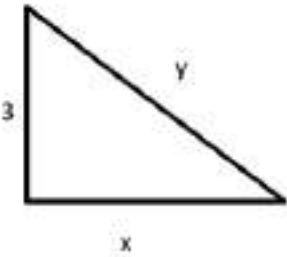


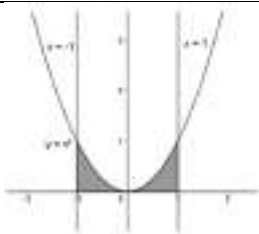
केन्द्रीय विद्यालय संगठन, बेंगलूरु संभाग
KENDRIYA VIDYALAYA SANGATHAN , BENGALURU REGION
प्रथम प्री-बोर्ड परीक्षा (2024-25)
FIRST PRE BOARD EXAMINATION (2024-25)
CLASS:XII
SUBJECT : MATHEMATICS
MARKING SCHEME

MAX MARKS:80
TIME : 3 HRS

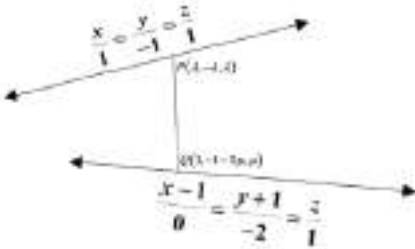
SECTION-A			
Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each .			
1. c) ± 12	6.c) 5	11.d) I quadrant	16. d) at every point of the line-segment joining the points (0.6, 1.6) and (3, 0)
2. a) 4	7. a) $C(A + B')$	12.d) $-\cot x - \tan x + c$	17. c) 1.5
3. a) $(-\infty, -4) \cup (0, \infty)$	8.b) 0.25	13.b) 0	18. c) 2
4.d) 8	9.c) $-\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}$	14.b) not defined , 2	19. C
5.b) $\frac{1}{x}$	10.b) $\frac{7}{3}$	15. b) $\frac{\pi}{2} < y < \pi$	20 . A

Q No	Expected Answers/Value Points	Marks
21	$-1 \leq 3x - 2 \leq 1$ $\Rightarrow \frac{1}{3} \leq x \leq 1$ or $\left[\frac{1}{3}, 1\right]$	1 1
22	$f'(x) = \frac{1 - \log x}{x^2}$, $\therefore f'(x) = 0 \Rightarrow \log x = 1 \Rightarrow x = e$ $f''(x) = \frac{2x \log x - 3x}{x^4}$, $f''(e) = -\frac{1}{e^3} < 0$, $x = e$ is a point of local maximum	1 $\frac{1}{2}$ $\frac{1}{2}$
23	a) Let $x = \sin A$ and $y = \sin B$ $y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1 \Rightarrow \sin A \cos B + \cos A \sin B = 1$ $\Rightarrow \sin(A+B) = 1$ $\Rightarrow A+B = \frac{\pi}{2}$ $\Rightarrow \sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$ Differentiating w.r.t x , we obtain $\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$ b) Let $y = \tan^{-1} x$ and $z = \log x$ $\frac{dy}{dx} = \frac{1}{1+x^2}$ and $\frac{dz}{dx} = \frac{1}{x}$ $\frac{dy}{dz} = \frac{\frac{1}{1+x^2}}{\frac{1}{x}} = \frac{x}{1+x^2}$	$\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2} + \frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$
24	a) $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12 \Rightarrow \vec{x} ^2 - \vec{a} ^2 = 12$ $\Rightarrow \vec{x} ^2 - 1 = 12$ $\Rightarrow \vec{x} ^2 = 13$ $\Rightarrow x = \sqrt{13}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

