SAHODAYA SCHOOL COMPLEX KOCHI

MODEL EXAMINATION 2024-25

CLASS XII **MATHEMATICS (041)** **MAX. MARKS- 80**

TIME- 3 HRS

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains **38** questions. All questions are **compulsory.**

(ii) This Question paper is divided into **five** Sections - A, B, C, D and E.

(iii) In **Section A**, Questions no. 1 to 18 are **multiple choice questions (MCQs)** and **Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.**

(iv) In **Section B**, Questions no. 21 to 25 are **Very Short Answer (VSA)**-type questions, **carrying 2 marks each.**

(v) In **Section C,** Questions no. **26** to **31** are **Short Answer (SA)-**type questions, carrying 3 marks each.

(vi) In **Section D,** Questions no. **32** to **35** are **Long Answer (LA)-**type questions, carrying 5 marks each.

(vii) In **Section E**, Questions no. **36** to **38** are **Case study-based questions**, **4 marks each.**

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.

(ix) Use of calculators is **not** allowed.

SECTION-A **[1x20=20]**

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

Q.1.If A is a square matrix of order 3 and |A|=6,then the value of |AdjA| is:

(A) 6 (B) 36 (C) 27 (D) 216

Q.2.If $[x-2 \quad 5+y]\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$ =0, then x+y =

(A) 0 (B) -2 (C) -1 (D) -3

Q.3.In which of these intervals is the function f(x) =$3x^2$-4x strictly decreasing?

(A) $(-\infty, 0)$ (B) (0,2) (C) $(\frac{2}{3}, \infty)$ (D) $(-\infty, \infty)$

Q.4.If $\begin{bmatrix} 6x & 8 \\ 3 & 2 \end{bmatrix}$ is singular matrix, then the value of x is

(A) 3 (B) -2 (C) 0 (D)2

Q.5.Integrating factor for the differential equation $(x\log x)\frac{dy}{dx}$ + y =2log x is

(A) log(logx) (B) logx (C) e^x (D) x

Q.6.The direction ratios of the line 3x+1 =6y-2 =1-z are

(A) 3,6,1 (B) 3,6,-1 (C) 2,1,6 (D) 2,1,-6

Q.7.If A is a square matrix such that A^2 =A, then $(I + A)^2$ -3A is

(A) I (B) 2A (C) 3I (D)A

Q.8.Let A and B be two given events such that P(A)= 0.6, P(B) =0.2 and P(A/B) =0.5. Then P(A'/B') is

(A) = $\frac{1}{10}$ (B) $\frac{3}{10}$ (C) $\frac{3}{8}$ (D) $\frac{6}{7}$

Q.9. If A = $\begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ and A = A′ then

(A) x=0 ,y=5 (B) x=y (C) x + y=5 (D) x-y=5

Q.10.If $\vec{a}$ and $\vec{b}$ are unit vectors, then what is the angle between $\vec{a}$ and $\vec{b}$ for $(\sqrt{3}\vec{a} - \vec{b})$ is an unit vector.

(A) 30^0 (B) 45^0 (C) 60^0 (D) 90^0

Q.11.Corner points of the feasible region determined by the system of linear constraints are (0,3), (1,1) and (3,0).Let Z= px+qy ,where p,q > 0.Condition on p and q so that the minimum of Z occurs at (3,0) and (1,1) is:

(A) P=2q (B) p=$\frac{q}{2}$ (C) p=3q (D) p=q

Q.12.Find the value of $\int \frac{dx}{\sin^2 x \cos^2 x}$

(A) tanx + cotx + C (B) tanx- cotx +C (C) tanx cotx +C (D)tanx-cot2x+C

Q.13.$\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\tan x}}$ dx is equal to

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\pi + 1$

Q.14.Degree of the differential equation$\left(\frac{d^2y}{dx^2}\right)^{3/2}$ =x is

(A) 2 (B) 3 (C) $\frac{3}{2}$ (D) not defined

Q.15.The domain of the function defined by f(x) =$\sin^{-1}\sqrt{x-1}$ is

(A) [1,2] (B)[-1,1] (C)[0,1] (D)[-1,0]

Q.16.The bounded feasible region of an LPP is always a-------------

(A) Convex polgon (B) Concave polygon (C) Either A or B (D) Neither A nor B

Q.17.Which of the following function is not continuous in the given domain ?

