

First Pre Board (2025-26)

Class: 12

Subject: Mathematics

Marking Scheme(SET-A)

Section A

1. (d) 5
2. (c) 24 [1]
3. (b) $\left[\frac{-\pi}{2}, 0\right]$ [1]
4. (a) 0 [1]
5. (b) 2 [1]
6. (c) 8 [1]
7. (d) $\frac{11}{4}$ [1]
8. (d) 1.5 [1]
9. (a) $\frac{\cos a}{\cos^2(a+y)}$ [1]
10. (d) $-16x$ [1]
11. (d) $[2, \infty)$ [1]
12. (b) $e^x \cdot \sec x + c$ [1]
13. (d) 0 [1]
14. (d) $\frac{4}{3}$ [1]
15. (b) 1 [1]
16. (a) 3 [1]
17. (c) $\left(\frac{1}{2}, \frac{1}{4}\right)$ [1]
18. (c) $a=3, b=5$ [1]
19. (a) [1]
20. (a) [1]

Section B

21. As f is continuous at $x = -1$

$$\begin{aligned}\therefore \lim_{x \rightarrow -1} f(x) &= f(-1) & [1] \\ \Rightarrow \lim_{x \rightarrow -1} \frac{x^2 - 2x - 3}{x + 1} &= f(-1) \\ & \frac{1}{2} \\ \Rightarrow \lim_{x \rightarrow -1} \frac{(x-1)(x-3)}{(x+1)} &= k & [1] \\ \Rightarrow -1 - 3 &= k \\ \Rightarrow k &= -4 & [1]\end{aligned}$$

[OR]

$$f(x) = x|x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases} \quad [1]$$

LHL: -

$$\lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0^-} \frac{(-x^2) - 0}{x - 0} = \lim_{x \rightarrow 0^-} (-x) = 0 \quad [1]$$

RHL: -

$$\lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0^+} \frac{x^2 - 0^2}{x - 0} = \lim_{x \rightarrow 0^+} (x) = 0 \quad [1]$$

$$\therefore \underline{\text{LHL}} = \underline{\text{RHL}} = f'(0)$$

$\therefore f$ is differentiable at $x = 0$

$\left[\frac{1}{2}\right]$

22. $\therefore A(\text{adj}A) = |A|I$

$\left[\frac{1}{2}\right]$

$|A| = 2(10 - 9) = 2 \times 1 = 2$

$[1]$

$\therefore A(\text{adj}A) = 2I$ or $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$\left[\frac{1}{2}\right]$

23. let $x = \tan\theta$

$\Rightarrow \theta = \tan^{-1}x$

$\left[\frac{1}{2}\right]$

$\therefore \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) = \sin^{-1}\left(\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}\right) = \sin^{-1}\left(\frac{\tan\theta}{\sec\theta}\right) = \sin^{-1}(\sin\theta) = \theta$

$[1]$

$\theta = \tan^{-1}x$

$\left[\frac{1}{2}\right]$

[OR]

$\therefore -1 \leq \sqrt{x-1} \leq$

$\left[\frac{1}{2}\right]$

$\Rightarrow 0 \leq x-1 \leq 1$

$[1]$

$1 \leq x \leq 2$

$\therefore \text{Domain is } x \in [1, 2]$

$\left[\frac{1}{2}\right]$

24. $f'(x) = 2x - 2a$

$\left[\frac{1}{2}\right]$

$\therefore f$ is an increasing function

$\therefore f'(x) \geq 0$

$\left[\frac{1}{2}\right]$

$\Rightarrow 2x - 2a \geq 0$

$\left[\frac{1}{2}\right]$

$\Rightarrow a \leq x$ as $x > 0, a \leq 0$

$\left[\frac{1}{2}\right]$

25. $\frac{x^2}{9} + \frac{y^2}{4} = 1$

$\Rightarrow y = \pm \frac{2}{3}\sqrt{9-x^2}$

$\left[\frac{1}{2}\right]$

$\therefore \text{Required area} = 2 \int_0^{\frac{3}{2}} \sqrt{9-x^2}$

$\left[\frac{1}{2}\right]$

$= \frac{4}{3} \left[\frac{x}{2} \sqrt{9-x^2} + \frac{9}{2} \sin^{-1} \frac{x}{3} \right]_0^{\frac{3}{2}}$

