MARKING SCHEME
PRE-BOARD EXAMINATION (2024-25)
CLASS : XII
SUBJECT: MATHEMATICS (041)

Time Allowed : 3 hours **Maximum Marks : 80**

GENERAL INSTRUCTIONS:

1. Evaluation is to be done as per instructions provided in the marking scheme. Marking scheme should be strictly adhered to and religiously followed. However, while evaluating, answer which are based on latest information or knowledge and/or are innovative they may be assessed for their correctness otherwise and marks to be awarded to them.

2. If a student has attempted an extra question, answer of the question deserving more marks should be retained and other answer scored out.

3. A full scale (0-80) has to be used. Please do not hesitate to award full marks if the answer deserve it.

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SECTION-A

1. (c) 0 — 1

2. (b) 8 — 1

3. (d) $\begin{bmatrix} -3 & -6 \\ 0 & -9 \end{bmatrix}$ — 1

4. (a) 1 — 1

5. (c) $P(\bar{A}\,\bar{B}) = [1 - P(A)][1 - P(B)]$ 1

6. (c) 2 1

7. (c) $2x^x$ 1

8. (d) 3^{y-x} 1

9. (c) 5 1

10. (a) 0 1

11. (c) $16\pi\ cm^2/cm$ 1

12. (c) $f(2a - x) = -f(x)$ 1

13. (b) 2 1

14. (a) $\dfrac{\sin x + \sin y}{x}$ 1

15. (c) $\hat{k}$ 1

16. (c) $2b^2$ 1

17. (d) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = 1$ 1

18. (a) $e^x(x^3 + x^2) + c$ 1

19. (a) Both A and R are true and R is the correct explanation of A. 1

20. (d) A is false but R is true 1

SECTION-B

21. (a) $\vec{r} = (\hat{i} - \hat{j} + 3\hat{k}) + \mu(-\hat{i} + 2\hat{j} + 2\hat{k})$

Direction ratio's of line are $<-1,2,2>$ 1

Direction cosine's of the line are $< \frac{-1}{3}, \frac{2}{3}, \frac{2}{3} >$ 1

OR

(b) $\vec{b} + \vec{c} = 0\hat{i} + 3\hat{j} + 3\hat{k}$ ½

$\vec{a} \times (\vec{b} + \vec{c}) = -3\hat{i} - 3\hat{j} + 3\hat{k}$ 1

So, $|\vec{a} \times (\vec{b} + \vec{c})| = 3\sqrt{3}$ ½

22 (a) $\sin^{-1}\left(\sin\frac{5\pi}{6}\right) = \frac{\pi}{6}, \cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \frac{5\pi}{6}, \tan^{-1}\left(\tan\frac{3\pi}{4}\right) = -\frac{\pi}{4}$ 1½

So, $\sin^{-1}\left(\sin\frac{5\pi}{6}\right) + \cos^{-1}\left(\cos\frac{7\pi}{6}\right) + \tan^{-1}\left(\tan\frac{3\pi}{4}\right) = \frac{3\pi}{4}$ ½

OR

(b) f = {(1, 4), (2, 8), (3, 12), (4, 16)} (one-one) 1

Since function is onto, so, B = range of f = {4, 8, 12, 16} 1

23. $y^2 = 2024\,x + 2025$

On differentiating w.r.t. x both the sides,

$$2y\frac{dy}{dx} = 2024 \Rightarrow y\frac{dy}{dx} = 1012$$ 1

again differentiaging w.r.t. x,

$$y\frac{d^2y}{dx^2} + \frac{dy}{dx}.\frac{dy}{dx} = 0$$

$$\Rightarrow y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$ 1

24. $f'(x) = 2(e^{2x} + e^{-2x}) + 1 - \frac{1}{1+x^2} = 2(e^{2x} + e^{-2x}) + \frac{x^2}{1+x^2}$ 1

Since, $\frac{x^2}{1+x^2} \geq 0$ and $2(e^{2x} + e^{-2x}) > 0$, so, $f'(x) > 0$

This f(x) is strictly increasing in its domain. 1

25. $I = \int \frac{\sin(x+a-2a)}{\sin(x+a)}dx = \int \left(\cos 2a - \sin 2a\frac{\cos(x+a)}{\sin(x+a}\right)dx$ 1

$I = x\cos 2a - \sin 2a \log_e \left|\sin(x+a)\right| + c$ 1

Note: If student attempted by substitution method, then they might yet

$I = (x+a)\cos 2a - \sin 2a \log|\sin(x+a)| + c$, which is also correct.

