SS4

PRE BOARD EXAMINATION - 2 (2024 - 25)

SUBJECT: Mathematics (SET I)

Time: 3 Hours

Maximum Marks: 80

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) **This Question paper contains 38 questions. All questions are** compulsory.

(ii) This Question paper is divided into five Sections - A, B, C, D and E.

(iii) In **Section A**, Questions no. **1 to 18** are multiple choice questions (**MCQs**) and Question no. **19** and **20** are **Assertion-Reason** based questions of **1 mark each.**

(iv) In **Section B**, Question no. **21 to 25** are Very Short Answer (VSA)-type questions, carrying **2 marks each.**

(v) In **Section C**, Questions no. **26 to 31** are Short Answer (**SA**)-type questions, carrying **3 marks each.**

(vi) In **Section D**, Questions no. **32 to 35** are Long Answer (**LA**)-type questions, carrying **5 marks each.**

(vii) In **Section E**, Questions no. **36 to 38** are **Case study-based questions**, carrying **4 marks each.**

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.

(ix) Use of calculators is not allowed.

	SECTION-A **(This section comprises of multiple choice questions (MCQs) of 1 mark each)** **Select the correct option (Question 1 - Question 18)**
1	What type of a relation is "greater than" in the set of real numbers? (a) only symmetric (b) only transitive (c) only reflexive (d) equivalence relation
2	$\cos^{-1}(\cos 4) =$ (a) 4 (b) $\pi - 4$ (c) $2\pi - 4$ (d) -4
3	If for a square matrix A of order 3x3 , A.(adj.A) = 17 I, then $\lvert adjA\rvert + \lvert A\rvert =$ (a) 289 (b) 17 (c) 17^3 (d) 306
4	Number of matrices of order 2 x 3 that are possible if each entry has to be only 2 or -3 (a) 6^2 (b) 2^6 (c) 96 (d) 32
5	If $A=\begin{bmatrix} a & c & -1 \\ b & 0 & 5 \\ 1 & -5 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then the value of $2a-(b+c)$ is (a) 0 (b) 1 (c) -10 (d) 10
6	If $xe^y = 1$, then the value of $\frac{dy}{dx}$ at $x = 1$ is (a) -1 (b) 1 (c) $-e$ (d) $-\frac{1}{e}$

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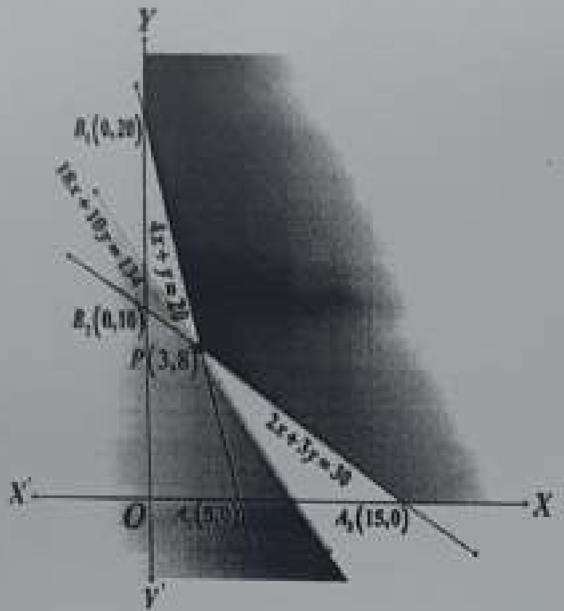
7 The corresponding objective function is: $Z = 18x + 10y$, which has to be minimized. The smallest value of the objective function Z is 134 and is obtained at the corner point $(3,8)$.



The optimal solution of the above linear programming problem ________

(A) does not exist as the feasible region is unbounded.

(B) does not exist as the inequality $18x+10y<134$ does not have any point in common with the feasible region.

(C) exists as the inequality $18x+10y>134$ has infinitely many points in common with the feasible region.

(D) exists as the inequality $18x+10y<134$ does not have any point in common with the feasible region.

