BALA VIDYA MANDIR SR SEC SCHOOL ADYAR
REVISION 1 EXAMINATION

SUBJECT: MATHEMATICS **TIME: 3 HRS**

DATE: 22.11.2024 **MAX MARKS:80**

CLASS: 12

Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains 38 questions. All questions are compulsory.

(ii) This Question paper is divided into five Sections - A, B, C, D and E.

(iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.

(iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.

(v) In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.

(vi) In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.

(vii) In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.

(ix) Use of calculators is not allowed.

SECTION-A [1 × 20 = 20]

(This section comprises of multiple-choice questions (MCQs) of 1 mark each)
Select the correct option (Question 1 - Question 18):

1. The solution of the differential equation $\frac{dy}{dx} + y = e^{-x}, y(0) = 1$ is

   (a) $y = e^{x}(x-1)$ (b) $y = xe^{x}$ (c) $y = xe^{x} + 1$ (d) $y = (x+1)e^{-x}$

2. Integrating factor of the differential equation $(x\log x)\frac{dy}{dx} + y = 2\log x$ is

   (a) $\log(\log x)$ (b) $\log x$ (c) e^{x} (d) x

3. The number of solutions of $\frac{dy}{dx} = \frac{y+1}{x-1}, when\ y(1) = 2$

   (a) one (b) two (c) infinte (d) three

4. The value of $\int_0^1 [3x]\, dx$

   (a) 1 (b) 2 (c) 0 (d) 3

5. **If the area bounded by the curve $y^2 = 4x$ and the line $y = mx$ is $\frac{8}{3}$ sq. units, then the value** of m is

   (a) 1 (b) 2 (c) 3 (d) 4

6. The area (in sq. units) enclosed by the ellipse $\frac{x^2}{[illegible]} + \frac{y^2}{16} = 1$ is

(a) 20π (b) 20π (c) 12π (d) $16\pi^2$

7. $\int[\sin(log x) + \cos(log x)]dx$ is equal to

(a) $\sin(log x) + C$ (b) $\cos(log x) + C$

(c) $x\cos(log x) + C$ (d) $x\sin(log x) + C$

8. $\int_0^{\frac{\pi}{8}} tan^2 2x\, dx$ is equal to

(a) $\frac{4-\pi}{8}$ (b) $\frac{4+\pi}{8}$ (c) $\frac{4-\pi}{4}$ (d) $\frac{4-\pi}{2}$

9. If the function f defined by $f(x) = \begin{cases} \frac{\log(1+ax) - \log(1-bx)}{x} & , x \neq 0 \\ k & , x = 0 \end{cases}$ is continous at $x = 0$,

then the value of k is

(a) a (b) a + b (c) a − b (d) b

10. If $x = cos\theta - cos2\theta, y = sin\theta - sin2\theta, then \frac{dy}{dx}$ $at\ \theta = \frac{\pi}{3}$ is

(a) 1 (b) $\sqrt{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{2}{\sqrt{3}}$

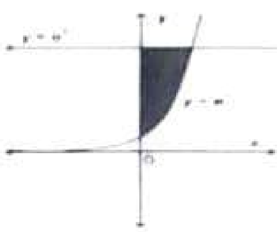
11. The local maximum value of $x + \frac{1}{x}$ is

(a) −2 (b) 2 (c) 3 (d) −3

12. In LPP, if the object function Z = ax + by has same maximum on two corner points of the feasible region, then the number of points at which maximum value of Z occurs is

(a) 0 (b) 2 (c) 3 (d) infinite

13. The shaded region shown in the fig. is bounded by the curve $y = e^x$, the y-axis and the line $y = e^2$ is



(a) $\int_1^{e^2} (e^2 - e^x)\, dx$ (b) $\int_1^2 (e^2 - e^x)\, dx$

(c) $\int_1^{e^2} e^x dx$ (d) $\int_1^2 e^x\, dx$

14. The value of $\sin^{-1}(cos\frac{33\pi}{5})$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{10}$ (c) $-\frac{\pi}{10}$ (d) $\frac{3\pi}{5}$

15. The value of $\tan(2\sin^{-1}\frac{1}{\sqrt{3}})$is

(a) $2\sqrt{2}$ (b) $2\sqrt{3}$ (c) $\sqrt{2}$ (d) $\frac{2}{\sqrt{3}}$

16. If an edge of a variable cube is increasing at the rate of 0.5 cm/sec, the rate of its surface area increasing when its edge is 12 cm, is (in cm^2/sec)

(a) 72 (b) 36 (c) 12 (d) 24

17. $\int_0^1 \log(\frac{1}{x} - 1)\, dx$ is equal to

(a) − 1 (b) 0 (c) 1 **(d) 2**

18. The maximum value of $f(x) = 3 + |x - 2|$ in $[-2, 5]$ is

(a) 7 (b) 8 (c) 3 (d) 1

In the following questions, a statement of Assertion (A) is followed by the statement of Reason(R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **ASSERTION(A)** : The area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is $\frac{9}{4}$ sq. units.

REASON(R) : The area of the region bounded by the curve $x = f(y)$, the y-axis and the ordinates $y = a$ and $y = b$ is $\int_a^b f(y)\,dy$.