SECTION-C

26. $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \Rightarrow A - \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

$\therefore A = \begin{bmatrix} 3 & 5 \\ -5 & -8 \end{bmatrix}$ 2

Now, $(A^T)^{-1} = \begin{bmatrix} 3 & -5 \\ 5 & -8 \end{bmatrix}^{-1} = \frac{1}{1}\begin{bmatrix} -8 & 5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} -8 & 5 \\ -5 & 3 \end{bmatrix}$ 1

OR

$AB = \begin{bmatrix} 3 & 7 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -2 & 7 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \Rightarrow \boxed{A^{-1} = B}$ 1½

Now $\begin{bmatrix} 3 & 7 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}C = BC$ ½

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 & 7 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 10 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\therefore x = 1, \ y = 1$ 1

27. $I = \int \frac{2}{(1+x)(1+x^2)} dx = \int \frac{(1+x^2)+(1+x)-x^2-x}{(1+x)(1+x^2)} dx = \int \left(\frac{1}{1+x} - \frac{x}{1+x^2} + \frac{1}{1+x^2} \right) dx$ 1½

(or any other partial fraction)

$I = \log|1+x| - \frac{1}{2}\log|1+x^2| + \tan^{-1} x + c$ 1½

OR

$$I = \int_1^4 (x-1)\, dx - \int_1^3 (x-3)\, dx + \int_3^4 (x-3)\, dx$$ 1

$$I = \left[\frac{(x-1)^2}{2}\right]_1^4 - \left[\frac{(x-3)^2}{2}\right]_1^3 + \left[\frac{(x-3)^2}{2}\right]_3^4$$ 1

$$I = \frac{9}{2} + \frac{4}{2} + \frac{1}{2} = 7$$ 1

28. $$\left.\begin{aligned} x = \sin^3 t \Rightarrow \frac{dx}{dt} = 3\sin^2 t \cos t \\ y = \cos^3 t \Rightarrow \frac{dy}{dx} = -3\cos^2 t \sin t \end{aligned}\right\} \frac{dy}{dx} = -\cot t$$ 1½

$$\frac{d^2y}{dx^2} = \operatorname{cosec}^2 t \frac{dt}{dx} = \frac{1}{3}\operatorname{cosec}^4 t \sec t$$

$$\left.\frac{d^2y}{dx^2}\right|_{t=\frac{\pi}{4}} = \frac{1}{3}(\sqrt{2})^4(\sqrt{2}) = \frac{4\sqrt{2}}{3}$$ ½

29. $\dfrac{dy}{dx} = \dfrac{y}{x} + \cos^2\left(\dfrac{y}{2x}\right) = f\left(\dfrac{y}{x}\right)$ (homogenous differential equation)

let $\dfrac{y}{x} = \cup \Rightarrow \dfrac{dy}{dx} = \cup + x\dfrac{dv}{dx}$ 1

Thus, $\cancel{v} + x\dfrac{dv}{dx} = \cancel{v} + \cos^2\left(\dfrac{v}{2}\right)$

$$\Rightarrow \int \sec^2\left(\frac{v}{2}\right) dv = \int \frac{1}{x}\, dx$$

$$\Rightarrow 2\tan\frac{v}{2} = \log|x| + c$$

$\therefore 2\tan\left(\frac{y}{2x}\right) = \log|x| + c$ (required general solution) 1½

Now, when $x = 1, y = \frac{\pi}{2}$, we get $2 = c$

Thus, $2\tan\left(\frac{y}{2x}\right) = \log|x| + 2$, is the required particular solution. ½

OR

$\frac{dy}{dx} + (2\tan x)y = \sin x$ (linear differential equation)

Integrating factor = $e^{\int 2\tan x\, dx} = \sec^2 x$ 1

Thus, solution of given differential equation is

$y.\sec^2 x = \int \frac{\sin x}{\cos x}.\frac{1}{\cos x} dx = \int \tan x \sec x \; dx$

$y.\sec^2 x = \sec x + c$ (required general solution) 1½

Now, when y = 0 , $x = \frac{\pi}{3}$, we get $0 = 2 + c \Rightarrow c = -2$

Thus, $y\sec^2 x = \sec x - 2$, is the required particular solution. ½

30. As, direction ratio's of live are proportional, so lines are parallel.

Say, $\overrightarrow{a_2}-\overrightarrow{a_1}=-2\hat{i}-\hat{j}+\hat{k},\ \vec{b}=2\hat{i}+3\hat{j}+6\hat{k},\ |\vec{b}|=7$ 1

