

KENDRIYA VIDYALAYA SANGATHAN AGRA REGION

FIRST PRE BOARD EXAMINATION 2024-25

CLASS XII

SUBJECT: MATHEMATICS

MARKING SCHEME

1	c	1
2	a	1
3	a	1
4	b	1
5	c	1
6	b	1
7	c	1
8	a	1
9	b	1
10	b	1
11	b	1
12	b	1
13	a	1
14	c	1
15	b	1
16	d	1
17	b	1
18	b	1
19	a	1
20	c	1
21	$C(x) = 0.000014x^3 + 0.005x^2 + 5x + 1100$ Marginal Cost = $dC(x)/dx = 0.000042x^2 + 0.010x + 5$ Marginal Cost of 200 items = Rs8.68 OR $f(x) = \sin 2x + 5$ $-1 \leq \sin 2x \leq 1$ i.e $4 \leq \sin 2x + 5 \leq 6$ Maximum value = 6 Minimum value = 4	1 1 1 1/2 1/2
22	Let V, S and r be volume, surface area and radius of a sphere at any time t,	

	$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = \frac{4}{3}\pi 3r^2 \frac{dr}{dt}$ $8 = 4\pi r^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{2}{\pi r^2}$ $S = 4\pi r^2 \Rightarrow \frac{dS}{dt} = \frac{16}{r}$ $\frac{dS}{dt} \text{ at } r=12 \Rightarrow 4/3 \text{ cm}^2/\text{sec.}$	1 1
23	$\int_1^2 \frac{1}{x(\log x)^4} dx,$ Let $\log x = t, 1/x dx = dt$ $\int_0^{\log 2} \frac{1}{(t)^4} dt = -1/3 (\log 2)^{-3}$	1/2 1/2 1
24	$f'(x) = 3x^2 - 6x + 3$ $= 3(x-1)^2 > 0, \text{ in every interval of } R.$ Therefore, the function f is increasing on R.	1 1
25	$\sin^{-1}(\cos \frac{\pi}{6}) = \sin^{-1}(\frac{\sqrt{3}}{2})$ $= \frac{\pi}{3}$ OR $-1 \leq (2x-3) \leq 1, 2 \leq 2x \leq 4,$ $x \in [1, 2]$	1 1 1 1
26	$\int \frac{dx}{5-4x-2x^2} = -1/2 \int \frac{dx}{\{(x+1)^2 - (\frac{\sqrt{7}}{2})^2\}}$ $= -1/2\sqrt{14} \log \left \frac{x+1-\frac{\sqrt{7}}{2}}{x+1+\frac{\sqrt{7}}{2}} \right + C$ OR $I = \int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx \dots\dots\dots(1)$ $I = \int_0^{\frac{\pi}{4}} \log(1 + \tan(\frac{\pi}{4} - x)) dx$ $I = \int_0^{\frac{\pi}{4}} \log(1 + \frac{1-\tan x}{1+\tan x}) dx$ $I = \int_0^{\frac{\pi}{4}} \log(\frac{2}{1+\tan x}) dx \dots\dots\dots(2)$ $(1) + (2)$ $2I = \int_0^{\frac{\pi}{4}} \log 2 dx$ $I = \frac{\pi}{8} \log 2$	2 1 1 1 1
27	$\frac{dy}{dx} = \frac{2\sin^{-1}x}{\sqrt{1-x^2}}$ $\sqrt{1-x^2} \frac{dy}{dx} = 2\sin^{-1}x$	1 1

	$(1 - x^2) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} - 2 = 0$	1												
28	$\int \frac{x^2}{(x^2 + 4)(x^2 + 9)} dx$ Let $x^2 = t$, Consider $\frac{x^2}{(x^2 + 4)(x^2 + 9)}$ $= \frac{A}{(t + 4)} + \frac{B}{(t + 9)}$ Solving A = -4/5, B = 9/5, $\int \frac{x^2}{(x^2 + 4)(x^2 + 9)} dx = -\frac{4}{5} \int \frac{1}{(x^2 + 4)} dx + \frac{9}{5} \int \frac{1}{(x^2 + 9)} dx$ $= -\frac{2}{5} \tan^{-1} \frac{x}{2} + \frac{3}{5} \tan^{-1} \frac{x}{3} + c$	1 1 1												
29	$n(S) = {}^6C_2 = 15$ Let X denote the larger of two numbers obtained $\therefore X = 3, 4, 5, 6, 7$ The Probability distribution is <table><tr><td>X</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>P(X)</td><td>1/15</td><td>2/15</td><td>3/15</td><td>4/15</td><td>5/15</td></tr></table>	X	3	4	5	6	7	P(X)	1/15	2/15	3/15	4/15	5/15	1 2
X	3	4	5	6	7									
P(X)	1/15	2/15	3/15	4/15	5/15									
30	$(\cos^2 x) \frac{dy}{dx} + y = \tan x$ $\frac{dy}{dx} + y \sec^2 x = \tan x \sec^2 x$ I.F. = $e^{\int \sec^2 x dx} = e^{\tan x}$ Sol. $y e^{\tan x} = \int e^{\tan x} \tan x \sec^2 x dx$ $y e^{\tan x} = e^{\tan x} (\tan x - 1) + c$ OR $(1 + e^{2x}) dy + (1 + y^2) e^x dx = 0$ $\frac{dy}{(1 + y^2)} = -\frac{e^x dx}{(1 + e^{2x})}$ on integrating $\tan^{-1} y = -\tan^{-1} e^x + c$	0.5 1 0.5 1 1 2												
31	Correct graph and shading <table><tr><td>Corner Point</td><td>Value of $z = 3x + 9y$</td></tr><tr><td>(0,10)</td><td>90</td></tr><tr><td>(5,5)</td><td>60 Min.</td></tr><tr><td>(15,15)</td><td>180</td></tr><tr><td>(0,20)</td><td>180</td></tr></table>	Corner Point	Value of $z = 3x + 9y$	(0,10)	90	(5,5)	60 Min.	(15,15)	180	(0,20)	180	1.5 1		
Corner Point	Value of $z = 3x + 9y$													
(0,10)	90													
(5,5)	60 Min.													
(15,15)	180													
(0,20)	180													

