Class XII Session 2024-25
Subject - Applied Mathematics
Sample Question Paper - 4

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This question paper contains five sections A, B, C, D and E. Each section is compulsory.
2. Section - A carries 20 marks weightage, Section - B carries 10 marks weightage, Section - C carries 18 marks weightage, Section - D carries 20 marks weightage and Section - E carries 3 case-based with total weightage of 12 marks.
3. **Section – A:** It comprises of 20 MCQs of 1 mark each.
4. **Section – B:** It comprises of 5 VSA type questions of 2 marks each.
5. **Section – C:** It comprises of 6 SA type of questions of 3 marks each.
6. **Section – D:** It comprises of 4 LA type of questions of 5 marks each.
7. **Section – E:** It has 3 case studies. Each case study comprises of 3 case-based questions, where 2 VSA type questions are of 1 mark each and 1 SA type question is of 2 marks. Internal choice is provided in 2 marks question in each case-study.
8. Internal choice is provided in 2 questions in Section - B, 2 questions in Section – C, 2 questions in Section - D. You have to attempt only one of the alternatives in all such questions.

Section A

1. If the matrix $\begin{bmatrix} 0 & -1 & 3x \\ 1 & y & -5 \\ -6 & 5 & 0 \end{bmatrix}$ is skew-symmetric, then **[1]**

a) x = 2, y = 0 b) x = 2, y = -1

c) x = -2, y = 0 d) x = -2, y = 1

2. Which of the following is an assumption underlying the use of the t-distribution? **[1]**

a) The sample size are drawn from a normally distributed population.

b) Sample standard deviation is an unbiased estimate of the population variance.

c) All of these

d) The variance of the population is known.

3. A certain sum of money amounts to ₹ 5832 in 2 years at 8% p.a. compound interest. The sum invested is **[1]**

a) ₹ 5280 b) ₹ 5400

c) ₹ 5200 d) ₹ 5000

4. Any feasible solution which maximizes or minimizes the objective function is called: **[1]**

a) An objective feasible solution b) A reasonable feasible solution

c) An optimal feasible solution

d) A regional feasible solution

5. For the curve $\sqrt{x}+\sqrt{y}=1, \frac{dy}{dx}$ at $\left(\frac{1}{4}, \frac{1}{4}\right)$ is **[1]**

a) 2

b) -1

c) -2

d) 1

6. In a binomial distribution, the probability of getting success is $\frac{1}{4}$ and standard deviation is 3. Then, its mean is **[1]**

a) 10

b) 6

c) 8

d) 12

7. A random variable 'X' has the following probability distribution: **[1]**

X	0	1	2	3	4	5	6	7
P(X)	0	2k	2k	3k	k^2	$2k^2$	$7k^2$	2k

The value of k is

a) -1

b) 1

c) $-\frac{1}{10}$

d) $\frac{1}{10}$

8. The degree of the differential equation $\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2 = x^2\log\left(\frac{d^2y}{dx^2}\right)$ is **[1]**

a) 1

b) 3

c) 2

d) not defined

9. A pipe A can fill a tank in 25 minutes and pipe B can empty the full tank in 50 minutes. The time taken by two pipes to fill the tank is: **[1]**

a) 20 minutes

b) 30 minutes

c) 50 minutes

d) 10 minutes

10. Solution of the differential equation $x\frac{dy}{dx}+2y = x^2$ is **[1]**

a) $y=\frac{x^2}{4}+C$

b) $y=\frac{x^4+C}{4x^2}$

c) $y=\frac{x^2+C}{4x^2}$

d) $y=\frac{x^2+C}{x^2}$

11. In what ratio must rice at ₹ 29.30 per kg be mixed with rice at ₹ 30.80 per kg so that the mixture be worth ₹ 30 per kg? **[1]**

a) 7:8

b) 3:8

c) 8:3

d) 8:7

12. x and b are real numbers. If b > 0 and |x| > b, then **[1]**

a) $x \in (-b, b)$

b) $x \in [-\infty, b)$

c) $x \in (-b, \infty)$

d) $x \in (-\infty, -b) \cup (b, \infty)$

13. In a 100 m race A and B are two participants. If A runs at 5 kilometer per hour and A gives B a start of 8 m and still beats him by 8 seconds, then the speed of B is: **[1]**

a) 5.15 km/hr

b) 4.4 km/hr

c) 4.14 km/hr

d) 4.25 km/hr

14. Corner points of the feasible region determined by the system of linear constraints (0, 3), (1, 1) and (3, 0). Let z = px + qy, where p, q > 0 . Condition on p and q so that the minimum of z occurs at (3, 0) and (1, 1) is **[1]**

a) p = 3q
b) p = 2q
c) p = q
d) 2p = q

15. The solution set of system of linear inequalities **[1]**
$2(x + 1) \leq x + 5, 3(x + 2) > 2 - x, x \in R$ is

a) [-1, 3)
b) (-1, 3)
c) [-1, 3]
d) (-1, 3]

16. The assumed hypothesis which is tested for rejection considering it to be true is called **[1]**

a) true hypothesis
b) simple hypothesis
c) null hypothesis
d) alternative hypothesis

17. If the marginal revenue function of a commodity is MR = $2x - 9x^2$, then the revenue function is **[1]**

a) 2 - 18x
b) $x^2 - 3x^3$
c) $2x^2 - 9x^3$
d) $18 + x^2 - 3x^3$

18. For the given five values 15, 24, 18, 33, 42, the three years moving averages are **[1]**

a) 19, 25, 33
b) 19, 25, 31
c) 19, 30, 31
d) 19, 22, 33

19. **Assertion (A):** If A = $\begin{bmatrix} 2 & 3 & -1 \\ 1 & 4 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$, then AB and BA both are defined. **[1]**

Reason (R): For the two matrices A and B, the product AB is defined, if number of columns in A is equal to the number of rows in B.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. The function f be given by $f(x) = 2x^3 - 6x^2 + 6x + 5$. **[1]**
Assertion (A): x = 1 is not a point of local maxima.
Reason (R): x = 1 is not a point of local minima.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. The Production of cement by a firm in year 1 to 9 is given below: **[2]**

Year	1	2	3	4	5	6	7	8	9
Production in (Tonnes)	4	5	5	6	7	8	9	8	10

Calculate the trend values for the above series by the 3-yearly moving average method.

