SAHODAYA SCHOOL COMPLEX - KOCHI.

XII - MATHEMATICS (2024-25).

ANSWER KEY (MODEL EXAMINATION)

Section A

[1 X 20 = 20]

1. (B) 36
2. (D) -3
3. (A) $(-\infty, 0)$
4. (D) 2
5. (B) $\log x$
6. (D) $2, 1, -6$
7. (A) I
8. (C) $\frac{3}{8}$
9. (B) $x = y$
10. (A) 30°
11. (B) $P = \frac{q}{2}$
12. (B) $\tan x - \cot x + C$
13. (C) $\frac{\pi}{4}$
14. (B) 3

15. (A) $[1,2]$

16. (A) Convex polygon

17. (D) $f(x) = \frac{x}{x+5}$

18. (C) $\log_e 6$ sq. units.

19. (C) A is true R is false

20. (A) Both (A) and (R) are true and (R) is the correct explanation of (A)

Section B $[2 \times 5 = 10]$

21 $\sin(2\tan^{-1}x) = 1$

$$\sin(2\tan^{-1}x) = \sin\frac{\pi}{2}$$

$$2\tan^{-1}x = \frac{\pi}{2}$$

$$\tan^{-1}x = \frac{\pi}{4}$$

$$x = \tan(\pi/4) = \underline{\underline{1}}$$

22. $f(x) = x^3 - 3x^2 + 6x - 100$

$$f'(x) = 3x^2 - 6x + 6$$

$$= 3x^2 - 6x + 3 + 3$$

$$= 3(x^2 - 2x + 1) + 3$$

$$= 3(x-1)^2 + 3 > 0$$

$f'(x) > 0$.

This show that $f(x)$ is increasing on R.

23(a) $u = \log(1+x^2)$ $\qquad$ $V = \tan^{-1} x$

$$\frac{du}{dx} = \frac{1}{1+x^2} \cdot 2x \qquad \frac{dv}{dx} = \frac{1}{1+x^2}$$

$$\frac{du}{dv} = 2x$$

OR

23(b) $x^y = y^x$

$$y \log x = x \log y$$

$$y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx} = x \cdot \frac{1}{y} \cdot \frac{dy}{dx} + \log y$$

$$\frac{dy}{dx} = \frac{(x \log y - y)\, y}{x\,(y \log x - x)}$$

24(a) $\vec{r} = \vec{a} - \vec{b}$

$$= -2\hat{i} + \hat{j} + 4\hat{k}$$

$$|\vec{r}| = \sqrt{4+1+16} = \sqrt{21}$$

Unit vector $\hat{r} = \dfrac{\vec{r}}{|\vec{r}|} = \dfrac{-2\hat{i}+\hat{j}+4\hat{k}}{\sqrt{21}}$

<u>OR</u>

24(b)

$|\vec{a}| = 3 \qquad |\vec{b}| = \dfrac{\sqrt{2}}{3}$

$|\vec{a}\times\vec{b}| = 1$

$1 = |\vec{a}||\vec{b}| \sin\theta$

$1 = 3 \times \dfrac{\sqrt{2}}{3} \sin\theta$

$\sin\theta = \dfrac{1}{\sqrt{2}}$

$\theta = \underline{\underline{\pi/4}}$

25. Let angle with z-axis be γ

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2 90 + \cos^2 60 + \cos^2\gamma = 1$

$\cos\gamma = \pm\dfrac{\sqrt{3}}{2}$

$\gamma = 30^\circ, 150^\circ$

Let x be the side of a rectangle and r be the radius of semicircle.

$$2x + 2r + \pi r = 10$$

$$2x + (2+\pi) r = 10 \quad \text{—— (1)}$$

Area of window, $A = 2xr + \frac{1}{2}\pi r^2$

$$= r\left[10 - (2+\pi) r\right] + \frac{1}{2}\pi r^2$$

$$= 10r - (2+\pi) r^2 + \frac{1}{2}\pi r^2$$

$$\frac{dA}{dr} = 10 - 2(2+\pi) + \pi r$$

$$= 10 - (4+\pi) r$$

For Max. Area, $\frac{dA}{dr} = 0$

$$\Rightarrow 10 - (4+\pi) r = 0.$$

$$\Rightarrow r = \frac{10}{4+\pi}$$

$$\left.\frac{d^2A}{dr^2}\right|_{r=\frac{10}{4+\pi}} = -(4+\pi) < 0.$$

Sub. r in (1)

$$x = \frac{10}{4+\pi} \text{ m.}$$

Dimensions are $\frac{20}{4+\pi}$ m, $\frac{10}{4+\pi}$ m

Section C

[3x6=18]