	Minimum value of Z is 60 at x = 5 and y = 5. OR Correct graph and shading <table border="1"><tr><th>Corner Point</th><th>Value of z = 400x + 300y</th></tr><tr><td>(20,0)</td><td>8000</td></tr><tr><td>(40,0)</td><td>16000</td></tr><tr><td>(40,160)</td><td>64000 Max.</td></tr><tr><td>(20,180)</td><td>62000</td></tr></table> Maximum value of Z is 64000 at x = 40 and y = 160.	Corner Point	Value of z = 400x + 300y	(20,0)	8000	(40,0)	16000	(40,160)	64000 Max.	(20,180)	62000	0.5 1.5 1 0.5
Corner Point	Value of z = 400x + 300y											
(20,0)	8000											
(40,0)	16000											
(40,160)	64000 Max.											
(20,180)	62000											
32	The line $y=3x+2$ meets X axis at $x=-2/3$ and its graph lies below x axis for $x \in (-1,-2/3)$ and above x axis for $x \in (-2/3,1)$ Required area of $\Delta ABC = \left \int_{-1}^{-2/3} (3x + 2) dx \right + \int_{-2/3}^1 (3x + 2) dx$ Calculation part Getting answer = $\frac{13}{3}$	1 2 1 1										
33	For proving Reflexive relation For proving Symmetric relation For proving Transitive relation Set of elements related to 1=$\{1,6,11\}$ OR For proving Reflexive relation For proving Symmetric relation For proving Transitive relation Since the relation is reflexive, symmetric and transitive so it is an equivalence relation.	1 1 2 1 1 1 2 1										
34	$AB = 6 I$ $A^{-1} = B/6$, given system of equations can be written, in matrix form, as follows $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$ $AX = C$ $X = A^{-1} C$ $x = 2$, $y = -1$, $z = 4$	1.5 1 1 1.5										
35	$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} = \lambda \dots\dots(i)$											

	and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = \mu \dots (ii)$ General point on line(i) and (ii) are $(3\lambda - 1, 5\lambda - 3, 7\lambda - 5) \dots (iii)$ $(\mu + 2, 3\mu + 4, 5\mu + 6) \dots (iv)$ Getting $3\lambda - \mu = 3 \dots (v)$ $5\lambda - 3\mu = 7 \dots (vi)$ $7\lambda - 5\mu = 11 \dots (vii)$ Solving (v),(vi) getting value of $\lambda = 1/2$ & $\mu = -3/2$ and satisfying in (vii) Point of intersection: $(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2})$ Comparing with standard equations $\vec{a_1} = 3\hat{i} + 5\hat{j} + 7\hat{k}$, $\vec{a_2} = -\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b_1} = \hat{i} - \hat{j} - \hat{k}$, $\vec{b_2} = 7\hat{i} - 6\hat{j} + \hat{k}$ $\vec{a_2} - \vec{a_1} = -4\hat{i} - 6\hat{j} - 8\hat{k}$ $\vec{b_1} \times \vec{b_2} = 4\hat{i} + 6\hat{j} + 8\hat{k}$ $(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = -116$ $\vec{b_1} \times \vec{b_2} = \sqrt{116}$ shortest distance $= \sqrt{116}$ units.	0.5 1 1.5 1 1 1 1 1.5 1 0.5 1
36	(i) 1/10 (ii) 31/100 (iii) 24/31 OR 7/31	1 1 2
37	(i) Position vector of vector AB $= -\hat{i} + 3\hat{j} - \hat{k}$ (ii) Distance between aeroplane and bird $= \sqrt{11}$ units (iii) Direction cosines of vector AB $= \langle \frac{-1}{\sqrt{11}}, \frac{3}{\sqrt{11}}, \frac{-1}{\sqrt{11}} \rangle$ OR direction angles $\langle \cos^{-1} \frac{-1}{\sqrt{11}}, \cos^{-1} \frac{3}{\sqrt{11}}, \cos^{-1} \frac{-1}{\sqrt{11}} \rangle$	1 1 2
38	(i) let the length = breadth = x and height = y A.T.P. $X^2y = 500$ Surface area $S = x^2 + 4xy$	

	$dS/dx = 2x - 2000/x^2$ For maxima or minima $dS/dx = 0$, $x = 10m$ at $x = 10$, $d^2S/dx^2 > 0$ hence surface area is minimum at $x = 10m$ Min. Surface area = $300m^2$	1
		1
	(ii) If $x = 10m$, then $y = 5m$ Volume of tank = $x^2y = 500m^3$ New Volume = $(2x)^2y = 2000m^3$ Increase in volume of tank = $1500m^3$ %increase in volume of the tank = 300%	1
		1