

KENDRIYA VIDYALAYA SANGATHAN, RAIPUR REGION

FIRST PRE-BOARD EXAM (2025-26)

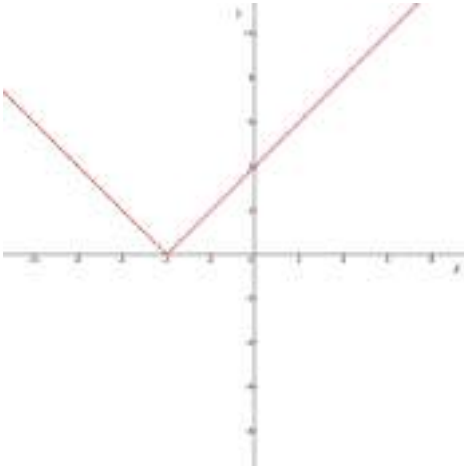
MARKING SCHEME

SUBJECT: MATHEMATICS

Q.NO.	ANSWER	TOTAL MARKS
1.	(A) $\begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{5} & 0 \\ 0 & 0 & \frac{1}{6} \end{bmatrix}$	1
2.	(B) 1	1
3.	(A) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{AC} = \vec{0}$	1
4.	(C) $\frac{\pi}{3}$	1
5.	(B) $\frac{1}{4}$	1
6.	(C) not defined	1
7.	(B) R is reflexive and transitive but not symmetric	1
8.	(D) 126	1
9.	(A) π	1
10.	(D) 2, 1	1
11.	(A) 0	1
12.	(C) $x + 2y \geq 76, 2x + y \leq 104, x, y \geq 0$	1
13.	(D) -1	1
14.	(A) $-\frac{1}{\log x} + c$	1
15.	(C) first order linear differential equation	1
16.	(B) ± 5	1
17.	(C) x^{-1}	1
18.	(C) z is maximum at (10, 50), minimum at (0, 0)	1
19.	(C) Assertion (A) is true, but Reason (R) is false	1
20.	(D) Assertion (A) is false, but Reason (R) is true	1
21.	Let $u(x) = \sin^2 x$ and $v(x) = e^{\cos x}$ Then, $\frac{du}{dx} = 2 \sin x \cos x$ (1/2 M) and $\frac{dv}{dx} = -\sin x e^{\cos x}$(1/2 M) Thus, $\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2 \sin x \cos x}{-\sin x e^{\cos x}} = -\frac{2 \cos x}{e^{\cos x}}$(1 M) OR Let $y = \tan^{-1} \left(\frac{\sin x}{1 + \cos x} \right)$ $\Rightarrow y = \tan^{-1} \left(\frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \right)$(1/2 M)	2

