



SECOND PRE-BOARD EXAMINATION, JANUARY 2025

CLASS: XII
SUBJECT: MATHEMATICS (041)
MAX. MARKS: 80

DATE: 6.01.2025
DURATION: 3 HOURS
SET: A


General Instructions:

1. This Question paper consists of 6 printed pages and contains 38 questions.
2. The question paper is divided into five sections-Section A, B, C, D and E.
Each section is compulsory. However, there are internal choices in some questions. Let
3. Section A has 18 MCQ's and 2 Assertion-Reason based questions of 1 mark each.
4. Section B has 5 Very Short Answer (VSA) type questions of 2 marks each.
5. Section C has 6 Short Answer (SA) type questions of 3 marks each.
6. Section D has 4 Long Answer (LA) type questions of 5 marks each.
7. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

Q NO.	SECTION-A This section comprises multiple choice questions (MCQs) of 1 mark each.	MARKS
1	$A = \{1, 2, 3, 4\}$. R is an equivalence relation on $A \times A$ defined by $(a, b) R (c, d)$ if $a + d = b + c$. Find the equivalence class $\{(1, 3)\}$ (a) $\{(1, 3), (2, 4)\}$ (b) $\{2, 4\}$ (c) $\{(2, 4)\}$ (d) $(2, 4)$	1
2	The domain of the function $f(x) = \sin^{-1}(x^2 - 4)$ is (a) $[-1, 1]$ (b) $[\sqrt{3}, \sqrt{5}]$ (c) $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$ (d) $[-\sqrt{5}, -\sqrt{3}) \cup (\sqrt{3}, \sqrt{5}]$	1
3	If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $ A^3 = 27$, then the value of α is (a) ± 2 (b) ± 1 (c) $\pm \sqrt{5}$ (d) $\pm \sqrt{7}$	1
4	$ A = 5$ where A is a matrix of order 2, then $ 4A^{-1} $ is equal to (a) $\frac{4}{5}$ (b) $\frac{16}{5}$ (c) $\frac{1}{80}$ (d) 40	1
5	If A is a matrix, $A = [a_{ij}]$ of order 3×3 is defined by $a_{ij} = \begin{cases} 2i + 3j, & i < j \\ 5, & i = j \\ 3i - 2j, & i > j \end{cases}$, then the number of elements in A more than 5 is (a) 4 (b) 3 (c) 5 (d) 6	1
6	If $ A = \begin{vmatrix} 0 & 3p & 4-p \\ -3p & 0 & p^2-1 \\ p-4 & 1-p^2 & 0 \end{vmatrix}$ then the value of $\text{adj}(\text{adj } A)$ is (a) 1 (b) 4 (c) 0 (d) -1	1

