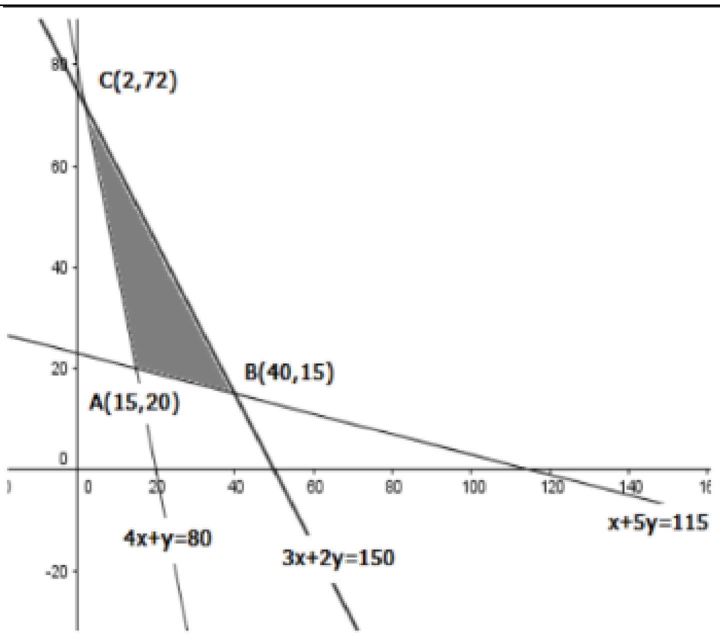
ANSWER KEY Pre Board Mock Test 06

1	(a)$x\frac{dy}{dx}+y=0$	1
2	(c)$\frac{1}{2}\ e^{2x}tanx+c$	1
3	(b) x^6/6 +c	1
4	(d) i$\neq j$	1
5	(c)0	1
6	(d)q=3p	1
7	(c)($\left(\pm\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}}\right)$)	1
8	(a)$\sqrt{3}$	1
9	(d)2and 4	1
10	(b)64	1
11	(a) $\frac{1}{detA}$	1
12	(b)$\frac{\pi}{3}$	1
13	(b)2	1
14	(c) 0	1
15	(d) Neither one-one nor onto	1
16	(b) p = q/2	1
17	(d) 1/y	1
18	(b) 4, not defined	1
19	(d)Assertion is false but Reason is true.	1
20	(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.	1
21	proving A^2 -9A +2I =O and finding $A^{-1}=\frac{9I-A}{2}$	2
22	Let $\sin^{-1}\frac{3}{4}=x \quad \therefore \sin x=\frac{3}{4} \quad \therefore \cos x=\frac{\sqrt{7}}{4}$ L H S = $\tan\frac{x}{2}=\sqrt{\frac{1-\cos x}{1+\cos x}}=\sqrt{\frac{1-\frac{\sqrt{7}}{4}}{1+\frac{\sqrt{7}}{4}}}=\frac{4-\sqrt{7}}{3}=RHS$	2
23	$\int\frac{(x^2+1)}{(x^2+2)(x^2+3)}dx\, x^2=t,$ = $\int\frac{t+1}{(t+2)(t+3)}dt$	2