22. Find the compound interest on ₹ 7000 at 6% p.a for 18 months compounded quarterly. [Use$(1.015)^6 = 1.093$] **[2]**

OR

A company ABC Ltd has raised funds in the form of 1,000 zero-coupon bonds worth ₹1,000 each. The company wants to set up a sinking fund for repayment of the bonds, which will be after 10 years. Determine the amount of the periodic contribution if the annualized rate of interest is 5%, and the contribution will be done half-yearly. Given that $(1.025)^{20} = 1.6386$.

23. Evaluate: $\int_{1}^{2} \frac{3x}{9x^2-1} dx$ **[2]**

24. Mrs. Dubey borrowed ₹500000 from a bank to purchase a car and decided to repay by monthly installments in 5 years. The bank charges interest at 8% p.a. compounded monthly. Calculate the EMI. (Given $(1.0067)^{60} = 1.4928$) **[2]**

OR

At what rate per cent, per annum compounded annually, will the sum of money become 4 times of itself in 2 years?

25. Find the remainder when 2^{100} is divided by 11. **[2]**

Section C

26. It is given that radium decomposes at a rate proportional to the amount present. If p % of the original amount of radium disappears in 1 year. What percentage of it will remain after 2l years? **[3]**

OR

Solve the differential equation: $x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x$

27. Consider a bond with a coupon rate of 10% charged annually. The par value is ₹2,000 and the bond has 5 years of maturity. The yield to maturity is 11%. What is the value of the bond. **[3]**

28. The marginal cost function of a product is given by MC = $\frac{x}{\sqrt{x^2+400}}$. Find the total cost and the average cost if the fixed cost is ₹ 1000. **[3]**

29. From a lot of 10 items containing 3 defectives, a sample of 4 items is drawn at random. Let the random variable X denote the number of defective items in the sample. If the items in the sample are drawn one by one without replacement, find: **[3]**

i. The probability distribution of X
ii. Mean of X
iii. Variance of X

OR

Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation and variance of X.

30. From the following data calculate the 4-yearly moving averages and determine the trend values. **[3]**

Years	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Value	50.0	36.5	43.0	44.5	38.9	38.9	32.6	41.7	41.1	33.8

31. Consider the following hypothesis test: **[3]**

$H_0 : \mu = 18$

$H_a : \mu \neq 18$

A sample of 48 provided a sample mean $\bar{x} = 17$ and a sample standard deviation S = 4.5

i. Compute the value of the test statistic.
ii. Use the t-distribution table to compute a range for the p-value.
iii. At $\alpha = 0.05$, what is your conclusion?
iv. What is the rejection rule using the critical value? What is your conclusion?

Section D

32. Two factories decided to award their employees for three values of **[5]**

a. adaptable to new techniques,

b. careful and alert in difficult situations and

c. keeping calm in tense situations, at the rate of ₹ x, ₹ y and ₹ z per person respectively. The first factory decided to honour respectively 2, 4 and 3 employees with a total prize money of ₹ 29000. The second factory decided to honour respectively 5, 2 and 3 employees with the prize money of ₹ 30500. If the three prizes per person together cost ₹ 9500, then

i. represent the above situation by a matrix equation and form linear equations using matrix multiplication.

ii. Solve these equations using matrices.

OR

Find the adjoint of the matrix A = $\begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence show that A (adj A) = |A| I_3.

33. In a 1000-metre race, A, B and C get Gold, Silver and Bronze medals respectively. If A beats B by 100 metres and B beats C by 100 metres, then by how many metres does A beat C? **[5]**

34. A box contains 4 red and 5 black marbles. Find the probability distribution of the red marbles in a random draw of three marbles. Also find the mean, variance and standard deviation of the distribution. **[5]**

OR

Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find E(X).

35. Find the amount of an annuity consisting of payment of ₹ 1000 at the end of every three months for 4 years at 8% per annum, compounded quarterly. [Use $(1.02)^{16}$ = 1.372] **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

There is a bridge whose length of three sides of a trapezium other than base are equal to 5cm:

(a) What is the value of DP?

(b) What is the area of the trapezium A(x)?

(c) A’(x) = 0 then what is the value of x?

OR

What is the value of A”(2.5)

37. **Read the text carefully and answer the questions:** **[4]**

The nominal rate of return is the amount of money generated by an investment before factoring in expenses such as taxes, investment fees, and inflation. If an investment generated a 10% return, the nominal rate would equal 10%. After factoring in inflation during the investment period, the actual return would likely be lower. However, the nominal rate of return has its merits since it allows investors to compare the performance of an investment irrespective of the different tax rates that might be applied for each investment.

(a) A person invests ₹10000 in 10% ₹100 shares of a company available at a premium of ₹25. Find his rate of return.

(b) A man invests ₹22500 in ₹50 shares available at 10% discount. If the dividend paid by the company is 12%, calculate his rate of return.

(c) A person invested ₹200000 in a fund for one year. At the end of the year, the investment was worth ₹216000. Calculate his rate of return.

OR

Balwant invests a sum of money in ₹50 shares paying 10% dividend quoted at 20% discount. If his annual dividend is ₹600, calculate his rate of return from the investment.

38. **Read the following text carefully and answer the questions that follow:** **[4]**

A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 hours on grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding/cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is ₹ 25 and that from a shade is ₹ 15.