7	Derivative of $e^{\sin^{-1}x}$ with respect to $\cos^{-1}x$ is (a) $-e^{\sin^{-1}x}$ (b) $e^{\cos^{-1}x}$ (c) $e^{\frac{-1}{\sqrt{1-x^2}}}$ (d) $2x e^{\frac{-1}{\sqrt{1-x^2}}}$	1
8	If $A = [a_{ij}]$ of order 3×3 and A_{ij} is the co-factor of a_{ij} such that $\sum_{i=1}^3 a_{i2} A_{i2} = -7$, then $A \cdot (\text{adj } A) = k I$, where I is identity matrix of order 3, then the value of k is (a) -7 (b) 49 (c) 7 (d) 343	1
9	A student observes an open-air honeybee nest on the branch of a tree, whose plane figure is parabolic in shape given by $x^2 = 4y$. Then the area (in sq. units) of the region bounded by the parabola $x^2 = 4y$ and the line $y = 4$ is: (a) $\frac{32}{3}$ (b) $\frac{64}{3}$ (c) $\frac{128}{3}$ (d) $\frac{256}{3}$	1
10	If $\int_0^2 2e^{2x} dx = \int_0^a e^x dx$, the value of a is: (a) 1 (b) 2 (c) 4 (d) $\frac{1}{2}$	1
11	If $\int_0^a \frac{1}{4+x^2} dx = \frac{\pi}{6}$, then the value of a is: (a) $\frac{\sqrt{3}}{2}$ (b) $2\sqrt{3}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$	1
12	For any two events A and B , if $P(\bar{A}) = \frac{1}{2}$, $P(\bar{B}) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{4}$, then $P(\bar{A}/\bar{B})$ equals (a) $\frac{3}{8}$ (b) $\frac{8}{9}$ (c) $\frac{5}{8}$ (d) $\frac{1}{4}$	1
13	The number of corner points of the feasible region determined by constraints $x \geq 0$, $y \geq 0$, $x + y \geq 4$ is (a) 0 (b) 1 (c) 2 (d) 3	1
14	The differential equation $\frac{dy}{dx} = F(x, y)$ will not be a homogeneous differential equation, if $F(x, y)$ is (a) $\cos x - \sin\left(\frac{y}{x}\right)$ (b) $\frac{y}{x}$ (c) $\frac{x^2+y^2}{xy}$ (d) $\cos^2\left(\frac{x}{y}\right)$	1
15	Write the sum of order and degree of the differential equation $\frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^4\right\} = 0$ (a) 4 (b) 3 (c) 5 (d) can not be determined	1
16	The unit vector perpendicular to both vectors $\hat{i} + \hat{k}$ and $\hat{i} - \hat{k}$ is (a) $2\hat{j}$ (b) \hat{j} (c) $\frac{\hat{i}-\hat{k}}{\sqrt{2}}$ (d) $\frac{\hat{i}+\hat{k}}{\sqrt{2}}$	1
17	If \vec{a} and \vec{b} are two vectors such that $ \vec{a} = 1$, $ \vec{b} = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{3}$, then the angle between $2\vec{a}$ and $-\vec{b}$ is (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{5\pi}{6}$ (d) $\frac{11\pi}{6}$	1

18	<p>For the Linear Programming Problem the objective function is $Z = 6x + 3y$ and the feasible region is the shaded region as shown in the graph.</p> <p>(Note: Figure is not to scale)</p> <p>Which of the following are the constraints that determine the given feasible region?</p> <p>(a) $x \geq 0, y \geq 0, 4x + y \geq 80, x + 5y \leq 115, 3x + 2y \leq 150$</p> <p>(b) $x \geq 0, y \leq 0, 4x + y \geq 80, x + 5y \geq 115, 3x + 2y \geq 150$</p> <p>(c) $x \geq 0, y \geq 0, 4x + y \geq 80, x + 5y \leq 115, 3x + 2y \geq 150$</p> <p>(d) $x \geq 0, y \geq 0, 4x + y \geq 80, x + 5y \geq 115, 3x + 2y \leq 150$</p>	1
	<p>Question numbers 19 and 20 are Assertion(A) and Reason(R) based questions carrying 1 mark each. Select the correct answer from the codes (a), (b), (c) and (d) as given below:</p> <p>(a) Both Assertion (A) and Reason (R) are true and reason (R) is the correct explanation of the Assertion (A).</p> <p>(b) Both Assertion (A) and Reason (R) are true but reason (R) is not the correct explanation of the Assertion (A).</p> <p>(c) Assertion (A) is true and Reason (R) is false.</p> <p>(d) Assertion (A) is false and Reason (R) is true</p>	
19	<p>A: $f(x) = e^{ x }$ is continuous for all $x \in \mathbb{R}$ but not differentiable at $x=0$</p> <p>R: If f, g are continuous functions then $f(g(x))$ is also continuous.</p>	1
20	<p>A: If $f(x) = a(x - \cos x)$ is strictly decreasing in \mathbb{R}, then a belongs to $(-\infty, 0)$</p> <p>R: A function $f(x)$ is said to be decreasing in an interval (a, b) if $x_1 < x_2$ then $f(x_1) \geq f(x_2)$</p>	1
SECTION-B This section comprises very short answer (VSA) type questions of 2 marks each.		
21	Evaluate $\sin^{-1} \sin\left(\frac{2\pi}{3}\right) + \cos^{-1} \cos\left(\frac{2\pi}{3}\right) + \tan^{-1} \tan\left(\frac{4\pi}{3}\right)$	2
22	<p>For what value of k, the function $f(x)$, given by</p> $f(x) = \begin{cases} \frac{1 - \cos 4x}{8x^2} & x \neq 0 \\ k & x = 0 \end{cases}$ <p>is continuous at $x=0$.</p> <p>OR</p> <p>If $y^x = e^{(y-x)}$, Prove that $\frac{dy}{dx} = \frac{(1 + \log y)^2}{\log y}$</p>	2

