{vmatrix} = 3[-3+2]-1[2+1]+2[4+3]$ $= -3-3+14=8$	1
	$A(\text{adj}A) = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$	½
	(OR)	
	$X = \begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$	½
	$X = \begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$	1
	$X = \begin{bmatrix} 1-4 & -2-12 \\ -1+5 & 2+15 \end{bmatrix} = \begin{bmatrix} -3 & -14 \\ 4 & 17 \end{bmatrix}$	½

Q.NO	ANSWER	MARKS
22	$\frac{30}{100}x + 5 = \frac{40}{100}(x+5)$	½
	$\frac{30x}{100} + 5 = \frac{40x}{100} + \frac{200}{100}$	½
	$\frac{10x}{100} = 5-2$	
	$x = 30$ L (Total volume)	½
	Quantity of water in original $= \frac{30}{100} \times 30 = 9$ L	½
23)	$-\int_{-5}^{0} x\,dx = -\frac{x^2}{2}\Big]_{-5}^{0} = \frac{25}{2}$	½
	$-\int_{-5}^{-2}(x+2)dx + \int_{-2}^{0}(x+2)dx$	½
	$= -\left[\frac{(x+2)^2}{2}\right]_{-5}^{-2} + \frac{(x+2)^2}{2}\Big]_{-2}^{0}$	
	$= -\frac{1}{2}[0-9] + \frac{1}{2}[4] = \frac{13}{2}$	½
	$\int_{-5}^{0}(x+5)dx = \frac{(x+5)^2}{2}\Big]_{-5}^{0} = \frac{1}{2}(25)$	½
	$\therefore \frac{25}{2} + \frac{13}{2} + \frac{25}{2} = \frac{63}{2}$	
24	p: Sel a white ball $= 1/3$	
	q: Sel black ball $= 2/3$	½
	$P(X \geq 4) = P(X=4) + P(X=5)$	
	$= {}^5C_4 p^4 q + p^5$	1
	$= p^4[5q + p] = \frac{1}{3^4}\left[\frac{10}{3} + \frac{1}{3}\right]$	
	$= \frac{11}{243}$	½

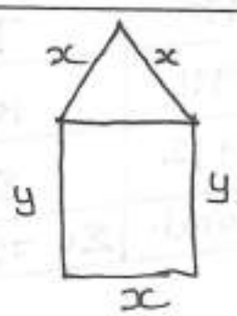
Q.NO	ANSWER	MARKS
	(OR)	
	$\mu = 12 \quad \sigma = 2$	
	a) $P(X < 7) \quad Z = \frac{x-\mu}{\sigma} = \frac{7-12}{2} = \frac{-5}{2}$	
	$= P(Z < -5/2) = 1 - P(Z < 5/2)$	1
	$= 1 - .9938 = 0.0062$	
	(b) $P(7 < x < 14) \quad Z = \frac{14-12}{2} = 1$	
	$= P\left(\frac{-5}{2} < Z < 1\right) = P(Z < 1) - P(Z < -5/2)$	1
	$= .8413 - 0.0062$	
	$= .8351$	
25)	$MC = 66 + \frac{400}{x+1}$	
	$C = 60x + 400 \log\|x+1\| + c$	1
	$x = 0 \quad C = 3000$	$\frac{1}{2}$
	$3000 = c$	
	$\therefore C = 60x + 400 \log\|x+1\| + 3000$	1/2
	Section C (3marks)	
26	given $n = 25 \quad \bar{x} = 237.6 \quad \mu = 225.4$	
	$S = 21.3 \quad H_0 : \mu = 225.4$	1/2
	$H_1 : \mu \neq 225.4$	
	$t = \frac{\bar{x}-\mu}{S/\sqrt{n}} = \frac{237.6 - 225.4}{21.3/\sqrt{25}}$	1/2
	$= \frac{12.2 \times 5}{21.3} = 2.86$	1/2 1
	Calculated t value > tabulated t value	1/2
	H_0 rejected. Advertising Campaign Successful.	1/2