$\left[\frac{1}{2}\right]$

$= \frac{4}{3} \left[\left(0 + \frac{9}{2} \sin^{-1} 1 - 0 \right) \right]$

$\left[\frac{1}{2}\right]$

$= 3\pi \text{ sq. units}$

$\left[\frac{1}{2}\right]$

Section C

26. Show that R is reflexive

$\left[\frac{1}{2}\right]$

Show that R is symmetric

$[1]$

Show that R is transitive

$[1]$

$\therefore R$ is reflexive, symmetric as well as transitive

$\therefore R$ is an equivalence relation

$\left[\frac{1}{2}\right]$

[OR]

for one-one:-

Let $x_1, x_2 \in R - \{2\}$ such that $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1-1}{x_1-2} = \frac{x_2-1}{x_2-2} \Rightarrow x_1 = x_2$

$[1]$

$\therefore f$ is one-one

$\left[\frac{1}{2}\right]$

For onto:-

$f(x) = y \Rightarrow \frac{x-1}{x-2} = y$

$\therefore x = \frac{2y-1}{y-1}$

$[1]$

$\therefore \text{Range of } f = R - \{1\} = \text{codomain } B$

∴ f is onto

$\left[\frac{1}{2}\right]$

27. $\log(\cos x)^y = \log(\cos y)^x$

$\Rightarrow y \log \cos x = x \log \cos y$

[1]

Differentiate both sides w.r.t x

$\Rightarrow y \frac{1}{\cos x} (-\sin x) + \log \cos x \frac{dy}{dx} = x \frac{1}{\cos x} (-\sin y) \frac{dy}{dx}$

[1]

$\Rightarrow (\log \cos x + x \tan y) \frac{dy}{dx} = \log \cos y + y \tan x$

$\Rightarrow \frac{dy}{dx} = \frac{\log \cos y + y \tan x}{\log \cos x + x \tan y}$

[1]

[OR]

$\frac{dy}{d\theta} = a \sin \theta, \frac{dx}{d\theta} = a(1 - \cos \theta)$

$\left[\frac{1}{2}\right]$

$\frac{dy}{dx} = \frac{\sin \theta}{1 - \cos \theta} = \cot \frac{\theta}{2}$

[1]

$\frac{d^2y}{dx^2} = -\frac{\operatorname{cosec}^2 \frac{\theta}{2}}{2a(1 - \cos \theta)}$

[1]

$\left. \frac{d^2y}{dx^2} \right|_{\theta=\frac{\pi}{3}} = -\frac{4}{a}$

$\left[\frac{1}{2}\right]$

28. $I = \int \frac{x}{1+\cos x} dx + \int \frac{\sin x}{1+\cos x} dx$

$I = \int \frac{x}{2\cos^2 \frac{x}{2}} dx + \int \frac{2\sin \frac{x}{2} \cos \frac{x}{2}}{2\cos^2 \frac{x}{2}} dx$

[1]

$I = \frac{1}{2} \int x \sec^2 \frac{x}{2} dx + \int \tan \frac{x}{2} dx$

$I = \left[\left(x \int \sec^2 \frac{x}{2} dx - \int \frac{d(x)}{dx} \int \sec^2 \frac{x}{2} dx \right) dx \right] + \int \tan \frac{x}{2} dx$

$I = \frac{1}{2} x \tan \frac{x}{2} - \frac{1}{2} \int 1 \left(2 \tan \frac{x}{2} \right) dx + \int \tan \frac{x}{2} dx$

[1]

$I = x \tan \frac{x}{2} - \frac{1}{2} \int \tan \frac{x}{2} dx + \frac{1}{2} \int \tan \frac{x}{2} dx$

[1]

$I = x \tan \frac{x}{2} + c$

$\therefore \int_0^{\frac{\pi}{4}} \frac{x+\sin x}{1+\cos x} dx = \left[x \tan \frac{x}{2} \right]_0^{\frac{\pi}{4}}$

$= \frac{\pi}{4} \tan \frac{\pi}{8} - 0 = \frac{\pi}{4} \tan \frac{\pi}{8}$

[1]

29. $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix} = 32\hat{i} - \hat{j} - 14\hat{k}$

[1]

∴ $\vec{d}$ is perpendicular to both $\vec{a}$ and $\vec{b}$

$\therefore \vec{d} = \lambda(\vec{a} \times \vec{b}) = \lambda(32\hat{i} - \hat{j} - 14\hat{k})$

$\left[\frac{1}{2}\right]$

∴ $\vec{c} \cdot \vec{d} = 18$ (given)

$\therefore (2\hat{i} - \hat{j} + 4\hat{k})(32\lambda\hat{i} - \lambda\hat{j} - 14\lambda\hat{k}) = 18$

