

**KENDRIYA VIDYALAYA SANGATHAN, PATNA REGION**

**PRE-BOARD I (2025-26)**

**Class: 12**

**F.M: 80**

**(SET-A)**

**Subject: Mathematics**

**Time: 3Hrs**

**General Instructions:**

1. All questions are compulsory.
2. The question paper consists 38 questions divided into 5 sections A, B, C, D & E.
3. Section A consist 18 MCQ and 2 ASSERTION-REASON BASED QUESTIONS carry 1 mark each, section B consist 5 questions carry 2 mark each, section C consist 6 questions carry 3 mark each, section D consist 4 questions carry 5 mark each, section E consist 3 questions (CASE BASED STUDY) carry 4 marks each.
4. There is no overall choice. However, and internal choice has been provided in 2 questions in section B, 3 questions in section C, 2 questions in sections D & 2 questions in sections E
5. Use of calculator is not permitted.

**Section A**

1. The maximum number of equivalence relation on the set  $A = \{1, 2, 3\}$  are  
(a) 1 (b) 2  
(c) 3 (d) 5
2. Set A has 3 elements and the set B has 4 elements, then the number of injective function that can be defined from A to B is  
(a) 144 (b) 12  
(c) 24 (d) 64
3. If  $y = \sin^{-1}x$ ,  $-1 \leq x \leq 0$  then the range of y is  
(a)  $(-\frac{\pi}{2}, 0)$  (b)  $[-\frac{\pi}{2}, 0]$   
(c)  $[-\frac{\pi}{2}, 0)$  (d)  $(-\frac{\pi}{2}, 0]$
4. If  $A = \begin{bmatrix} a & c & -1 \\ b & 0 & 5 \\ 1 & -5 & 0 \end{bmatrix}$  is a skew symmetric matrix, then the value of  $2a - (b+c)$  is  
(a) 0 (b) 1  
(c) -10 (d) 10
5. For a square matrix  $A$ ,  $A^2 - 3A + I = 0$  and  $A^{-1} = xA + yI$  then the value of  $x+y$  is  
(a) -2 (b) 2  
(c) 3 (d) -3
6. If  $|A|=2$ , where A is a  $2 \times 2$  matrix then  $|4A^{-1}|$  equals  
(a) 4 (b) 2  
(c) 8 (d)  $\frac{1}{32}$

7. The value of k for which  $f(x) = \begin{cases} 3x + 5, & x \geq 2 \\ kx^2, & x < 2 \end{cases}$  is a continuous function, is

- (a)  $\frac{-11}{4}$  (b)  $\frac{4}{11}$   
(c) 11 (d)  $\frac{11}{4}$

8. The function  $f(x)=[x]$ , where  $[x]$  denotes the greatest integer function, is continuous at

- (a) 4 (b) -2  
(c) 3 (d) 1.5

9. If  $\sin y = x \cdot \cos(a + y)$  then  $\frac{dy}{dx}$  is

- (a)  $\frac{\cos a}{\cos^2(a+y)}$  (b)  $\frac{-\cos a}{\cos^2(a+y)}$   
(c)  $\frac{\cos a}{\sin^2 y}$  (d)  $\frac{-\cos a}{\sin^2 y}$

10. If  $x = A \cos 4t + B \sin 4t$  then  $\frac{d^2x}{dt^2}$  is equal to

- (a) X (b)  $-x$   
(c)  $16x$  (d)  $-16x$

11. The function  $f(x) = x^2 - 4x + 6$  is increasing in the interval

- (a)  $(0, 2)$  (b)  $(-\infty, 2]$   
(c)  $[1, 2]$  (d)  $[2, \infty)$   
(e)

12.  $\int e^x [\sec x (1 + \tan x)] dx$  is

- (a)  $e^x \cdot \cos x + c$  (b)  $e^x \cdot \sec x + c$   
(c)  $e^x \cdot \tan x + c$  (d)  $e^x \cdot \cot x + c$

13.  $\int_{-1}^1 x|x| dx$  is

- (a)  $\frac{1}{6}$  (b)  $\frac{1}{3}$   
(c)  $-\frac{1}{6}$  (d) 0

14. Area of the region bounded by the curve  $y^2 = 4x$  and the x-axis between  $x=0$  and  $x=1$  is

- (a)  $\frac{2}{3}$  (b)  $\frac{8}{3}$   
(c) 3 (d)  $\frac{4}{3}$

15. The difference of order and degree of the differential equation  $\frac{d}{dx} \left( \frac{dy}{dx} \right)^3 = 0$  is

- (a) 0 (b) 1  
(c) 2 (d) 3  
(e)