Now, $\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right)\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -2 & -1 & 1\\ 2 & 3 & 6\end{vmatrix}=-9\hat{i}+14\hat{j}-4\hat{k}$ 1½

Distance between the lines = $\dfrac{|\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right)\times\vec{b}|}{|\vec{b}|}=\dfrac{\sqrt{293}}{7}$ units ½

31. As, $\sum_{x_i=0}^{3}p(X=x_i)=1\Rightarrow p+q=0.3$ ½

Now, mean = $\Sigma xp(x)=-8p^2+2p+2.1$ 1

$E(x)=\text{mean}=-8\left(p-\dfrac{1}{8}\right)^2+\dfrac{1}{8}+2.1$

Mean is maximum when $p-\dfrac{1}{8}=0\Rightarrow p=\dfrac{1}{8}$ 1

Thus, $\dfrac{1}{8}+q=\dfrac{3}{10}\Rightarrow q=\dfrac{3}{10}-\dfrac{1}{8}=\dfrac{7}{40}$

$\therefore \boxed{p=\dfrac{1}{8},q=\dfrac{7}{40}}$ ½

SECTION-D

32. (a) Let $y = f(x) = \frac{x-7}{x-5} \Rightarrow x = \frac{5y-7}{y-1}$

Range of f = R – {1}

Since f is onto, so, range of f = codomain of f

$\Rightarrow R - \{1\} = R - \{p\}$

$\Rightarrow \boxed{p = 1}$ 2½

Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in A$

$\Rightarrow 1 - \frac{2}{x_1 - 5} = 1 - \frac{2}{x_2 - 5}$

$\Rightarrow \boxed{x_1 = x_2}$

So, f is one-one function 2½

OR

(b) R = {(x, y) : (xy) is an irrational number}

Reflexive : As. (1, 1) $\notin$R since 1 is not irrational number

Thus, R is not reflexive. 1½

Symmetrix : Let (x, y) $\in$ R $\Rightarrow$ xy is an irrational

$\Rightarrow$ yx is also an irrational

Thus, $(y, x) \in R$

$\Rightarrow$ R is symmetric 1½

Transitive: As, $(\sqrt{2},1) \in R,$ since $\sqrt{2}$ is irrational.

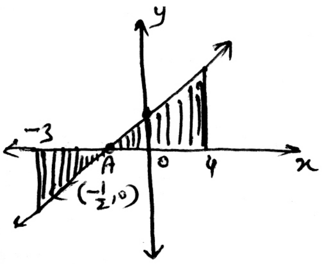
$(1,\sqrt{8}) \in R$, since $\sqrt{8} = 2\sqrt{2}$ is irrational.

but $(\sqrt{2},\sqrt{8}) \notin R,$ since $\sqrt{16} = 4$ is not irrational.

Thus, R is not transitive. 2

(Note: Any correct example for reference and transitive part must be accepted)

33.



1 mark for correct graph

Required area = $\int_{-3}^{4} y\, dx$ ½

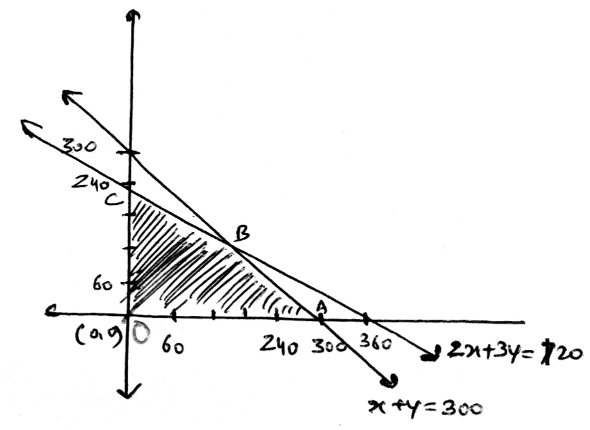
$$= \left| \int_{-3}^{-1/2} y\, dx \right| + \int_{-1/2}^{4} y\ dx$$ 1

$$= \left| \int_{-3}^{-1/2} (2x+1)\, dx \right| + \int_{-1/2}^{4} (2x+1)\ dx$$

$$=\left|\frac{(2x+1)^2}{4}\right|_{-3}^{-1/2}+\left[\frac{(2x+1)^2}{4}\right]_{-1/2}^{4}$$ 1½

$$=\left|\frac{0-25}{4}\right|+\left[\frac{81}{4}\right]=\frac{81+25}{4}=\frac{106}{4}=\frac{53}{2}\text{sq.units}$$ 1