$\left[\frac{1}{2}\right]$

$\Rightarrow (2) \times (32) + (1) \times (\lambda) + (4) \times (-14\lambda) = 18$

$\Rightarrow \lambda(64 + 1 - 56) = 18$

$\Rightarrow \lambda = \frac{18}{9}$

$\Rightarrow \lambda = 2$

$\left[\frac{1}{2}\right]$

$\therefore \vec{d} = 2(32\hat{i} - \hat{j} - 14\hat{k}) = 64\hat{i} - 2\hat{j} - 28\hat{k}$

$\left[\frac{1}{2}\right]$

30. Correct graph with shaded region

$\left[1\frac{1}{2}\right]$

Corner Points	Value of $Z=3x+9y$
(0,20)	180 → maximum
(0,10)	90
(5,5)	60

(15,15)

180 → maximum

∴ Z maximum = 180 at infinitely many points lying on the line joining points (0,20) & (15,15).

 $\left[\frac{1}{2} \right]$

31. $P(A) = \frac{2}{7}$ and $P(B) = \frac{4}{7}$

∴ $P(A') = 1 - \frac{2}{7} = \frac{5}{7}$ and $P(B') = 1 - \frac{4}{7} = \frac{3}{7}$

[1]

∴ $P(\text{one of them coming to school in time}) = P(A) \cdot P(B') + P(A') \cdot P(B)$

[1]

$= \frac{2}{7} \times \frac{3}{7} + \frac{5}{7} \times \frac{4}{7} = \frac{6}{49} + \frac{20}{49} = \frac{26}{49}$

[1]

[OR]

Let A and B be two events such that

A = Selection of committee having exactly 2 boys

B = Selection of committee having at least one girl

To find - $P(A/B)$

∴ $P(B) = \frac{{}^4C_1 \times {}^7C_3 + {}^4C_2 \times {}^7C_2 + {}^4C_3 \times {}^7C_1 + {}^4C_4}{11C_4}$

$\Rightarrow P(B) = \frac{140 + 126 + 28 + 1}{330} = \frac{295}{330} = \frac{59}{66}$

[1]

$\Rightarrow P(A \cap B) = \frac{{}^4C_2 \times {}^7C_2}{11C_4} = \frac{126}{330} = \frac{21}{55}$

[1]

∴ $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{21}{55}}{\frac{59}{66}} = \frac{21}{55} \times \frac{66}{59} = \frac{126}{295}$

[1]

Section D

32. $|A| = 1 \neq 0 \Rightarrow |A|^{-1}$ exists

[1]

$(\text{adj}A) = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}^T = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$

[1]

$A^{-1} = \frac{\text{adj}A}{|A|} = \frac{1}{1} \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$

[1]

In the given equations

$CX = B$ where $C = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix} = A^T, B = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$\Rightarrow X = C^{-1}B = (A^T)^{-1}B = [A^{-1}]^T B$

[1]

$\Rightarrow X = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} -30 + 16 + 14 \\ -20 + 8 + 7 \\ -40 + 16 + 21 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ 3 \end{bmatrix}$

∴ $x = 0, y = -5, z = 3$

[1]

33. Let $5x + 3 = A \cdot d(x^2 + 4x + 10) + B$

$A = \frac{5}{2}$ and $B = -7$

Now,

$I = \int \frac{\frac{5}{2}(2x+4)}{\sqrt{x^2+4x+10}} dx = \frac{5}{2} \int \frac{(2x+4)}{\sqrt{x^2+4x+10}} dx - 7 \int \frac{dx}{\sqrt{x^2+4x+10}}$

∴ $I = \frac{5}{2} I_1 - 7 I_2$

[1]

$I_1 = \int \frac{(2x+4)}{\sqrt{x^2+4x+10}} dx = \int \frac{dt}{\sqrt{t}}$ [where $t = x^2 + 4x + 10$]

$I_1 = 2\sqrt{t} + c_1$

[1]

$$I_1 = 2\sqrt{x^2 + 4x + 10} + c_1$$

$$I_2 = \int \frac{dx}{\sqrt{x^2 + 4x + 10}} = \int \frac{dx}{\sqrt{(x+2)^2 + (\sqrt{6})^2}} = \log(x+2) + \sqrt{x^2 + 4x + 10} + c_2 \quad [1]$$

$$\therefore I = \frac{5}{2} \times 2\sqrt{x^2 + 4x + 10} - 7 \log |(x+2) + \sqrt{x^2 + 4x + 10}| + \left(\frac{5}{2}c_1 - 7c_2\right)$$

$$\Rightarrow I = 5\sqrt{x^2 + 4x + 10} - 7 \log |(x+2) + \sqrt{x^2 + 4x + 10}| + C$$