	OR b) $\vec{a} + \vec{b} = 6\hat{i} - 2\hat{j} + (7 + \lambda)\hat{k}$ and $\vec{a} - \vec{b} = -4\hat{i} + (7 - \lambda)\hat{k}$ $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal if $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$ $\Rightarrow -24 + (7 + \lambda)(7 - \lambda) = 0$ $\lambda^2 = 25 \Rightarrow \lambda = \pm 5$	$\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$
25	$\vec{d}_1 = \vec{a} + \vec{b} = 4\hat{i} - 2\hat{j} - 2\hat{k}$, $\vec{d}_2 = \vec{a} - \vec{b} = -6\hat{j} - 8\hat{k}$ Area of the parallelogram $= \frac{1}{2} \vec{d}_1 \times \vec{d}_2 = \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & -2 \\ 0 & -6 & -8 \end{vmatrix} =$ $\frac{1}{2} 4\hat{i} + 32\hat{j} - 24\hat{k} = 2\hat{i} + 16\hat{j} - 12\hat{k} = \sqrt{404} = 2\sqrt{101}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$
26	$f(x) = \sin 3x \Rightarrow f'(x) = 3\cos 3x = 0 \Rightarrow x = \frac{\pi}{6}, \frac{\pi}{2}$ $f'(x) \geq 0$ for all $x \in \left[0, \frac{\pi}{6}\right] \Rightarrow f(x)$ is increasing on $\left[0, \frac{\pi}{6}\right]$ $f'(x) \leq 0$ for all $x \in \left[\frac{\pi}{6}, \frac{\pi}{2}\right] \Rightarrow f(x)$ is decreasing on $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$	1 1 1
27	 $x^2 + 3^2 = y^2$ When $y = 5$ then $x = 4$, now $2x \frac{dx}{dt} = 2y \frac{dy}{dt}$ $4(200) = 5 \frac{dy}{dt} \Rightarrow \frac{dy}{dt} = 160 \text{ cm/s}$	$\frac{1}{2}$ $\frac{1}{2}$ 1 1
28	a) $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d} \Rightarrow \vec{a} \times \vec{b} - \vec{a} \times \vec{c} = \vec{c} \times \vec{d} - \vec{b} \times \vec{d}$ $\Rightarrow \vec{a} \times \vec{b} - \vec{a} \times \vec{c} + \vec{b} \times \vec{d} - \vec{c} \times \vec{d} = 0$ $\Rightarrow \vec{a} \times (\vec{b} - \vec{c}) + (\vec{b} - \vec{c}) \times \vec{d} = 0$ $\Rightarrow \vec{a} \times (\vec{b} - \vec{c}) - \vec{d} \times (\vec{b} - \vec{c}) = 0$ $\Rightarrow (\vec{a} - \vec{d}) \times (\vec{b} - \vec{c}) = 0$ $\Rightarrow (\vec{a} - \vec{d})$ is parallel to $(\vec{b} - \vec{c})$ OR b) Any point on the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-a}{4}$ is of the form $(2\lambda + 1, 3\lambda + 2, 4\lambda + a)$ and any point on the line $\frac{x-4}{5} = \frac{y-1}{2} = z$ is of the form $(5\mu + 4, 2\mu + 1, \mu)$. Solving $2\lambda + 1 = 5\mu + 4$ and $3\lambda + 2 = 2\mu + 1$ to get $\lambda = -1$ and $\mu = -1$. The lines will be skew if, $4\lambda + a \neq \mu$ $4(-1) + a \neq (-1)$ $a \neq 3$, $\therefore a \in \mathbb{R} - \{3\}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2} + \frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
29	a) Let $x^{\frac{3}{2}} = t \Rightarrow \frac{3}{2} x^{\frac{1}{2}} dx = dt$	1

	$\int \sqrt{\frac{x}{1-x^3}} dx = \frac{2}{3} \int \frac{dt}{\sqrt{1-t^2}} dx$ $= \frac{2}{3} \sin^{-1} t + c = \frac{2}{3} \sin^{-1} \left(x^{\frac{3}{2}} \right) + c$ OR b) Using $\int_0^a f(x) dx = \int_0^a f(a-x) dx$ $\int_0^1 x(1-x)^n dx = \int_0^1 (1-x)x^n dx = \int_0^1 (x^n - x^{n+1}) dx$ $= \left. \frac{x^{n+1}}{n+1} \right _0^1 - \left. \frac{x^{n+2}}{n+2} \right _0^1$ $= \frac{1}{n+1} - \frac{1}{n+2}$	1 $\frac{1}{2} + \frac{1}{2}$ $1 + \frac{1}{2}$ $\frac{1}{2}$ 1												
30	Value of z at (15, 15) = 15p + q Value of z at (0, 20) = q 15p + q = q or p = 0	1 1 1												
31	a) $P(H) = \frac{3}{4}$ $P(T) = \frac{1}{4}$ <table border="1"><tr><td>X</td><td>0</td><td>1</td><td>2</td></tr><tr><td>P(X)</td><td>$\frac{9}{16}$</td><td>$\frac{6}{16}$</td><td>$\frac{1}{16}$</td></tr><tr><td>X P(X)</td><td>0</td><td>$\frac{6}{16}$</td><td>$\frac{2}{16}$</td></tr></table> Mean = $\frac{8}{16}$ or $\frac{1}{2}$ b) $P(A' \cup B') = \frac{1}{4} \Rightarrow 1 - P(A \cap B) = \frac{1}{4}$ $\Rightarrow P(A \cap B) = \frac{3}{4}$ Since $P(A \cap B) \neq 0$, A and B are not mutually exclusive. $P(A) \cdot P(B) = \frac{1}{2} \times \frac{7}{12} = \frac{7}{24} \neq P(A \cap B)$ A and B are not independent	X	0	1	2	P(X)	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$	X P(X)	0	$\frac{6}{16}$	$\frac{2}{16}$	$\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$ 1 1 1
X	0	1	2											
P(X)	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$											
X P(X)	0	$\frac{6}{16}$	$\frac{2}{16}$											
32	 Required area = $2 \int_0^1 x^2 dx$ $= 2 \left. \frac{x^3}{3} \right _0^1$ $= \frac{2}{3}$	1 1 2 1												
33	Assumes the number of litres of orange juice, beetroot juice and kiwi juice as x, y and z, respectively to frame equations as follows: $500x + 20y + 800z = 1860$ $2x + 5y + 3z = 22$ $100x + 120y + 200z = 760$ Writes the above system of equations in the matrix form using $AX = B$ as	1												