	$\Rightarrow y = \tan^{-1} \left(\frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} \right)$ $\Rightarrow y = \tan^{-1} \left(\tan \frac{x}{2} \right) \dots\dots\dots(1/2 \text{ M})$ $\Rightarrow y = \frac{x}{2} \dots\dots\dots(1/2 \text{ M})$ Hence, $\frac{dy}{dx} = \frac{1}{2} \dots\dots\dots(1/2 \text{ M})$	
22.	$\vec{AB} = (2-1)\hat{i} + (3-1)\hat{j} + (5-2)\hat{k} = \hat{i} + 2\hat{j} + 3\hat{k} \dots\dots\dots(1/2 \text{ M})$ $\vec{AC} = (1-1)\hat{i} + (5-1)\hat{j} + (5-2)\hat{k} = 4\hat{j} + 3\hat{k} \dots\dots\dots(1/2 \text{ M})$ $\text{Area of triangle ABC} = \frac{1}{2} \vec{AB} \times \vec{AC} = \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix} = \frac{1}{2} -6\hat{i} - 3\hat{j} + 4\hat{k} $ $= \frac{1}{2} \sqrt{(-6)^2 + (-3)^2 + 4^2} = \frac{1}{2} \sqrt{61} \text{ sq. units.} \dots\dots\dots(1 \text{ M})$	2
23.	$\cos^{-1} \left(-\frac{1}{\sqrt{2}} \right) + \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) + \tan^{-1} \left[\sin \left(-\frac{\pi}{2} \right) \right]$ $= \left(\pi - \frac{\pi}{4} \right) + \frac{\pi}{6} + \tan^{-1}(-1) \dots\dots\dots(1 \text{ M})$ $= \frac{3\pi}{4} + \frac{\pi}{6} - \frac{\pi}{4} \dots\dots\dots(1/2 \text{ M})$ $= \frac{9\pi + 2\pi - 3\pi}{12}$ $= \frac{8\pi}{12}$ $= \frac{2\pi}{3} \dots\dots\dots(1/2 \text{ M})$	2
24.	$f(x) = e^x - e^{-x} + x - \tan^{-1} x$ $f'(x) = e^x + e^{-x} + 1 - \frac{1}{1+x^2} \dots\dots\dots(1 \text{ M})$ $= e^x + e^{-x} + \frac{1+x^2-1}{1+x^2}$ $= e^x + e^{-x} + \frac{x^2}{1+x^2} > 0 \forall x \in R \dots\dots\dots(1 \text{ M})$ Hence, f is strictly increasing in its domain.	2
25.	Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = p\hat{i} + \hat{j} - 2\hat{k}$. A/Q, $\frac{\vec{a} \cdot \vec{b}}{ \vec{b} } = \frac{1}{3} \dots\dots\dots(1/2 \text{ M})$ $\Rightarrow \frac{(\hat{i} + \hat{j} + \hat{k}) \cdot (p\hat{i} + \hat{j} - 2\hat{k})}{ p\hat{i} + \hat{j} - 2\hat{k} } = \frac{1}{3}$ $\Rightarrow \frac{p-1}{\sqrt{p^2+5}} = \frac{1}{3} \dots\dots\dots(1/2 \text{ M})$ $\Rightarrow 4p^2 - 9p + 2 = 0 \dots\dots\dots(1/2 \text{ M})$	2

	$\Rightarrow p = 2 \text{ or } p = \frac{1}{4} \dots\dots\dots(1/2 \text{ M})$ OR $\overrightarrow{DV} = -5\vec{i} + 4\vec{j} + 7\vec{k} \dots\dots\dots(1/2 \text{ M})$ $\overrightarrow{DA} = 5\vec{i} + 2\vec{j} + 4\vec{k} \dots\dots\dots(1/2 \text{ M})$ $\angle VDA = \cos^{-1} \left(\frac{\overrightarrow{DV} \cdot \overrightarrow{DA}}{ \overrightarrow{DV} \overrightarrow{DA} } \right)$ $= \cos^{-1} \left(\frac{-25+8+28}{\sqrt{25+16+49} \sqrt{25+4+16}} \right) \dots\dots\dots(1/2 \text{ M})$ $= \cos^{-1} \left(\frac{11}{\sqrt{90} \sqrt{45}} \right) = \cos^{-1} \left(\frac{11}{45\sqrt{2}} \right) \dots\dots\dots(1/2 \text{ M})$	
26.	We have, $f(x) = 2x^3 - 15x^2 + 36x + 1$ $\Rightarrow f'(x) = 6x^2 - 30x + 36 \dots\dots\dots(1/2 \text{ M})$ Let $f'(x) = 0$ $\Rightarrow 6x^2 - 30x + 36 = 0$ $\Rightarrow 6(x-2)(x-3) = 0$ $\Rightarrow \text{Either } x = 3 \text{ or } x = 2 \dots\dots\dots(1/2 \text{ M})$ Now, $f(1) = 2(1)^3 - 15(1)^2 + 36(1) + 1 = 24$ $f(2) = 2(2)^3 - 15(2)^2 + 36(2) + 1 = 29$ $f(3) = 2(3)^3 - 15(3)^2 + 36(3) + 1 = 28$ $f(5) = 2(5)^3 - 15(5)^2 + 36(5) + 1 = 56 \dots\dots\dots(1 \text{ M})$ Absolute maximum is 56.(1/2 M) and Absolute minimum is 24.(1/2 M)	3
27.	$x = \left(t + \frac{1}{t}\right)^a$ $\Rightarrow \frac{dx}{dt} = a \left(t + \frac{1}{t}\right)^{a-1} \left(1 - \frac{1}{t^2}\right) \dots\dots\dots(1 \text{ M})$ Also, $y = a^{t+\frac{1}{t}}$ $\Rightarrow \frac{dy}{dt} = a^{t+\frac{1}{t}} \left(1 - \frac{1}{t^2}\right) \log a \dots\dots\dots(1 \text{ M})$ Now, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a^{t+\frac{1}{t}} \left(1 - \frac{1}{t^2}\right) \log a}{a \left(t + \frac{1}{t}\right)^{a-1} \left(1 - \frac{1}{t^2}\right)} = \frac{a^{t+\frac{1}{t}} \log a}{a \left(t + \frac{1}{t}\right)^{a-1}} \dots\dots\dots(1 \text{ M})$ OR Let $u = (\cos x)^x$ and $v = \cos^{-1} \sqrt{x} \dots\dots\dots(1/2 \text{ M})$ Then, $y = u + v$ $\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \dots\dots\dots(1/2 \text{ M})$	3