26. $\frac{dv}{dt} = 12 \text{ cm}^3/\text{sec}$

$h = \frac{1}{6} r \qquad h = 4 \text{cm}$

$\Rightarrow r = 24 \text{ cm}$

$V = \frac{1}{3} \pi r^2 h$

$= \frac{1}{3} \pi \times (6h)^2 h$

$= \frac{1}{3} \pi 36 h^3 = 12 \pi h^3$

$\frac{dv}{dt} = 12\pi \times 3h^2 \cdot \frac{dh}{dt}$

$12 = 36\pi h^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{12}{36\pi h^2}$

$\left.\frac{dh}{dt}\right|_{h=4} = \frac{1}{48\pi} \text{ cm/s}$

27. For the greatest possible light, area of window should be maximum.

28(a) Sum of vectors $\vec{r} = (2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$$

$$= \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+\lambda)^2 + 36 + 4}}$$

According to the given condition,

$$\frac{(\hat{i}+\hat{j}+\hat{k}) \cdot \{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}\}}{\sqrt{(2+\lambda)^2 + 36+4}} = 1$$

$$\Rightarrow 2+\lambda+6-2 = \sqrt{4+4\lambda+\lambda^2+40}$$

$$\Rightarrow \quad \lambda = 1$$

OR

28(b) $\dfrac{1-x}{3} = \dfrac{7y-14}{2\lambda} = \dfrac{5z-10}{11}$

$$\frac{-(x-1)}{3} = \frac{-7(y-2)}{2\lambda} = \frac{5(z-2)}{11}$$

$$\frac{x-1}{-105} = \frac{y-2}{10\lambda} = \frac{z-2}{77}$$

Direction ratios are $\langle -105, 10\lambda, 77 \rangle$

$$\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$$

$$\frac{-7(x-1)}{3\lambda} = \frac{y-5}{1} = \frac{-(z-6)}{5}$$

$$\frac{x-1}{3\lambda} = \frac{y-5}{-7} = \frac{z-6}{35}$$

direction ratios are $\langle 3\lambda, -7, 35 \rangle$

If the lines are perpendicular,

$$-105 \times 3\lambda + 10\lambda \times -7 + 77 \times 35 = 0$$

$$\underline{\underline{\lambda = 7}}$$

29(a) $\int x^2 \tan^{-1} x \, dx = \tan^{-1} x \cdot \frac{x^3}{3} - \int \frac{1}{1+x^2} \frac{x^3}{3} dx$

$$= \frac{x^3}{3} \tan^{-1} x - \frac{1}{3} \int \frac{x^3}{1+x^2} dx.$$

Consider. $\int \frac{x^3}{1+x^2} dx = \frac{1}{2} \int \left(\frac{t-1}{t}\right) dt$

put $1+x^2 = t$
$2x\,dx = dt$.

$$= \frac{1}{2} \int 1 - \frac{1}{t} \, dt$$

$$= \frac{1}{2} \left[t - \log|t|\right] + C$$

$$= \frac{1}{2}\left(1+x^2 - \log|1+x^2|\right) + C$$

$$\int x^2 \tan^{-1} x \, dx = \frac{x^3}{3}\tan^{-1} x - \frac{1}{6}\left[1+x^2 - \log|1+x^2|\right] + C$$

<u>OR</u>

29(b) $$I = \int_0^{\pi/4} \frac{1}{\sin x + \cos x}\, dx$$

$$= \int_0^{\pi/4} \frac{1}{\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right)}\, dx$$

$$= \frac{1}{\sqrt{2}} \int_0^{\pi/4} \frac{1}{\sin(x + \pi/4)}\, dx$$

$$= \frac{1}{\sqrt{2}} \int_0^{\pi/4} \operatorname{cosec}(x + \pi/4)\, dx$$

$$= \frac{1}{\sqrt{2}} \Big[\log|\operatorname{cosec}(x+\pi/4) - \cot(x+\pi/4)\Big]_0^{\pi/4}$$