If x is the number of lamps and y is the number of shades manufactured. Assuming that the manufacturer can sell all the lamps and shades that he produces.

i. In order to maximize the profit, what should be the objective function? (1)
ii. What are the constraints related to the given LPP: (1)
iii. The non-negative constraints associative to the given L.P.P are: (2)

OR

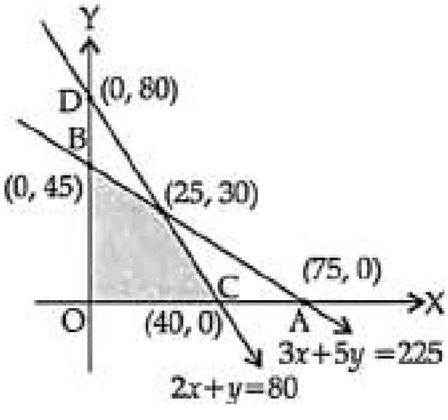
What are the vertices of feasible region of given L.P.P? (2)

OR

Read the following text carefully and answer the questions that follow:

A manufacturer produces two Models of bikes Model X and Model Y. Model X takes a 6 man hours to make per unit, while Model Y takes 10 man-hours per unit. There is a total of 450 man-hours available per week. Handling and Marketing costs are ₹ 2,000 and ₹ 1,000 per unit for Models X and Y respectively. The total funds available for these purposes are ₹ 80,000 per week. Profits per unit for Models X and Y are ₹ 1,000 and ₹ 500, respectively.

The feasible region of LPP is shown in the following graph.



i. Find the equation of line AB. (1)
ii. Find the equation of line CD. (1)
iii. Find the coordinates of point E. (2)

OR

How many bikes of model X and model Y should the manufacturer produce so as to yield a maximum profit? (2)

Solution

Section A

1. **(a)** x = 2, y = 0

   Explanation: Let A = $\begin{bmatrix} 0 & -1 & 3x \\ 1 & y & -5 \\ -6 & 5 & 0 \end{bmatrix}$, then A' = -A

   $\Rightarrow \begin{bmatrix} 0 & 1 & -6 \\ -1 & y & 5 \\ 3x & -5 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -3x \\ -1 & -y & 5 \\ 6 & -5 & 0 \end{bmatrix}$

   ⇒ -3x = -6 ⇒ x = 2, y = -y ⇒ 2y = 0 ⇒ y = 0

   ∴ x = 2, y = 0

   ∴ Option (x = 2, y = 0) is the correct answer.

2. **(a)** The sample size are drawn from a normally distributed population.

   Explanation: The sample size are drawn from a normally distributed population.

3.

   (d) ₹ 5000

   Explanation: Let sum invested be ₹ x, rate = 8%, time = 2 years

   Amount = ₹ 5832

   ∴ 5832 = $x\left(1+\frac{8}{100}\right)^2$

   ⇒ 5832 - x × $\left(\frac{27}{25}\right)^2$

   ⇒ $x=\frac{5832\times25\times25}{27\times27}$ = 5000

   ∴ Sum invested = ₹ 5000

4.

   (c) An optimal feasible solution

   Explanation: An optimal feasible solution

5.

   (b) -1

   Explanation: $\sqrt{x}+\sqrt{y}=1$

   Differentiating with respect to x,

   $\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\frac{dy}{dx}=0$

   $\frac{dy}{dx}=-\sqrt{\frac{y}{x}}$

   $\frac{dy}{dx}\left(\frac{1}{4^{-1}\frac{1}{4}}\right)=-\sqrt{\frac{\frac{1}{4}}{\frac{1}{4}}}=-1$

6.

   (d) 12

   Explanation: $p=\frac{1}{4},\sqrt{npq}=3$

   ⇒ $q=\frac{3}{4}$, npq = 9

   ⇒ Mean = np $=\frac{9}{q}$

   ⇒ Mean $=9\times\frac{4}{3}$ = 12

7.

   (d) $\frac{1}{10}$

   Explanation: 0 + 2k + 2k + 3k + k^2 + $2k^2$ + $7k^2$ + 2k = 1

   ⇒ $10k^2$ + 9k - 1 = 0 ⇒ (10k - 1) (k + 1) = 0

$\Rightarrow k=\frac{1}{10}$, -1 but k $\neq$ -1

$\Rightarrow k=\frac{1}{10}$

8.

(d) not defined

Explanation: As the term log $\left(\frac{d^2y}{dx^2}\right)$ is not a polynomial in $\frac{d^2y}{dx^2}$. So, the degree of the given differential equation is not defined.

9.

(c) 50 minutes

Explanation: Part of tank filled by A and B in 1 minute = $\frac{1}{25}-\frac{1}{50}$

= $\frac{2-1}{50}$

= $\frac{1}{50}$

$\because \frac{1}{50}$ part of tank is filled in 1 minute

$\therefore$ 1 part of tank is filled in 50 minute

Hence, time taken by two pipe to fill the tank = 50 minute

10.

(b) $y=\frac{x^4+\mathrm{C}}{4x^2}$

Explanation: $\frac{dy}{dx}+\frac{2}{x}$y = x $\Rightarrow$ I.F. = $e^{\int \frac{2}{x}dx}$ = $e^{2\log x}$ = x^2

$\therefore$ Solution is y $\cdot$ x^2 = $\int$ x $\cdot$ x^2 dx + C_1

y $\cdot$ x^2 = $\frac{x^4}{4}$ + C_1 $\Rightarrow$ y = $\frac{x^4+C}{4x^2}$

11.

(d) 8:7

Explanation: 8:7

12.

(d) x $\in$ ($-\infty$, -b) $\cup$ (b, ∞)

Explanation: x $\in$ ($-\infty$, -b) $\cup$ (b, ∞)

13.

(c) 4.14 km/hr

Explanation: A's Speed = $\frac{\text{Distance}}{\text{Time Travelled}}$

$\Rightarrow$ A's Speed = 5 kmph = $\frac{100\text{ m}}{\text{Time Travelled}}$

$\Rightarrow$ Total time taken by A to complete 100m = $\frac{100}{(\frac{5\times1000}{3600})}$ seconds = 72 seconds

$\Rightarrow$ B's Speed = $\frac{\text{Distance Travelled by B}}{\text{Time T aken byB}}$ = $\frac{\frac{(100-8)}{1000}}{\frac{(7+-8)}{3600}}$ kmph = $\frac{92\times36}{800}$ kmph = 4.14 kmph

14.

(d) 2p = q

Explanation: We have Z = px + qy, At (3, 0) Z = 3p ...(i)

At (1, 1) Z = p + q ...(ii)

Therefore, from (i) and (ii):

We have: p = $\frac{q}{2}$

2p = q

15.

(d) (-1, 3]

Explanation: (-1, 3]

16.

