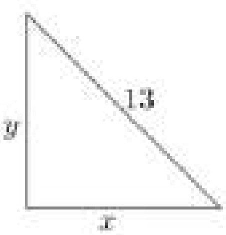
CRPF PUBLIC SCHOOL, ROHINI, DELHI
PRE-BOARD - 1 EXAMINATION (2025-26)
CLASS XII
MATHEMATICS (SET-A)
MARKING SCHEME

SECTION – A (MCQ) 1 Mark Questions	
Q1	(B) $\left[\frac{-\pi}{2}, 0\right]$
Q2	(A) -1
Q3	(D) 8
Q4	(B) ± 4
Q5	(B) 3
Q6	(D) 49
Q7	(C) $-(1+e)\sin e$
Q8	(B) $x = e^{-1}$
Q9	(A) $2(\sin x + x\cos\alpha) + C$
Q10	(D) $\sin^{-1}\left(\frac{e^x}{2}\right) + C$
Q11	(C) 1
Q12	(D) $\int_0^4 \sqrt{y}\, dy$
Q13	(C) order 2, degree not defined
Q14	(B) 18
Q15	(A) $90°$
Q16	(B) $a = \frac{b}{2}$
Q17	(C) 120
Q18	(D) $\frac{1}{5}$
Q19	(a) Note that $\cos^{-1}\left(\cos\frac{13\pi}{6}\right) = \cos^{-1}\left(\cos\left(2\pi+\frac{\pi}{6}\right)\right) = \cos^{-1}\left(\cos\frac{\pi}{6}\right) = \frac{\pi}{6}$. So, A and R both are true and R is the correct explanation of A.
Q20	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

SECTION – B (Very Short Answer (VSA)-type questions) 2 Marks Each		
Q21	Put $x = \tan^2\theta \Rightarrow \theta = \tan^{-1}\sqrt{x}$ RHS $= \frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$ $= \frac{1}{2}\cos^{-1}(\cos 2\theta)$ $= \frac{1}{2}(2\theta)$ $= \theta = \tan^{-1}\sqrt{x} =$ LHS **OR** Let $\tan^{-1}\frac{1}{2} = \theta \Rightarrow \tan\theta = \frac{1}{2}$ and $\cot^{-1}\frac{1}{3} = \alpha \Rightarrow \cot\alpha = \frac{1}{3}$ As $\sec^2\theta = 1+\tan^2\theta = 1+\left(\frac{1}{2}\right)^2 = \frac{5}{4}$, $\operatorname{cosec}^2\alpha = 1+\cot^2\alpha = 1+\left(\frac{1}{3}\right)^2 = \frac{10}{9}$ $\therefore \sec^2\left(\tan^{-1}\frac{1}{2}\right)+\operatorname{cosec}^2\left(\cot^{-1}\frac{1}{3}\right) = \sec^2\theta+\operatorname{cosec}^2\alpha = \frac{5}{4}+\frac{10}{9} = \frac{45+40}{36} = \frac{85}{36}$.	½ 1 ½
Q22	$\tan^{-1}\left(x^2+y^2\right) = a^2 \Rightarrow x^2+y^2 = \tan a^2$ **Differentiate both sides wrt x,** $2x+2y\frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = -\frac{x}{y}$ **OR** $\boxed{x^y = e^{x-y}}$ Taking log on both the sides, we get $\log x^y = \log e^{x-y}$ $\Rightarrow y\log x = (x-y)\log e$ $\Rightarrow y\log x = (x-y)$ $[\because \log e = 1$ $\Rightarrow y(1+\log x) = x$ or, $y = \frac{x}{(1+\log x)}$ Now $\frac{dy}{dx} = \frac{(1+\log x)\times 1 - x\left(0+\frac{1}{x}\right)}{(1+\log x)^2} = \frac{(1+\log x)-1}{(\log e+\log x)^2}$ Hence, $\frac{dy}{dx} = \frac{\log x}{\{\log(xe)\}^2}$.	½ 1 ½
Q23	$x^2+y^2 = 169$ Differentiate both sides w.r.t. t $2x\frac{dx}{dt}+2y\frac{dy}{dt} = 0$ $\Rightarrow 12(2)+5\left(\frac{dy}{dt}\right) = 0$ $[\because$ when $x = 12m, y = 5m]$ $\Rightarrow \frac{dy}{dt} = -\frac{24}{5}$ Hence, the height decreases at the rate of $\frac{24}{5}$ m/s  	½ 1 ½

