

KENDRIYA VIDYALAYA SANGATHAN, BHUBANESWAR REGION

PRE- BOARD EXAMINATION- 2024

CLASS-XII

SUBJECT- MATHEMATICS (041)

TIME- 3:00 Hrs

M.M. 80



General Instructions:

1. This paper contains 38 questions. All questions are compulsory.
2. This question paper is divided into five sections – A, B, C, D and E.
3. In Section A, Questions no.1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each. .
4. In Section B, Questions no. 21 to 25 are very short (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are short answer (SA)- type questions carrying 3 marks each.
6. In Sections D, Questions no. 32 to 35 are Long Answer (LA)- type questions, carrying 5 marks each.
7. In Section E, Question no. 36 to 38 are Case study- based questions, carrying 4 marks each.
8. There is no overall choice, however, an internal choice has been provided in 2 questions in section B,3 questions in section C, 2 questions in Section D and one subpart each in 2 questions of section E.
9. Use of calculator is not allowed.

Q.N O.	Section-A (MCQs)	
1	What is the domain of $\cos^{-1}(2x-3)$? a) $[-1,1]$ b) $(1,2)$ c) $(-1,1)$ d) $[1,2]$.	1
2	If $f(\alpha) = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix}$ then $f(\alpha)f(\beta) =$ a) $f(\alpha)$ b) $f(\alpha\beta)$ c) $f(\alpha + \beta)$ d) $f(\alpha - \beta)$	1
3	If $A = \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}$ then $A^2 - 4A + 7I$ is a) null matrix b) an identity matrix c) diagonal matrix d) none of these	1
4	If $A = \begin{pmatrix} 0 & 2 \\ 3 & -4 \end{pmatrix}$ and If $kA = \begin{pmatrix} 0 & 3a \\ 2b & 24 \end{pmatrix}$, then the values of k, a and b are a) -6, -12, -8 b) -6,-4, -9 c) -6, 4, 9 d) -6, 12, 18	1
5	If $ A = kA $ and A is a 2×2 matrix then sum of all possible values of k is a) 1 b) -1 c) 2 d) 0	1
6	If A is a skew symmetric matrix of order 3×3 and $ A = x$ then $(2025)^x =$ a) $\frac{1}{2025}$ b) 2025 c) $(2025)^2$ d) 1	1
7	If $y = e^{-x}$ then $\frac{d^2y}{dx^2} =$ a) $-y$ b) y c) x d) $-x$	1
8	The rate of change of area of a circle with respect to its radius at $r = 3cm$ is a) 3π b) 4π c) 6π d) 12π	1
9	$\int 3^{x+2} dx =$ a) $3^{x+2} + c$ b) $3^{x+2} \log 3 + c$ c) $\frac{3^{x+2}}{\log 3} + c$ d) $\frac{3^{x+2}}{2 \log 3} + c$	1
10	$\int_0^{\frac{\pi}{3}} \sec^2\left(\frac{\pi}{3} - x\right) dx =$ a) $\frac{1}{\sqrt{3}}$ b) $\sqrt{3}$ c) $-\sqrt{3}$ d) 1	1
11	The area of the curve $y = \sin x$ between 0 and π is a) 1 sq. unit b) 2 sq. unit c) 4 sq. unit d) 8 sq. unit	1

12.	The solution of differential equation $\frac{dy}{dx} + \frac{2y}{x} = 0$ is a) $y = \frac{c}{x^2}$ b) $x = \frac{c}{y^2}$ c) $xy = cx$ d) $y = c$	1
13.	The integrating factor of the differential equation $x \frac{dy}{dx} + 2y = x^2$ is a) $\frac{1}{x}$ b) x c) x^2 d) $\frac{1}{x^2}$	1
14.	The value of $\left(\hat{i} \times \hat{j} \right) \cdot \hat{k} + 2 \left(\hat{j} \times \hat{i} \right) \cdot \hat{k}$ is a) 1 b) -1 c) 2 d) -2	1
15.	Projection of $2\hat{i} + \hat{j}$ on the vector $\hat{i} - 2\hat{j}$ is a) 4 b) 0 c) -4 d) 2	1
16.	The maximum value of $z = 3x + 4y$ subject to constraints $x + y \leq 1$ and $x, y \geq 0$ is a) 7 b) 3 c) 4 d) 10	1
17.	The optimal value of the objective function is attained at the points a) given by intersection of inequation with y-axis only b) given by intersection of inequation with x-axis only c) given by corner points of the feasible region d) None of these.	1
18.	Two dice are thrown. If it is known that the sum of numbers on the dice was less than 5, the probability of getting a sum 3 is a) $\frac{1}{6}$ b) $\frac{2}{3}$ c) $\frac{1}{3}$ d) $\frac{5}{6}$	1
19.	The following question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) and has the following choice (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct choice. a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A. c) A is true but R is false. d) A is false but R is true. Assertion(A): Principal value of $\tan^{-1}(-1) = \frac{\pi}{4}$ Reason(R): $\tan^{-1}: \mathbb{R} \rightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	1
20.	The following question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) and has the following choice (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct choice. a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A. c) A is true but R is false. d) A is false but R is true. Assertion(A) : $ \sin x $ is continuous for all $x \in \mathbb{R}$ Reason(R) : $\sin x$ and $ x $ are continuous in \mathbb{R} .	1
Section-B (VSA)		
21.	Find the value of $\sin \left\{ 2 \cot^{-1} \left(-\frac{5}{12} \right) \right\}$	2
22.	Determine the values of the constants k so that the given function is continuous at $x = 0$. $f(x) = \begin{cases} \sin 3x & , \text{ if } x < 0 \\ \sin 5x & , \text{ if } x \geq 0 \end{cases}$	2
23.	Find $\frac{dy}{dx}$ where $x^6 y^5 = (x + y)^{11}$. OR Given $e^x + e^y = e^{x+y}$. Show that $\frac{dy}{dx} + e^{y-x} = 0$	2
24.	If $\vec{a} = 8\hat{j} + x\hat{k}$ and $\vec{b} = y\hat{i} - 2\hat{j} + \hat{k}$ are mutually perpendicular and $ \vec{a} = \vec{b} $, then find the values of x and y	2