$$= \frac{1}{\sqrt{2}}\left[\log(1-0) - \log(\sqrt{2}-1)\right]$$

$$= -\frac{1}{\sqrt{2}}\log(\sqrt{2}-1)$$

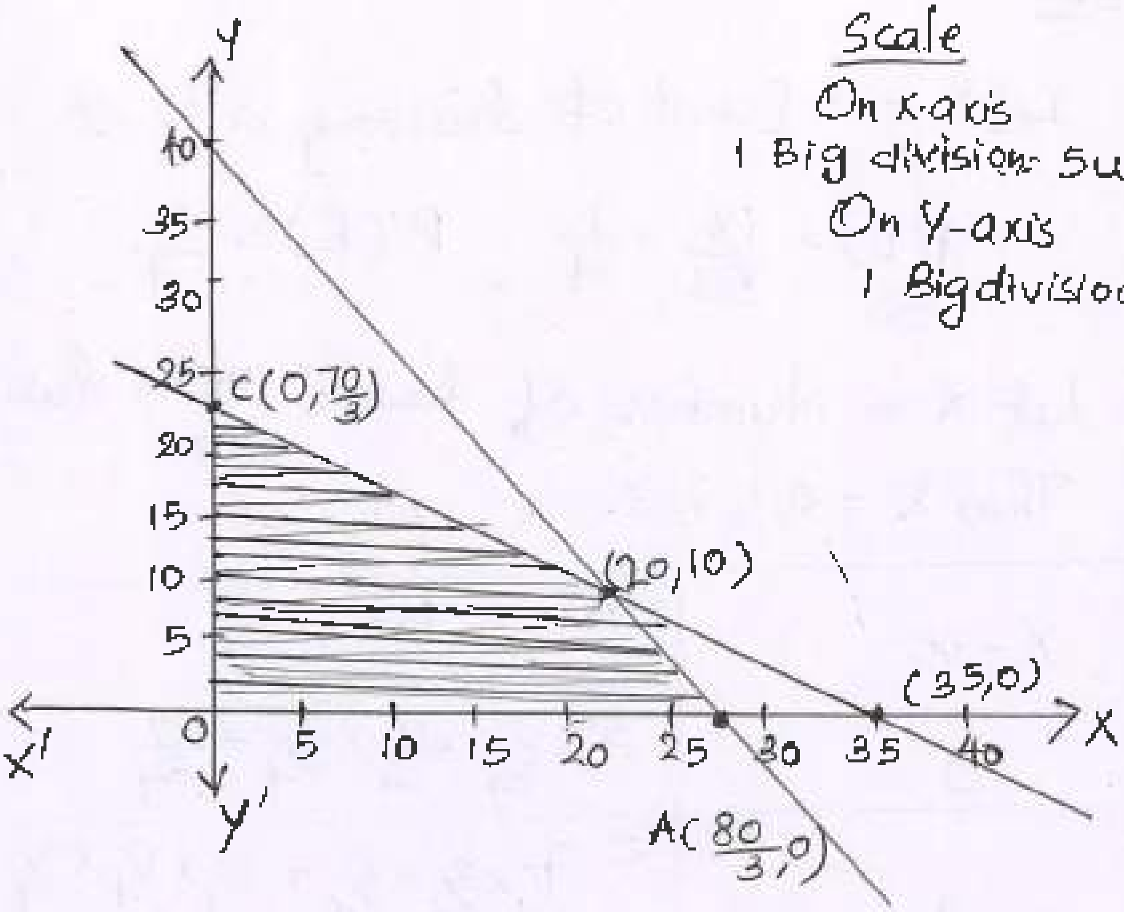
30. First we will convert the given inequations into equations, we obtain the following equations:

$3x+2y=80$; $2x+3y=70$, $x=0$, $y=0$.

The corner points are $(0,0)$, $A(\frac{80}{3},0)$ $B(20,10)$ and $C(0,\frac{70}{3})$

Corner points	$Z=15x+10y$
$(0,0)$	0
$(\frac{80}{3},0)$	400
$(20,10)$	400
$(0,70/3)$	$700/3$

The maximum value of the objective function Z is 400 which is at $A(80/3,0)$ and $B(20,10)$.



Scale
On x-axis
1 Big division = 5 units.
On Y-axis
1 Big division = 5 units.

31(a) Let E_1 be the event that a ball is drawn from first bag.

E_2 be the event that a ball is drawn from second bag

E be the event a black ball is choosen

$P(E_1) = \frac{1}{3}$ $P(E_2) = \frac{2}{3}$

$P(E) = P(E_1) \times P(E/E_1) + P(E_2) \cdot P(E/E_2)$

$= \frac{1}{3} \times \frac{3}{7} + \frac{2}{3} \times \frac{4}{7}$

$= \frac{11}{21}$

(6)

OR

31(b) Let E = Event of drawing a heart.