8 Corner points of the feasible region determined by the system of linear constraints are P(60,0), Q(120,0), R(60,30) and S(40,30). Let Z = 5x+10y be the objective function. The maximum value of Z occurs at

(a) P (b) Q (c) every point on QR (d) every point on PQ

9 A speaks truth is 90% cases. B speaks the truth in 70% cases,then the percentage of cases they likely to contradict each other in stating the same fact is

(a) 63% (b) 34% (c) 66% (d) 27%

10 The direction cosines of the vector $3\hat{i} - 4\hat{j} + 12\hat{k}$ is

(a) $\frac{3}{13}, \frac{4}{13}, \frac{12}{13}$ (b) $\frac{3}{\sqrt{13}}, \frac{-4}{\sqrt{13}}, \frac{12}{\sqrt{13}}$ (c) $\frac{-3}{\sqrt{13}}, \frac{4}{\sqrt{13}}, \frac{-12}{\sqrt{13}}$ (d) $\frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$

11 If inverse of matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then value of λ is

(a) −4 (b) 1 (c) 3 (d) 4

12	The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minima at x = (a) 2 (b) 1 (c) 0 (d) -2
13	$\int \frac{x+3}{x-1} dx$ (a) $x+4\log\|x+1\|+c$ (b) $x-4\log\|x-1\|+c$ (c) $x+4\log\|x-1\|+c$ (d) $4x-\log\|x-1\|+c$
14	$\int_{-1}^{1} log\left(\frac{2-x}{2+x}\right)dx$ (a) 2 log3 (b) $\frac{1}{2} log\ 3$ (c) $2 - log\ 3$ (d) 0
15	The area bounded by $y=x^2, y=1$ and $y=2$ is (a) $\frac{4}{3}(2\sqrt{2}-1)$ *sq.units* (b) $\frac{2}{3}(2\sqrt{2}-1)$ *sq.units* (c) $2(2\sqrt{2}-1)$ *sq.units* (d) $\frac{4}{3}(2\sqrt{2}+1)$ *sq.units*
16	The order of the differential equation $\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2=x^2\log\left(\frac{d^3y}{dx^3}\right)$ is : (a) 1 (b) 2 (c) 3 (d) Not defined
17	The integrating factor of $x\frac{dy}{dx}-y=x^4-3x$ is : (a) x (b) $\log x$ (c) $\frac{1}{x}$ (d) $-x$
18	The vector in the direction of the vector $\hat{i}-2\hat{j}+2\hat{k}$ that has magnitude 9 unit is : (a) $\hat{i}-2\hat{j}+2\hat{k}$ (b) $\frac{\hat{i}-2\hat{j}+2\hat{k}}{3}$ (c) $3(\hat{i}-2\hat{j}+2\hat{k})$ (d) $9(\hat{i}-2\hat{j}+2\hat{k})$

ASSERTION-REASON BASED QUESTIONS

In each of the following questions a statement of Assertion (A) is followed by a statement of Reason(R).

Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19 Assertion (A) : The function $f(x) = x^3 + 2x^2 - 1$, is continuous at $x = 1$

Reason (R): A function 'f' is continuous at $x = a$, if $\lim_{h\to 0} f(a-h) = \lim_{h\to 0} f(a+h) = f(a)$

20 Assertion(A) : If the matrix $P = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & 3 \end{bmatrix}$ is a symmetric matrix, then $a = \frac{-2}{3}$ and $b = \frac{3}{2}$

Reason (R) : If P is a symmetric matrix, then $P^T = P$

SECTION B

(This section comprises 5 very short answer (VSA) type questions of 2 marks each.)

21 Show that the line joining the points A (4, 7, 8), B (2, 3, 4) is parallel to the line joining the points C (2, 4,10), D (–2, –4, 2).

22 Solve : $\int \frac{dx}{x+\sqrt{x}}$.

23 Water is flowing out of a cylindrical tank at the rate of π cu. cm/sec. If the height of the water level is decreasing at the rate of 0.01 cm/sec, then find the radius of the tank

OR

Find the maximum value of $\left(\frac{1}{x}\right)^x$.