	<p style="text-align: center;">OR</p> <p>If $\vec{a} = 3, \vec{b} = 5, \vec{c} = 7$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ then find the angle between \vec{a} and \vec{b}.</p>	
25	Find a unit vector perpendicular to both of the vectors $\vec{p} + \vec{q}$ and $\vec{p} - \vec{q}$ where $\vec{p} = 2\hat{i} - \hat{j} + 2\hat{k}, \vec{q} = 3\hat{i} + 4\hat{j} + 5\hat{k}$.	2
	Section-C (SA)	
26	The area of an expanding rectangle is increasing at the rate of $48\text{cm}^2/\text{s}$. The length of the rectangle is always equal to square of breadth. At what rate, the length is increasing when breadth is 4.5cm.	3
27	Find the interval in which the function $f(x) = \tan^{-1}(\sin x + \cos x), x \in (0, \pi)$ is increasing or decreasing.	3
28	Evaluate $\int \left(\frac{1}{\log x} - \frac{1}{(\log x)^2} \right) dx$	3
29	Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from $P(1, 3, 3)$. <p style="text-align: center;">OR</p> <p>Find the equation of a line passing through $(1, 2, -4)$ and perpendicular to the lines $\vec{r} = 8\hat{i} + 2\hat{j} - 5\hat{k} + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$ $\vec{r} = 3\hat{i} - \hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.</p>	3
30	Solve the following LPP graphically. Maximize $Z = 10x + 15y$ Subject to constraints: $3x + y \leq 12, \quad x + 2y \leq 10$ and $x, y \geq 0$	3
31	Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that <i>i)</i> The youngest is a girl? <i>ii)</i> At least one is a girl? <p style="text-align: center;">OR</p> <p>Two defective bulbs are mixed with 8 good ones. Find the probability distribution of number of defective bulbs if two bulbs are drawn at random. What is the average number of defective bulbs drawn?</p>	3
	Section-D (LA)	
32	If $A = \begin{bmatrix} 2 & 4 & 6 \\ 3 & -6 & 9 \\ 10 & 5 & -20 \end{bmatrix}$. Find A^{-1} and hence solve the equations $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2, \quad \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5, \quad \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = -4$	5
33	If $x = \sin t$ and $y = \sin pt$ then prove that $(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + p^2 y = 0$. <p style="text-align: center;">OR</p> <p>If $x = a \cos \theta + b \sin \theta$ and $y = a \sin \theta - b \cos \theta$ then show that $y^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = 0$.</p>	5
34	Using integration, find the area bounded between two curves $x^2 = 4y$ and the line $x = 4y - 2$	5
35	Find the foot of the perpendicular from $A(1, 2, -3)$ on the line $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z-0}{-1}$. Also find the image of the point A in the line. <p style="text-align: center;">OR</p> <p>Find the value of 'a' so that the lines $\frac{x-1}{2} = \frac{y-a}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1}$ are intersecting lines. Also, find the point of intersection</p>	5

Section-E (Case-Based) (4 marks)		
36	<p>Rahul visited the amusement park along with his family. The amusement park had a huge swing, which attracted many children. He found that the swing traced the path of a parabola as given by $y = 3x^2$</p>  <p>Answer the following questions using the above information.</p> <p>(i) If $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 3x^2$, then check whether f is an injective function or not.</p> <p>(ii) Let $f: \mathbb{N} \rightarrow \mathbb{N}$ be defined by $f(x) = 3x^2$. Check whether f is a bijective function or not.</p> <p>(iii) Let $f: \{1, 2, 3, \dots\} \rightarrow \{3, 12, 27, \dots\}$ be defined by $f(x) = 3x^2$. Check whether the function f is bijective or not by giving suitable reason.</p> <p>OR</p> <p>Let $f: \mathbb{N} \rightarrow \mathbb{R}$ be defined by $f(x) = 3x^2$. Determine the range of the function f. Also find $f(3)$</p>	<p>1</p> <p>1</p> <p>2</p> <p>2</p>
37.	<p>The relation between the heights of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$, where x is the number of days exposed to</p>  <p>sunlight</p> <p>Answer the following based on above information:</p> <p>(i) Find the rate of growth of the plant with respect to sunlight.</p> <p>(ii) What are the number of days it will take for the plant to grow to the maximum height?</p> <p>(iii) What is the maximum height of the plant?</p> <p>OR</p> <p>What will be the height of the plant after 2 days?</p>	<p>1</p> <p>1</p> <p>2</p> <p>2</p>
38	<p>In answering a question on a multiple-choice test for class XII, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assume that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. Let E_1, E_2 and E be the events that the student knows the answer, guesses the answer and answers.</p> <p>Based on the above information, answer the following:</p> <p>(i) What is the value of $P(E_1)$?</p> <p>(ii) Find the value of $P(E E_1)$?</p> <p>(iii) Find the value of $\sum_{k=1}^2 P(E E_k)P(E_k)$</p> <p>OR</p> <p>What is the probability that the student knows the answer given that he answered it correctly?</p>	<p>1</p> <p>1</p> <p>2</p> <p>2</p>