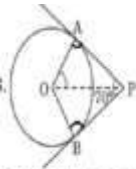
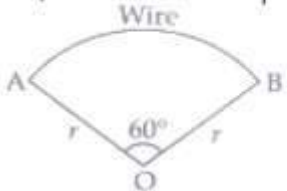


KENDRIYA VIDYALAYA SANGATHAN
BHOPAL REGION
10TH PREBOARD 1
MATHS SOLUTION (SET 1)
SECTION A

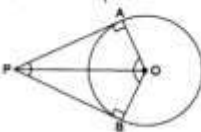
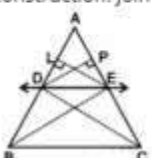
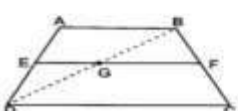
- | | | | | |
|---------------------|-----------------------------------|--------------------|-----------------------------|-----------------------------|
| 1. (b) 0 | 2. (b) $a < 0, b < 0$ and $c > 0$ | 3. (c) no solution | 4. (c) -14 | 5. (c) 3 |
| 6. (b) $2\sqrt{10}$ | 7. (a) (-1, 2) | 8. (a) 50^0 | 9. (d) 130^0 | 10. (b) $10\sqrt{2}$ |
| 11. (c) 9 | 12. (d) 1 | 13. (d) 100 m | 14. (c) 31.5 cm^2 | 15. (c) $2\pi/3 \text{ cm}$ |
| 16. (c) $2/7$ | 17. (d) $1/3$ | 18. (a) 15 | 19. (d) | 20. (a) |

SECTION B

21.	<p>The prime factorization of 2520 are $2520 = 2^3 \times 3^2 \times 5 \times 7$ -----(1) It is given that $2520 = 2^3 \times 3^p \times q \times 7$ -----(2) On comparing equation (1) and (2) we get $p = 2$ and $q = 5$</p> <p style="text-align: right;">OR</p> <p>Clearly, the maximum number of columns = HCF (612, 48).</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> $\begin{array}{r} 2 \overline{) 612} \\ 2 \overline{) 306} \\ 3 \overline{) 153} \\ 3 \overline{) 51} \\ 17 \end{array}$ </div> <div style="text-align: center;"> $\begin{array}{r} 2 \overline{) 48} \\ 2 \overline{) 24} \\ 2 \overline{) 12} \\ 2 \overline{) 6} \\ 3 \end{array}$ </div> </div> <p>Now, $612 = 2 \times 2 \times 3 \times 3 \times 17 = (2^2 \times 3^2 \times 17)$ and $48 = 2 \times 2 \times 2 \times 2 \times 3 = (2^4 \times 3)$. \therefore HCF (612, 48) = $(2^2 \times 3) = (4 \times 3)$ \therefore HCF (612, 48) = 12. \therefore Maximum number of columns in which they can march = 12.</p>	<p>0.5</p> <p>0.5</p> <p>1</p> <p>1</p> <p>0.5</p> <p>0.5</p>
22.	<p>In $\triangle ABE$, we have $DF \parallel AE$, then $\frac{BD}{AD} = \frac{BF}{FE}$ [By BPT] (i) In $\triangle ABC$, we have $DE \parallel AC$, then $\frac{BD}{AD} = \frac{BE}{EC}$ [By BPT] (2) From (i) and (2), We get $\frac{BF}{FE} = \frac{BE}{EC}$</p>	<p>0.5</p> <p>0.5</p> <p>1</p>
23.	<div style="display: flex; align-items: flex-start;"> <div style="flex: 1;">  <p>PA and PB are tangents to the circle. We know that the tangent to a circle is perpendicular to the radius through the point of contact.</p> </div> <div style="flex: 2;"> <p>Thus, $OA \perp PA$ and $OB \perp PB$ $\therefore \angle OBP = 90^0$ and $\angle OAP = 90^0$ In quadrilateral AOBP, $\angle OAP + \angle APB + \angle PBO + \angle BOA = 360^0$ (Sum of all interior angles of quadrilateral is 360^0) $90^0 + 70^0 + 90^0 + \angle BOA = 360^0$ $\angle BOA = 110^0$ In $\triangle OPA$ and $\triangle OPB$, $AP = BP$ (Tangents from a point outside the circle are equal in length) $OA = OB$ (Radii of the same circle) $OP = OP$ (Common side) $\therefore \triangle OPA \cong \triangle OPB$ (SSS congruence criterion) $\Rightarrow \angle POA = \angle POB$ $\Rightarrow \angle POA = \frac{1}{2} \angle AOB = \frac{110^0}{2} = 55^0$</p> </div> </div>	<p>0.5</p> <p>1</p> <p>0.5</p>

