

Time: 3 hours

Maximum Marks:80

General Instructions:

Read the following instructions very carefully and strictly follow them:

1. This Question paper contains **38 questions**. All questions are compulsory.
2. This Question paper is divided into **five Sections - A, B, C, D, and E**.
3. In **Section A**, Questions no. 1 to 18 are multiple choice questions (MCQs) with only one correct option, and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In **Section B**, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In **Section C**, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In **Section D**, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In **Section E**, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in **Section B**, 3 questions in **Section C**, 2 questions in **Section D**, and one subpart each in 2 questions of **Section E**.

SECTION-A

**This section comprises of multiple-choice questions (MCQs) of 1 mark each.
Select the correct option (Question 1 - Question 18)**

1. Given triangles with sides $T_1: 3,4,5$; $T_2: 5,12,13$; $T_3: 6,8,10$; $T_4: 4,7,9$ and a relation R in set of triangles defined as $R = \{(\Delta_1, \Delta_2): \Delta_1 \text{ is similar to } \Delta_2\}$. Which triangles belong to the same equivalence class?
(a) T_1 and T_2
(b) T_2 and T_3
(c) T_1 and T_3
(d) T_1 and T_4
2. Given set $A = \{1,2,3\}$ and a relation $R = \{(1,2), (2,1)\}$, then the relation R will be
(a) reflexive if $(1,1)$ is added.
(b) symmetric if $(2,3)$ is added.
(c) transitive if $(1,1)$ is added.
(d) symmetric if $(3,2)$ is added.
3. Principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$ is
(a) $\pi/3$
(b) $-\pi/3$
(c) $5\pi/6$
(d) $-\pi/6$
4. $\tan^{-1}\left\{\sin\left(\frac{-\pi}{2}\right)\right\}$ is equal to
(a) -1
(b) 1
(c) $\pi/2$
(d) $-\pi/4$

5. If a matrix has 6 elements, then number of possible orders of the matrix can be

- ☒ (a) 2
(c) 3

- (b) 4
☒ (d) 6

6. The value of $\lim_{x \rightarrow 0} \frac{\sqrt{\frac{1}{2}(1 - \cos 2x)}}{x}$

- ☒ (a) 1
(c) 0

- (b) -1
(d) does not exist

7. If $f(x) = \frac{\sin(e^{x-2}-1)}{\log(x-1)}$, $x \neq 2$ and $f(x)=k$ for $x=2$, then value of k for which f is continuous is

- ☒ (a) -2
(c) 0

- (b) -1
(d) 1

8. $y = x(x-3)^2$ decreases for the values of x given by:

- (a) $1 < x < 3$
(c) $x > 0$

- ☒ (b) $x < 0$
(d) $0 < x < 3/2$

9. The function $f(x) = 4\sin^3 x - 6\sin^2 x + 12\sin x + 100$ is strictly

- (a) increasing in $(\pi/6, 3\pi/2)$
☒ (c) decreasing in $[-\pi/2, \pi/2]$

- (b) decreasing in $(\pi/2, \pi)$
(d) decreasing in $[0, \pi/2]$

10. If $\frac{d}{dx} f(x) = g(x)$, then antiderivative of $g(x)$ is

- (a) $f(x)$
(c) $\frac{1}{2} [f(x)]^2$

- (b) $g(x)$
☒ (d) $\frac{1}{2} [g(x)]^2$

11. Given $\int 2^x dx = f(x) + C$, then $f(x)$ is

- (a) 2^x
☒ (c) $\frac{2^x}{\log_e 2}$

- (b) $2^x \log_e 2$
(d) $\frac{2^{x+1}}{x+1}$

12. Area bounded by the curve $y = \sin x$ and the x -axis between $x=0$ and $x=2\pi$ is

- ☒ (a) 2 sq units
(c) 3 sq units

- (b) 0 sq units
☒ (d) 4 sq units

13. The area of the region bounded by the curve $y = 1/x$, the x -axis and between $x=1$ to $x=6$ is

- (a) $1/36$ sq units
(c) $\log_e 6$ sq units

- ☒ (b) $1/6$ sq units
☒ (d) $-\log_e 6$ sq units

14. Degree of differential equation $\left(\frac{d^2 y}{dx^2}\right)^{\frac{2}{3}} = x$ is

- (a) 2
(c) 3

- (b) 1
☒ (d) $2/3$

15. $\int \sec^2(4-3x) dx$ is equal to

- ☒ (a) $\tan^2(4-3x) + C$
(c) $\frac{1}{4} \tan(4-3x) + C$

- (b) $x + \tan(4-3x) + C$
(d) $-\frac{1}{3} \tan(4-3x) + C$

16. Mathematically, a vector is defined as a

- (a) line segment
(c) line

- (b) directed line segment
(d) ray

17. Given $P(A)=0.4$, $P(B)=0.7$ and $P(B|A)=0.6$, then $P(A \cup B) =$

- (a) 1.1
(c) 0.46

- (b) 0.86
(d) 0.16

18. Let A and B be two given events such that $P(A)=0.6$, $P(B)=0.2$, and $P(A|B)=0.5$. Then $P(A|B^c) =$

- (a) 1
(c) $3/8$

- (b) $3/10$
(d) $6/7$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.)