20. **ASSERTION (A)**: The degree of the differential equation $\left(1 - \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}} = k\frac{d^2y}{dx^2}$ is 2.

REASON(R) : The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 - \sin\left(\frac{dy}{dx}\right) = 0$ is 2.

SECTION B [2 × 5 = 10]

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each)

21. Evaluate: $\int \frac{1}{x(x^2+1)}dx$.

22. Find the area of the region bounded by the curve $y = \log x$, the line $x = 2$ and the x-axis.

23. Find the intervals in which the function $f(x) = x^4 - 2x^2$ is strictly increasing or strictly decreasing.

OR

Find the intervals in which the function $f(x) = \tan x - 4x$, $x \epsilon (0, \frac{\pi}{2})$ is strictly increasing or strictly decreasing.

24. Find the particular solution of the differential equation $(x+1)\frac{dy}{dx} = 2e^{-y} - 1$, given $y = 0$ $when$ $x = 0$.

OR

Solve the differential equation $\frac{dy}{dx} + ay = e^{mx}$.

25. If $\frac{x}{x-y} = \log\left(\frac{a}{x-y}\right)$, $then$ $find$ $\frac{dy}{dx}$.

SECTION C [3 × 6 = 18]

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. Evaluate: $\int \frac{\sqrt{x^2+1}\,(\log(x^2+1) - 2\log x)}{x^4}dx$, $x > 0$

OR

Evaluate: $\int \frac{x^2+1}{x^4+1}\,dx$

27. Using integration, find the area of the region bounded by the curve $y = 20\cos 2x$ from the ordinates $x = \frac{\pi}{6}$ to $x = \frac{\pi}{4}$ and the x-axis.

28. Prove that $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right) = \frac{2b}{a}$.

29. Evaluate: $\int_0^{3/2} |x\cos\pi x|\,dx$.

OR

Evaluate: $\int_0^{\frac{\pi}{2}} \log(\sin x)\,dx$

30. Consider the following Linear Programming Problem:

Minimize $Z = x + 2y$.

Subject to $2x + y \geq 3, x + 2y \geq 6, x \geq 0, y \geq 0$.

Show graphically that the minimum of Z occurs at more than two points

31. A man, 2 *m* tall, walks at the rate of $1\frac{2}{3}$ *m/sec* towards a street light which is $5\frac{1}{3}$ *m* above the ground. At what rate is the tip of his shadow moving? What rate is the length of his shadow changing when he is $3\frac{1}{3}$ *m* from the base of light?

SECTION D [5 × 4 = 20]

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. Using the method of integration, find the area of the region bounded by the lines $2x + y = 4$, $3x - 2y = 6$ *and* $x - 3y + 5 = 0$

33. Solve the differential equation

$$e^{x/y}\left(1 - \frac{x}{y}\right) + \left(1 + e^{x/y}\right)\frac{dx}{dy} = 0, \text{ when } x = 0 \text{ and } y = 1$$

4. A straight line is drawn through the point P (1, 4). Find the least value of the sum of intercepts made by the line on the coordinate axes. Also find the equation of the line.

OR

A window is in the form of a rectangle surmounted by a semicircle. If the perimeter of the window is 10m, find the dimensions of the window so that the maximum possible light is admitted.

. Make a rough sketch of the region given below and find its area using methods of integration $\{(x, y): |x - 1| \leq y \leq \sqrt{5 - x^2}\}$.

OR

Make a rough sketch of the region given below and find its area using methods of integration $\{(x, y): 0 \leq y \leq x^2 + 3, 0 \leq y \leq 2x + 3, 0 \leq x \leq 3\}$

SECTION- E[$4 \times 3 = 12$]

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

36. Consider the curve $x^2 + y^2 = 16$ and line $y = x$ in the first quadrant

Based on the above information answer the following questions.

(i) Draw the rough sketch of the graph and shade the required area. [1Mark]

(ii) Find the point of intersection the curves. [1Mark]

OR

(ii) Express the required area as a definite integral using appropriate limits. [1Mark]

(iii) Find the area bounded by the two given curves, using integration. [2Marks]

37. A man has an expensive square shape piece of golden board of size 24 cm is to be made into a box without top by cutting from each corner and folding the flaps to form a box. Based on the above information answer the following questions.

(i) Find the volume of open box formed by folding up the flap, in terms of x. [1Mark]

(ii) Find the side of the square piece to be cut from each corner of the board to behold the maximum volume. [1Mark]

(iii) Find the maximum volume of open box. [2Marks]

OR

(iii) Find the largest value of the function $f(x) = sinx + \sqrt{3}cosx\ in[0, \pi]$. [2Marks]

38. Let f be a continuous function on the closed interval [a, b], then

$\int_a^b f(x)\,dx = \int_a^b f(a + b - x)\,dx$ and

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & if\ f(x) is\ even\ function \\ 0, & if\ f(x) is\ odd\ function \end{cases}$$

Based on the above information answer the following questions.

(i) Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (2sin|x| + cos|x|)dx$ [2Marks]

(ii) Evaluate: $\int_2^4 \frac{\log(x^2)}{\log(x^2) + \log(36 - 12x + x^2)} dx$ [2Marks]