Q24	$\overrightarrow{BA} = -6\hat{i} + 2\hat{j} + 3\hat{k}$ **Required unit vector of magnitude 21** $= 21 \times \left(\dfrac{-6\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{36+4+9}} \right)$ $= 3\left(-6\hat{i} + 2\hat{j} + 3\hat{k}\right)$ or $-18\hat{i} + 6\hat{j} + 9\hat{k}$	1 ½ ½
Q25	$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{vmatrix} = -2\hat{i} + 3\hat{j} + 7\hat{k}$ **Area of parallelogram** $= \dfrac{1}{2}\left|\vec{a} \times \vec{b}\right|$ $= \dfrac{1}{2}\sqrt{(-2)^2 + 3^2 + 7^2} = \dfrac{\sqrt{62}}{2}$ **OR** Let $\vec{d} = \vec{b} + \vec{c} = (2+\lambda)\hat{i} - 6\hat{j} + 2\hat{k}$ $\hat{d} = \dfrac{(2+\lambda)\hat{i} - 6\hat{j} + 2\hat{k}}{\sqrt{(2+\lambda)^2 + 40}}$ $\vec{a} \cdot \hat{d} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) \cdot \dfrac{(2+\lambda)\hat{i} - 6\hat{j} + 2\hat{k}}{\sqrt{(2+\lambda)^2 + 40}} = 1$ $\Rightarrow (2+\lambda) + 6 + 4 = \sqrt{(2+\lambda)^2 + 40} \Rightarrow \lambda = -5$	1 1 ½ ½ 1
	SECTION – C (Short Answer (SA)-type questions) 3 Marks Each	
Q26	$f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51 \Rightarrow f'(x) = 6x^3 - 12x^2 - 90x$ For critical points $f'(x) = 0$ $\Rightarrow 6x^3 - 12x^2 - 90x = 0$ $\Rightarrow 6x(x^2 - 2x - 15) = 0$ $\Rightarrow 6x(x-5)(x+3) = 0 \Rightarrow x = -3, 0, 5$  $f(x)$ is strictly increasing in $(-3,0)$ *and* $(5,\infty)$ $(f(x)$ is strictly decreasing in $(-\infty,-3)$ *and* $(0,5)$ Note: Closed intervals are also acceptable. **OR**	1 1 1

$$f(x) = x + \frac{1}{x}$$

$$\Rightarrow f'(x) = 1 - \frac{1}{x^2},\ f''(x) = \frac{2}{x^3}$$

For $f'(x) = 1 - \frac{1}{x^2} = 0 \Rightarrow x^2 = 1 \qquad \therefore x = -1, 1$

Note that $f''(x=1) = \frac{2}{1^3} = 2 > 0,\ f''(x=-1) = \frac{2}{(-1)^3} = -2 < 0$.

So, f is minimum at $x = 1$ and maximum at $x = -1$.

Therefore, the maximum value of f is $M = f(-1) = -1 + \frac{1}{-1} = -2$; and the minimum value of f is

$m = f(1) = 1 + \frac{1}{1} = 2$.

Hence, $(M - m) = -2 - 2 = -4$.

