COMMON PRE-BOARD EXAMINATION 2024-25

Subject: MATHEMATICS (041)

Class XII

Set - A

Time: 3 Hours   Roll: 06   Max Marks: 80
Date : 01.12.2024

General Instructions:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question Paper divided into five Sections – A, B, C, D and E.
2. Section **A** has 20 Multiple Choice Questions (MCQs) carrying 1 mark each.
3. Section **B** has 5 Very Short Answer (VSA)-type questions carrying 02 marks each.
4. Section **C** has 6 Short Answer (SA)-type questions carrying 03 marks each.
5. Section **D** has 4 Long Answer (LA)-type questions carrying 05 marks each.
6. Section **E** has 3 Case Based questions carrying 04 marks each.
7. There is no overall choice. However, an internal choice in 2 questions in section B, 3 questions of Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
8. Use of calculators is not allowed.

SECTION – A
(Each MCQ Carries 1 Mark)

1 If C_{ij} denotes the cofactor of element P_{ij} of the matrix $P = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & 2 & 4 \end{bmatrix}$. Then the value of $C_{31} \times C_{23}$:

a) 24   b) -24   c) -5   d) 5

2 If the amount of pollution content added in air in a city due to x diesel vehicles is given by $P(x) = 0.005x^3 + 0.02x^2 + 30x$, then the marginal increase in pollution content when 3 diesel vehicles are added is:

a) 0.450   b) 0.12   c) 30.255   d) 30

3 Value of 'k' for which $A = \begin{bmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix}$. is a singular matrix is:

a) 2   b) 4   c) 6   d) 8

4 If $y = \log\left(\frac{x^2}{e^2}\right)$, then $\frac{d^2y}{dx^2}$ is equal to:

a) $-\frac{1}{x}$ b) $-\frac{1}{x^2}$ c) $\frac{2}{x^2}$ d) $-\frac{2}{x^2}$

5 $\int_1^2 \frac{1}{x\sqrt{x^2-1}}\,dx =$ ________

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$

6 $\sin\left(\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right) =$ ________

a) 1 b) $\frac{1}{2}$ c) $\frac{1}{3}$ d) $\frac{1}{4}$

7 The value of the expression $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2$ is:

a) $\vec{a} \cdot \vec{b}$ b) $|\vec{a}| \cdot |\vec{b}|$ c) $|\vec{a}|^2 \; |\vec{b}|^2$ d) $(\vec{a} \cdot \vec{b})$

8 The number of solutions of the system of linear inequations $x + 2y \le 3$, $3x + 4y \le 12$, $x \ge 0$ and $y \ge 0$

a) Infinite b) 0 c) 2 d) 4

9 Differential of log [log (log (x^5))] with respect to x is:

a) $\frac{5}{x\log(x^5) \cdot \log(\log(x^5))}$ c) $\frac{5x^4}{\log(x^5) \cdot \log(\log(x^5))}$

b) $\frac{5}{x\log(\log(x^5))}$ d) $\frac{5x}{\log(x^5) \cdot \log(\log(x^5))}$

10 If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$ then the values of k, a and b respectively are:

a) -6, -12, -18 b) -6, -4, -9 c) -6, 4, 9 d) -6, 12, 18

11 If $\tan\left(\frac{x+y}{x-y}\right) = k$, then $\frac{dy}{dx}$ is equal to:

a) $\frac{-y}{x}$ b) $\frac{y}{x}$ c) $\sec^2\left(\frac{-y}{x}\right)$ d) $-\sec^2\left(\frac{-y}{x}\right)$

12 The maximum value of $z = 3x + 4y$ subject to the constraints $x \ge 0$, $y \ge 0$ and $x + y \le 1$ is:

a) 10 b) 7 c) 4 d) 3

13 The direction cosines of the lines $\frac{x-1}{2} = \frac{1-y}{3} = \frac{2z-1}{12}$ are:

a) $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$ b) $\frac{2}{\sqrt{157}}, \frac{-3}{\sqrt{157}}, \frac{12}{\sqrt{157}}$ c) $\frac{2}{7}, \frac{-3}{7}, \frac{-6}{7}$ d) $\frac{2}{7}, \frac{-3}{7}, \frac{6}{7}$

