केन्द्रीय विद्यालय संगठन, बेंगलूरु संभाग

KENDRIYA VIDYALAYA SANGATHAN , BENGALURU REGION

प्रथम प्री-बोर्ड परीक्षा (2024-25)

FIRST PRE BOARD EXAMINATION (2024-25)

CLASS:XII MAX MARKS:80

SUBJECT : MATHEMATICS TIME : 3 HRS

General Instructions :

Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains 38 questions. All questions are compulsory.

(ii) This Question paper is divided into five Sections - A, B, C, D and E.

(iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.

(iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.

(v) In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.

(vi) In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.

(vii) In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.

(ix) Use of calculators is not allowed

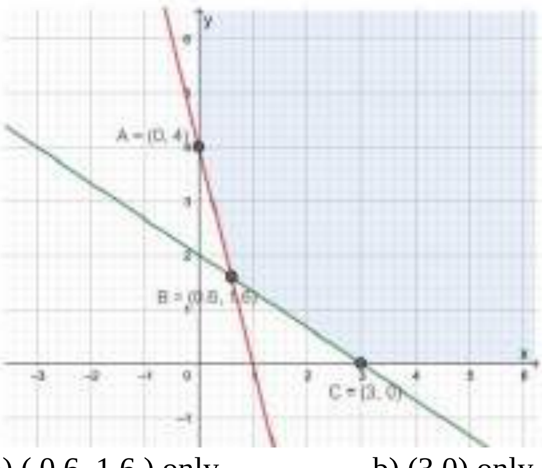
SECTION-A **[1× 20= 20]**

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. If $|adj\ A| = 144$, where A is a square matrix of order 3×3 , then $|A| =$
   a)12 b)-12 c) ± 12 d)16
2. If $\begin{bmatrix} 5 & 2x+3 \\ 3x-1 & x \end{bmatrix}$ is a symmetric matrix , then value of x is :
   a) 4 b) 3 c) 2 d) 1
3. The interval, in which function $y = x^3 + 6x^2 + 6$ is increasing is :
   a) $(-\infty, -4) \cup (0, \infty)$ b)$(-\infty, 4)$ c) $(-4,0)$
   d) $(-\infty, 0) \cup (4, \infty)$
4. If B is a non singular matrix of order 3 such that $B^2 = 2B$, then the value of $|B| =$
   a) -2 b) 2 c) 4 d) 8
5. The integrating factor of the differential equation $x\frac{dy}{dx} - y = x^2e^x$ is :
   a) x b) $\frac{1}{x}$ c) $-x$ d) e^{-x}
6. If the points $A(3, -2)$, $B(k, 2)$ and $C(8,8)$ are collinear , then the value of k is :
   a) 2 b) -3 c) 5 d) -4

7. If order of matrix A is 2×3 , of matrix B is 3×2 and of matrix C is 3×3 , then which one of the following is **not** defined ?
a) $C(A + B')$ b) $C(A + B')'$ c) BAC d) $CB + A'$
8. If A and B are two events such that $P(A) = 0.2$, $P(B) = 0.4$ and $P(A \cup B) = 0.5$, then the value of $P(A/B)$ is :
a) 0.1 b) 0.25 c) 0.5 d) 0.08
9. A unit vector perpendicular to the two vectors $\vec{a} = -2\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$ is given by :
a) $\hat{\imath} - \hat{\jmath}$ b) $\hat{\imath} + \hat{k}$ c) $-\frac{1}{\sqrt{2}}\hat{\imath} - \frac{1}{\sqrt{2}}\hat{\jmath}$ d) $\frac{1}{\sqrt{2}}\hat{\imath} - \frac{1}{\sqrt{2}}\hat{\jmath}$
10. What is the value of $\frac{projection\ of\ \vec{a}\ \text{on}\ \vec{b}}{projection\ of\ \vec{b}\ \text{on}\ \vec{a}}$ for vectors $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} - 6\hat{k}$ and $\vec{b} = 2\hat{\imath} - 2\hat{\jmath} + \hat{k}$
a) $\frac{3}{7}$ b) $\frac{7}{3}$ c) $\frac{4}{3}$ d) $\frac{4}{7}$
11. The feasible region , for the constraints $x \geq 0$, $y \geq 0$ and $x + y \leq 2$ lies in :
a) IV quadrant b) III quadrant c) II quadrant d) I quadrant
12. $\int \frac{\cos 2x}{\sin^2 x cos^2 x} dx$ is equal to :
a) $cotx + tanx + c$ b) $-cotx + tanx + c$
c) $cotx - tanx + c$ d) $-cotx - tanx + c$
13. If $f(x)$ is an odd function , then $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x) \cos^3 x\, dx$ equals :
a) $2\int_0^{\frac{\pi}{2}} f(x) \cos^3 x\, dx$ b) 0
c) $2\int_0^{\frac{\pi}{2}} f(x)\, dx$ d) $2\int_0^{\frac{\pi}{2}} 2\cos^3 x\, dx$
14. The degree and order of differential equation $y''^2 + \log(y') = x^5$ respectively are :
a) not defined , 5 b) not defined , 2
c) 5 , not defined d) 2,2
15. If $y = \cot^{-1} x$, $x < 0$, then
a) $\frac{\pi}{2} < y \leq \pi$ b) $\frac{\pi}{2} < y < \pi$ c) $-\frac{\pi}{2} < y < 0$ d) $-\frac{\pi}{2} \leq y < 0$
16. The corner points of the shaded unbounded feasible region of an LPP are (0, 4), (0.6, 1.6) and (3, 0) as shown in the figure. The minimum value of the objective function Z = 4x + 6y occurs at