Q27

Required Area

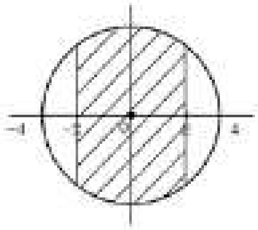
$= 4\int_0^2 \sqrt{16 - x^2}\,dx$ 1

$= 4\left[\frac{x}{2}\sqrt{4^2 - x^2} + \frac{4^2}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^2$ 1

$= 4\left[\left(\sqrt{12} - 0\right) + 8\left(\sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1} 0\right)\right]$

$= 4\left(2\sqrt{3} + 8 \times \frac{\pi}{6}\right)$

$= \left(8\sqrt{3} + \frac{16\pi}{3}\right)$ 1



Q28

Given D.E. is $\frac{dy}{dx} + \frac{2x}{1+x^2}\,y = \frac{4x^2}{1+x^2}$

Integrating factor is $e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = (1 + x^2)$ 1

Solution is $y(1 + x^2) = \int 4x^2 dx + C$

$y(1 + x^2) = \frac{4x^3}{3} + C$ 1

$y(0) = 0$ gives $C = 0$, hence solution is $y(1 + x^2) = \frac{4x^3}{3}$ 1

OR

$$\frac{dy}{dx} = \frac{y}{x} - \operatorname{cosec}\left(\frac{y}{x}\right)$$

Put $\frac{y}{x} = v$ i.e. $y = vx$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ ½

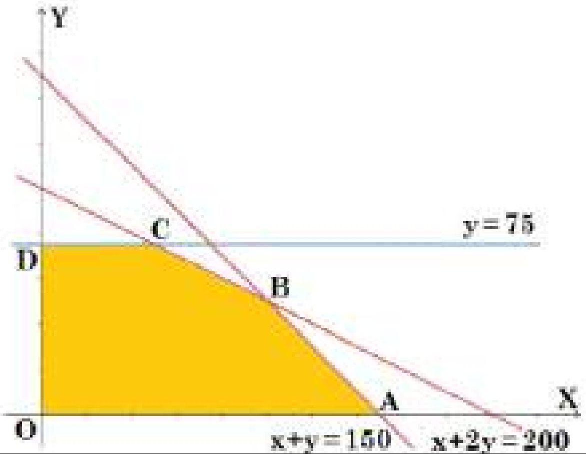
The differential equation reduces to

$v + x\frac{dv}{dx} = v - \operatorname{cosec} v$ ½

	$\sin v\,dv = -\frac{1}{x}dx \Rightarrow \int \sin v\,dv = -\int \frac{dx}{x}$ $\Rightarrow -\cos v = -\log\lvert x\rvert - C$ $\text{or, } \cos v = \log\lvert x\rvert + C'$ $\Rightarrow \cos\left(\frac{y}{x}\right) = \log\lvert x\rvert + C'$ $y=0, x=1 \text{ gives } C'=1$ $\therefore$ Solution is given by $\cos\left(\frac{y}{x}\right) = \log\lvert x\rvert + 1$	 1 ½ ½
Q29	**The given line is** $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9} = \lambda$ **and** $Q(2,4,-1)$ **Any random point on the line will be given by** $P(\lambda-5, 4\lambda-3, -9\lambda+6)$ **Since** $PQ = 7 \Rightarrow \sqrt{(\lambda-7)^2 + (4\lambda-7)^2 + (-9\lambda+7)^2} = 7$ $\Rightarrow 98(\lambda^2 - 2\lambda + 1) = 0 \Rightarrow \lambda = 1$ **Hence, the required point is** $P(-4,1,-3)$ **The equation of line PQ is** $\frac{x+4}{6} = \frac{y-1}{3} = \frac{z+3}{2}$ **or** $\frac{x-2}{6} = \frac{y-4}{3} = \frac{z+1}{2}$	 1 1 1
Q30	Consider the graph shown below.	

Corner points	Value of Z
A(150, 0)	150
B(100, 50)	250
C(50, 75)	275 ← Max.
D(0, 75)	225
O(0, 0)	0

Therefore, the maximum value of Z is 275.