(c) null hypothesis

Explanation: null hypothesis

17.

(b) x^2 - $3x^3$

Explanation: Given MR = $2x - 9x^2$

$\therefore \mathrm{R}(x) = \int \left(2x - 9x^2\right) dx$

$\Rightarrow$ R(x) = $x^2 - 3x^3 + k$

We know that when x = 0, R(x) = 0

$\Rightarrow$ 0 - 0 + k = 0 $\Rightarrow$ k = 0

$\therefore$ R(x) = $x^2 - 3x^3$

18.

(b) 19, 25, 31

Explanation: 3-years moving average are

$\frac{15+24+18}{3}, \frac{24+18+33}{3}, \frac{18+33+42}{3}$

i.e. $\frac{57}{3}, \frac{75}{3}, \frac{93}{3}$ i.e. 19, 25, 31

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: The given matrices are $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$

Order of A = 2×3; Order of B = 3×2

Since, number of columns in A is equal to the number of rows in B.

$\Rightarrow$ AB is defined.

Also, number of columns in B is equal to the number of rows in A.

$\therefore$ The product BA is also defined.

20.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: We have,

f(x) = $2x^3 - 6x^2 + 6x + 5$

$\Rightarrow$ f'(x) = $6x^2 - 12x + 6 = 6(x - 1)^2$

and f''(x) = 12(x - 1)

Now, f'(x) = 0 gives x = 1.

Also, f''(1) = 0.

Therefore, the second derivative test fails in this case.

So, we shall go back to the first derivative test.

Using first derivatives test, we get x = 1 is neither a point of local maxima nor a point of local minima and so it is a point of inflexion.

Section B

21. To calculate the trend values, we make the following table

Year	**Production (in Tonnes)**	**Three yearly moving totals**	**Three yearly moving averages**
1	4	-	-
2	5	14	4.67
3	5	16	5.33
4	6	18	6
5	7	21	7
6	8	24	8
7	9	25	8.33
8	8	27	9
9	10	-	-

22. P = ₹ 7000, r = 6% p.a. = 1.5% quarterly n = 18 months = 6 quarters

$\therefore$ C.I. = 7000 $\left[\left(1 + \frac{1.5}{100}\right)^6 - 1\right]$

= $7000[(1.015)^6 - 1]$

= $7000(1.093 - 1)$

= 7000×0.093

= ₹ 651

OR

Sinking Fund, A = ₹1,000 $\times$ 1000 = ₹1,000,000, r = 5% or 0.05, No. of years, n = 10 years and No. of payments per year, m = 2 (Half Yearly)

Periodic Contribution, P = $\frac{A\times\left(\frac{r}{m}\right)}{\left[\left(1+\left(\frac{r}{m}\right)\right)^{n\times m}\right]-1}$

P = $\frac{1{,}000{,}000\times\left(\frac{0.05}{2}\right)}{\left[\left(1+\left(\frac{0.05}{2}\right)\right)^{10\times 2}\right]-1}$

= $\frac{1{,}000{,}000\times 0.025}{1.6386-1}$

= $\frac{25{,}000}{0.6386}$

= ₹39,148.136 $\sim$ ₹39,148

Therefore, the company will be required to contribute a sum of ₹39,148 half-yearly in order to build the sinking fund to retire the zero-coupon bonds after 10 years.

23. Put $9x^2 - 1 = t \Rightarrow 18x\,dx = dt \Rightarrow 3x\,dx = \frac{1}{6}dt$.

When x = 1, t = $9.1^2 - 1 = 8$ and when x = 2, t = $9.2^2 - 1 = 35$.

$\therefore \mathrm{I} = \frac{1}{6}\int_8^{35}\frac{1}{t}dt = \frac{1}{6}[\log|t|]_8^{35} = \frac{1}{6}(\log 35 - \log 8) = \frac{1}{6}\log\frac{35}{8}$.

24. Given P = ₹500000, n = 12 $\times$ 5 = 60 months, i = $\frac{8}{1200}$ = 0.0067

$\therefore$ EMI = $\frac{\mathrm{P}\times i(1+i)^n}{(1+i)^n-1} = \frac{500000\times 0.0067\times(1.0067)^{60}}{(1.0067)^{60}-1}$

= $\frac{500000\times 0.0067\times 1.4928}{0.4928}$ = ₹10147.89

OR

Interest for 1 year = ₹(4320 - 4000) = ₹ 320

Let rate of interest be r%

$\therefore \frac{4000\times r\times 1}{100} = 320 \Rightarrow r = 8$

$\therefore$ Rate of interest = 8%

$\therefore$ Amount after 3 years = $4000\left(1+\frac{8}{100}\right)^3 = 4000(1.08)^3 = 4000 \times 1.259$ = ₹ 5036

25. We know that if a $\equiv$ b (mod m) and $0 \leq b \leq m$, then b is the remainder when a is divided by m. Therefore, to find the remainder when 2^{100} is divided by 11, its is sufficient to find an integer b such that $2^{100} \equiv b \pmod{11}$, where $0 \leq b \leq 11$

Now,

$2^1 \equiv 2 \pmod{11}$

$\Rightarrow 2^2 \equiv 2 \times 2 = 4 \pmod{11}$

$\Rightarrow 2^3 \equiv 2 \times 4 = 8 \pmod{11}$

$\Rightarrow 2^4 \equiv 2 \times 8 \equiv 5 \pmod{11}$ [$\because 2^4 \equiv 16 \pmod{11}$ and $16 \equiv 5 \pmod{11}$ $\therefore 2^4 \equiv 5 \pmod{11}$]

$\Rightarrow 2^5 \equiv 2 \times 5 \equiv 10 \pmod{11}$

$\Rightarrow 2^5 \equiv -1 \pmod{11}$ [$\because 10 \equiv -1 \pmod{11}$]

$\Rightarrow (2^5)^{20} \equiv (-1)^{20} \pmod{11}$

$\Rightarrow 2^{100} \equiv 1 \pmod{11}$

Hence, 1 is the remainder when 2^{100} is divided by 11.