14 Position vector of the midpoint of the line segment AB is $3\hat{i} + 2\hat{j} - 3\hat{k}$. If position vector of the point A is $2\hat{i} + 3\hat{j} - 4\hat{k}$, then position vector of the point B can be given as:

a) $4\hat{i} + \hat{j} - 2\hat{k}$ b) $5\hat{i} + 5\hat{j} - 7\hat{k}$ c) $\frac{\hat{i}}{2} - \frac{\hat{j}}{2} + \frac{\hat{k}}{2}$ d) $\frac{5\hat{i}}{2} + \frac{5\hat{j}}{2} - \frac{7\hat{k}}{2}$

15 If $|A| = |kA|$, where A is a square matrix of order 2, then the sum of all possible values of k is:

a) 2 b) 1 c) 0 d) -1

16 The area bounded by the curve $y = x^2$, x-axis and the lines $x = -1$ and $x = 1$ is:

a) 0 b) $\frac{1}{6}$ sq. units c) $\frac{1}{3}$ sq. units d) $\frac{2}{3}$ sq. units

17 Assume that in a family each child is equally likely to be a boy or girl. A family with three is chosen at random. The probability that the eldest child is a girl given that the family has at least one girl is:

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{2}{3}$ d) $\frac{4}{7}$

18 If P and Q are the points (1, 2, 3) and (4, 5, 6), respectively then the magnitude of the vector PQ is:

a) $\sqrt{14}$ b) $\sqrt{27}$ c) $\sqrt{63}$ d) $\sqrt{77}$

Directions: In the following 2 questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

(A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false and R is True

19 **Assertion (A):** If $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ then $|\text{adj}\,(\text{adj}\,A)| = 16$.

Reason (R): $|\text{adj}\,(\text{adj}\,A)| = |A|^{(n-1)^2}$.

20 **Assertion (A):** If a line makes angles α, β and γ with the positive direction of the coordinate axes then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 1$.

Reason (R): The sum of the squares of the direction cosine of a line is 1.

SECTION – B

(Each Question Carries 2 Marks)

21 Find the maximum profit that a company can make, if the profit function is given by $p(x) = 41 - 72x - 18x^2$.

22 (a) If $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$ then show that $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are perpendicular to each other.

- OR -

(b) Find the value of λ and μ if $\vec{a} \times \vec{b} = 0$, where $\vec{a} = 2\hat{i} + 6\hat{j} + 27\hat{k}$ and $\vec{b} = \hat{i} + \lambda\hat{j} + \mu\hat{k}$.

23 (a) Solve for 'x' if $\sin[\cot^{-1}(x+1)] = \cos(\tan^{-1}x)$.

- OR -

(b) Find the value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$.

24 If the angle between the lines $\frac{x-5}{\alpha} = \frac{y+2}{-5} = \frac{z+\frac{24}{5}}{\beta}$ and $\frac{x}{1} = \frac{y}{0} = \frac{z}{1}$ is $\frac{\pi}{4}$, then find the relation between α and β.

25 If $y = \sqrt{\tan\sqrt{x}}$, then find $\frac{dy}{dx}$ at $x = \frac{\pi^2}{16}$.

SECTION – C

(Each Question Carries 3 Marks)

26. Evaluate $\int \frac{2x}{(x^2+1)+(x^2+2)}\,dx$.

27. (a) The probability that it rains today is 0.4. If it rains today, the probability that it will rain tomorrow is 0.8. If it does not rain today, the probability that it will rain tomorrow is 0.7.

If. P_1: denotes the probability that it does not rain today.

P_2: denotes the probability that it will not rain tomorrow, if it rains today.

P_3: denotes the probability that it will rain tomorrow, if it does not rain today.

P_4: denotes the probability that it will not rain tomorrow, if it does not rain today

then, (i) Find the value of $P_1 \times P_4 - P_2 \times P_3$

(ii) Calculate the probability of raining tomorrow.

- OR -

(b) Find the mean number of defective items in a sample of two items drawn one-by-one without replacement from an urn containing 6 items, which include 2 defective items. Assume that the items are identical in shape and size.