Q31	$P(\text{at most one of them will solve the problem})$ $= P(\text{none of them solves the problem}) + P(\text{only one of them solves the problem})$ $= \left(\frac{1}{2}\times\frac{2}{3}\times\frac{1}{3}\times\frac{4}{5}\right) + \left(\frac{1}{2}\times\frac{2}{3}\times\frac{1}{3}\times\frac{4}{5} + \frac{1}{2}\times\frac{1}{3}\times\frac{1}{3}\times\frac{4}{5} + \frac{1}{2}\times\frac{2}{3}\times\frac{2}{3}\times\frac{4}{5} + \frac{1}{2}\times\frac{2}{3}\times\frac{1}{3}\times\frac{1}{5}\right)$ $= \frac{19}{45}$	 2½ ½

OR

	E : 'a total of 8' and F : 'red die resulted in a number less than 4' *i.e.*, $E = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$ and $F = \{(x, y) : x \in \{1, 2, 3, 4, 5, 6\}, y \in \{1, 2, 3\}\}$ *i.e.*, $F = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2), (6, 3)\}$ Hence, $E \cap F = \{(5, 3), (6, 2)\}$, $P(E) = 5/36$, $P(F) = 18/36$, $P(E \cap F) = 2/36$ $\therefore$ Required probability $= P(E/F)$ $= \frac{P(E \cap F)}{P(F)} = \frac{2/36}{18/36} = \frac{2}{18} = \frac{1}{9}$.	
	SECTION – D (Long Answer (LA)-type questions) 5 Marks Each	
Q32	$\|A\| = 3(-2) - 4(1) + 1(5) = -5 \neq 0 \Rightarrow A^{-1}$ exists.	1
	$A_{11} = -2, A_{12} = -1, A_{13} = 3$ $A_{21} = -1, A_{22} = 2, A_{23} = -1$ $A_{31} = 5, A_{32} = -5, A_{33} = -5$ $adjA = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$	1 ½
	$A^{-1} = \frac{1}{\|A\|} adjA = -\frac{1}{5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$	1
	Given system of equations can be written as $AX = B$, where $B = \begin{bmatrix} 2000 \\ 2500 \\ 900 \end{bmatrix}$	
	$X = A^{-1}B$	½
	$= -\frac{1}{5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}\begin{bmatrix} 2000 \\ 2500 \\ 900 \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} -2000 \\ -1500 \\ -1000 \end{bmatrix} = \begin{bmatrix} 400 \\ 300 \\ 200 \end{bmatrix}$	½
	$\therefore x = 400, y = 300$ and $z = 200$	½

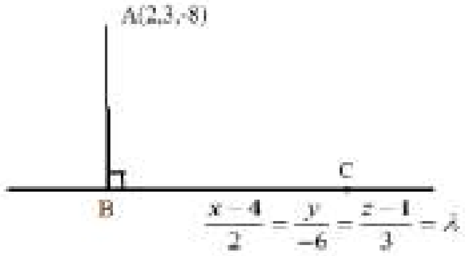
Q33	Let $x=\sin A, y=\sin B \Rightarrow A=\sin^{-1}x, B=\sin^{-1}y$	1
	$\therefore \sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$	
	$\Rightarrow \cos A+\cos B=a(\sin A-\sin B)$	
	$\Rightarrow 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)=2a\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$	1
	$\Rightarrow \cot\left(\frac{A-B}{2}\right)=a \Rightarrow A-B=2\cot^{-1}a$	1
	$\Rightarrow \sin^{-1}x-\sin^{-1}y=2\cot^{-1}a$	½
	differentiate both sides wrt x,	
	$\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=0$	1½
	$\Rightarrow \frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$	
	OR	
	$x=a\left(\cos\theta+\log\tan\frac{\theta}{2}\right)$	
	$\Rightarrow \frac{dx}{d\theta}=a\left(-\sin\theta+\frac{1}{\tan\frac{\theta}{2}}\times\sec^2\frac{\theta}{2}\times\frac{1}{2}\right)$	½
	$=a\left(-\sin\theta+\frac{1}{\sin\theta}\right)=a\left(\frac{1-\sin^2\theta}{\sin\theta}\right)$	½
	$\frac{dx}{d\theta}=a\cot\theta\cos\theta$	½
	Also, $y=\sin\theta \Rightarrow \frac{dy}{d\theta}=\cos\theta$	½
	$\therefore \frac{dy}{dx}=\frac{\tan\theta}{a}$	1
	Differetiating wrt x,	
	$\frac{d^2y}{dx^2}=\frac{\sec^2\theta}{a}\times\frac{d\theta}{dx}$	
	$=\frac{\sec^3\theta\tan\theta}{a^2}$	1
	$\left.\frac{d^2y}{dx^2}\right]_{\text{at }\theta=\frac{\pi}{4}}=\frac{2\sqrt{2}}{a^2}$	1

