NAVODAYA VIDYALAYA SAMITI

Pre Board-I Examination (2025-26)

MARKING SCHEME

Class :XII **Subject : MATHAMATICS** **Set – 1**

Time : 180 MIN **Maximum Marks**

Q. No .	**Section – A** Question	Marks
1.	C	1
2.	C	1
3.	B	1
4.	C	1
5.	B	1
6.	A	1
7.	C	1
8.	B	1
9.	B	1
10.	D	1
11.	A	1
12.	D	1
13.	D	1
14.	B	1
15.	**A**	1
16.	C	1
17.	B	1
18.	C	1
19 & 20		
19	A	1
20	A **Section - B**	1

21.

ANS: Given x = 10(t – sin t) ⇒ $\frac{dx}{dt}$ = 10(1 – cos t) y = 12(1 – cos t) ⇒ $\frac{dy}{dx}$ = 12 sin t

$$\therefore \quad \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{12 \sin t}{10(1 - \cos t)}$$

$$= \frac{12 \times 2 \sin \frac{t}{2} \cos \frac{t}{2}}{10 \times 2 \sin^2 \frac{t}{2}} = \frac{6}{5} \cot \frac{t}{2}$$ 1

$$\left.\frac{dy}{dx}\right]_{t=\frac{2\pi}{3}} = \frac{6}{5} \cot \frac{\pi}{3} = \frac{6}{5} \times \frac{1}{\sqrt{3}} = \frac{6}{5\sqrt{3}}$$ 1

OR

$y = \log(x + \sqrt{a^2 + x^2})$

$\frac{dy}{dx} = \frac{1}{(x + \sqrt{a^2+x^2})} \cdot \frac{d}{dx}\left(x + \sqrt{a^2 + x^2}\right)$ 1

$\frac{dy}{dx} = \frac{1}{(x + \sqrt{a^2+x^2})} \cdot \left(1 + \frac{2x}{2\sqrt{a^2+x^2}}\right) = \frac{1}{(x + \sqrt{a^2+x^2})} \cdot \left(\frac{(x + \sqrt{a^2+x^2})}{\sqrt{a^2+x^2}}\right) = \frac{1}{\sqrt{a^2+x^2}}$ 1

22.

$\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right) = \tan^{-1}\left(\frac{1 - \tan x}{1 + \tan x}\right)$ 1

$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} - x\right)\right) = \frac{\pi}{4} - x$ 1

23.

$$\frac{dy}{dx} = 2x(x - 2) \frac{d}{dx} [x(x - 2)]$$

$$\frac{dy}{dx} = 4x(x - 2)(x - 1)$$

$\frac{dy}{dx} = 0 \Rightarrow x = 0, 1, 2$ 1

Interval	sign of $\frac{dy}{dx} = 4x(x-2)(x-1)$ ...(i)	Nature of $y = f(x)$
$(-\infty, 0]$	Take $x = -1$ (say). Then from (i), $\frac{dy}{dx} = (-)(-)(-) = (-)$ (or $= 0$ at $x = 0$) i.e., ≤ 0	$\therefore$ $f(x)$ is decreasing ↓
$[0, 1]$	Take $x = \frac{1}{2}$ (say). Then from (i), $\frac{dy}{dx} = (+)(-)(-) = (+)$ (or $= 0$ at $x = 0, x = 1$) i.e., ≥ 0	$\therefore$ $f(x)$ is increasing ↑
$[1, 2]$	Take $x = 1.5$ (say). Then from (i), $\frac{dy}{dx} = (+)(-)(+)$	$\therefore$ $f(x)$ is decreasing ↓

1

24.

$\int \sqrt{1 + \sin 2x}\, dx = \int \sqrt{(\sin x + \cos x)^2}\, dx$ 1

$\int (\sin x + \cos x)\, dx$ = -$\cos x$ + $\sin x$ +c 1

25.