24.	<p>We have, $\cos (A - B) = \frac{\sqrt{3}}{2}$ $\Rightarrow \cos(A-B) = \cos 30^\circ$ $A - B = 30^\circ \dots\dots(i)$ Again, $\sin (A + B) = \frac{\sqrt{3}}{2}$ $\Rightarrow \sin(A+B) = \sin 60^\circ$ $A + B = 60^\circ \dots\dots(ii)$ Adding, (i) and (ii), $2A = 90^\circ$ $\therefore A = 45^\circ$ Put $A = 45^\circ$ in (ii), $B = 60^\circ - A = 60^\circ - 45^\circ = 15^\circ$ Therefore, $A = 45^\circ$ and $B = 15^\circ$</p> <p style="text-align: right;">OR</p> $2(\sin^2 45^\circ + \cot^2 30^\circ) - 6(\cos^2 45^\circ - \tan^2 30^\circ)$ $\left[\left(\frac{1}{\sqrt{2}} \right)^2 + (\sqrt{3})^2 \right] - 6 \left[\left(\frac{1}{\sqrt{2}} \right)^2 - \left(\frac{1}{\sqrt{3}} \right)^2 \right]$ $= 7 - 1 = 6$	<p>0.5</p> <p>0.5</p> <p>0.5</p> <p>0.5</p> <p>1</p> <p>1</p>
25.	<p>Arc is a part of circle that makes 60° between radii at end points A and B of wire. So, it form the shape of a sector.</p>  <p>$r = ?$ $l = 20$ cm \therefore Length of arc $l = \frac{2\pi r\theta}{360^\circ}$ $\Rightarrow 20 \text{ cm} = \frac{2 \times \pi \times r \times 60^\circ}{360^\circ}$ $\Rightarrow 2\pi r = 20 \times 6$ $\Rightarrow r = \frac{120}{2\pi} = \frac{60}{\pi} \text{ cm}$ Hence, radius $(r) = \frac{60}{\pi} \text{ cm}$.</p>	<p>0.5</p> <p>0.5</p> <p>1</p>
SECTION C		
26.	<p>Let us prove $\sqrt{5}$ irrational by contradiction. Let us suppose that $\sqrt{5}$ is rational. It means that we have co-prime integers a and b ($b \neq 0$) Such that $\sqrt{5} = a/b$ Squaring both sides, we get $5b^2 = a^2 \dots (1)$ It means that 5 is factor of a^2. Hence, 5 is also factor of a by Theorem. ... (2) If, 5 is factor of a, it means that we can write $a = 5c$ for some integer c. Substituting value of a in (1) $5b^2 = 25c^2 \Rightarrow b^2 = 5c^2$ It means that 5 is factor of b^2. Hence, 5 is also factor of b by Theorem. ... (3) From (2) and (3), we can say that 5 is factor of both a and b. But, a and b are co-prime. Therefore, our assumption was wrong. $\sqrt{5}$ cannot be rational. Hence, it is irrational.</p>	<p>0.5</p> <p>0.5</p> <p>0.5</p> <p>0.5</p> <p>1</p>