Q34	$I=\int\frac{dx}{\sin x+\sin 2x}$	
	$=\int\frac{dx}{\sin x(1+2\cos x)}$	½
	$=\int\frac{\sin x}{\sin^2 x(1+2\cos x)}dx$	½
	$=\int\frac{\sin x}{(1-\cos x)(1+\cos x)(1+2\cos x)}dx$	½
	Put $\cos x=t\Rightarrow\sin x\,dx=dt$	½
	$I=-\int\frac{dt}{(1-t)(1+t)(1+2t)}$	
	$=-\frac{1}{6}\int\frac{dt}{1-t}+\frac{1}{2}\int\frac{dt}{1+t}-\frac{4}{3}\int\frac{dt}{1+2t}$	1
	$=\frac{1}{6}\log\lvert 1-t\rvert+\frac{1}{2}\log\lvert 1+t\rvert-\frac{2}{3}\log\lvert 1+2t\rvert+C$	1½
	$=\frac{1}{6}\log\lvert 1-\cos x\rvert+\frac{1}{2}\log\lvert 1+\cos x\rvert-\frac{2}{3}\log\lvert 1+2\cos x\rvert+C$	½
	OR	
	$I=\int_0^{\pi/4}\frac{\sin x\cos x}{\cos^4 x+\sin^4 x}dx$	
	dividing numerator and denominator by $\cos^4 x$,	
	$I=\int_0^{\pi/4}\frac{\tan x\sec^2 x}{1+\tan^4 x}dx$	1
	Put $\tan^2 x=t\Rightarrow 2\tan x\sec^2 x\,dx=dt$	1
	when $x=0, t=0$; when $x=\frac{\pi}{4}, t=1$	
	$\Rightarrow I=\frac{1}{2}\int_0^1\frac{dt}{1+t^2}$	1½
	$=\frac{1}{2}\left[\tan^{-1}t\right]_0^1$	1
	$=\frac{\pi}{8}$	½
Q35	Given lines are	
	$l_1:\frac{x-1}{4}=\frac{y-2}{6}=\frac{z-3}{8}$ and $l_2:\frac{x-2}{2}=\frac{y-4}{3}=\frac{z-5}{4}$	
	DRs of two lines are 4, 6, 8 and 2, 3, 4	
	Here $\frac{4}{2}=\frac{6}{3}=\frac{8}{4}=2$, So, lines are parallel	1
	Given lines in vector form are	
	$l_1:\vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\lambda(4\hat{i}+6\hat{j}+8\hat{k})$ and $l_2:\vec{r}=2\hat{i}+4\hat{j}+5\hat{k}+\lambda(2\hat{i}+3\hat{j}+4\hat{k})$	
	Here	
	$\vec{a_1}=\hat{i}+2\hat{j}+3\hat{k}, \vec{a_2}=2\hat{i}+4\hat{j}+5\hat{k}, \vec{b}=2\hat{i}+3\hat{j}+4\hat{k}$	1
	$\vec{a_2}-\vec{a_1}=\hat{i}+2\hat{j}+2\hat{k}$	1

