

Time 3 hrs.

Class - 12 -C

Max Marks 80

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GENERAL INSTRUCTIONS :-

1. This question paper contains five sections :- A, B, C, D and E. Each section is compulsory.
2. In Section A – Question numbers 1 to 18 are Multiple Choice Questions (MCQs) type and Question numbers 19 and 20 are Assertion-Reason based questions of 1 mark each.
3. In Section B - Question numbers 21 to 25 are very short answer type questions of 2 marks each.
4. In Section C - Question numbers 26 to 31 are short answer type questions of 3 marks each.
5. In Section D - Question numbers 32 to 35 are long answer type questions of 5 marks each.
6. In Section E & Question numbers 36 to 38 are case study questions of 4 marks each.

[SECTION - A]

Q-1 $\int x \log x \, dx = ?$

(a) $\frac{x^2}{2} \log x + \frac{x}{4} \log x - \frac{x^2}{4} + C$

(b) $\frac{x^2}{2} \log x - \frac{x}{4} \log x - \frac{x^2}{4} + C$

(c) $\frac{x^2}{2} \log x - \frac{x}{4} - C$

(d) $\frac{x^2}{2} \log x - \frac{x^2}{4} + C$

Q-2 I.F. of the differential equation $\frac{dy}{dx} + y \tan x - \sec x = 0$ is :

(a) $\cos x$

(b) $\sec x$

(c) $e^{\cos x}$

(d) $e^{\sec x}$

Q-3 If area of a triangle with vertices (3, 2), (-1, 4) and (6, k) is 7 sq units, then possible values of k are :

(a) 3, -4

(b) 3, 4

(c) -3, 4

(d) -3, -4

[P. T. O.]

Q-4 Solution of the differential equation $\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$ is:

- (a) $y = \sin^{-1} y = \sin^{-1} x + C$ (b) $\sin^{-1} y - \sin^{-1} x = C$
 (c) $\sin^{-1} y + \sin^{-1} x = C$ (d) None of these

Q-5 The projection of the vector $\vec{a} = 3\hat{i} - \hat{j} - 2\hat{k}$ on $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ is:

- (a) $\sqrt{14}/2$ (b) $14/\sqrt{2}$
 (c) $\sqrt{14}$ (d) 7

Q-6 Corner points of the feasible region for an LPP are $(0, 2)$, $(3, 0)$, $(6, 0)$, $(6, 8)$ and $(0, 5)$. Let $F = 4x + 6y$ be the objective function then Maximum of F - Minimum of $F =$

- (a) 60 (b) 48
 (c) 42 (d) 18

Q-7 Corner points are $(0, 0)$, $(0, 8)$, $(2, 7)$, $(5, 4)$ and $(6, 0)$ maximum profit $P = 3x + 2y$ occurs at point:

- (a) $(2, 7)$ (b) $(5, 4)$
 (c) $(6, 0)$ (d) $(0, 8)$

Q-8 The angle between the lines $\frac{x+4}{1} = \frac{2-y}{-2} = \frac{z+2}{3}$ and $\frac{x}{-2} = \frac{y-1}{3} = \frac{z}{-1}$ is:

- (a) $\cos^{-1} \frac{1}{14}$ (b) $\cos^{-1} \frac{3}{14}$
 (c) $\cos^{-1} \frac{1}{17}$ (d) none of these

Q-9 If $y = e^{x^2 + x + 1}$ then $\frac{dy}{dx}$ is:

- (a) $\frac{y^2}{1-y}$ (b) $\frac{y^2}{y-1}$
 (c) $\frac{y}{1-y}$ (d) $\frac{-y}{1-y}$

Q-10 An urn contains 10 black & 5 white balls. Two balls are drawn from urn one after the other without replacement, then probability that both drawn balls are black is:

- (a) $2/7$ (b) $1/7$
 (c) $5/7$ (d) $3/7$

Q-11 If $y = \log(x + \sqrt{x^2 + a^2})$ then $\frac{dy}{dx}$ is:

- (a) $\frac{1}{2(x + \sqrt{x^2 + a^2})}$ (b) $\frac{-1}{\sqrt{x^2 + a^2}}$
 (c) $\frac{1}{\sqrt{x^2 + a^2}}$ (d) none of these

Q-12 The radius of a circle is increasing at the uniform rate of 3 cm/s. At the instant when the radius of the circle is 2 cm, its area increases at the rate of _____:

- (a) 12π sq. cm/s (b) 10π sq. cm/s
 (c) 11π sq. cm/s (d) -12π sq. cm/s

Q-13 If $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{j} + p\hat{j} + q\hat{k}) = 0$ then p and q are:

- (a) $p = 6, q = 27$ (b) $p = 3, q = 27/2$
 (c) $p = 6, q = 27/2$ (d) $p = 3, q = 27$

Q-14 Value of $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 15^\circ & \cos 15^\circ \end{vmatrix}$ is:

- (a) 1 (b) $\frac{1}{2}$
 (c) $\frac{\sqrt{3}}{2}$ (d) none of these

Q-15 If a and b be two vectors such that $|\vec{a}| = |\vec{b}| = \sqrt{2}$ and $\vec{a} \cdot \vec{b} = -1$ then the angle between \vec{a} and \vec{b} is:

- (a) $\pi/3$ (b) $\pi/4$
 (c) $2\pi/3$ (d) None of these

Q-16 The area of a triangle formed by vertices O, A and B where

$$\vec{AO} = \vec{i} + 2\vec{j} + 3\vec{k} \text{ and } \vec{OB} = -3\vec{i} - 2\vec{j} + \vec{k} \text{ is:}$$