	Now, $u = (\cos x)^x$ $\Rightarrow \log u = \log(\cos x)^x$ $\Rightarrow \log u = x \log(\cos x)$ $\Rightarrow \frac{1}{u} \frac{du}{dx} = \log(\cos x) - x \tan x$ $\Rightarrow \frac{du}{dx} = (\cos x)^x \{\log(\cos x) - x \tan x\} \dots\dots\dots(1 \text{ M})$ Also, $v = \cos^{-1} \sqrt{x}$ $\Rightarrow \frac{dv}{dx} = -\frac{1}{2\sqrt{x-x^2}} \dots\dots\dots(1/2 \text{ M})$ Now, $\frac{dy}{dx} = (\cos x)^x \{\log(\cos x) - x \tan x\} - \frac{1}{2\sqrt{x-x^2}} \dots\dots\dots(1/2 \text{ M})$	
28.	 \dots\dots\dots(1 \text{ M}) Reqd. Area = $\int_{-8}^{-4} y dx + \int_{-4}^0 y dx \dots\dots\dots(1/2 \text{ M})$ $= \int_{-8}^{-4} -(x+4) dx + \int_{-4}^0 (x+4) dx \dots\dots\dots(1/2 \text{ M})$ $= \left[-\frac{x^2}{2} - 4x \right]_{-8}^{-4} + \left[\frac{x^2}{2} + 4x \right]_{-4}^0 \dots\dots\dots(1/2 \text{ M})$ $= \left\{ \left[-\frac{(-4)^2}{2} - 4(-4) \right] - \left[-\frac{(-8)^2}{2} - 4(-8) \right] \right\} + \left[\frac{0^2}{2} + 4(0) - \frac{(-4)^2}{2} - 4(-4) \right]$ $= \{ [-8 + 16] - [-32 + 32] \} + [0 - 8 + 16]$ $= 16 \dots\dots\dots(1/2 \text{ M})$	3
29.	Let $\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$ $\vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k}$, $\vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$ $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = -3\hat{i} + 3\hat{k} \dots\dots\dots(1 \text{ M})$ $\vec{a}_2 - \vec{a}_1 = \hat{i} - 3\hat{j} - 2\hat{k} \dots\dots\dots(1/2 \text{ M})$	3

$$(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = -9 \dots\dots\dots(1/2 \text{ M})$$

$$\text{Shortest Distance} = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right| \dots\dots\dots(1/2 \text{ M})$$

$$= \left| \frac{-9}{\sqrt{18}} \right| = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2} \text{ units} \dots\dots\dots(1/2 \text{ M})$$

OR

$$\text{Let } \vec{b}_1 = -3\hat{i} + 2\hat{j} + 2\hat{k} \text{ and } \vec{b}_2 = 3\hat{i} + \hat{j} - 7\hat{k} \dots\dots\dots(1/2 \text{ M})$$

Now,

$$\vec{b} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & 2 \\ 3 & 1 & -7 \end{vmatrix} = -16\hat{i} - 15\hat{j} - 9\hat{k} \dots\dots\dots(1 \text{ M})$$