Q.NO	ANSWER	MARKS
27	$\frac{1}{\sqrt{3}}\int \frac{dx}{\sqrt{x^2-\frac{x}{3}-\frac{1}{3}}}$	½
	$\frac{1}{\sqrt{3}}\int \frac{dx}{\sqrt{x^2-2x\cdot\frac{1}{6}+\frac{1}{36}-\frac{1}{36}-\frac{1}{3}}}$	
	$\frac{1}{\sqrt{3}}\int \frac{dx}{\sqrt{(x-\frac{1}{6})^2-(\frac{\sqrt{13}}{6})^2}}$	1
	$\frac{1}{\sqrt{3}}\log\left\lvert x-\frac{1}{6}+\sqrt{(x-\frac{1}{6})^2-(\frac{\sqrt{13}}{6})^2}\right\rvert+C$	1
	$\frac{1}{\sqrt{3}}\log\left\lvert\frac{6x-1}{6}+\sqrt{x^2-\frac{x}{3}-\frac{1}{3}}\right\rvert+C$	½
	(OR)	
	$I=\int_2^4 \frac{\log(x^2)}{\log x^2+\log(6-x)^2}dx$ — ①	½
	$I=\int_2^4 \frac{\log(6-x)^2}{\log(6-x)^2+\log x^2}$ — ②	1
	① + ②	
	$2I=\int_2^4 dx.$	½
	$2I=x\Big]_2^4=2$	1
	$I=1$	
28.	$y=e^{2x}(a+bx)$ — ①	
	$\frac{dy}{dx}=e^{2x}\cdot b+(a+bx)e^{2x}\cdot 2$	1
	$\frac{dy}{dx}-2y=be^{2x}$ — ② from ①	½
	$\frac{d^2y}{dx^2}-2\frac{dy}{dx}=2be^{2x}$	½
	$\frac{d^2y}{dx^2}-2\frac{dy}{dx}=2\left[\frac{dy}{dx}-2y\right]$ from 2	½
	$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=0$	½

Q.NO	ANSWER	MARKS
29.	$y = \log(x+\sqrt{x^2+1})$ —①	
	$\frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+1}}\left[1+\frac{1}{2\sqrt{x^2+1}}(2x)\right]$	1
	$\frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+1}} \times \frac{\sqrt{1+x^2}+x}{2\sqrt{1+x^2}}$	
	$\sqrt{1+x^2}\,\frac{dy}{dx} = 1$	
	$(1+x^2)\left(\frac{dy}{dx}\right)^2 = 1$	½
	$(1+x^2)\,2\frac{dy}{dx}\cdot\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 2x = 0$	1
	$\Rightarrow \quad (1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$	½
30	Number of tickets from 1 to 200 divisible by 20 = 10	½
	$p = \frac{10}{200} = \frac{1}{20} \qquad q = \frac{19}{20} \qquad \lambda = np = 20\times\frac{1}{20} = 1$	½
	$P(X\le 4) = P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)$	½
	$= e^{-\lambda} + e^{-\lambda}\lambda + \frac{e^{-\lambda}\lambda^2}{2} + \frac{e^{-\lambda}\lambda^3}{6} + \frac{e^{-\lambda}\lambda^4}{24}$	½
	$= e^{-1}\left[1+1+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}\right]$	½
	$= 0.368 \times \frac{65}{24} = .9967$	½
31	Cost of new machine 5,25,000 Scrap value = 25000 money req for new machine after 20 years = 5,00,000 $A = 500000 \quad i = 0.05 \quad n = 20$	1