	$\begin{bmatrix} 500 & 20 & 800 \\ 2 & 5 & 3 \\ 100 & 120 & 200 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1860 \\ 22 \\ 760 \end{bmatrix}$ Finds $A = 110000 \neq 0$ and hence writes that A is non-singular and has a unique solution. Finds adj A as: $\begin{bmatrix} 640 & 92000 & -3940 \\ -100 & 20000 & 100 \\ -260 & -58000 & 2460 \end{bmatrix}$ Finds A^{-1} using A and adj A as: $A^{-1} = \frac{1}{110000} \begin{bmatrix} 640 & 92000 & -3940 \\ -100 & 20000 & 100 \\ -260 & -58000 & 2460 \end{bmatrix}$ Writes that $X = A^{-1}B$ and finds X as $\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ Concludes that 2 litres of orange juice, 3 litres of beetroot juice and 1 litre of kiwi juice should go into the mixture.	$\frac{1}{2}$ 2 $\frac{1}{2}$ 1
34	a) $(ax + b)e^{\frac{y}{x}} = x \Rightarrow \frac{y}{x} = \log\left(\frac{x}{a+bx}\right) = \log x - \log(a + bx)$ differentiating w.r.t. x, $\Rightarrow \frac{x \frac{dy}{dx} - y}{x^2} = \frac{1}{x} - \frac{b}{a+bx} = \frac{a}{x(a+bx)}$ $\Rightarrow \frac{x \frac{dy}{dx} - y}{x} = \frac{a}{a+bx} \quad \text{---(1)}$ $\Rightarrow x \frac{dy}{dx} - y = \frac{ax}{a+bx}$ Again differentiating w.r.t x, $\Rightarrow x \frac{d^2y}{dx^2} + \frac{dy}{dx} - \frac{dy}{dx} = \frac{(a+bx)a - ax.b}{(a+bx)^2} = \frac{a^2}{(a+bx)^2}$ $\Rightarrow x \frac{d^2y}{dx^2} = \frac{a^2}{(a+bx)^2} = \left[\frac{a}{a+bx}\right]^2 = \left[\frac{x \frac{dy}{dx} - y}{x}\right]^2 \text{ from (1)}$ $\Rightarrow x^3 \frac{d^2y}{dx^2} = \left(x \frac{dy}{dx} - y\right)^2.$ b) $(x - a)^2 + (y - b)^2 = c^2$ Differentiating both sides w.r.t x, $2(x - a) + 2(y - b) \frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = -\frac{x - a}{y - b}$ Again differentiating w.r.t. x $\frac{d^2y}{dx^2} = -\left[\frac{(y - b) - (x - a) \frac{dy}{dx}}{(y - b)^2}\right]$ $= -\left[\frac{(y - b) - (x - a)\left\{-\frac{x - a}{y - b}\right\}}{(y - b)^2}\right] = -\frac{c^2}{(y - b)^3}$ $1 + \left(\frac{dy}{dx}\right)^2 = 1 + \left[-\frac{x - a}{y - b}\right]^2 = \frac{c^2}{(y - b)^2}$	$\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$ 1 1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2} + \frac{1}{2}$ $\frac{1}{2}$