$$3^2 + 4^2 + 5^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$9 + 16 + 25 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$50 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$ 1

$$2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = -50$$

$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = \frac{-50}{2}$$ 1

$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -25$$

	OR $(a + \lambda b) \cdot c = 0$ $((2-\lambda)\mathbf{i} + (2+2\lambda)\mathbf{j} + (3+\lambda)\mathbf{k}) \cdot (3\mathbf{i} + \mathbf{j} + 0\mathbf{k}) = 0$ $(2-\lambda)(3) + (2+2\lambda)(1) + (3+\lambda)(0) = 0$ $6 - 3\lambda + 2 + 2\lambda = 0$ $8 - \lambda = 0$ $\lambda = 8$	
Q. No.	**Section - C**	Marks
26.	$x\sqrt{1+y} + y\sqrt{1+x} = 0$ $x\sqrt{1+y} = -y\sqrt{1+x}$ $x^2(1+y) = y^2(1+x)$ $x^2 - y^2 = y^2x - x^2y$ $(x+y)(x-y) = -xy\,(x-y)$ $x + y = -xy\,;$ $\quad x = -y - xy$ $y(1+x) = -x;$ $\quad y = -\frac{x}{1+x}$ $\quad [\,since, x \neq y]$ $\frac{dy}{dx} = -\left\{\frac{(1+x)\times 1 - x(0+1)}{(1+x)^2}\right\}$ $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$ OR If $\sin y = x \sin(a+y)$, prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ Solution: We have, $\sin y = x \sin(a+y)$ $x = \frac{\sin y}{\sin(a+y)}$ Differentiating both sides with respect to y, we get $\frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y \cos(a+y)}{\sin^2(a+y)}$ $\frac{dx}{dy} = \frac{\sin(a+y-y)}{\sin^2(a+y)}$ $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{\sin^2(a+y)}{\sin a}$	1 1 1
27.	$\int \frac{1-\sin 2x}{x+\cos^2 x}dx = \int \frac{1}{t}dt$ $= \log\lvert t \rvert + C = \log\lvert x + \cos^2 x \rvert + C$ Let $x + \cos^2 x = t$ $\Rightarrow (1 - 2\cos x \sin x)\,dx = dt$ $\Rightarrow (1 - \sin 2x)\,dx = dt$	1 1 1
28.	The given differential equation $e^x tany\, dx + (1 - e^x)sec^2 y\, dy = 0$ $\Rightarrow e^x tany\, dx = (e^x - 1)sec^2 y\, dy$ Separating the variables $\frac{e^x}{(e^x - 1)}dx = \frac{sec^2 y\, dy}{tany}$ On integrating both sides we get $\int \frac{e^x}{(e^x-1)}dx = \int \frac{sec^2 y\, dy}{tany} \Rightarrow \log\lvert e^x - 1\rvert - log\lvert tany\rvert + log\lvert C\rvert$ OR $\frac{dy}{dx} + sec^2 x \,.\, y = tanx \,.\, sec^2 x$ which is of the form $\frac{dy}{dx} + P\,y = Q$ Here P = $sec^2 x$ Q= $tanx \,.\, sec^2 x$ Integrating factor (I.F) = $e^{\int P\,dx} = e^{\int sec^2 x\, dx} = e^{tan\, x}$ The solution of the linear differential equation is given by	1 1 1 1 1

$y\,(I.F) = \int [Q\,.\,(I.F)]\,dx + C$

$\Rightarrow y\,e^{\tan x} = \int \tan x\,.\sec^2 x\, e^{\tan x}\,dx + C$ ----(1)

Put $\tan x = t \quad \sec^2 x\ dx = dt$

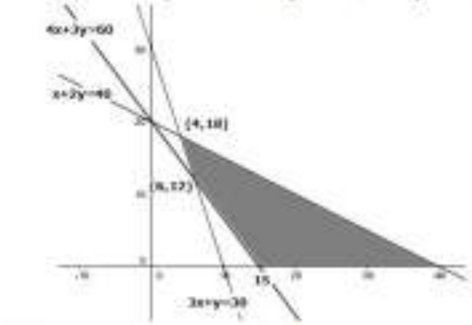
$y\,e^{\tan x} = \int t\,e^t\,dt + C \qquad \Rightarrow y\,e^{\tan x} = t\,e^t - e^t + C$

$\Rightarrow y\,e^{\tan x} = e^t(t-1) + C \quad \Rightarrow y\,e^{\tan x} = e^{\tan x}(\tan x - 1) + C$

29.

Subject to $x + 2y \le 40, 3x + y \ge 30, 4x + 3y \ge 60, x, y \ge 0$ 1

2



Corner Pts	Z=20x+10y
(15,0)	300
(40,0)	800
(4,18)	260
(6,12)	240

imum Value of Z is 240 at (6,12)

30.