$(\vec{a_2}-\vec{a_1})\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 2\\ 2 & 3 & 4\end{vmatrix} = 2\hat{i}-\hat{k}$ — 1 ½

So, shortest distance $= \frac{|(\vec{a_2}-\vec{a_1})\times\vec{b}|}{|\vec{b}|} = \frac{\sqrt{5}}{\sqrt{29}}$ — ½

OR



Any point on line is $(2\lambda+4, -6\lambda, 3\lambda+1)$ for some λ. Let $B(2\lambda+4, -6\lambda, 3\lambda+1)$

d.r. of $AB = <2\lambda+2, -6\lambda-3, 3\lambda+9>$

d.r. of $BC = <2, -6, 3>$

$AB \perp BC \Rightarrow 2(2\lambda+2)-6(-6\lambda-3)+3(3\lambda+9)=0$

$\Rightarrow \lambda = -1$

$\therefore B(2,6,-2)$

Now, $AB=\sqrt{(2-2)^2+(6-3)^2+(-2+8)^2}=\sqrt{45}$ or $3\sqrt{5}$ units

SECTION – E (Case Study Based Questions) 4 Marks Each

Q36

Ans (i)	The number of relations $= 2^{4\times3} = 2^{12}$	1
Ans (ii)	Since, S_2 and S_3 have been assigned the same judge J_2, the function is not one-one. Hence, it is not bijective.	1
(iii) (a)	There cannot exist any one-one function from S to J as n(S) > n(J). Hence, the number of one-one functions from S to J is 0. OR	2
(iii) (b)	To make R_1 reflexive and not symmetric we need to add the following ordered pairs: $(S_1,S_1),(S_2,S_2),(S_3,S_3),(S_4,S_4)$	2

Q37

(i) $\pi r^2+2\pi rh=75\pi \Rightarrow h=\frac{75-r^2}{2r}$, $\therefore V=\pi r^2h=\frac{\pi}{2}(75r-r^3)$

(ii) $\frac{dV}{dr}=\frac{\pi}{2}(75-3r^2)$

(iii) $\frac{dV}{dr}=0\Rightarrow r=5, \left.\frac{d^2V}{dr^2}\right]_{r=5}=\frac{\pi}{2}(-6r)<0$ $\therefore$ volume is maximum when $r=5$

Or

False,

$\frac{dV}{dr}=0\Rightarrow r=5, \left.\frac{d^2V}{dr^2}\right]_{r=5}=\frac{\pi}{2}(-6r)<0$ $\therefore$ volume is maximum when $r=5$

As volume is maximum at $r=5\Rightarrow h=\frac{75-5^2}{2(5)}=5\Rightarrow h=r$

Q38	E_1 :customer avails loan on fixed rate E_2 :customer avails loan on floating rate E_3 :customer avails loan on variable rate A:the person defaults on the loan $P(E_1)=\frac{1}{10}, P(E_2)=\frac{2}{10}, P(E_3)=\frac{7}{10}$ $P(A\mid E_1)=\frac{5}{100}, P(A\mid E_2)=\frac{3}{100}, P(A\mid E_3)=\frac{1}{100}$ $(i) P(A)=P(E_1).P(A\mid E_1)+P(E_2).P(A\mid E_2)+P(E_3).P(A\mid E_3)$ $=\frac{1}{10}\times\frac{5}{100}+\frac{2}{10}\times\frac{3}{100}+\frac{7}{10}\times\frac{1}{100}$ $=\frac{18}{1000}$ or $\frac{9}{500}$ $(ii) P(E_3\mid A)=\frac{P(E_3).P(A\mid E_3)}{P(E_1).P(A\mid E_1)+P(E_2).P(A\mid E_2)+P(E_3).P(A\mid E_3)}$ $=\frac{\frac{7}{10}\times\frac{1}{100}}{\frac{18}{1000}}$ $=\frac{7}{18}$	 1 1 1 1