Section C

26. Let A_0 be the original amount of radium and A be the amount of radium at any time t. Then, the rate of decomposing of radium is $\frac{dA}{dt}$. It is given that

$\frac{dA}{dt} \propto A$

$\Rightarrow \frac{dA}{dt} = -\lambda A$, where λ is a positive constant

$\Rightarrow \frac{dA}{A} = -\lambda dt$

$\Rightarrow \log A = -\lambda t + C$ ...(i)

At t = 0, we have A = A_0. Putting t = 0 and A = A_0 in (i), we get

$\log A_0 = 0 + C \Rightarrow C = \log A_0$

Putting C = $\log A_0$ in (i), we get

$\log A = -\lambda t + \log A_0$

$\Rightarrow \log\left(\frac{A}{A_0}\right) = -\lambda t$ ...(ii)

It is given that p% of the original amount of radium disintegrates in l years. This means that the amount of radium present att = l is $\left(A_0 - \frac{p}{100} \times A_0\right) = \left(1 - \frac{p}{100}\right) A_0$. Putting A = $A_0 \left(1 - \frac{p}{100}\right)$ and t = l in (ii), we get

$\log\left(1 - \frac{p}{100}\right) = -\lambda l \Rightarrow \lambda = -\frac{1}{l}\log\left(1 - \frac{p}{100}\right)$

Substituting the value of λ in (ii), we get

$\log\left(\frac{A}{A_0}\right) = \frac{t}{l}\log\left(1 - \frac{p}{100}\right)$ ...(iii)

Let A be the amount of radium available after 2l years.

Putting t = 2l in (iii), we get

$\log\left(\frac{A}{A_0}\right) = 2\log\left(1 - \frac{p}{100}\right)$

$\Rightarrow \frac{A}{A_0} = \left(1 - \frac{p}{100}\right)^2$

$\Rightarrow \frac{A}{A_0} \times 100 = \left(1 - \frac{p}{100}\right)^2 \times 100$ [Multiplying both sides by 100]

$\Rightarrow \frac{A}{A_0} \times 100 = \left(10 - \frac{p}{10}\right)^2$

OR

The given differential equation is

$x \log x\frac{dy}{dx} + y = \frac{2}{x} \log x$

$\Rightarrow \frac{dy}{dx} + \frac{1}{x\log x}y = \frac{2}{x^2}$ ...(i)

This is a linear differential equation of the form

$\frac{dy}{dx} + Py = Q$, where $P = \frac{1}{x\log x}$ and $Q = \frac{2}{x^2}$

$\therefore$ I.F. = $e^{\int Pdx} = e^{\int \frac{1}{x\log x}dx} = e^{\int \frac{1}{t}dt}$, where t = log x

$\Rightarrow$ I.F. = $e^{\log t} = t = \log x$

Multiplying both sides of (i) by I.F. = log x, we get

$\log x\frac{dy}{dx} + \frac{1}{x}y = \frac{2}{x^2}\log x$

Integrating both sides with respect to x, we get

$y \log x = \int \frac{2}{x^2} \log x\, dx + C$ [Using: y(I.F.) = $\int$ Q (I.F.) dx + c]

$\Rightarrow y\log x = 2\int \underset{I}{\log x}\ \underset{II}{x^{-2}}dx + C$

$\Rightarrow y\log x = 2\left\{\log x\left(\frac{x^{-1}}{-1}\right) - \int \frac{1}{x}\left(\frac{x^{-1}}{-1}\right)dx\right\} + C$

$\Rightarrow y\log x = 2\left\{-\frac{\log x}{x} + \int x^{-2}dx\right\} + C$

$\Rightarrow y\log x = 2\left\{-\frac{\log x}{x} - \frac{1}{x}\right\} + C$

$\Rightarrow y\log x = -\frac{2}{x}(1 + \log x) + C$, which gives the required solution.

27. Face value C = ₹2,000

Coupon rate i_d = 10% annually or 0.1

Therefore R = C $\times$ i_d = 2,000 $\times$ 0.1 = ₹200

No. of periods before redemption (n) = 5

Yield rate i = 11% or 0.11

Therefore,

$V = R\left|\frac{1-(1+i)^{-n}}{i}\right| + C(1+i)^{-n}$

$= 200\left[\frac{1-(1+0.11)^{-5}}{0.11}\right] + 2000(1+0.11)^{-5}$

$= 200\left[\frac{1-(1.11)^{-5}}{0.11}\right] + 2000(1.11)^{-5}$

$= 200\left|\frac{1-0.593451}{0.11}\right| + 2000(0.593451)$

= 200 (3.6959) + 1186.902 - 739.18 + 1186.902

= 1926.08

Therefore, the value of the bond is ₹1,927.

28. Let C(x) be the total cost of x units of the product and MC be the marginal cost, then

MC = $\frac{x}{\sqrt{x^2+400}}$ (given)

As MC = $\frac{d}{dx}$ (C(x)), so $\frac{d}{dx}$ (C(x)) = $\frac{x}{\sqrt{x^2+400}}$

$\therefore$ C(x) = $\int \frac{x}{\sqrt{x^2+400}} dx$ (put $\sqrt{x^2+400}$ = t i.e. x^2 + 400 = t^2 $\Rightarrow$ 2x dx = 2t dt i.e. x dx = t dt)

$= \int \frac{tdt}{t} = \int$ 1 dt = t + k, k is constant of integration

$\Rightarrow$ C(x) = $\sqrt{x^2+400}$ + k.

Given fixed cost (in ₹) = 1000 i.e. when x = 0, C(x) = 1000

$\Rightarrow$ 1000 = $\sqrt{0^2+400}$ + k $\Rightarrow$ 1000 = 20 + k $\Rightarrow$ k = 980

$\therefore$ C(x) = $\sqrt{x^2+400}$ + 980

Average cost = $\frac{C(x)}{x} = \frac{\sqrt{x^2+400}}{x} + \frac{980}{x}$.