(A) f(x) =x^2+x-7, $x\in R$ (B)f(x) = $e^x, x \in R$ (C) f(x) =$\log_e x$, $x> 0$ (D)f(x)=$\frac{x}{x+5}$,$x\in R$

Q.18.The area of the region bounded by the curve y=$\frac{1}{x}$, the x -axis and between x=1 to x=6 is

(A) $\frac{1}{36}$ sq units (B) $\frac{1}{6}$ sq units (C)$\log_e 6$ sq units (D)-$\log_e 6$ sq units

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.)

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (A) is false but (R) is true.

Q.19.Assertion (A) : If y = $\sin^{-1}(6x\sqrt{1-9x^2})$, then $\frac{dy}{dx}=\frac{6}{\sqrt{1-9x^2}}$

Reason(R) :$\sin^{-1}(6x\sqrt{1-9x^2})=3\sin^{-1}(2x)$

Q.20.Assertion (A): Solution of the differential equation $(1+x^2)\frac{dy}{dx}+y=\tan^{-1}x$ is

$y e^{\tan^{-1}x} = (\tan^{-1}x-1)\, e^{\tan^{-1}x}+C$

Reason (R) : The differential equation of the form $\frac{dy}{dx}$ +Py =Q , where P, Q be the functions of x or constant, is a linear type differential equation.

SECTION B **[2X5=10]**

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

Q.21.Solve for x: $\sin(2\tan^{-1}x)=1$.

Q.22.Show that the function f(x) = $x^3-3x^2+6x-100$ is increasing on R.

Q.23.(a) Differentiate $\log(1+x^2)$ with respect to $\tan^{-1}x$.

OR

Q.23.(b) If $x^y = y^x$, then find $\frac{dy}{dx}$.

Q.24.(a) If $\vec{a}$ = $\hat{\imath}$ +2$\hat{\jmath}$ -$\hat{k}$ and $\vec{b}$ =3$\hat{\imath}$ +$\hat{\jmath}$ -5$\hat{k}$, find a unit vector in the direction of $\vec{a}$ - $\vec{b}$.

OR

Q.24.(b) Let the vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}|$=3 and $|\vec{b}|$ = $\frac{\sqrt{2}}{3}$, then axb is a unit vector, find the angle between $\vec{a}$ and $\vec{b}$.

Q.25.If a line makes angles 90^0 and 60^0respectively with the positive directions of x and y axes, find the angle which it makes with the positive direction of z axis.

SECTION C **[3x6=18]**

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

Q.26.Sand is pouring from a pipe at the rate of 12 cm^3/s.The falling sand forms a cone on a ground in such a way that the height of the cone is always one- sixth of the radius of the base. How fast is the height of sand cone increasing when the height is 4 cm.

Q.27.A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10m.Find the dimensions of the window to admit maximum light through the whole opening.

Q.28.(a) The scalar product of the vector $\hat{\imath}$ +$\hat{\jmath}$ +$\hat{k}$ with the unit vector along the sum of vectors 2$\hat{\imath}$ +4$\hat{\jmath}$ -5$\hat{k}$ and $\lambda\hat{\imath}$ +2$\hat{\jmath}$+ 3$\hat{k}$ is equal to one. Find the value of λ.

OR

Q.28.(b) Find the value of λ, so that the lines $\frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{5z-10}{11}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other.