SOL: Given $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

A Unit vector perpendicular to $\vec{a}$ and $\vec{b} = \hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$ 1

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}$$

$= \hat{i}(-14+14) - \hat{j}(2-21) + \hat{k}(-2+21) = 19\hat{j} - 19\hat{k}$ 1

$|\vec{a} \times \vec{b}| = \sqrt{(19)^2 + (19)^2} = 19\sqrt{2}$

A Unit vector perpendicular to $\vec{a}$ and $\vec{b} = \hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \frac{19\hat{j} - 19\hat{k}}{19\sqrt{2}} = \frac{\hat{j} - \hat{k}}{\sqrt{2}}$

1

31

(i) P(A and B): $P(A \text{ and } B) = P(A) \cdot P(B) = 0.3 \cdot 0.6 = 0.18$ 1

(ii) P(A and not B): $P(A \text{ and not } B) = P(A) \cdot P(\text{not } B) = 0.3 \cdot 0.4 = 0.12$

(iii) P(A or B): 1

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 0.3 + 0.6 - 0.18 = 0.9 - 0.18 = 0.72$

(iv) P(neither A nor B): 1

$P(\text{neither } A \text{ nor } B) = P(\text{not } A \text{ and not } B) = P(\text{not } A) \cdot P(\text{not } B) = 0.7 \cdot 0.4 = 0.28$

OR

$$P(\text{neither solves}) = P(A') \times P(B') = \frac{1}{2} \times \frac{2}{3} = \frac{2}{6} = \frac{1}{3}$$

The probability that the problem is solved is $1 - P(\text{neither solves})$.

$$P(\text{problem is solved}) = 1 - \frac{1}{3} = \frac{2}{3}$$

2

$$P(\text{exactly one solves}) = P(A)P(B') + P(B)P(A') = \frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

1

Section - D

32.

Shortest Distance between the lines $= \frac{|(\vec{a_2}-\vec{a_1})\cdot(\vec{b_1}\times\vec{b_2})|}{|\vec{b_1}\times\vec{b_2}|}$ 1

$\vec{a_1} = -\hat{i}-\hat{j}-\hat{k}$ $\quad \vec{a_2} = 3\hat{i}+5\hat{j}+7\hat{k}$ $\quad \vec{a_2}-\vec{a_1} = 4\hat{i}+6\hat{j}+8\hat{k}$

$\vec{b_1} = 7\hat{i}-6\hat{j}+\hat{k}$ $\quad \vec{b_2} = \hat{i}-2\hat{j}+\hat{k}$

$$\vec{b_1}\times\vec{b_2} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 7 & -6 & 1\\ 1 & -2 & 1\end{vmatrix} = \hat{i}(-6+2)-\hat{j}(7-1)+\hat{k}(-14+6) = -4\hat{i}-6\hat{j}-8\hat{k}$$ 2

$|\vec{b_1}\times\vec{b_2}| = \sqrt{16+36+64} = \sqrt{116}$

$SD = \frac{|(\vec{a_2}-\vec{a_1})\cdot(\vec{b_1}\times\vec{b_2})|}{|\vec{b_1}\times\vec{b_2}|} = \left|\frac{(4\hat{i}+6\hat{j}+8\hat{k})\cdot(-4\hat{i}-6\hat{j}-8\hat{k})}{\sqrt{116}}\right|$

$SD = \left|\frac{(4)(-4)+(6)(-6)+(8)(-8)}{\sqrt{116}}\right| = \left|\frac{-16-36-64}{\sqrt{116}}\right| = \frac{116}{\sqrt{116}} = \sqrt{116} = 2\sqrt{29}$ 2

OR

Sol: Let P(1,2,1) be the given point and L be the foot of the perpendicular from P to the given line AB (as shown in the figure above). 1

Let's put $\frac{x-3}{1} = \frac{y+1}{2} = \frac{z-1}{3} = \lambda$. Then, $x = \lambda+3, y = 2\lambda-1, z = 3\lambda+1$

Let the coordinates of the point L be $(\lambda+3, 2\lambda-1, 3\lambda+1)$.