27.	<p>Let the required polynomial be $ax^2 + bx + c$ and let its zeroes be α and β</p> <p>Then, $\alpha + \beta = \frac{1}{4} = -\frac{b}{a}$ and $\alpha\beta = -1 = \frac{c}{a}$</p> <p>If $a = 4$, then $b = -1$ and $c = -4$</p> <p>So, one quadratic polynomial which satisfies the given conditions is $4x^2 - x - 4$</p> <p>Or</p> <p>If α and β zeroes of the polynomials then standard quadratic polynomial is given by $x^2 - (\alpha + \beta)x + \alpha\beta$, where $\alpha + \beta = \frac{1}{4}$ and $\alpha\beta = -1$ [Given]</p> <p>Now, we have,</p> $x^2 - (\alpha + \beta)x + \alpha\beta$ $= x^2 - \left(\frac{1}{4}\right)x + (-1)$ $= \frac{1}{4}(4x^2 - x - 4)$ <p>Required polynomial is $4x^2 - x - 4$</p>	1.5
		1.5
		1
		1
		1
28.	<p>When 3 coins are tossed simultaneously, all possible outcomes are HHH, HHT, HTH, THH, HTT, THT, TTH, TTT.</p> <p>Total number of possible outcomes = 8.</p> <p>i. Let E_1 be the event of getting exactly 2 heads.</p> <p>Then, the favourable outcomes are HHT, HTH, THH.</p> <p>Number of favourable outcomes = 3.</p> <p>$\therefore P(\text{getting exactly 2 heads}) = P(E_1) = \frac{3}{8}$.</p> <p>ii. Let E_2 be the event of getting at least 2 heads.</p> <p>Then, E_2 is the event of getting 2 or 3 heads.</p> <p>So, the favourable outcomes are HHT, HTH, THH, HHH.</p> <p>Number of favourable outcomes = 4.</p> <p>$\therefore P(\text{getting at least 2 heads}) = \frac{4}{8} = \frac{1}{2}$.</p> <p>iii. Let E_3 be the event of getting at most 2 heads.</p> <p>Then, E_3 is the event of getting 0 or 1 head or 2 heads.</p> <p>So, the favourable outcomes are TTT, HTT, THT, TTH, HHT, HTH, THH.</p> <p>Number of favourable outcomes = 7.</p> <p>$\therefore P(\text{getting at most 2 heads}) = P(E_3) = \frac{7}{8}$.</p>	1
		1
		1
29.	<p>Let the fixed charges be Rs x and additional charges be Rs. y per km.</p> <p>Then charges paid for journey of 12km = $x + 12y$</p> <p>Atq:</p> $x + 12y = 89 \quad \text{-----(i)}$ <p>And, charges paid for journey of 20km = $x + 20y$</p> <p>Atq:</p> $x + 20y = 145 \quad \text{-----(ii)}$ <p>Subtracting eq.(i) from (ii)</p> $x + 20y - (x + 12y) = 145 - 89$ $\Rightarrow 8y = 56$ $\Rightarrow y = 7$ <p>By eq.(i) $x + 12 \times 7 = 89$</p> $\Rightarrow x = 5$ <p>Now, charges paid for a journey of 30 km = $x + 30y = 5 + 30 \times 7 = \text{Rs. } 215$</p> <p>OR</p> <p>Let tens digit be x and unit digit be y.</p> <p>Then the number = $10x + y$</p> <p>and the number formed by interchanging the digits = $10y + x$</p> <p>Given that: $x + y = 15$ -----(i)</p> <p>and</p> $10y + x = 10x + y + 9$ $\Rightarrow 9x - 9y = -9$ $\Rightarrow x - y = -1 \quad \text{-----(ii)}$ <p>Solving eq (i) and (ii) $x = 7$ and $y = 8$</p> <p>So the number = $10 \times 7 + 8 = 78$</p>	1
		1
		0.5
		0.5
		1
		0.5
		0.5