	$\frac{t + 1}{(t + 2)(t + 3)} = \frac{A}{(t + 2)} + \frac{B}{(t + 3)}$ $t + 1 = A\,(t + 3) + B\,(t + 2)$ $Let\colon\ t = -2, \quad -1 = A + 0, \quad A = -1,$ $t = -3, \quad -2 = 0 - B, \quad B = 2$ $= \int \frac{t+1}{(t+2)(t+3)} dt = \int \frac{-1}{(t+2)} dt + \int \frac{2}{(t+3)} dt$ $= -log\lvert t + 2\rvert + 2\ log\lvert t + 3\rvert + c$ $= 2log\lvert x^2 + 3\rvert - \ log\lvert x^2 + 2\rvert + c$ **OR** I= $\int \frac{cosx}{(1-sinx)(2-sinx)} dx$ Put $sinx = \ t \ \Rightarrow cosx\ dx = \ dt$ $\therefore I = \int \frac{dt}{(1-t)(2-t)}$ $\Rightarrow A = 1, B = -1$ $Hence\ I = \int \frac{1}{(1-t)(2-t)} dt = \int \left(\frac{1}{(1-t)} - \frac{1}{2-t}\right) dx$ $= \int \frac{1}{1-t} dx - \int \frac{dt}{2-t} = -log\lvert 1-t\rvert + log\lvert 2-t\rvert + C$ $= log\left\lvert\frac{2-t}{1-t}\right\rvert + C$ $= log\left\lvert\frac{2-sinx}{1-sinx}\right\rvert + C$	
24	$\lim_{x\to3^+} f(x) = \lim_{x\to3^-} f(x) = f(3)$ f(3) = 4 $\lim_{x\to3^+} f(x) = \lim_{x\to3^+} 3x - 5 = 4$ $\lim_{x\to0^-} f(x) = \lim_{x\to3^-} x + k = 3 + k$ 3+k = 4 , k = 1	**2**
25	the volume of a cube with radius “x” is given by V = x^3 and surface area = $6x^2$ Hence, the rate of change of volume “V’ with respect to the time “t” is given by: $\frac{dV}{dt}$ = 9 cm^3/s $9 = \frac{dV}{dt} = \frac{d}{dt}x^3 = 3x^2.\frac{dx}{dt}$, By using the chain rule $\frac{dx}{dt} = \frac{3}{x^2}$ $\frac{ds}{dt} = \frac{d(6x^2)}{dt} = 12x\frac{dx}{dt} = 12x\frac{3}{x^2} = \frac{36}{x}$ At x= 10cm ,$\frac{ds}{dt}$ = 3.6 cm^2/s **OR** p(x) = 41-72x-18x^2 P’(x) = -72 -36 P’’ = -36 For maximum or minina or critical points p’(x) =0 -72 -36x =0 , x=-2 x=-2, p’’ (-2) = -36 <0 hence x= -2 is a point of local maxima, maximum profit = 113	**2**
26		



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1

Corner pts	Z=6x+3y
A(15,20)	150
B(40,15)	285
C(2,72)	228

27

$I = \int_0^{\frac{\pi}{4}} \log(1+tanx)dx$

$I = \int_0^{\frac{\pi}{4}} \log\left(1+\tan\left(\frac{\pi}{4}-x\right)\right)dx$

$I = \int_0^{\frac{\pi}{4}} \log\left(1+\frac{tan\frac{\pi}{4}-tanx}{1+tan\frac{\pi}{4}tanx}\right)dx$

$I = \int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1+tanx}\right)dx = \int_0^{\frac{\pi}{4}} \log(2)\,dx - I$

$2I = \int_0^{\frac{\pi}{4}} \log(2)\,dx = \log 2\left(\frac{\pi}{4}-0\right) = \frac{\pi}{4}log2$

$I = \frac{\pi}{8}log2$

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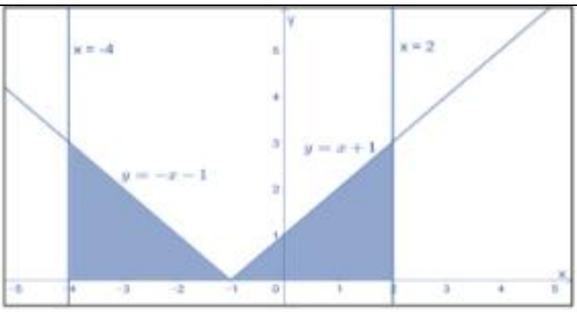


