केन्द्रीय विद्यालय संगठन, भोपाल संभाग

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कक्षा- XII Class-XII-2024-2025-Set-2

CLASS - XII **Maximum Marks : 80**

SUBJECT – MATHEMATICS (041) **Time Allowed : 3 Hours**

Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains **38** questions. **All** questions are **compulsory.**

(ii) This Question paper is divided into **five** Sections - **A, B, C, D** and **E.**

(iii) In **Section A**, Questions no. **1** to **18** are **multiple choice questions (MCQs)** and Questions no. **19** and **20** are **Assertion-Reason based** questions of **1 mark each**.

(iv) In **Section B,** Questions no. **21** to **25** are **Very Short Answer (VSA)-type** questions, carrying **2 marks each**.

(v) In **Section C,** Questions no. **26** to **31** are **Short Answer (SA)-type** questions, carrying **3 marks each**.

(vi) In **Section D,** Questions no. **32** to **35** are **Long Answer (LA)-type** questions, carrying **5 marks each**.

(vii) In **Section E,** Questions no. **36** to **38** are **Case study-based questions,** carrying **4 marks each.**

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart in each question of Section E.

(ix) Use of calculators is **not** allowed.

SECTION:-A **Multiple Choice Questions (MCQs)**	
Q.1	Let ‘f ’ : R – {2} → R – {1} be a function defined by f (x) = $\frac{x-1}{x-2}$,then ‘f ’ is (a) into function (b) many one function (c) bijective function (d) many one, into function.
Q.2	$Sin\left[\frac{\pi}{3}+Sin^{-1}(\frac{1}{2})\right]$ is equal to (a) 1 (b) 1/ 2 (c) 1/ 3 (d) 1/ 4
Q.3	Assume ***X, Y, Z, W*** and ***P*** are matrices of order **2** × ***n*** **, 3** × ***k*** **, 2** × ***p*** **, n** × ***3*** and ***p*** × ***k,*** respectively. Then the restriction on ***n, k*** and ***p*** so that ***PY + WY*** will be defined are: (a) ***k* = 3, *p* = *n*** (b) ***k*** **is arbitrary, p = 2** (c) ***p*** **is arbitrary,** ***k*** **= 3** **(**d) ***k*** **= 2, p = 3**
Q.4	Number of symmetric matrices of order 3 × 3 with each entry 1 or – 1 is (a) 512 (b) 64 (c) 8 (d) 4
Q.5	If $\begin{vmatrix}4 & 1\\2 & 1\end{vmatrix}^2 = \begin{vmatrix}3 & 2\\1 & x\end{vmatrix} - \begin{vmatrix}x & 3\\-2 & 1\end{vmatrix}$, then the value of x is: **(a)** 6 **(b)** 3 **(c)** 7 **(d)**1

Q.6	If A is a non-singular square matrix of order 3 such that $\|A\|$ = 3, then value of $\|2A^T\|$ is (a) 3 (b) 6 (c) 12 (d) 24
Q.7	If $x = sin^3t, y = cos^3t$ then $\frac{dy}{dx}$ (a) $\tan t$ (b) $\cot t$ (c) $-\tan t$ (d) $-\cot t$
Q.8	The function $f: R \rightarrow Z$ defined by $\boldsymbol{f(x)} = [x]$**;** where [ ] denotes the greatest integer function, is (a)Continuous at $\boldsymbol{x} = \mathbf{2.5}$ but not differentiable at $\boldsymbol{x} = \mathbf{2.5}$ (b) Not Continuous at $\boldsymbol{x} = \mathbf{2.5}$ but differentiable at $\boldsymbol{x} = \mathbf{2.5}$ (c) Not Continuous at $\boldsymbol{x} = \mathbf{2.5}$ and not differentiable at $\boldsymbol{x} = \mathbf{2.5}$ (d)Continuous as well as differentiable at $\boldsymbol{x} = \mathbf{2.5}$
Q.9	$\int_0^{2\pi} cosec^7 x\ \boldsymbol{dx}$ = (a) **0** (b)**1** (c) **4** (d) **2**π
Q.10	If m and n respectively are the order and degree of the differential equation $\frac{d}{dx}\left(\frac{dy}{dx}\right)^4 = 0$ then m + n is (a) 1 (b) 2 (c) 3 (d) 4
Q.11	The value of '$\boldsymbol{n}$' , such that the differential equation $\boldsymbol{x^n}\frac{\boldsymbol{dy}}{\boldsymbol{dx}}= \boldsymbol{y}(\boldsymbol{logy} - \boldsymbol{logx} + \mathbf{1})$; (**where** $\boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{R^+}$) is homogeneous, is (a) **0** (b)**1** (c) **2** (d) **3**
Q.12	If $\|\vec{a}\|$ = 3, $\|\vec{b}\|$ = 4 and $\|\vec{a} + \vec{b}\|$ =5, then $\|\vec{a} - \vec{b}\|$ = (a) **3** (b) **4** (c) **5** (d) **8**
Q.13	The vector of the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9 is (a) $\hat{i} - 2\hat{j} + 2\hat{k}$ (b) $\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$ (c) $3(\hat{i} - 2\hat{j} + 2\hat{k})$ (d) $9(\hat{i} - 2\hat{j} + 2\hat{k})$
Q.14	The projection of the vector $2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\hat{i} + 2\hat{j} + \hat{k}$ is (a) $10/\sqrt{6}$ (b) $10/\sqrt{3}$ (c) $5/\sqrt{6}$ (d) $5/\sqrt{3}$
Q.15	If a line makes angles α, β, γ with x-axis, y-axis and z-axis respectively, then $sin^2\alpha + sin^2\beta + sin^2\gamma =$ (a) 1 (b) 2 (c) 0 (d) 3