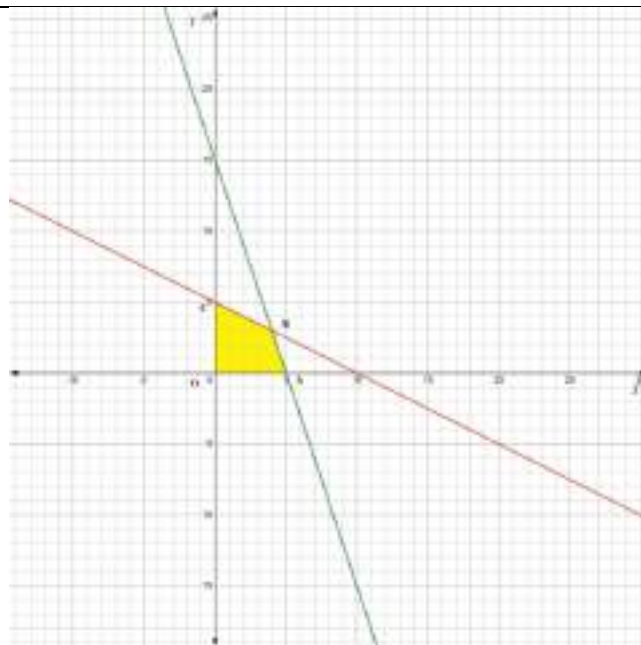
$$\text{Let } \vec{a} = 3\hat{i} - 4\hat{j} + 7\hat{k} \dots\dots\dots(1/2 \text{ M})$$

Therefore, required vector equation of the line is

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

$$\text{i.e. } \vec{r} = 3\hat{i} - 4\hat{j} + 7\hat{k} + \lambda(-16\hat{i} - 15\hat{j} - 9\hat{k}) \dots\dots\dots(1 \text{ M})$$

30.



\dots\dots\dots(1\frac{1}{2} \text{ M})

Corner Points	Z= 3x + 2y
O (0, 0)	0
A (5, 0)	15
B (4, 3)	18
C (0, 5)	10

Table: (1 M)

Reqd. Max Value is 18 obtained at point B (4, 3) \dots\dots\dots(1/2 M)

3

31.	Let A and B be the set of students who read Hindi and English respectively. Then, $P(A) = \frac{60}{100} = 0.6$, $P(B) = \frac{40}{100} = 0.4$, $P(A \cap B) = \frac{20}{100} = 0.2$ (a) $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$ $= 1 - P(A) - P(B) + P(A \cap B)$ $= 1 - 0.6 - 0.4 + 0.2 = 0.2$(1 M) (b) $P(B A) = \frac{P(A \cap B)}{P(A)} = \frac{0.2}{0.6} = \frac{1}{3}$.....(1 M) (c) $P(A B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.4} = \frac{1}{2}$.....(1 M) OR $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ (i) P (the problem is solved) = $P(A \cup B)$ $= P(A) + P(B) - P(A \cap B)$(1/2 M) $= P(A) + P(B) - P(A)P(B)$ $= \frac{1}{2} + \frac{1}{3} - \frac{1}{2} \cdot \frac{1}{3}$(1/2 M) $= \frac{3+2-1}{6}$ $= \frac{4}{6}$ $= \frac{2}{3}$(1/2 M) (ii) P (exactly one of them solves the problem) = $P(A \cap B') + P(A' \cap B)$ $= P(A) - P(A \cap B) + P(B) - P(A \cap B)$(1/2 M) $= P(A) + P(B) - 2 P(A)P(B)$(1/2 M) $= \frac{1}{2} + \frac{1}{3} - 2 \cdot \frac{1}{2} \cdot \frac{1}{3}$ $= \frac{3+2-2}{6}$ $= \frac{3}{6}$ $= \frac{1}{2}$(1/2 M)	3
32.	Let $I = \int [\sqrt{\cot x} + \sqrt{\tan x}] dx$ $\Rightarrow I = \int \sqrt{\tan x} \left[\frac{\sqrt{\cot x}}{\sqrt{\tan x}} + 1 \right] dx$ $\Rightarrow I = \int \sqrt{\tan x} [\cot x + 1] dx$(1 M) Let $\sqrt{\tan x} = z$ $\Rightarrow \tan x = z^2$	