34. OABCO is the correct shaded region, where 0(0,0), A(300,0), B(180,120) and C(0,240)



3 marks for correct graph

Now, $Z_0 = 0 + 0 = 0$,

$Z_A = 300 + 0 = 300$

$Z_B = 180 + 240 = 420$

$Z_C = 0 + 480 = 480$

Maximum value of Z = 480 at x = 0 , y = 240 2

35. (a) dr. of first given line = <3, –16, 7>

dr. of second given line = <3, 8, –5> 1

Thus, dr. of a line ⊥ to both these lines are <24, 35, 72>

$$\text{As, } \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix} = 24\hat{i} + 36\hat{j} + 72\hat{k}$$ 2

Thus equation of required line is

$$\frac{x-1}{24} = \frac{y-2}{36} = \frac{z+4}{72} \text{ or } \frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$$ 1

and $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ 1

OR

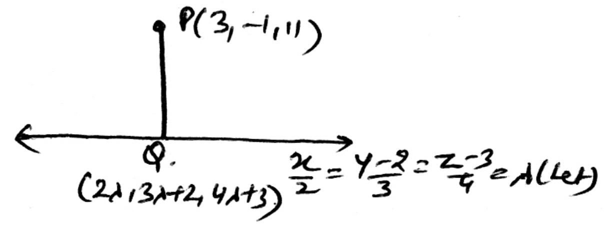
(b) dr. of PQ = <2λ –3, 3λ + 3, 4λ – 8) 1

dr. of line = <2, 3, 4>

So, $2(2\lambda - 3) + 3(3\lambda + 3) + 4(4\lambda - 8) = 0 \Rightarrow \boxed{\lambda = 1}$ 2½

Thus foot of the drawn from p to the line = (2, 5, 7) 1

Equation of PQ is $\frac{x-3}{1} = \frac{y+1}{-6} = \frac{z-11}{4}$ 1



½ mark for figure

36. (i) $5x + 7y = 310$

$7x + 5y = 290$

(ii) $\underset{A}{\begin{bmatrix} 5 & 7 \\ 7 & 5 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{B}{\begin{bmatrix} 310 \\ 290 \end{bmatrix}}$ 1

(iii) (a) $A\times = B \Rightarrow X = A^{-1}B = \frac{1}{-24}\begin{bmatrix} 5 & -7 \\ -7 & 5 \end{bmatrix}\begin{bmatrix} 310 \\ 290 \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \end{bmatrix}$

$\therefore x = 20,\ y = 30$ 2

OR

(b) $|A| = -24,\ |\text{adj}A| = |A|,\ |a^{-1}| = \frac{1}{|A|}, |A^T| = A|A$ 1

Thus, $\frac{|2A^T| + |\text{adj } A|}{|A^{-1}|} = \frac{4|A| + |A|}{\frac{1}{|A|}} = 5|A|^2 = 5 \times 576 = 2880$ 1

37. (i) Volume = $x^2 y = 36 \Rightarrow y = \frac{36}{x^2}$

Outer surface area = $xy + 2x^2 + 2xy = 3xy + 2x^2 = 3x\left(\frac{36}{x^2}\right) + 2x^2$

$A(x) = \frac{108}{x} + 2x^2$ 1

(ii) $\frac{dA}{dx} = \frac{-108}{x^2} + 4x$ 1

(iii) (a) For minimum surface area, $4x = \frac{108}{x^2} \Rightarrow x = 3$

$\frac{d^2A}{dx^2} = \frac{216}{x^3} + 4 > 0 \ (\text{at } x = 3)$

Thus height = 3m when outside surface area is minimum 2

OR

(b) $\frac{dA}{dx} = 0 \Rightarrow x = 3, \frac{d^2A}{dx^2} > 0$ at $x = 3$, so, outside surface area is minimum when x = 3, thus $y = \frac{36}{9} = 4m$ 2

38. (i) $P(\text{sleeps well}) = P(W \cap S_w) + P(D \cap S_w) + P(L \cap S_w)$

$= 0.5 \times 0.7 + 0.2 \times 0.8 + 0.3 \times 0.3$

$= 0.6$ 2

(ii) $P\left(\frac{\text{did not end in a draw}}{\text{sleep well}}\right) = \frac{0.5 \times 0.7 + 0.3 \times 0.3}{0.6} = \frac{0.44}{0.60} = \frac{11}{15}$ 2