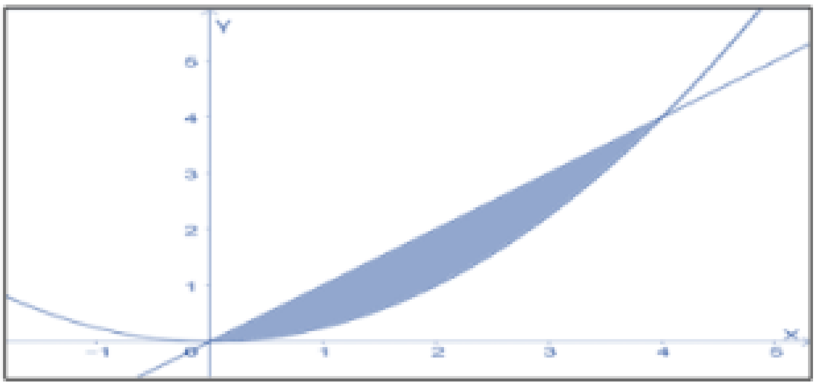
$\int_{-4}^{2}\lvert x+1\rvert dx= \int_{-4}^{-1}(-x-1)\,dx + \int_{-1}^{2}(x+1)\,dx$

$= -\left.\frac{(x+1)^2}{2}\right]_{-4}^{-1} + \left.\frac{(x+1)^2}{2}\right]_{-1}^{2}$

$= -\left(0-\frac{9}{2}\right)+\left(\frac{9}{2}-0\right)=9$

It represent the area of shaded region bounded by the curve $y=\lvert x+1\rvert$, $x-$ axis and the lines $x=-4$ and $x=2$

OR



Required Area = $\int_0^4 x\,dx - \int_0^4 \frac{x^2}{4}\,dx$

$= \left.\frac{x^2}{2}\right]_0^4 - \frac{1}{12}[x^3]_0^4$

$= \frac{1}{2}(16-0) - \frac{1}{12}(64-0) = 8 - \frac{16}{3} = \frac{8}{3}$ sq. units

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P(AUB)= 3/5 P(B) = p P(A) = 1/2

P(A UB)= P(A) + P(B) -P(A ∩B)

P(A U B)= p + 1/2 - 3/5 = p - 1/10

3/5 = 1/2 + p -P(A∩ B)

(i) If A and B are mutually exclusive, then A ∩ B =ϕ

P(A∩ B)=0 p = 1/10

(ii) If A and B are independent, then P(A ∩B)= P(A) P(B)