$$\Rightarrow \sec^2 x \, dx = 2z \, dz$$

$$\Rightarrow dx = \frac{2z \, dz}{\sec^2 x}$$

$$\Rightarrow dx = \frac{2z \, dz}{1 + \tan^2 x}$$

$$\Rightarrow dx = \frac{2z \, dz}{1 + z^4} \dots\dots\dots(1 \text{ M})$$

$$\text{Now, } I = \int z \left[\frac{1}{z^2} + 1 \right] \frac{2z \, dz}{1 + z^4}$$

$$\Rightarrow I = \int \left[\frac{1+z^2}{z^2} \right] \frac{2z^2 \, dz}{1 + z^4}$$

$$\Rightarrow I = 2 \int \left[\frac{1+z^2}{1+z^4} \right] dz$$

$$\Rightarrow I = 2 \int \left[\frac{\frac{1}{z^2} + 1}{\frac{1}{z^2} + z^2} \right] dz$$

$$\Rightarrow I = 2 \int \left[\frac{\frac{1}{z^2} + 1}{2 + \left(z - \frac{1}{z}\right)^2} \right] dz \dots\dots\dots(1.5 \text{ M})$$

$$\text{Let } t = z - \frac{1}{z}$$

$$\Rightarrow dt = \left(1 + \frac{1}{z^2} \right) dz \dots\dots\dots(1/2 \text{ M})$$

Hence,

$$I = 2 \int \frac{dt}{2 + t^2}$$

$$\Rightarrow I = 2 \int \frac{dt}{(\sqrt{2})^2 + t^2}$$

$$\Rightarrow I = 2 \cdot \frac{1}{\sqrt{2}} \tan^{-1} \frac{t}{\sqrt{2}} + c$$

$$\Rightarrow I = \sqrt{2} \tan^{-1} \left(\frac{\sqrt{\tan x} - \sqrt{\cot x}}{\sqrt{2}} \right) + c \dots\dots\dots(1 \text{ M})$$

OR

$$\text{Let } I = \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{25 - 16 + 16 \sin 2x} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{25 - 16(1 - \sin 2x)} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{25 - 16(\sin^2 x + \cos^2 x - 2 \sin x \cos x)} dx$$

$$\Rightarrow I = \frac{1}{16} \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{\frac{25}{16} - (\sin x - \cos x)^2} dx \dots\dots\dots(2 \text{ M})$$

Let $\sin x - \cos x = t$

$$\Rightarrow (\cos x + \sin x) dx = dt$$

When $x = 0, t = \sin 0 - \cos 0 = 0 - 1 = -1$

When $x = \frac{\pi}{4}, t = \sin \frac{\pi}{4} - \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0 \dots\dots\dots(1 \text{ M})$

Then,

$$I = \frac{1}{16} \int_{-1}^0 \frac{1}{\left(\frac{5}{4}\right)^2 - (t)^2} dt$$

$$\Rightarrow I = \frac{1}{16} \left[\frac{1}{2 \cdot \frac{5}{4}} \log \left| \frac{\frac{5}{4} + t}{\frac{5}{4} - t} \right| \right]_{-1}^0$$

$$\Rightarrow I = \frac{1}{40} \left[\log \left| \frac{5+4t}{5-4t} \right| \right]_{-1}^0$$

$$\Rightarrow I = \frac{1}{40} \left[\log 1 - \log \frac{1}{9} \right]$$

$$\Rightarrow I = \frac{1}{40} \log 9 \dots\dots\dots(2 \text{ M})$$

33.

$$2ye^{\frac{x}{y}} dx + \left(y - 2xe^{\frac{x}{y}} \right) dy = 0$$

$$\Rightarrow \frac{dx}{dy} = \frac{2xe^{\frac{x}{y}} - y}{2ye^{\frac{x}{y}}}$$

$$\Rightarrow \frac{dx}{dy} = \frac{x}{y} - \frac{1}{2} e^{-\frac{x}{y}} \dots\dots\dots(\text{Eq. 1}) \dots\dots\dots(1/2 \text{ M})$$

$$\text{Let } F(x, y) = \frac{x}{y} - \frac{1}{2} e^{-\frac{x}{y}}$$

$$\text{Then } F(\lambda x, y \lambda) = \frac{\lambda x}{\lambda y} - \frac{1}{2} e^{-\frac{\lambda x}{\lambda y}} = \lambda^0 \left(\frac{x}{y} - \frac{1}{2} e^{-\frac{x}{y}} \right) \dots\dots\dots(1/2 \text{ M})$$