[OR]

$$I = \int_0^\pi \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx \quad (i) \quad [1]$$

$$I = \int_0^\pi \frac{\pi - x}{a^2 \cos^2(\pi - x) + b^2 \sin^2(\pi - x)} dx$$

$$I = \int_0^\pi \frac{\pi - x}{a^2 \cos^2 x + b^2 \sin^2 x} dx \quad (ii) \quad [1]$$

Adding (i) and (ii)

$$I + I = \int_0^\pi \frac{\pi - x + x}{a^2 \cos^2 x + b^2 \sin^2 x} dx$$

$$2I = \int_0^\pi \frac{\pi}{a^2 \cos^2 x + b^2 \sin^2 x} dx$$

$$I = \frac{\pi}{2} \times 2 \int_0^{\frac{\pi}{2}} \frac{1}{a^2 \cos^2 x + b^2 \sin^2 x} dx \quad \left[\because \int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx \right] \quad [1]$$

$$I = \pi \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{a^2 + b^2 \tan^2 x} dx \quad (\text{dividing numerator and denominator by } \cos^2 x)$$

$$I = \frac{\pi}{b} \int \frac{dt}{a^2 + (t)^2} \quad \left[\begin{array}{l} \text{where } t = b \tan x \\ \text{when } x = 0, t = 0 \\ \text{when } x = \frac{\pi}{2}, t = \infty \end{array} \right] \quad [1]$$

$$I = \frac{\pi}{b} \left[\frac{1}{a} \tan^{-1} \frac{t}{a} \right]_0^\infty$$

$$I = \frac{\pi}{ab} [\tan^{-1} \infty - \tan^{-1} 0]_0^\infty$$

$$I = \frac{\pi}{ab} \left[\frac{\pi}{2} - 0 \right]$$

$$I = \frac{\pi^2}{ab} \quad [1]$$

34. $x \frac{dy}{dx} + x \cos^2 \left(\frac{y}{x} \right) = y$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \cos^2 \left(\frac{y}{x} \right) \quad (i) \quad [1]$$

$$\text{Let } y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx} \quad (ii) \quad \left(\frac{1}{2} \right)$$

$$\therefore v - \cos^2 \left(\frac{y}{x} \right) = v + x \frac{dv}{dx}$$

$$\Rightarrow \sec^2 v dv = -\frac{dx}{x} \quad [1]$$

$$\Rightarrow \int \sec^2 v dv = -\int \frac{dx}{x} \quad \left(\frac{1}{2} \right)$$

$$\Rightarrow \tan v = -\log|x| + c$$

$$\Rightarrow \tan \frac{y}{x} = -\log|x| + c \quad [1]$$

Putting $x = 1$ & $y = \frac{\pi}{4}$ we get $c = 1$

$$\therefore \text{Particular solution is } \tan \frac{y}{x} = -\log|x| + 1 \quad [1]$$

[OR]

$$\Rightarrow (x^2 + 1) \frac{dy}{dx} + 2xy = \sqrt{x^2 + 4}$$

$$\Rightarrow \frac{dy}{dx} + \frac{2x}{x^2+1} = \frac{\sqrt{x^2+4}}{x^2+1} \quad [1]$$

$$\Rightarrow P = \frac{2x}{x^2+1} \& Q = \frac{\sqrt{x^2+4}}{x^2+1}$$

$$\therefore I.F = e^{\int P dx} = e^{\int \frac{2x}{x^2+1} dx} = e^{\log|x^2-1|} = x^2 - 1 \quad [1]$$

$\therefore$ Solution of the differential equation is

$$y(I.F) = \int Q(I.F) dx \quad \left(\frac{1}{2}\right)$$

$$\Rightarrow y(x^2 + 1) = \int \frac{(\sqrt{x^2+4})(x^2-1)}{(x^2-1)} dx$$

$$\Rightarrow y(x^2 + 1) = \int \sqrt{x^2 + 4} dx$$

$$\Rightarrow y(x^2 + 1) = \int \sqrt{x^2 + 2^2} dx \quad \left(\frac{1}{2}\right)$$

$$\Rightarrow y(x^2 + 1) = \frac{x}{2} \sqrt{x^2 + 2^2} + \frac{4}{2} \log|x + \sqrt{x^2 + 2^2}| + c$$

$$\Rightarrow y(x^2 + 1) = \frac{x}{2} \sqrt{x^2 + 2^2} + 2 \log|x + \sqrt{x^2 + 2^2}| + c$$