29. It is clear that X can assume values 0, 1, 2, 3 such that,

P(X = 0) = $\frac{{}^7C_4}{{}^{10}C_4} = \frac{1}{6}$, P(X = 1) = $\frac{{}^3C_1 \times {}^7C_3}{{}^{10}C_4} = \frac{1}{2}$

P(X = 2) = $\frac{{}^3C_2 \times {}^7C_2}{{}^{10}C_4} = \frac{3}{10}$, and P(X = 3) = $\frac{{}^3C_3 \times {}^7C_1}{{}^{10}C_4} = \frac{1}{30}$

Therefore, the probability distribution of X is as follows:

X	0	1	2	3
P(X)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

Computation of mean and variance :

x_i	P(X = x_i) = p_i	p_ix_i	$p_ix_i^2$
0	$\frac{1}{6}$	0	0
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
2	$\frac{3}{10}$	$\frac{3}{5}$	$\frac{6}{5}$
3	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{3}{10}$
		$\Sigma p_ix_i = \frac{12}{10}$	$\Sigma p_ix_i^2 = 2$

Thus, we have $\Sigma p_ix_i = \frac{12}{10} = \frac{6}{5}$ and $\Sigma p_ix_i^2 = 2$

$\therefore \overline{X}$ = Mean = $\Sigma p_ix_i = \frac{6}{5}$

and, Var(X) = $\Sigma p_ix_i^2 - (\Sigma p_ix_i)^2$ = $2 - \left(\frac{6}{5}\right)^2$ = $2 - \frac{36}{25}$ = $\frac{14}{25}$

Hence, Mean = $\frac{6}{5}$ and Variance = $\frac{14}{25}$

OR

Let X be a random variable denoting the number of sixes in throwing a die two times. Then, X can take values 0, 1, 2.

Now,

P(X = 0) = P(six does not appear on any of die) = $\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

P(X = 1) = P(six appears atleast once of the die) = $\left(\frac{1}{6} \times \frac{5}{6}\right) + \left(\frac{1}{6} \times \frac{5}{6}\right) = \frac{10}{36} = \frac{5}{18}$

P(X = 2) = P(six does appear on both of die) = $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

Hence, the required probability distribution is,

X	0	1	2
P(X)	$\frac{25}{36}$	$\frac{5}{18}$	$\frac{1}{36}$

Computation of mean and variance

x_i	p_i = P(X = x_i)	p_ix_i	$p_ix_i^2$
0	$\frac{25}{36}$	0	0
1			

	$\frac{10}{36}$	$\frac{10}{36}$	$\frac{10}{36}$
2	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{4}{36}$
		$\Sigma p_i x_i = \frac{12}{36}$	$\Sigma p_i x_i^2 = \frac{14}{36}$

Thus, we have

$\Sigma p_i x_i = \frac{12}{36} = \frac{1}{3}$ and $\Sigma p_i x_i^2 = \frac{7}{18}$

$\therefore$ E(X) = $\Sigma p_i x_i = \frac{1}{3}$ and, Var(X) = $\Sigma p_i x_i^2 - (\Sigma p_i x_i)^2$ = $\frac{7}{18} - \frac{1}{9} = \frac{5}{18}$

Hence, E(X) = $\frac{1}{3}$ and Var(X) = $\frac{5}{18}$

30. Calculation of Trend values by it four yearly Moving Averages:

Year	**Value**	**4-yearly centered Moving Total**	**4-yearly Moving Average (Trend values)**	**4-yearly centered Moving Average**
2012	50.0	-		
2013	36.5	-		
		174.0	43.5	
2014	43.0	-		42.12
		162.9	40.73	
2015	44.5	-		41.03
		165.3	41.33	
2016	38.9	-		40.03
		154.9	38.73	
2017	38.9	-		38.38
		152.1	38.03	
2018	32.6	-		38.31
		154.3	38.58	
2019	41.7	-		37.94
		149.2	37.3	
2020	41.1	-		
2021	33.8	-		

31. Given μ_0 = 18, n = 48, $\bar{x}$ = 17, S = 4.5

i. t = $\frac{\bar{x}-\mu_0}{\frac{S}{\sqrt{n}}} = \frac{17-18}{\frac{4.5}{\sqrt{48}}}$

= $\frac{-1\times\sqrt{48}}{4.5}$ = -1.54

$\therefore$ t = -1.54

and degrees of freedom = 48 - 1 = 47.

ii. $\because$ t = -1.54 < 0

So, p-value of -1.54 = 2 $\times$ Area under the t-distribution curve to the left of t

= 2 $\times$ Area under the t-distribution curve to the right of t

From the t-distribution table, we find that t = 1.54 lies between 1.300 and 1.678 for which area lies between 0.05 and 0.10, so, p-values lies between 2 $\times$ 0.05 and 2 $\times$ 0.10 i.e. between 0.10 and 0.20.

So, 0.10 < p-value < 0.20

iii. $\because$ p-value > 0.05

So, do not reject H_0.

iv. Reject H_0 if $t \le t_{\frac{\alpha}{2}}$ or $t \ge t_{\frac{\alpha}{2}}$.

Here, t = -1.54 and $t_{\frac{\alpha}{2}} = t_{0.025}$

From the table, $t_{0.025}$ = 2.012 with df = 47

∵ -1.54 > -2.012

So, do not reject H_0

Section D

32. Let x, y, z be the prize amount per person for adaptability, carefulness and calmness respectively

Accoring to question,

2x + 4y + 3z = 29000

5x + 2y + 3z = 30500

x + y + z = 9500

These three equations can be written as

$$\begin{bmatrix} 2 & 4 & 3 \\ 5 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 29000 \\ 30500 \\ 9500 \end{bmatrix}$$

A X = B

|A| = 2(2 - 3) - 4(5 - 3) + 3(5 - 2)

= 2(-1) - 4(2) + 3(3)

= - 2 - 8 + 9

= - 1

Hence, the unique solution given by x = $A^{-1}B$

C_{11} = $(-1)^{1+1}$ (2 - 3) = -1

C_{12} = $(-1)^{1+2}$ (5 - 3) = -2

C_{13} = $(-1)^{1+3}$ (5 - 2) = 3

C_{21} = $(-1)^{2+1}$ (4 - 3) = -1

C_{22} = $(-1)^{2+2}$ (2 - 3) = -1

C_{23} = $(-1)^{2+3}$ (2 - 4) = -2

C_{31} = $(-1)^{3+1}$ (12 - 6) = 6

C_{32} = $(-1)^{3+2}$ (6 - 15) = -9

C_{33} = $(-1)^{3+3}$ (4 - 20) = -16

Adj A = $\begin{bmatrix} -1 & -2 & 3 \\ -1 & -1 & 2 \\ 6 & 9 & -16 \end{bmatrix}^T = \begin{bmatrix} -1 & -1 & 6 \\ -2 & -1 & 9 \\ 3 & 2 & -16 \end{bmatrix}$