28. Evaluate $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\,dx$.

29. Solve the following linear programming problem graphically:

Minimize: $z = 13x - 15y$ subject to the constraints $x + y \le 7$, $2x - 3y + 6 \ge 0$, $x, y \ge 0$.

30. (a) Evaluate $\int_1^3 |x^2 - 2x|\,dx$.

- OR -

(b) Evaluate $\int_0^1 \sqrt{5 - 4x - x^2}\,dx$.

31. (a) Find the particular solution of the differential equation $x\frac{dy}{dx} - y = x^2 . e^x$, given $y(1) = 0$.

- OR -

(b) Find the particular solution of the differential equation $x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$, $x \ne 0$ given that $y = \frac{\pi}{4}$ when $x = 1$.

SECTION – D

(Each Question Carries 5 Marks)

32. (a) Show that the relation S in the set R of real numbers defined as $S = \{(a, b) : a, b \in R \text{ and } a \le b^3\}$ is neither reflexive nor symmetric and nor transitive.

- OR -

(b) Let $A = R - \{2\}$ and $B = R - \{1\}$. If $f : A \to B$ is a function defined by $f(x) = \frac{x-1}{x-2}$, then show that 'f' is one-one and onto.

33 Find the area of the region bounded by curve $4x^2 = y$ and the line $y = 8x + 12$, using integration.

34 The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others and some others (say z) for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others. Using matrix method, find the number of awardees of each category.

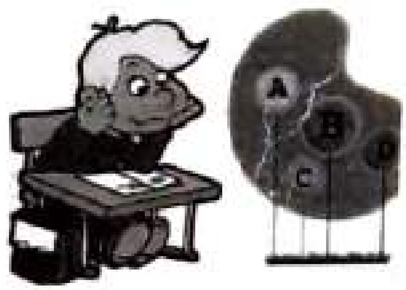
35 (a) Find the shortest distance between the lines
$\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 4\hat{j} - 5\hat{k})$.

- OR -

(b) Find the equations of the diagonals of the parallelogram PQRS whose vertices are P(4, 2, -6), Q(5, -3, 1), R(12, 4, 5) and S(11, 9, -2). Use these equations to find the point of intersection of diagonals.

SECTION – E
(CASE STUDY - Each Question Carries 4 Marks)

36 In a test, you either guesses or copies or knows the answer to a multiple-choice question with four choice. The probability that you make a guess is $\frac{1}{3}$, you copy the answer is $\frac{1}{6}$. The probability that your answer is correct, given that you guess it, is $\frac{1}{8}$. And also, the probability that you answer is correct, given that you copy it, is $\frac{1}{4}$.
Based on these information's, answer the following questions.



(i) The probability that you know the answer. [1m]

(ii) Find the probability that your answer is correct given that you guess it and the probability that your answer is correct given that you know the answer. [1m]

(iii) (a) Find the probability that you know the answer given that you correctly answered it. [2m]

- OR -

(iii) (b) Find the total probability of correctly answered the question. [2m]

37 Shalini wants to prepare a handmade gift box for her friend's birthday at home. For making lower part of box, she takes a square piece of cardboard of side 20cm. If 'x' cm be the length of each side of the square cardboard which is to be cut off from corners of the square piece of side 20cm and Volume of the box is 'V' then, answer the following questions.

(i) Find the value of 'x' for which $\frac{dV}{dx} = 0$. [2m]

(ii) Shalini is interested in maximizing the volume of the box. So, what should be the side of the square to be cut off so that the volume of the box is maximum? [2m]

38 The temperature of a person during an intestinal illness is given by $f(x) = \frac{-x^2}{10} + mx + \frac{493}{50}$, $0 \le x \le 12$. m being a constant, where $f(x)$ is the temperature in °F at x days.

Then answer the following question.

i) Is the function differentiable in the interval (0, 12)? Justify your answer. [1m]

ii) If 6 is the critical point of the function, then find the value of the constant. [1m]

iii) (a) Find the intervals in which the function is strictly increasing /strictly decreasing. [2m]

\- OR -

(b) Find the points of local maximum / local minimum, if any, in the interval (0, 12) and corresponding local maximum/local minimum. [2m]