16. The value of P for which the vectors  $2\hat{i} + p\hat{j} + \hat{k}$  and  $-4\hat{i} - 6\hat{j} + 26\hat{k}$  are perpendicular, is

- (a) 3 (c) -3  
(c)  $\frac{-17}{3}$  (d)  $\frac{17}{3}$

17. If the points  $P(a, b, 0)$  lies on the line  $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z+3}{4}$ , then  $(a, b)$  is

- (a)  $(1, 2)$  (b)  $(\frac{1}{2}, \frac{2}{3})$   
(c)  $(\frac{1}{2}, \frac{1}{4})$  (d)  $(0, 0)$

18. The objective function  $Z = ax + by$  of an LPP has maximum value 42 at  $(4, 6)$  and minimum value 19 at  $(3, 2)$ , which of the following is true?

- (a)  $a=9, b=1$  (b)  $a=9, b=2$   
(c)  $a=3, b=5$  (d)  $a=5, b=3$

#### ASSERTION-REASON BASED QUESTIONS

Question number 19 & 20 are Assertions(A) and Reason(R) based question carrying 1 mark each. The two statements are given, one labelled Assertion(A) & one labelled Reason(R). Select the correct answer from the option given below.

- (a) Both A and R are true and R is the correct explanation of A.  
(b) Both A and R are true but R is not the correct explanation of A.  
(c) A is true but R is false.  
(d) A is false but R is true.

19. Assertion(A): In a linear programming problem if the feasible region is empty, then the linear programming problem has no solution.

Reason(R): A feasible region is defined as the region that satisfies all the constraints.

20. Assertion(A): If  $P(A) = \frac{1}{2}$ ,  $P(A \cap B) = \frac{1}{3}$ , then  $P(A/B) = \frac{2}{3}$

Reason(R):  $P(B/A) = \frac{P(A \cap B)}{P(A)}$

#### Section B

21. Find  $k$  so that  $f(x) = \begin{cases} \frac{x^2-2x-3}{x+1}, & x \neq -1 \\ k, & x = -1 \end{cases}$  is continuous at  $x = -1$

[OR]

Check the differentiability of the function  $f(x) = x|x|$  at  $x = 0$

22. If  $A = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 2 & 3 \\ 3 & 3 & 5 \end{bmatrix}$ , then find  $A(\text{adj}A)$

23. Simplify  $\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$

[OR]

Find the domain of  $\sin^{-1} \sqrt{x-1}$

24. Find the values of  $a$  for which  $f(x) = x^2 - 2ax + b$  is an increasing function for  $x > 0$ .

25. Calculate the area of the region bounded by the curve  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  and the x-axis using Integration.

### Section C

26. Show that the relation  $R$  in the set  $A = \{x \in \mathbb{Z}: 0 \leq x \leq 12\}$  given by  
 $R = \{(a, b): a, b \in \mathbb{Z}, |a - b| \text{ is divisible by } 3\}$  is an equivalence relation.

[OR]

Let  $A = \mathbb{R} - \{2\}$  and  $B = \mathbb{R} - \{1\}$ . If  $f: A \rightarrow B$  is a function defined by  $f(x) = \frac{x-1}{x-2}$ ,  
 show that  $f$  is a bijective function.

27. Find  $\frac{dy}{dx}$ , if  $(\cos x)^y = (\cos y)^x$

[OR]

If  $x = a(\theta - \sin \theta)$ ,  $y = a(1 - \cos \theta)$ ,  $a > 0$ , then find  $\frac{d^2y}{dx^2}$  at  $\theta = \frac{\pi}{3}$ .

28. Find  $\int_0^{\frac{\pi}{4}} \frac{x + \sin x}{1 + \cos x} dx$ .

29. Let  $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$ ,  $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$  and  $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$ . Find a vector  $\vec{d}$   
 which is perpendicular to both  $\vec{a}$  and  $\vec{b}$  and  $\vec{c} \cdot \vec{d} = 18$ .

30. Solve the following linear programming problem graphically:

Maximise  $Z = 3x + 9y$

Subject to constraints:

$$x + y \geq 10, x + 3y \leq 60, x \leq y, x \geq 0, y \geq 0.$$

31. The probability of two students A and B coming to school in time are  $\frac{2}{7}$  and  $\frac{4}{7}$   
 respectively. Assuming that the events 'A coming on time' and 'B coming on time'  
 are independent. Find the probability of only one of them coming to school on  
 time

[OR]

A committee of 4 students is selected at random from a group consisting of 7 boys  
 and 4 girls. Find the probability that there are exactly 2 boys in the committee,  
 given that at least one girl must be there in the committee.