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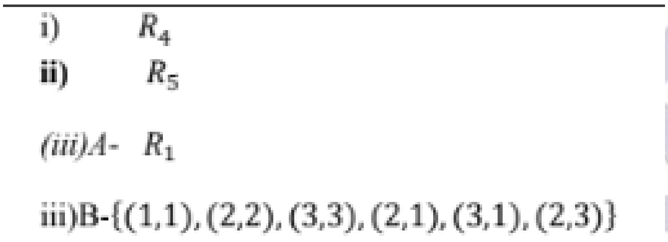
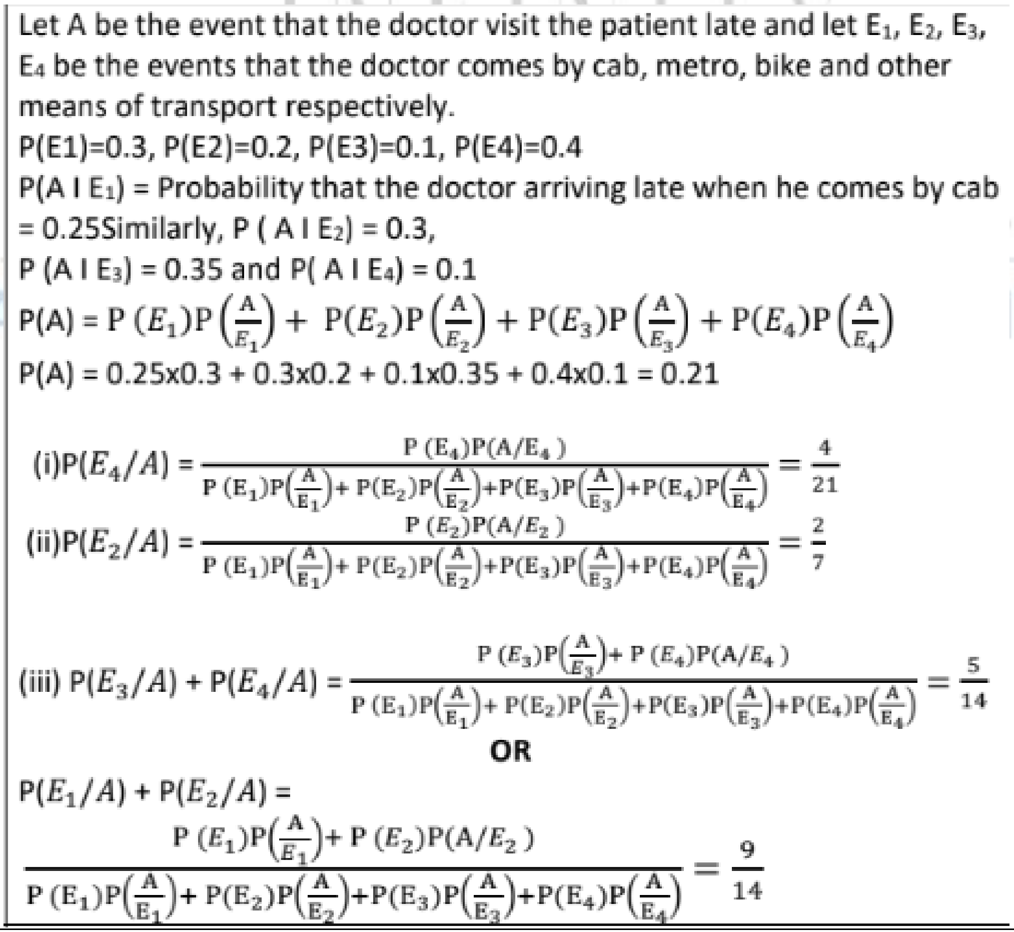
	3/5 = ½ + p – p/2 P = 1/5 **OR** Let E be the event that 'number 4 appears at least once' and F be the event that 'the sum of the numbers appearing is 6. Then, E = {(4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (1,4), (2,4), (3,4), (5,4), (6,4)} F = {(1,5), (2,4), (3,3), (4,2), (5,1)} P(E) = 11/36 and P(F) = 5/36 , E∩F = {(2,4), (4,2)}, P(E∩F) = 2/36 P(E/F) = $\frac{P(E\cap F)}{P(F)} = \frac{2}{5}$	
30	$\int_1^4 [\lvert x-1\rvert + \lvert x-2\rvert + \lvert x-3\rvert]\,dx$ $\int_1^4 \lvert x-1\rvert dx + \int_1^4 \lvert x-2\rvert dx + \int_1^4 \lvert x-3\rvert dx$ $= \int_1^4 (x-1)dx + \int_1^2 -(x-2)dx + \int_2^4 (x-2)dx + \int_1^3 -(x-3)\,dx + \int_3^4 (x+3)\,dx$ For calculation $\int_1^4 [\lvert x-1\rvert + \lvert x-2\rvert + \lvert x-3\rvert]\,dx = \frac{19}{2}$	**3**
31	e^x tan y dx + (1-e^x) $\sec^2$ y dy =0 (1-e^x) $\sec^2$ y dy = - e^x tan y dx $\frac{\sec^2 y}{\tan y}\,dy = \frac{-e^x}{1-e^x}dx$ log |tany|= log |1-e^x| + log|c| log |tany|- log |1-e^x| = log|c| Taking exponential both sides tany = c (1-e^x)	**3**
32	the product of two matrices is 7 I And the inverse of $\begin{bmatrix}1 & 2 & -3\\ 3 & 2 & -2\\ 2 & -1 & 1\end{bmatrix}$ is 1/7 of $\begin{bmatrix}0 & 1 & 2\\ -7 & 7 & -7\\ -7 & 5 & -4\end{bmatrix}$ Let X = $\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ and B = $\begin{bmatrix}6\\ 3\\ 2\end{bmatrix}$ using X= A^{-1}B , we get x=1,y=-5 and z=-5.	**2** **1** **2**
33		**1** **1**