23	Evaluate: $\int_0^1 \log\left(\frac{1}{x} - 1\right) dx$	2
24	<p>Position vectors of the points A, B and C as shown in the figure below are \vec{a}, \vec{b} and \vec{c} respectively.</p>  <p>If $\vec{AC} = \frac{5}{4}\vec{AB}$, express \vec{c} in terms of \vec{a} and \vec{b}.</p> <p>OR</p> <p>Check whether the lines given by equations $x = 2\lambda + 2$, $y = 7\lambda + 1$, $z = -3\lambda - 3$ and $x = -\mu - 2$, $y = 2\mu + 8$, $z = 4\mu + 5$ are perpendicular to each other or not.</p>	2
25	Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.	2
<p style="text-align: center;">SECTION-C</p> <p style="text-align: center;">This section comprises short answer (SA) type questions of 3 marks each.</p>		
26	A vertical tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Semi vertical angle is $\tan^{-1}(0.5)$. water is poured into it at a constant rate of 5 cubic metre per hour. Find the rate at which the level of water is rising at the instant when the depth of water in the tank is 4m. (Use $\pi = \frac{22}{7}$)	3
27	<p>Evaluate $\int_1^3 (x-1 + x-2 + x-3) dx$</p> <p>OR</p> <p>Find $\int \frac{e^x}{5-4e^x-e^{2x}} dx$</p>	3
28	Find the particular solution of the differential equation $\frac{dy}{dx} + y \cot x = 2 \cos x$, given that $y\left(\frac{\pi}{2}\right) = 0$.	3
29	<p>If vectors \vec{a}, \vec{b} and $2\vec{a} + 3\vec{b}$ are unit vectors, then find the angle between \vec{a} and \vec{b}.</p> <p>OR</p> <p>If \vec{a}, \vec{b} vectors of equal magnitude and α is the angle between them, then</p> <p>prove that $\frac{ \vec{a} + \vec{b} }{ \vec{a} - \vec{b} } = \cot(\alpha/2)$</p>	3
30	<p>Solve the following Linear Programming Problem graphically:</p> <p>Maximise $Z = 4x + y$</p> <p>subject to the constraints :</p> <p>$x + y \leq 50$</p> <p>$3x + y \leq 90$</p> <p>$x \geq 0, y \geq 0$</p>	3
		3