24 Evaluate : $\sin\left(\cos^{-1}\frac{3}{5} + \operatorname{cosec}^{-1}\frac{13}{5}\right)$

25 Prove that $|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 - |\vec{a} \cdot \vec{b}|^2$

OR

Find the value of p, if $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + 3\hat{j} + p\hat{k}) = 0$

SECTION C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26 If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

27	Find the intervals in which the function $f(x)$ is strictly increasing or strictly decreasing : $$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$$
28	Evaluate: $\int \left(\sin^{-1}x\right)^2 dx$ **OR** Evaluate: $\int_0^{\pi} \frac{x}{1+\sin x} dx$
29	Find the foot of perpendicular drawn from the point $P(2, -1, 5)$ *to the line* $l: \frac{11-x}{-10} = \frac{y+2}{-4} = \frac{z+8}{-11}$ **OR** If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{j} - \hat{k}$, then find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$
30	Maximize $Z = 7x + 10y$, subject to constraints $4x + 6y \le 240$, $6x + 3y \le 240$, $x \ge 10$, $x \ge 0$, $y \ge 0$
31	An urn contains 3 green and 5 white balls. Three balls are drawn one by one without replacement.. Find the probability distribution of the number of green balls drawn. **OR** Given that the events A and B are such that $P(A) = \frac{1}{3}$, $P(B) = p$, and $P(A \cup B) = \frac{2}{5}$. Find p if A and B are (i) mutually exclusive (ii) independent.
	SECTION D **(This section comprises 4 long answer (LA) type questions of 5 marks each.)**
32	Using integration find the area of the region in the first quadrant enclosed by x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.
33	Find the product of the matrices $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations $x + y + 2z = 1$, $3x + 2y + z = 7$, $2x + y + 3z = 2$.
34	Solve : $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$ **OR** Solve : $(x^2 - 2y^2)dx + 2xy\,dy = 0$, *given* $y(1) = 1$

35 An aeroplane is flying along the line $\vec{r} = \lambda(\hat{i} - \hat{j} + \hat{k})$; *where* λ is a scalar and another aeroplane is flying along the line $\vec{r} = \hat{i} - \hat{j} + \mu(-2\hat{j} + \hat{k})$; where μ is a scalar. At what points on the lines should they reach, so that the distance between them is the shortest?. Also find the possible distance between them.

OR

Find the equation of the line in vector form which is passing through the point $(3, 0, -4)$ and parallel to the line $\frac{x-1}{5} = \frac{3-y}{2} = \frac{z+1}{4}$. Also find the distance between these two parallel lines.

SECTION E

(This section comprises 3 case-study/passage-based questions of 4 marks each with subparts.
The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively.
The third case study question has two subparts of 2 marks each)

36 Three shooters are training in an academy. Shooter A can hit the target 4 times in 5 shots, Shooter B can hit 3 times in 4 shots and Shooter C can hit the target 2 times in 3 shots.

Based on the above information, answer the following questions.

1.Find the probability that all three will hit the target? (1)

2.What is the probability that B,C will hit and A will miss the target? (1)

3.What is the probability that at least one of them will hit the target? (2)

OR

3. What is the probability that any two of them will hit the target?

37 Sonam visited an Exhibition along with her family. The Exhibition had a huge swing, which attracted many children. Sonam found that the swing traced the path of a parabola given by $y = x^2$

Based on the above information answer the following questions

(i) If $f: R \to R$ be defined by $f(x) = x^2$, then check whether f is an injective function, give reason (1)

(ii) Let $f: N \to N$ be defined by $f(x) = x^2$, then check whether f is surjective, give suitable reason. (1)

(iii) Let $f: Z \to \{1, 4, 9, \ldots\}$. Is function f is bijective or not, give reason. (2)

OR

(iii) Let $f: N \to R$ be defined by $f(x) = x^2$. Is f bijective, give reason.

38 A company makes open cylindrical tanks for water storage. Given S is the surface area of the open cylindrical tank where r and h are radius and height of the tank respectively.

Based on this information, answer the following questions.

(i) Find the volume of the tank **V** in terms of **S** and **r**. (2)

(ii) When the volume is maximum, find a relation between **h** and **r**. (2)