	Evaluate : $\int_0^{\pi} \frac{x\tan x}{\sec x+\tan x)} dx$ $I=\int_0^{\pi} \frac{(\pi-x)\tan(\pi-x)}{\sec(\pi-x)+\tan(\pi-x)} dx$ $\int_0^{\pi} \frac{(\pi-x)\tan x}{\sec x+\tan x} dx$ $I=\int_0^{\pi} \frac{x}{1+\sin x} dx$ $= \int_0^{\pi} \frac{\pi-x}{1+\sin(\pi-x)} dx$ $= \int_0^{\pi} \frac{\pi-x}{1+\sin x} dx$ $I+I = \int_0^{\pi} \frac{\pi}{1+\sin x} dx$ $= \pi \int_0^{\pi} \frac{1-\sin x}{\cos^2 x} dx$ $= \pi \int_0^{\pi} (\sec^2 x - \sec x \tan x) dx$ $= 2\pi$ $\Rightarrow I = \pi$	1 1 1
34	$2x + 4y + 3z = 29000$ $5x + 2y + 3z = 30500$ $x + y + z = 9500$ matrix method AX=B $X=A^{-1}B$ IAI = -1 adj A= $\begin{bmatrix} -1 & -1 & 6 \\ -2 & -1 & 9 \\ 3 & 2 & -16 \end{bmatrix}$ calculation X= 2500, y= 3000 and z =4000	1 1/2 2 1½
35		1 1/2 1 1/2 1/2