30.	<p>We have, $p = \sin\theta + \cos\theta$ and $q = \sec\theta + \operatorname{cosec}\theta$</p> <p>$\therefore \text{LHS} = q(p^2 - 1) = (\sec\theta + \operatorname{cosec}\theta) \{(\sin\theta + \cos\theta)^2 - 1\}$</p> <p>$= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right) \{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1\}$</p> <p>$= \left(\frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta}\right) (1 + 2\sin\theta\cos\theta - 1)$</p> <p>$= \left(\frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta}\right) (2\sin\theta\cos\theta) = 2(\sin\theta + \cos\theta) = 2p = \text{RHS}$</p>	1 1 1																																																
31.	<p>$\angle OAP = 90^\circ$(1) [Angle between tangent and radius through the point of contact is 90°]</p> <p>$\angle OBP = 90^\circ$(2) [Angle between tangent and radius through the point of contact is 90°]</p> <p>\therefore OAPB is quadrilateral</p>  <p>$\therefore \angle APB + \angle AOB + \angle OAP + \angle OBP = 360^\circ$ [Angle sum property of a quadrilateral]</p> <p>$\Rightarrow \angle APB + \angle AOB + 90^\circ + 90^\circ = 360^\circ$ [From (1) and (2)]</p> <p>$\Rightarrow \angle APB + \angle AOB = 180^\circ$</p> <p>$\Rightarrow \angle APB$ and $\angle AOB$ are supplementary</p> <p style="text-align: right;">OR</p> <p>We know that the lengths of tangents drawn from an exterior point to a circle are equal</p> <p>$\therefore AE = AF = x$ cm (say);</p> <p>$BD = BF = 6$ cm;</p> <p>$CD = CE = 8$ cm</p> <p>And so, $AB = AF + BF = (x + 6)$ cm; $BC = BD + CD = 14$ cm;</p> <p>$CA = CE + AE = (x + 8)$ cm.</p> <p>Join OE and OF and also OA, OB and OC.</p> <p>$\therefore \text{ar}(\triangle ABC) = \text{ar}(\triangle OAB) + \text{ar}(\triangle OBC) + \text{ar}(\triangle OCA)$</p> <p>$\Rightarrow 63 = \left(\frac{1}{2} \times AB \times OF\right) + \left(\frac{1}{2} \times BC \times OD\right) + \left(\frac{1}{2} \times CA \times OE\right)$</p> <p>$\Rightarrow 63 = \left\{\frac{1}{2} \times (x + 6) \times 3\right\} + \left\{\frac{1}{2} \times 14 \times 3\right\} + \left\{\frac{1}{2} \times (x + 8) \times 3\right\}$</p> <p>$\Rightarrow 63 = \frac{3}{2} \times (2x + 28) \Rightarrow x = 7$</p> <p>$\therefore AB = (x + 6)$ cm $= (7 + 6)$ cm $= 13$ cm</p>	0.5 0.5 0.5 1 0.5 0.5 1 1 0.5																																																
SECTION D																																																		
32.	<table><thead><tr><th>Class</th><th>x_i</th><th>f_i</th><th>$u_i = \frac{x_i - 97.5}{5}$</th><th>$f_i u_i$</th><th>c.f.</th></tr></thead><tbody><tr><td>85 - 90</td><td>87.5</td><td>10</td><td>-2</td><td>-20</td><td>10</td></tr><tr><td>90 - 95</td><td>92.5</td><td>12</td><td>-1</td><td>-12</td><td>22</td></tr><tr><td>95 - 100</td><td>97.5</td><td>15</td><td>0</td><td>0</td><td>37</td></tr><tr><td>100 - 105</td><td>102.5</td><td>14</td><td>1</td><td>14</td><td>51</td></tr><tr><td>105 - 110</td><td>107.5</td><td>12</td><td>2</td><td>24</td><td>63</td></tr><tr><td>110 - 115</td><td>112.5</td><td>7</td><td>3</td><td>21</td><td>70</td></tr><tr><td></td><td></td><td></td><td></td><td>27</td><td></td></tr></tbody></table> <p>Mean $= 97.5 + \left(5 \times \frac{27}{70}\right) = 99.4$</p> <p>$f = 15$, c.f. = 22, $l = 95$</p> <p>Median class : 95 - 100</p> <p>Median $= 95 + \frac{5}{15} (35 - 22) = 99.3$</p>	Class	x_i	f_i	$u_i = \frac{x_i - 97.5}{5}$	$f_i u_i$	c.f.	85 - 90	87.5	10	-2	-20	10	90 - 95	92.5	12	-1	-12	22	95 - 100	97.5	15	0	0	37	100 - 105	102.5	14	1	14	51	105 - 110	107.5	12	2	24	63	110 - 115	112.5	7	3	21	70					27		2.5 1 0.5 1
Class	x_i	f_i	$u_i = \frac{x_i - 97.5}{5}$	$f_i u_i$	c.f.																																													
85 - 90	87.5	10	-2	-20	10																																													
90 - 95	92.5	12	-1	-12	22																																													
95 - 100	97.5	15	0	0	37																																													
100 - 105	102.5	14	1	14	51																																													
105 - 110	107.5	12	2	24	63																																													
110 - 115	112.5	7	3	21	70																																													
				27																																														
33.	<p>Given : A triangle ABC, $DE \parallel BC$, intersecting AB at D and AC at E.</p> <p>To Prove: $\frac{AD}{DB} = \frac{AE}{EC}$</p> <p>Construction: Join BE, CD and draw $EL \perp AD$ and $DP \perp AE$.</p>  <p>Proof: $\triangle BDE$ and $\triangle CDE$ are on the same base DE and between the same parallel lines BC and DE,</p> <p>Hence $\text{ar}(\triangle BDE) = \text{ar}(\triangle CDE)$(i)</p> <p>Now, $\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle BDE)} = \frac{\frac{1}{2} \cdot AD \times EL}{\frac{1}{2} \cdot BD \times EL} = \frac{AD}{BD}$(ii)</p> <p>Similarly, $\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle CDE)} = \frac{\frac{1}{2} \cdot AE \times DP}{\frac{1}{2} \cdot EC \times DP} = \frac{AE}{EC}$(iii)</p> <p>From (i), eq (iii) becomes</p> <p>$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle BDE)} = \frac{AE}{EC}$ (iv)</p> <p>From (ii) and (iv) we get,</p> <p>$\Rightarrow \frac{AD}{BD} = \frac{AE}{EC}$</p> <p>Hence proved</p> <p>Consider the given trapezium ABCD. Join BD intersecting EF at G.</p>  <p>It is proved above that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. So,</p> <p>In $\triangle DAB$, $EG \parallel AB$,</p> <p>$\therefore \frac{AE}{DE} = \frac{BG}{GD}$(v)</p> <p>In $\triangle BCD$, $GF \parallel DC$</p> <p>$\therefore \frac{BG}{GD} = \frac{BF}{FC}$(vi)</p> <p>From (v) and (vi) we get,</p> <p>$\frac{AE}{DE} = \frac{BF}{FC}$</p> <p>Hence proved</p>	Given, to prove, diagram (1) 1 1 1																																																