	Hence, the given differential equation is homogeneous differential equation. Let $\frac{x}{y} = v$ $\Rightarrow x = yv$ $\Rightarrow \frac{dx}{dy} = v + y \frac{dv}{dy} \dots\dots\dots(1/2 \text{ M})$ Putting in Eq. 1, we get $v + y \frac{dv}{dy} = v - \frac{1}{2}e^{-v}$ $\Rightarrow y \frac{dv}{dy} = -\frac{1}{2}e^{-v}$ $\Rightarrow 2e^v dv = -\frac{dy}{y}$ $\Rightarrow 2e^v = -\log y + c$ $\Rightarrow 2e^{\frac{x}{y}} = -\log y + c \dots\dots\dots(2 \text{ M})$ Given , $x = 0, y = 1$, we get $2e^0 = -\log 1 + c$ i.e. $c = 2 \dots\dots\dots(1 \text{ M})$ Hence, the required particular solution is $2e^{\frac{x}{y}} = -\log y + 2 \dots\dots\dots(1/2 \text{ M})$	
34.	Let $\frac{x}{1} = \frac{y-2}{2} = \frac{z-3}{3} = \lambda$ $\Rightarrow x = \lambda, y = 2\lambda + 2, z = 3\lambda + 3 \dots\dots\dots(1/2 \text{ M})$ Then $A(\lambda, 2\lambda + 2, 3\lambda + 3)$ is a point on the given line. Let the coordinates of P' be (x_1, y_1, z_1). Since, A is the midpoint of the line segment PP', we get $\lambda = \frac{1+x_1}{2} \Rightarrow x_1 = 2\lambda - 1 \dots\dots\dots(1/2 \text{ M})$ $2\lambda + 2 = \frac{3+y_1}{2} \Rightarrow y_1 = 4\lambda + 1 \dots\dots\dots(1/2 \text{ M})$ $3\lambda + 3 = \frac{2+z_1}{2} \Rightarrow z_1 = 6\lambda + 4 \dots\dots\dots(1/2 \text{ M})$ Direction ratios of the line PP' are $(1 - x_1, 3 - y_1, 2 - z_1) \dots\dots\dots(1/2 \text{ M})$ Since, the given line is perpendicular to PP'. Therefore, $1.(1 - x_1) + 2.(3 - y_1) + 3.(2 - z_1) = 0 \dots\dots\dots(1/2 \text{ M})$ $\Rightarrow (2 - 2\lambda) + 2(2 - 4\lambda) + 3(-2 - 6\lambda) = 0$ $\Rightarrow \lambda = 0 \dots\dots\dots(1/2 \text{ M})$ Hence, $P'(x_1, y_1, z_1) \equiv (-1, 1, 4) \dots\dots\dots(1/2 \text{ M})$ Equation of PP' is	

$$\frac{x-1}{1-x_1} = \frac{y-3}{3-y_1} = \frac{z-2}{2-z_1} \dots\dots\dots(1/2 \text{ M})$$

$$\text{i.e. } \frac{x-1}{2} = \frac{y-3}{2} = \frac{z-2}{-2} \dots\dots\dots(1/2 \text{ M})$$

OR

Given,

Equation of CD is $\frac{x-4}{1} = \frac{y+7}{-2} = \frac{z-8}{2}$

Let $\vec{a}_1 = 4\hat{i} - 7\hat{j} + 8\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$

Again, let $\vec{a}_2 = -\hat{i} + 2\hat{j} + \hat{k}$

$\vec{a}_2 - \vec{a}_1 = -5\hat{i} + 9\hat{j} - 7\hat{k} \dots\dots\dots(1 \text{ M})$

$$\vec{b} \times (\vec{a}_2 - \vec{a}_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ -5 & 9 & -7 \end{vmatrix} = -4\hat{i} - 3\hat{j} - \hat{k} \dots\dots\dots(1 \text{ M})$$