	$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ *and* $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ $\overrightarrow{a_1}=\hat{\imath}+2\hat{\jmath}+3\hat{k}$, $\overrightarrow{a_2}=2\hat{\imath}+4\hat{\jmath}+5\hat{k}$ $\overrightarrow{b_1}=2\hat{\imath}+3\hat{\jmath}+4\hat{k}$, $\overrightarrow{b_2}=3\hat{\imath}+4\hat{\jmath}+5\hat{k}$ $\overrightarrow{a_2}-\overrightarrow{a_1}=\hat{\imath}+2\hat{\jmath}+2\hat{k}$ $\overrightarrow{b_1}\times\overrightarrow{b_2}=-\hat{\imath}+2\hat{\jmath}-\hat{k}$ $\lvert\overrightarrow{b_1}\times\overrightarrow{b_2}\rvert=\sqrt{6}$ $\overrightarrow{(b_1}\times\overrightarrow{b_2)}.\overrightarrow{(a_2}-\overrightarrow{a_1})=1$ S.D. = d = $\left\lvert\frac{\overrightarrow{(b_1}\times\overrightarrow{b_2)}.\overrightarrow{(a_2}-\overrightarrow{a_1})}{\lvert\overrightarrow{b_1}\times\overrightarrow{b_2}\rvert}\right\rvert$ Shortest distance = d = $\frac{1}{\sqrt{6}}$ The lines do not intersect **OR** Eq. of line $\vec{r}=\vec{a}+\lambda\vec{b}$ Line passes through (1,2,-4) and let (a,b,c) be the direction ratio of line then Eq of line is $\vec{r}=\hat{\imath}+2\hat{\jmath}-4\hat{k}+\lambda(a\hat{\imath}+b\hat{\jmath}+c\hat{k})$ Line is perpendicular to the lines	1/2 1/2 1/2 1/2
36	 i) R_4 **ii)** R_5 *(iii)A-* R_1 iii)B-{(1,1), (2,2), (3,3), (2,1), (3,1), (2,3)}	1 1 2
37	 Let A be the event that the doctor visit the patient late and let E_1, E_2, E_3, E_4 be the events that the doctor comes by cab, metro, bike and other means of transport respectively. P(E1)=0.3, P(E2)=0.2, P(E3)=0.1, P(E4)=0.4 P(A I E_1) = Probability that the doctor arriving late when he comes by cab = 0.25Similarly, P (A I E_2) = 0.3, P (A I E_3) = 0.35 and P(A I E_4) = 0.1 P(A) = $P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)+P(E_3)P\left(\frac{A}{E_3}\right)+P(E_4)P\left(\frac{A}{E_4}\right)$ P(A) = 0.25x0.3 + 0.3x0.2 + 0.1x0.35 + 0.4x0.1 = 0.21 (i)P(E_4/A) = $\frac{P(E_4)P(A/E_4)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)+P(E_3)P\left(\frac{A}{E_3}\right)+P(E_4)P\left(\frac{A}{E_4}\right)}=\frac{4}{21}$ (ii)P(E_2/A) = $\frac{P(E_2)P(A/E_2)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)+P(E_3)P\left(\frac{A}{E_3}\right)+P(E_4)P\left(\frac{A}{E_4}\right)}=\frac{2}{7}$ (iii) P(E_3/A) + P(E_4/A) = $\frac{P(E_3)P\left(\frac{A}{E_3}\right)+P(E_4)P(A/E_4)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)+P(E_3)P\left(\frac{A}{E_3}\right)+P(E_4)P\left(\frac{A}{E_4}\right)}=\frac{5}{14}$ **OR** P(E_1/A) + P(E_2/A) = $\frac{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P(A/E_2)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)+P(E_3)P\left(\frac{A}{E_3}\right)+P(E_4)P\left(\frac{A}{E_4}\right)}=\frac{9}{14}$	1 1 2
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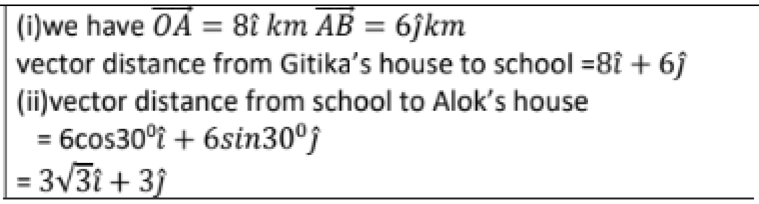
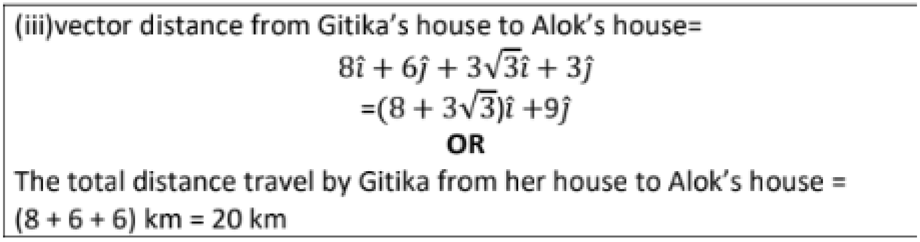
	(i)we have $\overline{OA} = 8\hat{\imath}\ km\ \overline{AB} = 6\hat{\jmath}km$ vector distance from Gitika's house to school =$8\hat{\imath} + 6\hat{\jmath}$	1
	(ii)vector distance from school to Alok's house = $6\cos30^0\hat{\imath} + 6sin30^0\hat{\jmath}$ = $3\sqrt{3}\hat{\imath} + 3\hat{\jmath}$	1
	(iii)vector distance from Gitika's house to Alok's house= $8\hat{\imath} + 6\hat{\jmath} + 3\sqrt{3}\hat{\imath} + 3\hat{\jmath}$ =$(8 + 3\sqrt{3})\hat{\imath}$ +$9\hat{\jmath}$ **OR** The total distance travel by Gitika from her house to Alok's house = (8 + 6 + 6) km = 20 km	2