a) (0.6 ,1.6) only b) (3,0) only c) (0.6 , 1.6) and (3,0) only
d) at every point of the line-segment joining the points (0.6, 1.6) and (3, 0)
17. The function $f(x) = [x]$, where $[x]$ is the greatest integer function that is less than or equal to x , is continuous at :
a) 4 b) -2 c) 1.5 d) 1

18. The area (in sq.units) of the region bounded by the curve $y = x, x - axis$, $x = 0$ and $x = 2$ is :

a) $\frac{3}{2}$ b) $\frac{1}{2}\log 2$ c) 2 d) 4

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.)

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).
(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(C) (A) is true but (R) is false.
(D) (A) is false but (R) is true.

19. ASSERTION (A) : The function $f(x) = |x - 6|cosx$ is differentiable in $R - \{6\}$.
REASON (R) : If a function f is continuous at a point c then it is also differentiable at that point .

20. ASSERTION (A) : $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{6}$
REASON (R) : $cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

SECTION B [2 × 5 =10]

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Find the domain of $\cos^{-1}(3x - 2)$.

22. Given that $f(x) = \frac{\log x}{x}$, find the point of local maximum of $f(x)$.

23. (a) If $y\sqrt{1 - x^2} + x\sqrt{1 - y^2} = 1$, then prove that $\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$.

OR

23 . (b) Find the derivative of $\tan^{-1} x$ with respect to $logx$ (where $x \in (1, \infty)$)

24. a) Find $|\vec{x}|$ if $(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 12$, where $\vec{a}$ is a unit vector.

OR

24. b) If $\vec{a} = \hat{\imath} - \hat{\jmath} + 7\hat{k}$ and $\vec{b} = 5\hat{\imath} - \hat{\jmath} + \lambda\hat{k}$, find value of λ so that the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal.

25. The two co initial sides of a parallelogram are $2\,\hat{\imath} - 4\hat{\jmath} - 5\hat{k}$ and $2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$. Find the diagonals and use them to find the area of the parallelogram .

SECTION C [3 × 6 =18]

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. Find the intervals in which the function given by $f(x) = \sin 3x$, $x \in \left[0, \frac{\pi}{2}\right]$ is
a) increasing b) decreasing

27. A kite is flying at a height of 3 metres and 5 metres of string is out. If the kite is moving away horizontally at the rate of 200 cm/s, find the rate at which the string is being released.

28. a) If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, show that $\vec{a} - \vec{d}$ is parallel to $\vec{b} - \vec{c}$.

OR

28. b) Find the value(s) of $\boldsymbol{a}$ so that the following lines are skew :
$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-a}{4}$, $\frac{x-4}{5} = \frac{y-1}{2} = z$

29. a) Evaluate $\int \sqrt{\frac{x}{1-x^3}}\,dx \qquad , x \in (0,1)$

OR

29. b) Evaluate $\int_0^1 x(1-x)^n\,dx \quad$ where $n \in N$

30. The corner points of the feasible region determined by the system of linear constraints in an LPP are $(0,10), (5,5), (15,15)$ and $(0,20)$. Let $z = px + q$,$p, q > 0$ be the objective function , then find the relation between p and q so that the maximum of z occurs at both the points $(15,15)$ and $(0,20)$.