- (a) $3\sqrt{5}$ sq. units (b) $5\sqrt{5}$ sq. units
(c) $6\sqrt{5}$ sq. unit (d) 4 sq. units

Q-17 If $A = \{a\}_{n=1}^{\infty}$ and $B = \{b\}_{n=1}^{\infty}$ then $AB = BA$ then:

- (a) $n=p$ (b) $n=p, m=q$
(c) $m=n, p=q$ (d) $m=q$

Q-18 Let $X = \{-1, 0, 1\}$ $Y = \{0, 2\}$ and a function $f: X \rightarrow Y$ defined by $y = 2x^2$ is:

- (a) one-one onto (b) one-one into
(c) many-one onto (d) many-one into

In question numbers 19 and 20, two statements are given, one Assertion (A) and other Reason (R). Select the correct answer from the following options:

- (a) Both Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of the Assertion (A).
(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

Q-19 Assertion (A): $\vec{a} = \vec{i} + \vec{j} + 2\vec{k}$ is perpendicular to $\vec{b} = -\vec{i} + \vec{j}$.

Reason (R): Two vectors \vec{a} and \vec{b} are perpendicular to each other if $\vec{a} \cdot \vec{b} = 0$

Q-20 Assertion (A): $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) = \frac{2\pi}{3}$

Reason (R): $\sin^{-1}(\sin(\theta)) = \theta$ if $\theta \in [-\pi/2, \pi/2]$

[SECTION - B]

Q-21 If $y = x^{m+n}$ then find $\frac{dy}{dx}$.

Q-22 Evaluate: $\int \frac{2x}{\sqrt{1-x^2-x^4}} dx$.

OR

Evaluate: $\int \sin 4x \sin 8x dx$

Q-23 If E and F are independent events then show that \bar{E} and \bar{F} are independent events.

Q-24 Write $y = \tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$, $x \neq 0$ in the simplest form.

Q-25 If $y = (\sin^{-1} x)^2$ then prove that $(1-x^2)y_1 - xy_2 = 2$

[SECTION - C]

Q-26 Show that the differential equation:

$$\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0 \text{ is homogeneous and solve it.}$$

Q-27 Find the image of the point (1, 6, 3) in the line:

$$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$

Q28 Find Maximum $z = 5x + 2y$ subject to the following constraints:

$$x - 2y \leq 2, \quad 3x + 2y \leq 12, \quad -3x + 2y \leq 3, \quad x \geq 0, \quad y \geq 0$$

Q29 If $f(x) = \begin{cases} \frac{x-4}{|x-4|} + a & \text{if } x < 4 \\ a+b & \text{if } x = 4 \\ \frac{x-4}{|x-4|} + b & \text{if } x > 4 \end{cases}$ is continuous at $x = 4$ find a and b

Q-30. Evaluate : $\int \frac{x \sin x}{1 + \cos^2 x} dx$

OR

Evaluate : $\int_0^{\pi} \frac{1}{1 + \sqrt{\tan x}} dx$

Q-31. Find the intervals in which the function f given by :

$$f(x) = \sin x + \cos x, \quad 0 \leq x \leq 2\pi$$

- (a) strictly increasing (b) strictly decreasing

[SECTION - D]

Q-32. Two schools A and B decided to award prizes to their students for three games hockey (x), cricket (y) and tennis (z). School A decided to award a total of Rs. 11000 for the three games to 5, 4 and 3 students respectively, while school B decided to award Rs. 10700 for the three games to 4, 3 and 5 students respectively. Also, the three prizes together amount to Rs. 2700. Using matrix method, Find out the prize amount for hockey, cricket & tennis.

Q-33. A line passes through $(2, -1, 3)$ and is perpendicular to the lines :

$$\frac{x-1}{2} = \frac{y-1}{-2} = \frac{z+1}{1} \quad \text{and} \quad \frac{x-2}{1} = \frac{y+1}{2} = \frac{z+3}{2}$$

Obtain its equation in vector and Cartesian form.

Q-34. Show that function f in $A = R - \{2/3\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto.

OR

Show that the relation R in the set $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$ given by $R = \{a, b\} : |a-b|$ is a multiple of 4 is an equivalence relation. Find the set of all elements related to 1.

Q-35. Using integration, find the area of the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$ included between the lines $x = -2$ and $x = 2$

[P. T. O.]

[SECTION - E]

Q-36. Case Study - 1

$$\int e^x [f(x) + f'(x)] dx = e^x f(x) + C$$

answer the following questions.

2+2

(i) Evaluate : $\int e^x \left(\frac{1 - \sin x}{1 - \cos x} \right) dx$

(ii) Evaluate : $\int e^x \left(\frac{x-3}{(x-1)^2} \right) dx$

Q-37. Case Study - 2

3+1

Shyam is making a project of Maths. He has a piece of wire of length 28 m. Wire is to be cut into two pieces. One of the pieces is to be made into a circle and other into a square.

- (i) What should be the length of the two pieces so that the combined area of circle and square is minimum ?
(ii) Find the side of this square.

Q-38. Case Study - 3

A doctor is to visit a patient. From past experience, it is known that the probabilities that he will come by train, bus, scooter or car are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$, if he comes by train, bus and scooter respectively, but if he comes by car, he will not be late :

2+2

- (i) When doctor comes and he is late, what is the probability that he has come by train?
(ii) When doctor comes and he is late, what is the probability that he has come by scooter?