Summer Fields School

KAILASH COLONY, NEW DELHI-110048

PRE BOARD -1 EXAMINATION

CLASS –XII –SET 2(2025)

SUB: -MATHEMATICS CODE: -041

Time -3 Hrs **M.M-80**

General Instructions:

Read the following instructions very carefully and strictly follow them :

1) This question paper contains 38 questions. All questions are compulsory.
2) This question paper is divided into five Sections - A, B, C, D and E.
3) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.
4) In Section B, Questions no. 21 to 25 are very short answer (VSA) type questions, carrying2 marks each.
5) In Section C, Questions no. 26 to 31 are short answer (SA) type questions, carrying 3 marks each.
6) In Section D, Questions no. 32 to 35 are long answer (LA) type questions carrying 5 marks each.
7) In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each.
8) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in SectionE.

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. For a non-singular matrix X , if $X^2 = I$, then X^{-1} is equal to :
   (A) X (C) I
   (B) X^2 (D) O

2. The cofactor of the element a_{32} in the determinant $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix}$ is :
   (A) ±5 C) 5
   (B) -5 ((D) 0

3. If A is an identity matrix of order n , then A(AdjA) is a/an :
   (A) identity matrix (C) zero matrix
   (B) row matrix (D) skew symmetric matrix

4. If the matrix $A = [a_{ij}]_{2\times2}$ is such that $a_{ij} = \begin{cases} 1, \text{ if } i \neq j \\ 0, \text{ if } i = j \end{cases}$, then $A + A^2$ is equal to:
   (A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
   (B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

5. The domain of $f(x) = \cos^{-1}(2x)$ is :

(A) $[-1,1]$ (C) $[-2,2]$

(B) $\left[0,\frac{1}{2}\right]$ (D) $\left[-\frac{1}{2},\frac{1}{2}\right]$

6. $\int \frac{3\cos\sqrt{x}}{\sqrt{x}}\,dx$ is equal to :

(A) $-6\sin\sqrt{x} + C$ (C) $6\cos\sqrt{x} + C$

(B) $-6\cos\sqrt{x} + C$ (D) $6\sin\sqrt{x} + C$

7. If $\frac{d}{dx} f(x) = 3x^2 - \frac{3}{x^4}$ such that $f(1) = 0$, then $f(x)$ is :

(A) $6x + \frac{12}{x^5}$ (C) $x^3 + \frac{1}{x^3} - 2$

(B) $x^4 - \frac{1}{x^3} + 2$ (D) $x^3 + \frac{1}{x^3} + 2$

8. In an LPP, corner points of the feasible region determined by the system of linear constraints are $(1,1)$, $(3,0)$ and $(0,3)$. If $Z = ax + by$, where $a, b > 0$ is to be minimized, the condition on a and b, so that the minimum of Z occurs at $(3,0)$ and $(1,1)$, will be :

(A) $a = 2b$ (C) $a = 3b$

(B) $a = \frac{b}{2}$ (D) $a = b$

9. If $x = t^3$ and $y = t^2$, then $\frac{d^2y}{dx^2}$ at $t = 1$ is :

(A) $\frac{3}{2}$ (C) $-\frac{3}{2}$

(B) $-\frac{2}{9}$ (D) $-\frac{2}{3}$

10. The area bounded by the parabola $x^2 = y$ and the line $y = 1$ is :

(A) $\frac{2}{3}$ sq unit (C) $\frac{4}{3}$ sq units

(B) $\frac{1}{3}$ sq unit (D) 2 sq units

11. If the rate of change of volume of a sphere is twice the rate of change of its radius, then the surface area of the sphere is :

(A) 1 sq unit (C) 3 sq units

(B) 2 sq units (D) 4 sq units

12. If $|\vec{a}| = 1, |\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 2$, then the value of $|\vec{a} + \vec{b}|$ is :

(A) 9 (C) -3

(B) 3 (D) 2

13. Two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$. The angle between the two vectors is :

(A) 30° (C) 45°

(B) 60° (D) 90°

14. A coin is tossed three times. The probability of getting at least two heads is :

(A) $\frac{1}{2}$ (C) $\frac{1}{8}$

(B) [illegible] (D) [illegible]

15. The maximum value of $Z = 3x + 4y$ subject to the constraints $x + y \le 1$, $x, y \ge 0$ is:

(A) 3 (C) 7

(B) 4 (D) 0

16. The general solution of the differential equation $\frac{dy}{dx} = 2x \cdot e^{x^2+y}$ is :

(A) $e^{x^2+y} = C$ (C) $e^{x^2} = e^y + C$

(B) $e^{-x^2} + e^{-y} = C$ (D) $e^{x^2-y} = C$

17. If ' m ' and ' n ' are the degree and order respectively of the differential equation $1 + \left(\frac{dy}{dx}\right)^3 = \frac{d^2y}{dx^2}$ then the value of $(m + n)$ is :

(A) 4 (C) 2

(B) 3 (D) 5

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion** (A) : $f(x) = [x]$, $x \in \mathbb{R}$, the greatest integer function is not differentiable at $x = 2$.

Reason (R) : The greatest integer function is not continuous at any integral value.

20 Consider the function $f : R \to R$, defined as $f(x) = x^3$.

Assertion (A) : f(x) is a one-one function.