i.



Let BC be the tower of height h and CD be the water tank of height h_1

In $\triangle ABD$, we have

$$\tan 45^\circ = \frac{BD}{AB}$$

$$\Rightarrow 1 = \frac{h+h_1}{40}$$

$$\Rightarrow h + h_1 = 40 \quad \dots (1)$$

In $\triangle ABC$, we have

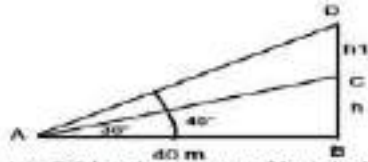
$$\tan 30^\circ = \frac{BC}{AB}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{40}$$

$$\Rightarrow h = \frac{40}{\sqrt{3}} = \frac{40\sqrt{3}}{3} = 23.1 \text{ m}$$

Thus height of the tower is 23.1 m.

ii.



Let BC be the tower of height h and CD be the water tank of height h_1

In $\triangle ABD$, we have

$$\tan 45^\circ = \frac{BD}{AB}$$

$$\Rightarrow 1 = \frac{h+h_1}{40}$$

$$\Rightarrow h + h_1 = 40 \quad \dots (1)$$

In $\triangle ABC$, we have

$$\tan 30^\circ = \frac{BC}{AB}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{40}$$

$$\Rightarrow h = \frac{40}{\sqrt{3}} = \frac{40\sqrt{3}}{3} = 23.1 \text{ m}$$

Thus height of the tower is 23.1 m.

Substituting the value of h in (1), we have

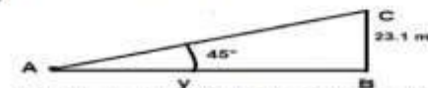
$$23.1 + h_1 = 40$$

$$\Rightarrow h_1 = 40 - 23.1$$

$$= 6.9 \text{ m}$$

Thus height of the tank is 6.9 m.

iii.



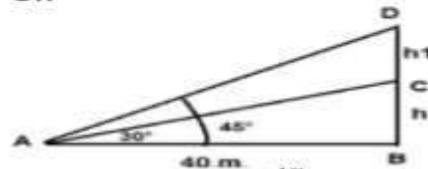
In the $\triangle ABC$ if $\angle CAB = 45^\circ$ then

$$\cot 45^\circ = \frac{y}{23.1} = 1$$

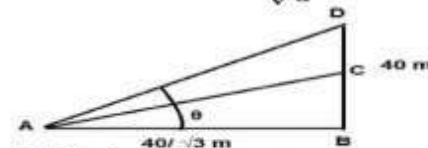
$$y = 23.1 \text{ m}$$

Thus the angle of elevation will be 45° at 23.1 m.

OR



$$\text{Given that } AB = \frac{40}{\sqrt{3}}$$



In the $\triangle ABD$

$$\cot \theta = \frac{AB}{BD} = \frac{\frac{40}{\sqrt{3}}}{40}$$

$$\cot \theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = 60^\circ$$

Hence the angle of elevation would be 60° .

1

1

2

2