Q.NO	ANSWER	MARKS
	$A = R\left[\frac{(1+i)^n - 1}{i}\right] \Rightarrow 500000 = R\left[\frac{(1.05)^{20} - 1}{.05}\right]$	½
	$\frac{500000 \times 0.05}{(1.05)^{20} - 1}$	½
	$\frac{25000}{1.655} = 15105.74$	1
	(OR)	
	$CAGR = \left[\left(\frac{FV}{BV}\right)^{1/n} - 1\right] \times 100$	
	$.2247 = \left(\frac{30000}{20000}\right)^{1/n} - 1$	1
	$1.2247 = \left(\frac{3}{2}\right)^{1/n}$	½
	$\frac{1}{n} \log 1.5 = \log 1.2247$	½
	$n = \frac{\log 1.5}{\log 1.2247} = \frac{0.1761}{.0882}$	1
	$n = 1.996 \simeq 2$ years	
	Section D (5 marks)	
32	$a+b+c=0$ — ①	1½
	$4a+2b+c=-2$ — ②	
	$9a+3b+c=-6$ — ③	
	$D = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{vmatrix} = -2 \quad D_1 = \begin{vmatrix} 0 & 1 & 1 \\ -2 & 2 & 1 \\ -6 & 3 & 1 \end{vmatrix} = 2$	2
	$D_2 = \begin{vmatrix} 1 & 0 & 1 \\ 4 & -2 & 1 \\ 9 & -6 & 1 \end{vmatrix} = -2 \quad D_3 = \begin{vmatrix} 1 & 1 & 0 \\ 4 & 2 & -2 \\ 9 & 3 & -6 \end{vmatrix} = 0$	
	$a = -1 \quad b = 1 \quad c = 0$	1
	$f(x) = -x^2 + x.$	½
	OR	

Q.NO	ANSWER	MARKS
	$\|A\| = 10$	1
	$A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$	2
	$A'X = B$ $X = (A')^{-1}B$ $X = (A^{-1})'B$	½
	$\begin{bmatrix} x \\ y \\ 3 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$	1½
	$\begin{bmatrix} x \\ y \\ 3 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 18 \\ 04 \\ 14 \end{bmatrix}$ $x = 9/5 \quad y = 2/5 \quad 3 = 7/5$	1
33	$A = A_0 e^{(0.04)t}$ $\quad K = \frac{4}{100} = 0.04$	1
	$t = 0 \quad A = 10{,}000$	½
	$10000 = A_0 e^{(.04)t}$	½
	$A = 10000\, e$	½
	$t = 10 \quad A = ?$	1
	$A = 10000\, e^{0.4}$	
	$A = 10000 \times 1.49182$	1½
	$= 14918.2$	
34	$10 = 3x + 2y$	½
	$A = \frac{\sqrt{3}}{4}x^2 + xy$	½
	$A = \frac{\sqrt{3}}{4}x^2 + x\left[\frac{10-3x}{2}\right]$	
	$A = \frac{\sqrt{3}}{4}x^2 + 5x - \frac{3x^2}{2}$	½



Q.NO	ANSWER	MARKS

$$\frac{dA}{dx} = \frac{\sqrt{3}x}{2} + 5 - 3x$$ — 1

$$\frac{dA}{dx} = 0 \qquad 5 = x\left[3 - \frac{\sqrt{3}}{2}\right]$$ — 1/2

$$\frac{10}{6-\sqrt{3}} = x$$

$$\frac{d^2A}{dx^2} = \frac{\sqrt{3}}{2} - 3 < 0$$ — 1

A is max $x = \frac{10}{6-\sqrt{3}}$ m. — 1/2

$$3x + 2y = 10$$

$$2y = 10 - \frac{30}{6-\sqrt{3}}$$

$$y = 5 - \frac{15}{6-\sqrt{3}} = \frac{30 - 5\sqrt{3} - 15}{6-\sqrt{3}}$$ — 1/2

$$y = \frac{15 - 5\sqrt{3}}{6-\sqrt{3}} \qquad x = \frac{10}{6-\sqrt{3}} \text{ m.}$$

35

Year(t)	Production(y)	$x = t_i - 1967$	x^2	xy	Trend Value
1962	2	−5	25	−10	1.73
1963	4	−4	16	−16	2.42
1964	3	−3	9	−9	3.11
1965	4	−2	4	−8	3.8
1966	4	−1	1	−4	4.49
1967	2	0	0	0	5.18
1968	4	1	1	4	5.87
1969	9	2	4	18	6.56
1970	7	3	9	21	7.25
1971	10	4	16	40	7.94
1972	8	5	25	40	8.63
Total	$\Sigma y = 57$	$\Sigma x = 0$	$\Sigma x^2 = 110$	$\Sigma xy = 76$	