Q.16	The direction ratios of the line 6x – 2 = 3y + 1 = 2z – 2 are: (a) 6, 3, 2 (b) 1, 1, 2 (c) 1, 2, 3 (d) 1, 3, 2
Q.17	The bounded feasible region of an LPP is always a _________________ (a) Convex Polygon (b) Concave Polygon (c) Either (a) or (b) (d)Neither (a) nor (b)
Q.18	For any two events ***A*** and ***B*** , if ***P (A***) $=\frac{1}{2}$, ***P (B***) $=\frac{2}{3}$and ***P(A*** ∩ B) = $\frac{1}{4}$,then $P\left(\frac{A}{B}\right)$ equals: (a) $\frac{3}{8}$ (b) $\frac{8}{9}$ (c) $\frac{5}{8}$ (d) $\frac{1}{4}$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.)

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

Q.I9	**Assertion (A):** Every scalar matrix is a diagonal matrix. **Reason (R):** In a diagonal matrix, all the diagonal elements are 0.
Q.20	**Assertion (A)**: The max. value of Z = x + 3y subject to constraints 2x + y ≤ 20, x + 2y ≤ 20, x ≥ 0, y ≥ 0 is 30 . Corner points of feasible region are (0,0),(10,0),(0,10)and(20/3,20/3)} **Reason(R)**: The variables that are present in the LPP are called decision variables.

SECTION:- B
Very Short Answer (VSA)-type

Q.21	Find the value of the expression sin $[cot^{-1}\{cos(tan^{-1}1)\}]$
Q.22	Find the relationship between *a* and *b* so that the function *f* defined by f(x) = $\begin{cases} ax+1\ if\ x\le 3 \\ bx+3\ if\ x>3 \end{cases}$ is continuous at *x* = 3.
Q.23	(a) Find the derivative of $\mathbf{tan^{-1}}$ ***x*** with respect to **log x;** (where ***x*** ∈ (**1,** ∞)). **OR** (b) Differentiate the following function with respect to x : $(\cos x)^x$; (where $x \in (0, \pi/2)$).
Q.24	(a) If vectors $\vec{a}$ = 2î + 2ĵ + 3k̂ , $\vec{b}$ = - î + 2ĵ + k̂ and $\vec{c}$ = 3î + ĵ are such that $\vec{b}$ + λ$\vec{c}$ is

	perpendicular to $\vec{a}$, then find the value of λ. **OR** (b) A person standing at **O** (**0, 0, 0**) is watching an aeroplane which is at the coordinate point ***A*** (**4, 0, 3**). At the same time he saw a bird at the coordinate point ***B***(**0, 0, 1**) Find the direction cosines of $\overrightarrow{BA}$.
Q.25	Find the vector equation of the line passing through (1, 2, 3) and parallel to the vector -4i+2j and show that it is perpendicular to the z-axis.
SECTION:- C **Short Answer (SA)-type**	
Q.26	(a) Evaluate the following integral $\int_{-1}^{2}\lvert x^3 - x\rvert dx$ OR (b) Evaluate: $\int_0^{\pi} \frac{xsinx}{1+ cos^2x} dx$
Q.27	Evaluate: $\int \frac{\sqrt{1-sinx}}{(1+cosx)}\ e^{-x/2}\ dx$
Q.28	Evaluate the following integral $\int \frac{cosx}{(1 - sinx)(2 - sinx\ )}\ dx$
Q.29	(a) Find a particular solution of the differential equation$(tan^{-1}y - x)\frac{dy}{dx} = 1 + y^2$, given that $y = 0$ when $x = 0$. OR (b) Solve the following differential equation $\left[xy\ sin\left(\frac{y}{x}\right) - x^2cos\left(\frac{y}{x}\right)\right]\ dy\ = \left[xy\ cos\left(\frac{y}{x}\right) + y^2sin\left(\frac{y}{x}\right)\right]\ dx$
Q.30	Solve the following problem graphically: Minimise and Maximise Z = 3*x* + 9*y* subject to the constraints: x **+ 3**y **≤ 60,** x + y **≥ 10,** x ≤ y, x ≥ ***0,*** y ≥ ***0***
Q.31	(a) The probability that it rains today is 0.4. If it rains today, the probability that it will rain tomorrow is 0.8. If it does not rain today, the probability that it will rain tomorrow is 0.7. If P_1: denotes the probability that it does not rain today. P_2: denotes the probability that it will not rain tomorrow, if it rains today. P_3: denotes the probability that it will rain tomorrow, if it does not rain today. P_4: denotes the probability that it will not rain tomorrow, if it does not rain today. (i) Find the value of $P_1 \times P_4 - P_2 \times P_3$. (ii) Calculate the probability of raining tomorrow.