X = $A^{-1}B = \frac{1}{|A|}$ (Adj A)B

X = $\begin{bmatrix} 1 & 1 & -6 \\ - & 1 & -9 \\ -3 & -2 & 16 \end{bmatrix} \begin{bmatrix} 29000 \\ 30500 \\ 9500 \end{bmatrix}$

X = $\begin{bmatrix} 29000 + 30500 - 57000 \\ 58000 + 30500 - 85500 \\ -87000 - 61000 + 152000 \end{bmatrix}$

$$\begin{bmatrix} \text{x} \\ \text{y} \\ \text{z} \end{bmatrix} = \begin{bmatrix} 2500 \\ 3000 \\ 4000 \end{bmatrix}$$

Hence, x = 2500, y = 3000 and z = 4000

OR

Given, A = $\begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

Let A_{ij} be the co-factor of an element a_{ij} of |A|. Then, co-factors of elements of |A| are

$A_{11} = (-1)^{1+1} \begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} = (1 - 4) = -3$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & -2 \\ 2 & 1 \end{vmatrix} = -(2 + 4) = -6$

$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 1 \\ 2 & -2 \end{vmatrix} = (-4 - 2) = -6$

$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & -2 \\ -2 & 1 \end{vmatrix} = -(-2 - 4) = 6$

$A_{22} = (-1)^{2+2} \begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} = (-1 + 4) = 3$

$A_{23} = (-1)^{2+3} \begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = -(2 + 4) = -6$

$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & -2 \\ 1 & -2 \end{vmatrix} = (4 + 2) = 6$

$A_{32} = (-1)^{3+2} \begin{vmatrix} -1 & -2 \\ 2 & -2 \end{vmatrix} = -(2 + 4) = -6$

$A_{33} = (-1)^{3+3} \begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} = (-1 + 4) = 3$

Clearly, the adjoint of the matrix A is given by

adj A = $\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$

Now, |A| = $\begin{vmatrix} 1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{vmatrix}$

= -1 (1 - 4) + 2 (2 + 4) - 2 (-4 - 2)

= -1(-3) + 2(6) - 2(-6)

= 3+ 12+ 12= 27

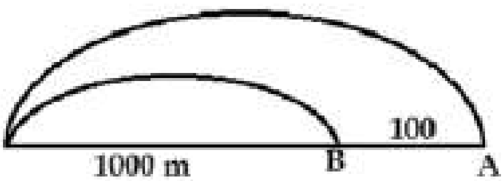
and A.(adj A) = $\begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$

= $\begin{bmatrix} 3+12+12 & -6-6+12 & -6+12-6 \\ -6-6+12 & 12+3+12 & 12-6-6 \\ -6+12-6 & 12-6-6 & 12+12+3 \end{bmatrix}$

= $\begin{bmatrix} 27 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 27 \end{bmatrix} = 27 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

= $27I_3 = |A|I_3$

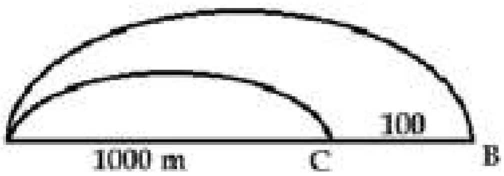
33. Distance covered by A = 1000 m



Distance covered by B = 900 m

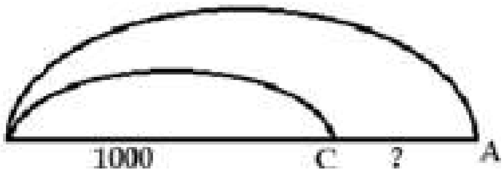
Speed of A: speed of B = 10 : 9

Distance covered by B = 1000



Distance covered by C = 900

Speed of B: Speed of C = 10 : 9



$\therefore$ A : B : C = 100 : 90 : 81

= 1000 : 900 : 81

A : B = 10 : 9

10 : 9.

When A covers 1000 meter C covers 810 metes

$\therefore$ Required distance cover = 1000 - 810

= 190 metre.

34. Total number of marbles in the box = 4 + 5 = 9.

Three marbles are drawn at random from the box.

Let X denote the number of red marbles drawn, then X can take values 0, 1, 2, 3.

P(0) = P(3 black marbles) $= \frac{^5C_3}{^9C_3} = \frac{5.4.3}{1.2.3} \times \frac{1.2.3}{9.8.7} = \frac{5}{42}$,

P(1) = P(1 red marble and 2 black marbles) $= \frac{^4C_1 \times {}^5C_2}{^9C_3} = \frac{4}{1} \times \frac{5.4}{1.2} \times \frac{1.2.3}{9.8.7} = \frac{20}{42}$,

P(2) = P(2 red marbles and 1 black marble) $= \frac{^4C_2 \times {}^5C_1}{^9C_3} = \frac{4.3}{1.2} \times \frac{5}{1} \times \frac{1.2.3}{9.8.7} = \frac{15}{42}$,

P(3) = P(3 red marbles) $= \frac{^4C_3}{^9C_3} = \frac{4\cdot3\cdot2}{1\cdot2\cdot3} \times \frac{1\cdot2\cdot3}{9\cdot8\cdot7} = \frac{2}{42}$.

$\therefore$ Probability distribution of the number of red marbles drawn is $\begin{pmatrix} 0 & 1 & 2 & 3 \\ \frac{5}{42} & \frac{20}{42} & \frac{15}{42} & \frac{2}{42} \end{pmatrix}$.