Q29.(a) Find $\int x^2 \ tan^{-1}x$ dx

OR

Q.29.(b) Evaluate $\int_0^{\frac{\pi}{4}} \frac{1}{sinx+cosx}$dx

Q.30.Solve the following problem graphically:

Maximize Z= 15x + 10y subject to the constraints:3x+ 2y ≤ 80, 2x+3y≤ 70, x≥ 0,y≥ 0

Q.31.(a) There are two bags, one of which contains 3 black and 4 white balls, while the other contains 4 black and 3 white balls. A fair die is tossed, if the face 1 or 3 turns up ,a ball is taken from the first bag, and if any other face turns up a ball is chosen from the second bag. Find the probability of choosing a black ball?

OR

Q.31.(b) Three cards are drawn successively with replacement from a well-shuffled deck of 52 cards. A random variable denotes the number of hearts in the three cards drawn. Find the mean of X.

SECTION D **[5x4=20]**

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

Q.32. Find the area of the region included between the parabola $y = \frac{3}{4}x^2$ and the line $3x-2y+12=0$.

Q.33.If $A=\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1}. Using A^{-1}, solve the following system of equations:

$2x-3y+5z=11$, $3x+2y-4z=-5$, $x+y-2z=-3$.

Q.34.(a)Find the shortest distance between the lines $\vec{r} = (\hat{\imath} + 2\hat{\jmath} + \hat{k}) + \lambda(\hat{\imath} - \hat{\jmath} + \hat{k})$ and $\vec{r} = (2\hat{\imath} - \hat{\jmath} - \hat{k}) + \mu(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$, where λ and μ are parameters.

OR

Q.34.(b) Find the vector and cartesian equations of a line passing through (1,2,-4) and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

Q.35.(a)Let A= {1,2,3,............9} and R be the relation in AXA defined by (a,b) R(c,d) if a+d =b+c , for (a,b), (c,d) ∈ AXA .Prove that R is an equivalence relation, also obtain the equivalence class [(2,5)].

OR

Q.35.(b) Let N be the set of all natural numbers and let R be a relation on NXN defined by (a,b) R (c,d) if and only if ad= bc for all (a,b),(c,d)∈ NXN. Show that R is an equivalence relation on NXN.

SECTION- E **[4x3=12]**

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study-1

Q.36. The relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation $y= 4x - \frac{1}{2}x^2$, where x is the number of days exposed to sunlight.

Answer the following based on above information:

i) Find rate of growth of the plant with respect to sunlight. [1 mark]

ii) What are the number of days it will take for the plant to grow to the maximum height?

[1 mark]

iii)What is the maximum height of the plant? [2 marks]

OR

What will be the height of the plant after 2 days? [2 marks]

Case Study-2

Q.37. Polio drops are delivered to 50K children in a district. The rate at which polio drops are given is directly proportional to the number of children who have not been administered the drops. By the end of 2^{nd} week half the children have been given the polio drops. How many will have been given the drops by the end of 3^{rd} week can be estimated using the solution to the differential equation $\frac{dy}{dx}$ = k(50-y) where x denotes the number of weeks and y the number of children who have been given the drops.

i) State the order of the above given differential equation. [1 mark]

ii) Which method of solving a differential equation can be used to solve $\frac{dy}{dx}$ =k(50-y)?

[1 mark]

iii) Find the solution of the differential equation $\frac{dy}{dx}$ =k(50-y)? [2 marks]

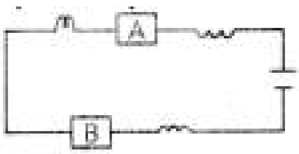
OR

Find the value of 'c' the constant of integration, in the particular solution given that y(0) = 0 and k= 0.049. [2 marks]

Case Study-3

Q.38. An electric circuit includes a device that gives energy to the charged particles constituting the current, such as a battery or a generator; devices that use current, such as lamps, electric motors, or computers; and the connecting wires or transmission lines.

An electric circuit consists of two subsystems say A and B as shown below:



For previous testing procedures, the following probabilities are assumed to be known. P(A fails) = 0.2, P(B fails alone) = 0.15, P(A and B fail) = 0.15 Based on the above information answer the following questions:

i)) Find the probability that the whole of the electric system fails? [2marks]

ii) Find the conditional probability that B fails when A has already failed. [2marks]