35. For the given lines

$$\vec{a_1} = -\hat{i} + \hat{j} + 9\hat{k}, \vec{b_1} = 2\hat{i} + \hat{j} - 3\hat{k}$$

$$\vec{a_2} = 3\hat{i} - 15\hat{j} + 9\hat{k}, \vec{b_2} = 2\hat{i} - 7\hat{j} + 5\hat{k}$$

$$\therefore \vec{a_1} - \vec{a_2} = 4\hat{i} - 16\hat{j} \quad [1]$$

$$\vec{b_1} \times \vec{b_2} = -16\hat{i} - 16\hat{j} - 16\hat{k} \quad [2]$$

$$|\vec{b_1} \times \vec{b_2}| = 16\sqrt{3} \quad [1]$$

$$\therefore S.D = \left| \frac{(\vec{a_1} - \vec{a_2}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

$$\Rightarrow S.D = \left| \frac{(4\hat{i} - 16\hat{j}) \cdot (-16\hat{i} - 16\hat{j} - 16\hat{k})}{16\sqrt{3}} \right| = \frac{192}{16\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3} \quad [1]$$

36. (i) Volume of tank = $250m^3$

$$\Rightarrow x^2 \times h = 250m^3$$

$$\Rightarrow h = \frac{250}{x^2} \quad \left(\frac{1}{2}\right)$$

$$\therefore \text{total cost of digging tank} = 40,000h^2 + 5,000x^2$$

$$\Rightarrow C(x) = 40,000 \left(\frac{250}{x^2}\right)^2 + 5,000x^2$$

$$\Rightarrow C(x) = 40,000 \times \frac{62,500}{x^4} + 5,000x^2 \quad \left(\frac{1}{2}\right)$$

$$(ii) \frac{dC(x)}{dx} = 40,000 \times \frac{62,500}{x^4} + 5,000x^2$$

$$\Rightarrow \frac{dC(x)}{dx} = \frac{-4 \times 40,000 \times 62,500}{x^5} + 10,000x \quad [1]$$

$$(iii) \text{For maximum/minimum cost } \frac{dC(x)}{dx} = 0$$

$$\Rightarrow x = 10m \quad [1]$$

$$\Rightarrow \frac{d^2C(x)}{dx^2} \text{ at } x=10 > 0$$

$$\therefore C(x) \text{ is minimum at } x = 10 \quad [1]$$

[OR]

$$C(x) = 5,000x^2 + \frac{2,50,00,00,000}{x^4}$$

$$C'(x) = 10,000 - \frac{(10)^{10}}{x^5} \quad [1]$$

For increasing $C'x > 0$

$$\Rightarrow x > 10$$

So, for $C'(x)$ to be increasing $x > 10$ and $C'(x) < 0, \forall x \in (0, 10)$.

$$\therefore C(x) \text{ is not increasing for } x > 0. \quad [1]$$

37. (i) $\frac{x}{3} = \frac{y}{-4} = \frac{z}{1}$ path is straight line [1]

(ii) After 5sec position of the missile be

$$\begin{aligned}x &= 3t = 3 \times 5 = 15 \\y &= -4t = -4 \times 5 = -20 \\z &= t = 5\end{aligned}$$

$\therefore$ Point is (15, -20, 5) [1]

Its distance from origin is $\sqrt{650}km$. [1]

(iii) Height of the missile from the ground = $\sqrt{(5-5)^2(-8+8)^2(10-0)^2} = 10km$ [1]

[OR]

Given lines are perpendicular if $2 \times (-2) + 3(-1) + 7 \times k = 0$

$\Rightarrow k = 1$ [1]

38. Let E_1, E_2, E_3 be the events that a customer avails loan on fixed rate, floating rate, variable rate respectively. Also let E be the events that the person defaults on the loan.

Now, $P(E_1) = 10\% = \frac{1}{10}, P(E_2) = 20\% = \frac{2}{10}, P(E_3) = 70\% = \frac{7}{10}$

Also, $P(E|E_1) = 5\%, P(E|E_2) = 3\%, P(E|E_3) = 1\%$.

(i) $P(E_1) = P(E_1)P(E|E_1) + P(E_2)P(E|E_2) + P(E_3)P(E|E_3)$ [2]

$\Rightarrow P(E) = 0.018$ [2]

(ii) By using Baye's theorem, $(E_3|E) = \frac{(E_3|E)}{P(E_1)P(E|E_1) + P(E_2)P(E|E_2) + P(E_3)P(E|E_3)}$ [2]

$\Rightarrow (E_3|E) = \frac{7}{18}$ [2]