31	<p>Four bad oranges are accidentally mixed with sixteen good oranges. Find the probability distribution of the number of bad oranges in a random draw of two oranges (without replacement). Also find the mean of the distribution.</p> <p>OR</p> <p>Probability that two shooters A and B hit a target is $\frac{1}{3}$ and $\frac{2}{5}$ respectively. If each shoots at the target, what is the probability that</p> <p>(i) The target is hit.</p> <p>(ii) Exactly one of them hits the target</p>	
<p align="center">SECTION-D</p> <p align="center">This section comprises long answer type questions (L.A) of 5 marks each</p>		
32.	<p>Find the values of a and b, if the function $f(x) = \begin{cases} ax^2 + b, & x < 1 \\ 2x + 1, & x \geq 1 \end{cases}$ is differentiable at $x=1$</p> <p>OR</p> <p>If $x = a(\cos t + t \sin t)$, $y = a(\sin t - t \cos t)$ $0 < t < \frac{\pi}{2}$</p> <p>Find (i) $\frac{d^2x}{dt^2}$ (ii) $\frac{d^2y}{dt^2}$ (iii) $\frac{d^2y}{dx^2}$.</p>	5
33.	<p>If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations</p> <p>$2x - 3y + 5z = 11$</p> <p>$3x + 2y - 4z = -5$</p> <p>$x + y - 2z = -3$.</p>	5
34	<p>If A_1 denotes the area of the region bounded by $y^2 = 4x$, $x = 1$ and x-axis in the first quadrant and A_2 denotes the area of the region bounded by $y^2 = 4x$, $x = 4$, find $A_1 : A_2$.</p>	5
35	<p>Find the co-ordinates of the foot of the perpendicular drawn from the point $(2, 3, -8)$ to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$. Also find the perpendicular distance of the given point from the line.</p> <p>OR</p> <p>Find the shortest distance between the lines L_1 and L_2 given below</p> <p>L_1: The line passing through $(2, -1, 1)$ and parallel to $\frac{x}{1} = \frac{y}{1} = \frac{z}{3}$.</p> <p>$L_2$: The line in Cartesian form, $\frac{y-1}{2} = \frac{z+2}{-1}$; $x=1$</p>	5
<p align="center">SECTION-E</p> <p align="center">This section comprises 3 case study-based questions of 4 marks each.</p>		
<p align="center">Case Study - 1</p>		
36.	<p>An organization conducted a bike race under two different categories -Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for final round. Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$ where B,G respectively, represent the set of boys, girls selected.</p> <p>Use the above information, answer the following questions:</p> <p>1) How many one-one functions are possible from G to B</p> <p>2) Is the function $f: B \rightarrow G$ defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$ a (i) one-one, (ii) onto function? Justify.</p>	1

	<p>3) If relation $R : G \rightarrow G$ defined by $R = \{ (x, y) : x \text{ and } y \text{ are students of same sex; } x, y \in G \}$. write all possible the reflexive relations as set of ordered pairs.</p> <p style="text-align: center;">OR</p> <p>If R is a relation on B given by $R = \{ (b_1, b_2), (b_2, b_1) \}$, how many minimum pairs to be added to make it an equivalence relation. List them.</p>	<p>1</p> <p>2</p>
<p>37.</p>	<p style="text-align: center;">Case Study - 2</p> <div data-bbox="295 481 1300 750" data-label="Image"> </div> <p>A rectangular visiting card is to contain 24 sq. cm of printed matter with dimension x, y as marked in the figure. The margin at the top and bottom of the card are to be 1 cm and the margin on the left and right are to be $1\frac{1}{2}$ cm. On the basis of the above information</p> <p>(i) Write the expression for the area A of the card in terms of x</p> <p>(ii) Find the value of x for which $\frac{dA}{dx} = 0$</p> <p>(iii) Find A if the area is least</p> <p style="text-align: center;">OR</p> <p>(iii) The perimeter of the card having least area.</p>	<p>1</p> <p>1</p> <p>2</p>

Case Study - 3

38.

In an office three employees James, Sophia and Oliver process incoming copies of a certain form. James processes 50% of forms, Sophia process 20% and Oliver the remaining 30% of forms. James has an error rate of 0.06, where as the error rate of Sophia and Oliver is 0.04, 0.03 respectively.

Based on the above information, answer the following questions

- (i) Find the probability that Sophia processed and committed error
- (ii) Find the probability of committing error in processing the form
- (iii) The manager of the company selects a form at random from the days output of processed form. If the selected form has an error, find the probability that the form is not processed by James.

OR

- (iv) Let E be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that James, Sophia and Oliver processed the form.

Find the value of $\sum_{i=1}^3 P(E_i / E)$.

1

1

2