So, direction ratios of PL are $(\lambda+3-1, 2\lambda-1-2, 3\lambda+1-1)$ *i.e.*, $(\lambda+2, 2\lambda-3, 3\lambda)$

Direction ratios of the given line are 1,2 and 3, which is perpendicular to PL. Therefore, we have, 2

$$(\lambda+2)\cdot 1 + (2\lambda-3)\cdot 2 + 3\lambda\cdot 3 = 0 \Rightarrow 14\lambda = 4 \Rightarrow \lambda = 2/7$$

Then, $\lambda+3 = \frac{2}{7}+3 = \frac{23}{7}$; $2\lambda-1 = 2\left(\frac{2}{7}\right)-1 = -\frac{3}{7}$; $3\lambda+1 = 3\left(\frac{2}{7}\right)+1 = \frac{13}{7}$

Therefore, coordinates of the point L are $\left(\frac{23}{7}, -\frac{3}{7}, \frac{13}{7}\right)$. 2

33.

$$\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$$

$$= \begin{bmatrix}-4+4+8 & 4-8+4 & -4-8+12\\ -7+1+6 & 7-2+3 & -7-2+9\\ 5-3-2 & -5+6-1 & 5+6-3\end{bmatrix}$$

$$= \begin{bmatrix}8 & 0 & 0\\ 0 & 8 & 0\\ 0 & 0 & 8\end{bmatrix} = 8I$$

$$\text{If } A = \begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$$ 2

$$\text{then } A^{-1} = \frac{1}{8}\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}$$

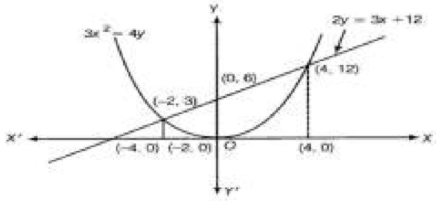
$$\therefore X = A^{-1}B = \frac{1}{8}\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}\begin{bmatrix}4\\ 9\\ 1\end{bmatrix}$$ 2

$$X = \frac{1}{8}\begin{bmatrix}-16+36+4\\ -28+9+3\\ 20-27-1\end{bmatrix} = \frac{1}{8}\begin{bmatrix}24\\ -16\\ -8\end{bmatrix} = \begin{bmatrix}3\\ -2\\ -1\end{bmatrix}$$ 1

$\Rightarrow x = 3, y = -2, z = -1$

34.

Solution:



Required are $= \int_{-2}^{4}\left(\frac{3x+12}{2} - \frac{3x^2}{4}\right) dx$ 2

$=\left[\frac{3x^2}{4} + 6x - \frac{x^3}{4}\right]_{-2}^{4} = \frac{3\times4^2}{4} + 6\times4 - \frac{4^3}{4} - \left[\frac{3\times4}{4} - 12 + \frac{8}{4}\right]$ 3

$=12 + 24 - 16 - 3 + 12 - 2 = 27$ sq units

35.

$$\frac{x^2+1}{(x-1)^2(x+3)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+3}$$ 1

Set $x = 1$: $1^2 + 1 = B(1+3) \Longrightarrow 2 = 4B \Longrightarrow B = \frac{1}{2}$

Set $x = -3$: $(-3)^2 + 1 = C(-3-1)^2 \Longrightarrow 10 = 16C \Longrightarrow C = \frac{10}{16} = \frac{5}{8}$

Set $x = 0$: $0^2 + 1 = A(-1)(3) + B(3) + C(-1)^2 \Longrightarrow 1 = -3A + 3B + C$

Substitute the values of B and C:

$1 = -3A + 3\left(\frac{1}{2}\right) + \frac{5}{8} \Longrightarrow 1 = -3A + \frac{12}{8} + \frac{5}{8} \Longrightarrow 1 = -3A + \frac{17}{8}$ 2

$3A = \frac{17}{8} - 1 \Longrightarrow 3A = \frac{9}{8} \Longrightarrow A = \frac{3}{8}$

$$\int \frac{x^2+1}{(x-1)^2(x+3)}\,dx = \int\left(\frac{3/8}{x-1} + \frac{1/2}{(x-1)^2} + \frac{5/8}{x+3}\right)dx$$

We integrate each term separately:

$$\int \frac{3/8}{x-1}\,dx = \frac{3}{8}\ln|x-1|$$

$$\int \frac{1/2}{(x-1)^2}\,dx = \frac{1}{2}\int (x-1)^{-2}dx = \frac{1}{2}\,\frac{(x-1)^{-1}}{-1} = -\frac{1}{2(x-1)}$$

$$\int \frac{5/8}{x+3}\,dx = \frac{5}{8}\ln|x+3|$$

Combining these results and adding the constant of integration, C_{int}: 2

$$\int \frac{x^2+1}{(x-1)^2(x+3)}\,dx = \frac{3}{8}\ln|x-1| - \frac{1}{2(x-1)} + \frac{5}{8}\ln|x+3| + C_{int}$$