	OR (b) A and B throw a die alternatively till one of them gets a '6' and wins the game. Find their respective probabilities of winning, if A starts first.
SECTION:- D **(Long answer type questions (LA) of 5 marks each)**	
Q.32	Find the product of matrices $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ with the help of it, solve the following system of linear equations: $x - y + z = 4$ $x - 2y - 2z = 9$ $2x + y + 3z = 1$
Q.33	(a) A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic meter per hour. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4 m. OR (b) Find the intervals in which the function f given by f (x) = sin x + cos x, $0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.
Q.34	Draw a sketch of the following region and find its area: $\{(x, y) : 0 \leq y \leq x^2, 0 \leq y \leq x, 0 \leq x \leq 2\}$.
Q.35	(a) Find the image of the point (**1,2, 1**) with respect to the line $\frac{x-3}{1} = \frac{y+1}{2} = \frac{z-1}{3}$. Also find the equation of the line joining the given point and its image. OR (b) By computing the shortest distance determine whether the given lines intersect or not. $\vec{r} = (1-t)\hat{\imath} + (t-2)j + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{\imath} + (2s-1)j - (2s+1)\hat{k}$
SECTION:- E **(This section comprises of 3 case-study/ passage-based question of 4 marks each with two sub-parts. First two case study)**	
Q.36	**Case Study-1** An organization conducted bike race under 2 different categories-boys and girls. In all, there were **250** participants. Among all of them finally three from Category **1** and two from Category **2** were selected for the final race. Ravi forms two sets ***B*** and ***G*** with these participants for his college project. Let ***B* = {*b*₁**, b₂, b₃}, G = { g₁, g₂} where ***B*** represents the set of boys selected and ***G*** the set of girls who were selected for the final race. Ravi decides to explore these sets for various types of relations and functions. On the basis of the above information, answer the following questions: **(i)** Ravi wishes to form all the relations possible from ***B*** to ***G*** . How many such relations are possible? [**1 *Mark***]

(ii) Write the smallest equivalence relation on **G**. [**1** ***Mark***]

(iii) (a) Ravi defines a relation from **B to B** as $R_1 = \{(b_1, b_2), (b_2, b_1)\}$. Write the minimum ordered pairs to be added in R_1 so that it becomes

(A) reflexive but not symmetric

(B) reflexive and symmetric but not transitive. [**2** ***Marks***]

OR

(iii) (b) If the track of the final race (for the biker $\boldsymbol{b_1}$) follows the curve

$x^2 = 4y$; (where $0 \le x \le 20\sqrt{2}$ & $0 \le y \le 200$), then state whether the track represents a one-one and onto function or not. (Justify). [**2** ***Marks***]

Q.37 **Case Study-2**

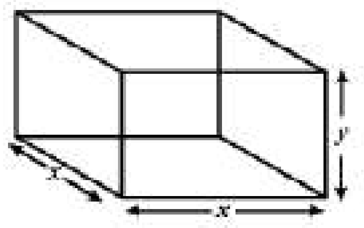
Suppose that the reliability of a HIV test is specified as follows: Of people having HIV, 90% of the test detect the disease but 10% go undetected. Of people free of HIV, 99% of the test are judged HIV–ive but 1% are diagnosed as showing HIV+ive. From a large population of which only 0.1% have HIV.

(i) one person is selected at random, given the HIV test, What is the probability that the person is correctly tested for HIV? [2 ***Marks***]

(iii)(a) one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV+ive. What is the probability that the person actually has HIV? [**2** ***Marks***]

Q.38 **Case-Study 3**: Read the following passage and answer the questions given below

Anuja wants to make a project for State level Science Exhibition. For this she wants to make metal box with square base and vertical sides to contain of 1024 cm^3 water material for top and bottom costs ₹ 5 per cm^2 and material for sides costs ₹2.5 per cm^2.



(i) Find the volume of the box. (1 mark)

(ii) What is the cost of the box in terms of x? (1 mark)

(iii)(a) Find the least cost of the box. (2 mark)

OR

(iii)(b) Find the dimensions of the box having minimum surface area. (2 mark)

===== ALL THE BEST=====