- (A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is not the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

19. **Assertion (A):** The domain of $\cos^{-1}(3x-1)$ is $[0, 2/3]$.

Reason (R): Domain of $\cos^{-1}x$ is $[-1, 1]$.

20. **Assertion (A):** A Linear programming problem is to optimise objective functions.

Reason (R): Maximum or minimum of objective function lies at the corner points of bounded feasible region.

SECTION-B

This section comprises of 5 very short answer (VSA) type questions of 2 marks each.

21. Find the scalar and vector components of the vector with initial point (2, 1) and terminal point (-5, 7).

[2]

22A. If $A = \begin{bmatrix} 3 & 0 & 1 \\ 2 & 1 & 5 \\ 1 & -1 & 0 \end{bmatrix}$, find $(A^3 - 12A)$.

[2]

OR

22B. If $A = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find $(A + 2B)^t$.

23. Evaluate: $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(\frac{\pi}{2}\right)\right)$

[2]

24. Show if the function f given by $f(x) = 2x^2 - 3x + 4$, $x \in \mathbb{R}$ is increasing on \mathbb{R} or not.

[2]

25A. A and B take turns in throwing two dice; the first to throw 9 is awarded a prize. If A throws first, what is the chance that B gets the prize? [2]

OR

25B. Two dice are thrown. Find the probability that the numbers that appeared have a sum of 8 if it is known that the second die always exhibits 4.

SECTION C

This section comprises of 6 short answer (SA) type questions of 3 marks each.

✓ 26. If $\begin{bmatrix} x+3y & y \\ 7-x & 4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$, find the values of x and y . [3]

✓ 27A. Show that the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-4\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear. [3]

OR

27B. Show that the points A $(-2\hat{i} + 3\hat{j} + 5\hat{k})$, B $(\hat{i} + 2\hat{j} + 3\hat{k})$ and C $(7\hat{i} - \hat{k})$ are collinear.

✓ 28. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1} and hence prove that $A^2 - 4A - 5I = 0$. [3]

✓ 29A. If $x = at^2$, $y = 2at$, then find $\frac{d^2y}{dx^2}$ at $t=2$. [3]

OR

29B. If $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ find $\frac{d^2y}{dx^2}$.

30. Find the area bounded by the lines $y=2x+1$, $y=3x+1$, and $x=4$ using integration. [3]

31A. Evaluate $\int \frac{x}{\sqrt{x+a}\sqrt{x+b}} dx$

OR

31B. Evaluate $\int \log(1+x^2) dx$ [3]

SECTION D

This section comprises of 4 long answer (LA) type questions of 5 marks each

32. Solve the differential equation: $y + \frac{d}{dx}(xy) = x(\sin x + x)$ [5]

33A. Find $\int \frac{2+\sin 2x}{1+\cos 2x} e^x dx$ [5]

OR

33B. Evaluate $\int_0^{\frac{\pi}{4}} \frac{1}{\sin x + \cos x} dx$

✓ 34. Solve the following linear programming problem graphically [5]

Maximise $Z = x + 2y$

Subject to constraints

$$x + 2y \geq 100, 2x - y \leq 0, 2x + y \leq 200, x \geq 0, y \geq 0$$

- 35A. Find the shortest distance between the lines

[5]

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$$

OR

- 35B. Find the distance from the point $P(3, -8, 1)$ to the line $\frac{x-3}{3} = \frac{y+7}{-1} = \frac{z+2}{5}$

SECTION E

This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (I), (II), (III) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each

36. Read the following passage and answer the following questions:

A city's traffic management department is planning to optimize traffic flow by analyzing the connectivity between various traffic signals. The city has five major spots labelled A, B, C, D, and E.



The department has collected the following data regarding one-way traffic flow between spots:

1. Traffic flows from A to B, A to C, and A to D.
2. Traffic flows from B to C and B to E.
3. Traffic flows from C to E.
4. Traffic flows from D to E and D to C.

The department wants to represent and analyze this data using relations and functions. Use the given data to answer the following questions:

- (i) Is the traffic flow reflexive? Justify. [1]
- (ii) Is the traffic flow transitive? Justify. [1]
- (iii) (a) Represent the relation describing the traffic flow as a set of ordered pairs. Also state the domain and range of the relation.

OR

- (iii) (b) Does the traffic flow represent a function? Justify your answer. [2]

37. Read the following passage and answer the questions given below.

For the diagnosis of Tuberculosis (TB), testing is very important. On testing, the probability that a person is diagnosed correctly, when a person is actually suffering from TB is 0.99. The

probability that doctor diagnoses incorrectly that a person is suffering from TB is 0.001. In a certain city it was detected that there is 0.001 chance that a person suffers from TB.



- (i) If the population of city is 200,000, then how many persons are expected to suffer from TB? [1]
- (ii) What is the probability that a person is diagnosed correctly for TB? [1]
- (iii) (a) What is the probability that a person actually has a TB, when he is diagnosed to have TB?

OR

- (b) Find the probability that error occurred in diagnosing the TB. [2]

38. Read the following passage and answer the questions given below.

XYZ Ltd. wants to manufacture right circular cylindrical dustbin with height H and radius R which is open at the top and has given surface area.



- (i) Find the surface area of dustbin in terms of radius at critical point where its volume is maximum. [2]
- (ii) Find the relationship between height and radius of the dustbin, when the volume is maximum. [2]