### Section D

32. If  $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$  then find  $A^{-1}$ . Hence solve the system of linear equations:

$$x - 2y = 10; \quad 2x - y - z = 8 \text{ and } -2y + z = 7$$

33. Evaluate:  $\int \frac{5x+3}{\sqrt{x^2+4x+10}} dx$

[OR]

Evaluate:  $\int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx$

34. Find the particular solution of the differential equation  $x \frac{dy}{dx} + x \cos^2 \frac{y}{x} = y$ ; given that when  $x = 1, y = \frac{\pi}{4}$ .

[OR]

Find the general solution of the differential equation:

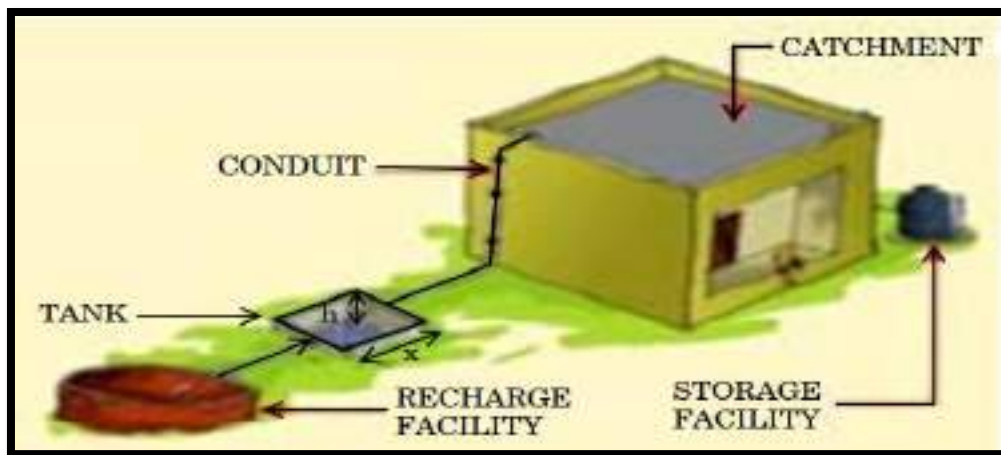
$$(x^2 + 1) \frac{dy}{dx} + 2xy = \sqrt{x^2 + 4}$$

35. Find the shortest distance between the lines:  $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$  and  $\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$ .

### Section E (CASE STUDY)

36. In order to set up a rain water harvesting system, a tank to collect rain water is to be dug. The tank should have a square base and a capacity of  $250 \text{ m}^3$ . The cost of land is ₹ 5,000 per square metre and cost of digging increases with depth and for the whole tank, it is ₹40,000  $h^2$ , where h is the depth of the tank in metres. x is the side of the square base of the tank in metres.

#### ELEMENTS OF A TYPICAL RAIN WATER HARVESTING SYSTEM



Based on the above information, answer the following questions:

- Find the total cost C of digging the tank in terms of x.
- Find  $\frac{dc}{dx}$ .
- Find the value of x for which cost C is minimum.

[OR]

Check whether the cost function  $C(x)$  expressed in terms of x is increasing or not, Where  $x > 0$ .

37. The equation of motion of a missile are  $x = 3t$ ,  $y = -4t$ ,  $z = t$ , where the time 't' is given in seconds, and the distance is measured in kilometres.



- (i) Write the path of the missile.
- (ii) Find the distance of the missile from the starting point (0, 0, 0) in 5 seconds.
- (iii) If the position of the missile at a certain instant of the time is (5, -8, 10). Find the height of the missile from the ground? (Ground considered as  $xy$  – plane)

[OR]

Find the value of  $k$  for which the lines

$$\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-1}{k} \text{ and } \frac{x-2}{-2} = \frac{y-3}{-1} = \frac{z-5}{7} \text{ are perpendicular?}$$

38. A bank offers loans to its customers on different types of interest namely, fixed rate, floating rate, and variable rate. From the past data with the bank, it is known that a customer avails loan on fixed rate, floating rate, or variable rate with probabilities 10%, 20%, and 70% respectively. A customer after availing loan can pay the loan or default on loan repayment. The bank data suggests that the probability that a person defaults on loan after availing it at fixed rate, floating rate, and variable rate is 5%, 3%, and 1% respectively.



Based on the above information, answer the following:

- (i) What is the probability that a customer after availing the loan will default on the loan repayment?
- (ii) A customer after availing the loan, defaults on loan repayment. What is the probability that he availed the loan at a variable rate of interest?

.....THE END.....