OR

$$\int_0^{\frac{\pi}{2}} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$$

Divide the numerator and denominator by $\cos^2 x$:

$$\int_0^{\frac{\pi}{2}} \frac{\sec^2 x\,dx}{a^2 + b^2\tan^2 x}$$ 1

Let $u = \tan x$, so $du = \sec^2 x\,dx$. The limits of integration change from $x = 0$ to $u = \tan(0) = 0$ and from $x = \frac{\pi}{2}$ to $u = \tan(\frac{\pi}{2}) = \infty$.

$$\int_0^{\infty} \frac{du}{a^2 + b^2u^2} = \frac{1}{b^2}\int_0^{\infty}\frac{du}{(a/b)^2 + u^2}$$ 2

Using the standard integral formula $\int \frac{dx}{c^2+x^2} = \frac{1}{c}\arctan(\frac{x}{c})$, we get:

$$\frac{1}{b^2}\left[\frac{b}{a}\arctan\left(\frac{bu}{a}\right)\right]_0^{\infty} = \frac{1}{ab}(\arctan(\infty) - \arctan(0)) = \frac{1}{ab}\left(\frac{\pi}{2} - 0\right) = \frac{\pi}{2ab}$$ 2

36	(i) Given that $\frac{dv}{dt} = 100$ and $v = \frac{1}{3}\pi r^2 h$ or $v = \frac{2}{9}\pi r^3$ So, $\frac{dv}{dt} = 100 = \frac{2}{3}\pi r^2 \frac{dr}{dt}$ implies $\frac{dr}{dt} = \frac{150}{\pi r^2}$ Now area of the base is $A = \pi r^2$ and $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ Therefore, rate of change in area of base at radius = 5, is $\frac{dA}{dt} = 60\ cm^2$ /minute (ii) Given that $\frac{dh}{dt} = \frac{4}{11}$ and $h = 7$ Then $v = \frac{1}{3}\pi r^2 h = \frac{3\pi h^3}{4}$ and hence, $\frac{dv}{dt} = \frac{9}{4}\pi h^2 . \frac{dh}{dt}$ $\frac{dv}{dt} = 2 \times 7 \times 9 = 126\ cc/min$	2 2
37.	(i)Let $(l_1, l_2) \in R \Rightarrow l_1 \parallel l_2 \Rightarrow l_2 \parallel l_1 \Rightarrow (l_2, l_1) \in R \Rightarrow$ R is a symmetric relation. (ii) Let $(l_1, l_2) \in R \Rightarrow l_1 \parallel l_2$ and Let $(l_2, l_3) \in R \Rightarrow l_2 \parallel l_3$ Since $l_1 \parallel l_2$ and $l_2 \parallel l_3, l_1 \parallel l_3 \Rightarrow (l_1, l_3) \in R$ Hence R is a transitive relation. (iii)(a) The set is $\{l : l\ is\ a\ line\ of\ type\ y = 3x + c, c \in R\}$ (b) Let $(l_1, l_2) \in S \Rightarrow l_1 \perp l_2 \Rightarrow l_2 \perp l_1 \Rightarrow (l_2, l_1) \in S$ $\Rightarrow$ S is symmetric. Let $(l_1, l_2) \in S \Rightarrow l_1 \perp l_2$ and Let $(l_2, l_3) \in S \Rightarrow l_2 \perp l_3$ $l_1 \perp l_2$ and $l_2 \perp l_3 \Rightarrow l_1 \parallel l_3$ $\Rightarrow (l_1, l_3)$ $is\ not\ an\ element\ of\ S.$ S is not a transitive relation.	1 1 2
38.	$(a) A: Cab B: Metro C: Bike D: other$ $E: Late arrival$ $P(E) = P(A).P(E/A) + P(B).P(E/B) + P(C).P(E/C) + P(D).P(E/D)$ $= \frac{3}{10}.\frac{1}{4} + \frac{1}{5}.\frac{1}{3} + \frac{1}{10}.\frac{1}{12} + \frac{2}{5}.\frac{1}{10} = \frac{114}{600}$ $(b) P(B/E) = \frac{P(B).P(E/B)}{P(E)} = \frac{1/15}{114/600} = 40/114 = 20/57$	2 2