31. a) A coin is biased so that the head is three times as likely to occur as tail . If the coin is tossed twice , find the probability distribution of number of tails . Also find the mean of the distribution.

OR

31. b) Let A and B be the events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and P(not A or not B)= $\frac{1}{4}$. Find whether A and B are

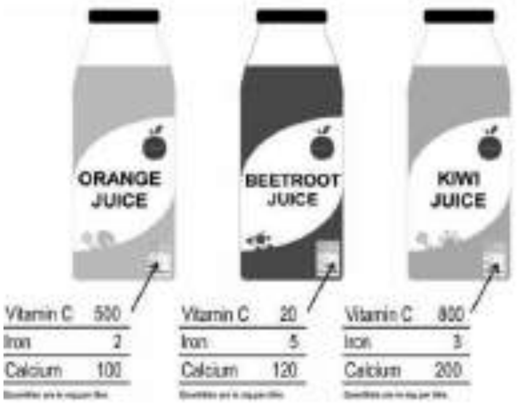
(i) mutually exclusive (ii) independent

SECTION D **[5 × 4 =20]**

(This section comprises of 4 long answer (LA) type questions of 5 marks each.)

32. Using integration , find the area of the region bounded by the curve $y = x^2, x = -1, x = 1$ and the $x - axis$.

33. Sravan is a nutritionist. He wants to create a mixture of orange juice, beetroot juice and kiwi juice that can provide 1860 mg of vitamin C, 22 mg of iron and 760 mg of calcium. The quantity of each nutrient per litre of juice is shown below.



Using the matrix method, find how many litres of each juice Sravan should add into the mixture.

34. a) If $(ax+b)e^{\frac{y}{x}} = x$, then show that $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$.

OR

34. b) If $(x-a)^2 + (y-b)^2 = c^2$, for some $c > 0$, prove that $\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ is a constant independent of a and b.

35. a) Find the coordinates of the image of the point $(1,6,3)$ with respect to the line $\vec{r} = (\hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$, where λ is a scalar . Also find the distance of the image from the y −axis.

OR

35. (b) An aeroplane is flying along the line $\vec{r} = \lambda(\hat{\imath} - \hat{\jmath} + \hat{k})$ where λ is a scalar and another aeroplane is flying along the line $\vec{r} = \hat{\imath} - \hat{\jmath} + \mu(-2\hat{\jmath} + \hat{k})$ where μ is a scalar. At what points on the lines should they reach, so that the distance between them is the shortest? Find the shortest possible distance between them.

SECTION E **[4 × 3 =12]**

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

36. Students of a school are taken to a railway museum to learn about railways heritage and its history .

An exhibit in the museum depicted many rail lines on the track near the railway station . Let L be the set of all rail lines on the railway track and R be the relation on L defined by

R $=\{(l_1, l_2): l_1$ is parallel to $l_2\}$

On the basis of above information , answer the following questions :

(i) Find whether the relation R is symmetric or not.

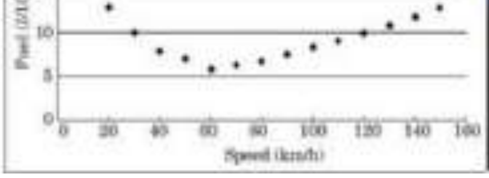
(ii) Find whether the relation R is transitive or not .

(iii) a) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$, then find the set of rail lines in R related to it .

OR

(iii) b) Let S be the relation defined by S $=\{(l_1, l_2): l_1$ is perpendicular to $l_2\}$. Check whether the relation S is symmetric and transitive .

37. Over speeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds , fuel mileage usually decreases rapidly at speeds above 80 km/h



The relation between fuel consumption F(l/100km) and speed V (km/h) under some constraints is given as $F = \frac{V^2}{500} - \frac{V}{4} + 14$.
On the basis of the above information , answer the following questions :

(i) Find F, when V = 40 km/h.

(ii) Find $\frac{dF}{dV}$.

(iii) a) Find the speed V for which fuel consumption F is minimum .

OR

(iii) b) Find the quantity of fuel required to travel 600 km at the speed V at which $\frac{dF}{dV} = -0.01$.

38. A Shopkeeper sells three types of flower seeds A_1, A_2 and A_3 . They are sold as a mixture where the proportions are $4:4:2$ respectively . The germination rates of the three types of seeds are $45\%, 60\%$ and 35% .
Based on the above information , answer the following questions :

(i) What is the probability of a randomly chosen seed to germinate ?

(ii) What is the probability that a randomly selected seed is of type A_1 , given that it germinates ?

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