	$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{c^3}{(y-b)^3}$ $\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{\frac{c^3}{(y-b)^3}}{-\frac{c^2}{(y-b)^3}} = -c, \text{ which is independent of } a \text{ and } b$	1/2 1
35	a) Let P(1,6,3) be the given point, and let ' ' L be the foot of the perpendicular from ' ' P to the given line AB (as shown in the figure below). The coordinates of a general point on the given line are given by  Coordinates of L is $(\lambda, 1 + 2\lambda, 2 + 3\lambda)$ So the direction ratios of PL are $\lambda - 1, 2\lambda - 5, 3\lambda - 1$ Since $PL \perp AB$, $1(\lambda - 1) + 2(2\lambda - 5) + 3(3\lambda - 1) = 0$ $\Rightarrow \lambda = 1$ $\Rightarrow$ Coordinates of L are (1,3,5) Let Q (α, β, γ) be the image, then L is the midpoint of PQ $\left(\frac{\alpha+1}{2}, \frac{\beta+6}{2}, \frac{\gamma+3}{2}\right) = (1,3,5) \Rightarrow \alpha = 1, \beta = 0, \gamma = 7$ $\therefore$ Image of P(1,6,3) is (1,0,7) The distance of (1,0,7) from y axis is $\sqrt{1^2 + 7^2} = \sqrt{50} = 5\sqrt{2}$ units OR b)  Let the coordinates of P be $(\lambda, -\lambda, \lambda)$ and Q be $(1, -2\mu - 1, \mu)$ PQ is perpendicular to both the lines, direction ratio of PQ are $\lambda - 1, -\lambda + 2\mu + 1, \lambda - \mu$, Since PQ is perpendicular to both the lines $1(\lambda - 1) + (-1)(-\lambda + 2\mu + 1) + 1(\lambda - \mu) = 0 \Rightarrow 3\lambda - 3\mu = 2 \quad \text{--(1)}$ $0(\lambda - 1) + (-2)(-\lambda + 2\mu + 1) + 1(\lambda - \mu) = 0 \Rightarrow 3\lambda - 5\mu = 2 \quad \text{--(2)}$ Solving (1) and (2) we get $\lambda = \frac{2}{3}$ and $\mu = 0$ $\therefore P\left(\frac{2}{3}, -\frac{2}{3}, \frac{2}{3}\right)$ and $Q(1, -1, 0)$	1/2 1/2 1/2 1/2 1 1

	So the required shortest distance is $\sqrt{\left(1 - \frac{2}{3}\right)^2 + \left(-1 + \frac{2}{3}\right)^2 + \left(-\frac{2}{3}\right)^2}$ $= \sqrt{\frac{2}{3}} \text{ units}$	1
36	(i) Let $(l_1, l_2) \in R \Rightarrow l_1$ is parallel to $l_2 \Rightarrow l_2$ is parallel to l_1 $\Rightarrow (l_2, l_1) \in R$, $\therefore R$ is symmetric. (ii) Let $(l_1, l_2), (l_2, l_3) \in R \Rightarrow l_1$ is parallel to l_2 and l_2 is parallel to l_3 $\Rightarrow l_1$ is parallel to l_3 $\Rightarrow (l_1, l_3) \in R$, $\therefore R$ is transitive. (iii) a) The set of rail lines in R related to the line $y = 3x + 2$ is the set of all lines parallel to it, $\{l: l \text{ is a line of type } y = 3x + c, c \in R\}$ OR (iii) b) Let $(l_1, l_2) \in S \Rightarrow l_1 \perp l_2 \Rightarrow l_2 \perp l_1 \Rightarrow (l_2, l_1) \in S$ $\therefore R$ is symmetric. Let $(l_1, l_2), (l_2, l_3) \in R \Rightarrow l_1 \perp l_2$ and $l_2 \perp l_3$ $\Rightarrow l_1$ is parallel to l_3 $\therefore R$ is not transitive.	1 1 2 1 1
37	(i) When $V = 40 \text{ km/h}$, $F = \frac{1600}{500} - \frac{40}{4} + 14 = \frac{36}{5} \text{ l/100km}$ (ii) $\frac{dF}{dV} = \frac{2V}{500} - \frac{1}{4} = \frac{V}{250} - \frac{1}{4}$ (iii) a) For minimum $\frac{dF}{dV} = 0 \Rightarrow \frac{V}{250} - \frac{1}{4} = 0 \Rightarrow V = 62.5 \text{ km/h}$ Also $\frac{d^2F}{dV^2} = \frac{1}{250} > 0$ Hence F is minimum when $V = 62.5 \text{ km/h}$ OR (iii) b) $\frac{dF}{dV} = -0.01 \Rightarrow \frac{V}{250} - \frac{1}{4} = -0.01 \Rightarrow V = 60 \text{ km/h}$ $F = \frac{3600}{500} - \frac{60}{4} + 14 = 6.2 \text{ l/100km}$ Quantity of fuel required for 600 km = $6.2 * 6 = 37.2 \text{ l}$	1 1 1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$
38	(i) Probability of a randomly chosen seed to germinate $= \frac{4}{10} \times \frac{45}{100} + \frac{4}{10} \times \frac{60}{100} + \frac{2}{10} \times \frac{35}{100} = \frac{490}{1000} = 0.49$ (ii) Probability that the randomly selected seed is of type of A_1, given that it germinates = $\frac{\frac{4}{10} \times \frac{45}{100}}{\frac{490}{1000}} = \frac{18}{49}$	$1 \frac{1}{2} + \frac{1}{2}$ 1 + 1