Reason (R): f(x) is a one-one function, if co-domain = range.

SECTION B

21. Find the value of λ, if the points $(-1, -1, 2)$, $(2, 8, \lambda)$ and $(3, 11, 6)$ are collinear.

OR

$\vec{a}$ and $\vec{b}$ are two co-initial vectors forming the adjacent sides of a parallelogram such that $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$. Find the area of the parallelogram.

22 Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$.

OR

Prove that :

$$\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x \in [0,1]$$

23 If $y = a(\sin t - t\cos t)$ and $x = a(\cos t + t\sin t)$, then find $\frac{dy}{dx}$

24 Find the least value of a such that the function $f(x) = x^2 + ax + 1$ is strictly increasing on (1,2).

25 Find the angle between the lines

$$\vec{r} = (3 + 2\lambda)\hat{i} - (2 - 2\lambda)\hat{j} + (6 + 2\lambda)\hat{k} \text{ and}$$
$$\vec{r} = (2\hat{j} - 5\hat{k}) + \mu(6\hat{i} + 3\hat{j} + 2\hat{k})$$

SECTION C

26 Find :

$$\int \sqrt{4x^2 - 4x + 10}\,dx$$

OR

Evaluate:

$$\int_0^{\pi} \frac{x\sin x}{1 + \cos^2 x}dx$$

27 Find the maximum slope of the curve $y = -x^3 + 3x^2 + 9x - 30$.

28 Solve the following L. P. P. graphically:
Maximize $Z = 4x + y$
Subject to the constraints
$x + y \le 50, 3x + y \le 90, x \ge 0, y \ge 0$

29 Four students of class XII are given a problem to solve independently. Their chances of solving the problem respectively are $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}$ and $\frac{1}{5}$. Find the probability that at most one of them will solve the problem.

30 Find the general solution of the differential equation

$$(2x^2 + y)dx = xdy$$

OR

For the differential equation $\frac{dy}{dx} - \frac{y}{x} + \text{cosec}\left(\frac{y}{x}\right) = 0$, find the particular solution, given that $y = 0$ when $x = 1$.

31 If $\hat{a}, \hat{b}$ and $\hat{c}$ are unit vectors such that $\hat{a}\cdot\hat{b} = \hat{a}\cdot\hat{c} = 0$ and the angle between $\hat{b}$

and $\vec{c}$ is $\frac{\pi}{6}$, then prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$.

SECTION D

32. If $y = \cos x^2 + \cos^2 x + \cos^2(x^2) + \cos(x^x)$, find $\frac{dy}{dx}$

OR

Find the intervals in which the function given by $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is

(i) strictly increasing

(ii) strictly decreasing.

33. Using integration find the area of the region bounded by the parabola $x^2 = 4y$ and the line $x = 4y - 2$.

34. If $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the given system of equations

$3x + 4y + 7z = 14; 2x - y + 3z = 4; x + 2y - 3z = 0.$

35. Find the shortest distance between the lines l_1 and l_2 given by :

$$l_1: \vec{r} = -\hat{i} + \hat{j} - 9\hat{k} + \lambda(2\hat{i} + \hat{j} - 3\hat{k})$$
$$\text{and } l_2: \vec{r} = 3\hat{i} - 15\hat{j} + 9\hat{k} + \mu(2\hat{i} - 7\hat{j} + 5\hat{k})$$

OR

Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$ intersect. Also, find their point of intersection.

SECTION E

Case Study - 1

36. Two persons are competing for a position on the Managing Committee of an organisation. The probabilities that the first and the second person will be appointed are 0.5 and 0.6 respectively. Also, if the first person gets appointed, then the probability of introducing waste treatment plant is $0 \cdot 7$ and the corresponding probability is $0 \cdot 4$, if the second person gets appointed.

Based on the above information, answer the following questions :

(i) What is the probability that the waste treatment plant is introduced?

(ii) After the selection, if the waste treatment plant is introduced, what is the probability that the first person had introduced it?

Case Study - 2

37. During the festival season, there was a mela organized by the Resident Welfare Association at a park, near the society. The main attraction of the mela was a huge swing installed at one corner of the park. The swing is traced to follow the path of a parabola given by $x^2 = y$. Based on the above information, answer the following questions :

(i) Let $f : N \to R$ is defined by $f(x) = x^2$. What will be the range ?

(ii) Let $f: N \to N$ is defined by $f(x) = x^2$. Check if the function is injective or not.

(iii) Let $f: \{1,2,3,4 \ldots\} \to \{1,4,9,16 \ldots\}$, $f(x) = x^2$. Prove that the function is bijective.

OR

(iii) Let $f: \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x^2$. Show that f is neither injective nor surjective.

Case Study – 3

38 A window is in the form of a rectangle surmounted by an equilateral triangle on its length. Let the rectangular part have length and breadth x and y metres respectively. Based on the given information, answer the following questions :

(i) If the perimeter of the window is 12 m , find the relation between x and y .

(ii) Using the expression obtained in (i), write an expression for the area of the window as a function of x only.

(iii) Find the dimensions of the rectangle that will allow maximum light through the window. (use expression obtained in (ii))

OR

If it is given that the area of the window is $50m^2$, find an expression for its perimeter in terms of x .