2 1/2

Set-1

Q.NO	ANSWER	MARKS
	$a = \frac{\Sigma y}{n} = \frac{57}{11} = 5.18$ $\quad b = \frac{\Sigma xy}{\Sigma x^2} = \frac{76}{110} = 0.69$	1
	$\therefore$ Eq of trend line $y = 5.18 + 0.69x$	½
	b) $y = 5.18 + (0.69)(13) = 14.15$.	1
	(OR).	

Yearly Quarterly		Small Scale Industry	4-Quarterly moving total	4 Quarterly mov Avg	4-Year Centred moving Avg
	I	39			
	II	47			
	III	20	162	40.5	44.125
	IV	56.	191	47.5	49.25
	I	68	203	50.75	56.5
	II	59	249	62.25	64.25
	III	66	265	66.25	68.75
	IV	72	285	71.25	71.375
	I	88	286	71.5	70.75
	II	60	280	70	69.375
	III	60	275	68.75	
	IV	67			

5

Section E (4 marks)

36 Minimize $Z = 30x + y + 39500$

$x + y \leq 7000$

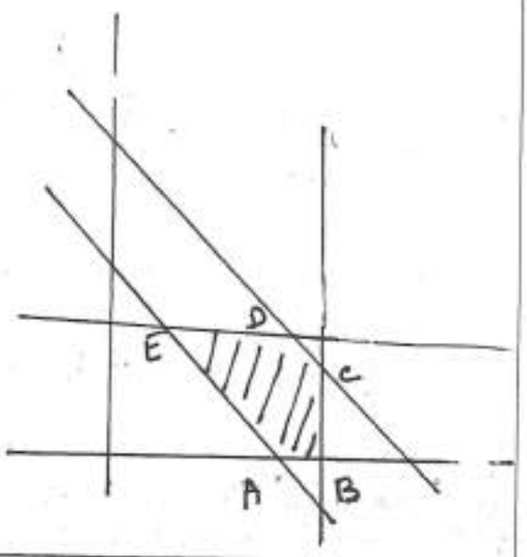
$x \leq 4500$

$y \leq 3000.$

$x + y \geq 3500$

$x \geq 0 \quad y \geq 0$

2

Q.NO	ANSWER	MARKS
	Corner pt Z A(3500,0) 50000 B(4500,0) 53000 C(4500,2500) 55500. D(4000,3000) 54500 E(500,3000) 44000 ← min Z = 44000 	2
37	(i) Total Number of payment $25 \times 12 = 300$ (ii) rate of interest/month $\frac{9}{1200} = 0.0075$ (iii) $EMI = \frac{Pi}{1-(1+i)^{-n}}$ $= \frac{2000000 \times 0.0075}{1-(1.0075)^{-300}}$ $= \frac{15000}{1-0.1062} = \frac{15000}{.8938} = 16782.27$ OR. Total Int $= nEMI - P$ $= 300 \times 16782.27 - 2000000$ $= 5034681 - 20000000$ $= \underline{3034681}$	1 1 2 2.
38.	x_i P_i 0 0.2 1 K 2 2K 3 2K. $\Sigma P_i = 1$ $5K + 0.2 = 1$ $5K = 0.8$	
12	a) $K = \frac{8}{50} = \frac{4}{25}$	1

Set-1

Q.NO	ANSWER	MARKS
b)	$P(X=2) = 2K = \frac{8}{25}$	1
c)	$P(X \geq 2) = P(x=2) + P(x=3) = 4K$ $= \frac{16}{25}$ OR $P(X \leq 2) = 1 - P(x=3)$ $= 1 - 2K = 1 - \frac{8}{25} = \frac{25-8}{25}$ $= \frac{17}{25}$	2

Q.NO	ANSWER	MARKS