We construct the following table:

x_i	p_i	p_ix_i	$p_ix_i^2$
0	$\frac{5}{42}$	0	0
1	$\frac{20}{42}$	$\frac{20}{42}$	$\frac{20}{42}$
2	$\frac{15}{42}$	$\frac{30}{42}$	$\frac{60}{42}$
3	$\frac{2}{42}$	$\frac{6}{42}$	$\frac{18}{42}$
Total		$\frac{56}{42}$	$\frac{98}{42}$

Mean $= \Sigma p_ix_i = \frac{56}{42} = \frac{4}{3}$;

Variance $= \frac{98}{42} - \left(\frac{4}{3}\right)^2$

$= \frac{7}{3} - \frac{16}{9} = \frac{5}{9}$

$\therefore$ Standard deviation $= \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$.

OR

Given: first six positive integers.

Two numbers can be selected at random (without replacement) from the first six positive integer in 6 $\times$ 5 = 30 ways.

X denotes the larger of the two numbers obtained. Hence, X can take any value of 2, 3, 4, 5 or 6.

For X = 2, the possible observations are (1, 2) and (2, 1)

$\Rightarrow P(X) = \frac{2}{30} = \frac{1}{15}$

For X = 3, the possible observations are (1, 3), (3, 1), (2, 3) and (3, 2).

$\Rightarrow P(X) = \frac{4}{30} = \frac{2}{15}$

For X = 4, the possible observations are (1, 4), (4, 1), (2, 4), (4, 2), (3, 4) and (4, 3).

$\Rightarrow P(X) = \frac{6}{30} = \frac{1}{5}$

For X = 5, the possible observations are (1, 5), (5, 1), (2, 5), (5, 2), (3, 5), (5, 3), (5, 4) and (4, 5).

$\Rightarrow P(X) = \frac{8}{30} = \frac{4}{15}$

For X = 6, the possible observations are (1, 6), (6, 1), (2, 6), (6, 2), (3, 6), (6, 3) (6, 4), (4, 6), (5, 6) and (6, 5).

$\Rightarrow P(X) = \frac{10}{30} = \frac{1}{3}$

Hence, the required probability distribution is,

X	2	3	4	5	6
P(X)	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{4}{15}$	$\frac{1}{3}$

Therefore E(X) = $2 \times \frac{1}{15} + 3 \times \frac{2}{15} + 4 \times \frac{1}{5} + 5 \times \frac{4}{15} + 6 \times \frac{1}{3}$

$\Rightarrow E(X) = \frac{14}{3}$

35. Each annuity = ₹ 1000,

r = 8% p.a. = 2% per quarter $\Rightarrow$ i = 0.02

n = 4 $\times$ 4 = 16 quarters

$\therefore$ Amount of annuity

= $\frac{1000}{0.02}$ [(1 + 0.02)^{16} -1]

= 50000[(1.02)^{16} - 1]

= 50000(1.372 - 1)

= 50000 $\times$ 0.372 = ₹ 18600

$\therefore$ Amount of annuity = ₹ 18,600

Section E

36. **Read the text carefully and answer the questions:**

There is a bridge whose length of three sides of a trapezium other than base are equal to 5cm:

(i) $\sqrt{25 - x^2}$

(ii) (x + 5)$\sqrt{25 - x^2}$

(iii) 2.5, -5

OR

$-\frac{15}{\sqrt{18.75}}$

37. **Read the text carefully and answer the questions:**

The nominal rate of return is the amount of money generated by an investment before factoring in expenses such as taxes, investment fees, and inflation. If an investment generated a 10% return, the nominal rate would equal 10%. After factoring in inflation during the investment period, the actual return would likely be lower.

However, the nominal rate of return has its merits since it allows investors to compare the performance of an investment irrespective of the different tax rates that might be applied for each investment.

(i) 8%

(ii) $13\frac{1}{3}$%

(iii) 8%

OR

12.5%

38. i. Since profit from the sale of a lamp = ₹25

And profit from the sale of a shade = ₹15

The associative objective function is Max. Z = 25x + 15y

ii.

	Lamp (x)	**Shade (y)**	
Cutting/grinding	2	2	12
Sprayer	3	3	20

So, constraints are:

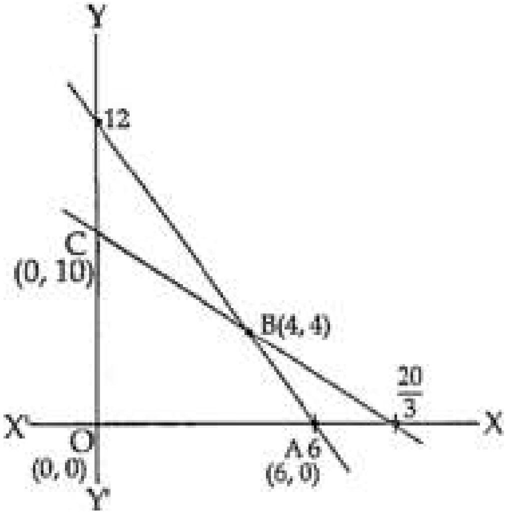
2x + y $\leq$ 12

3x + 2y $\leq$ 20

iii. The non-negative conditions are: $x \geq 0$, $y \geq 0$

OR

Vertices of feasible region are O(0, 0), A(6, 0), B(4, 4), and C (0, 10).



OR

i. From the given graph OA = 75 and OB = 45

The equation of line AB is $\frac{x}{75} + \frac{y}{45} = 1$

i.e., 3x + 5y = 225

ii. From the given graph OC = 40 and OD = 80.

The equation of line CD is $\frac{x}{40} + \frac{y}{80} = 1$

i.e., 2x + y = 80

iii. On solving the equations of lines AB and CD, we get the coordinates of point E i.e., (25, 30).

OR

The objective function for given L.P.P. is Z = 1000x + 500y

From the shaded feasible region, it is clear that coordinates of comer points are (0, 0), (40, 0), (25, 30) and (0, 45)

Corner points	**Value of Z = 1000x + 500y**
(0, 0)	0
(40, 0)	40,000 $\leftarrow$ Maximum
(25, 30)	25,000 + 15,000 = 40,000 $\leftarrow$ Maximum
(0, 45)	22,500

So, the manufacturer should produce 25 bikes of model X and 30 bikes of model Y to get a maximum profit of ₹40,000.