Distance between AB and CD is

$$d = \frac{|\vec{b} \times (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}|}$$

$$= \frac{|-4\hat{i} - 3\hat{j} - \hat{k}|}{|\hat{i} - 2\hat{j} + 2\hat{k}|}$$

$$= \frac{\sqrt{(-4)^2 + (-3)^2 + (-1)^2}}{\sqrt{1^2 + (-2)^2 + 2^2}}$$

$$= \frac{\sqrt{26}}{3} \text{ units} \dots\dots\dots(2 \text{ M})$$

Area of the parallelogram is $d \cdot |AB| = \frac{\sqrt{26}}{3} \cdot |2\hat{i} - 4\hat{j} + 4\hat{k}|$

$$= \frac{\sqrt{26}}{3} \cdot 6 = 2\sqrt{26} \text{ sq. units} \dots\dots\dots(1\text{M})$$

35. $\begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix} = \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix} = 7I \dots(\text{Eq. 1}) \dots\dots\dots(2 \text{ M})$

The given system of equations can be written as

$$\begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$$

$\Rightarrow AX = B \dots\dots\dots(1/2 \text{ M})$

Where $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$

	$\Rightarrow X = A^{-1}B \dots\dots\dots(1/2 \text{ M})$ From Eq. 1, we have $A^{-1} = \frac{1}{7} \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix} \dots\dots\dots(1/2 \text{ M})$ Then, $X = \frac{1}{7} \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix} \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 7 \\ -35 \\ -35 \end{bmatrix} \dots\dots\dots(1 \text{ M})$ $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ -5 \end{bmatrix}$ Hence, $x = 1, y = -5, z = -5 \dots\dots\dots(1/2 \text{ M})$	
36.	(i) $P(\text{the plane will not crash}) = P(E_2) = 1 - P(E_1) = 1 - 0.0000001 = 0.9999999$ (ii) $P(A E_1) + P(A E_2) = 0.95 + 1 = 1.95$ (iii) (a) $P(A) = P(E_1)P(A E_1) + P(E_2)P(A E_2)$ $\quad \quad \quad = 0.0000001 \times 0.95 + 0.9999999 \times 1$ $\quad \quad \quad = 0.000000095 + 0.9999999 = 0.999999995$ OR (iii) (b) $P(E_2 A) = \frac{P(E_2)P(A E_2)}{P(A)} = \frac{0.9999999}{0.999999995} = \frac{999999900}{999999995}$	
37.	(i) R_4 1 (ii) R_5 1 (iii) (a) R_1 and R_3 1+1 OR (iii) (b) Required pairs to be added to make the relation R_2 as an equivalence relation are: $(1,1),(2,2),(3,3),(2,1),(3,1)$ and $(2,3)$ 2m	
38.	(i) $F = \frac{(40)^2}{500} - \frac{(40)}{4} + 14 = \frac{1600}{500} - 10 + 14 = \frac{16}{5} + 4 = \frac{36}{5}$ (ii) $F = \frac{V^2}{500} - \frac{V}{4} + 14$ $\Rightarrow 1 = \frac{2V}{500} \frac{dV}{dF} - \frac{1}{4} \frac{dV}{dF}$ $\Rightarrow 1 = \left(\frac{2V-125}{500} \right) \frac{dV}{dF}$ $\Rightarrow \frac{dV}{dF} = \frac{500}{2V-125}$ (iii) (a) Let $\frac{dF}{dV} = 0$ $\Rightarrow \frac{2V-125}{500} = 0$ $\Rightarrow 2V = 125$	

$$\Rightarrow V = \frac{125}{2} = 62.5 \text{ km/h}$$

$$\text{Also, } \frac{d^2F}{dV^2} = \frac{1}{125} > 0$$

Therefore, V is minimum at $V = 62.5 \text{ km/h}$

OR

$$\text{(iii) (b) Given, } \frac{dF}{dV} = -0.01$$

$$\Rightarrow \frac{2V-125}{500} = -0.01$$

$$\Rightarrow 2V - 125 = -5$$

$$\Rightarrow 2V = 120$$

$$\Rightarrow V = 60 \text{ km/hr}$$

Then,

$$F = \frac{(60)^2}{500} - \frac{60}{4} + 14$$

$$\Rightarrow F = \frac{3600}{500} - 15 + 14$$

$$\Rightarrow F = \frac{31}{5} = 6.2 \text{ litre/100 km}$$

Therefore, Fuel consumption for 600 km is $\frac{6.2}{100} \times 600 = 37.2 \text{ litres}$