$P(E) = \frac{13}{52} = \frac{1}{4}$, $P(\bar{E}) = \frac{3}{4}$.

Let X = Number of hearts in a draw.

Then X = 0, 1, 2, 3.

$X = x_i$	P's
0	$3/4 \times 3/4 \times 3/4 = \frac{27}{64}$
1	$1/4 \times 3/4 \times 3/4 + 3/4 \times 1/4 \times 3/4 + 3/4 \times 3/4 \times 1/4 = \frac{27}{64}$
2	$1/4 \times 1/4 \times 3/4 + 1/4 \times 3/4 \times 1/4 + 3/4 \times 1/4 \times 1/4 = \frac{9}{64}$
3	$1/4 \times 1/4 \times 1/4 = \frac{1}{64}$

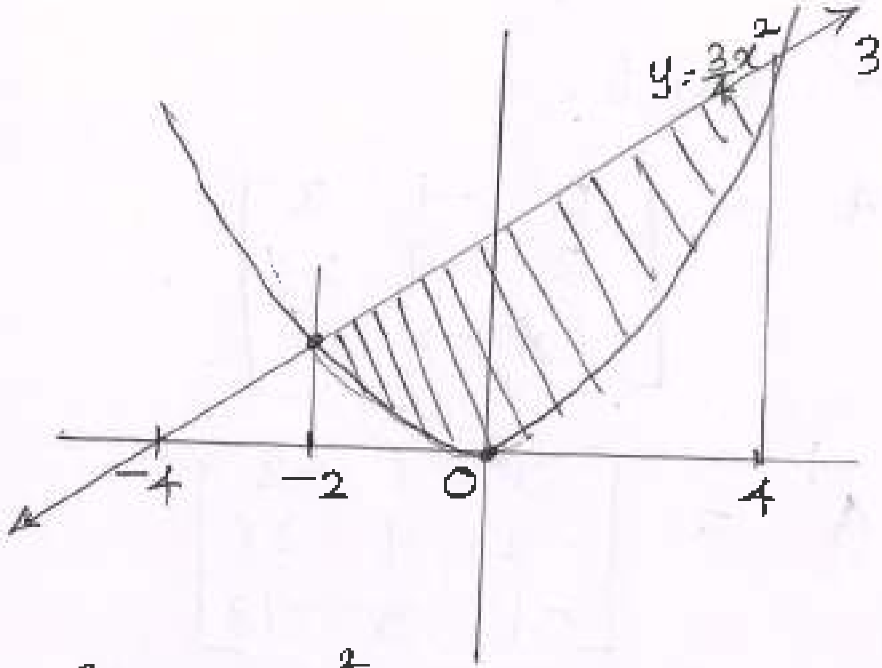
$$\text{Mean}(X) = \sum p_i x_i$$

$$= 0 \times \frac{27}{64} + 1 \times \frac{27}{64} + 2 \times \frac{9}{64} + 3 \times 1/64 = \underline{\underline{3/4}}$$

Section D

[$5 \times 4 = 20$]

32.



$$3x - \frac{3}{2}x^2 + 12 = 0$$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

$$x = -2 \ , x = 4.$$

$$A = \int_{-2}^{4} \left\{ \frac{3x+12}{2} - \frac{3}{4}x^2 \right\} dx$$

$$= \left[\frac{3x^2}{4} + 6x - \frac{x^3}{4} \right]_{-2}^{4} = 27 \text{ sq. units.}$$

33. $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

$|A| = -1 \neq 0$

A^{-1} exists.

$$\text{adj } A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

Now $AX = B$

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$x = 1, y = 2, z = 3.$

34(a) $\vec{a_1} = \hat{i} + 2\hat{j} + \hat{k}$ $\quad \vec{b_1} = \hat{i} - \hat{j} + \hat{k}$

$\vec{a_2} = 2\hat{i} - \hat{j} - \hat{k}$ $\quad \vec{b_2} = 2\hat{i} + \hat{j} + 2\hat{k}$

$$\vec{a_2} - \vec{a_1} = \hat{i} - 3\hat{j} - 2\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$

$$= -3\hat{i} + 0\hat{j} + 3\hat{k}$$

$$|\vec{b_1} \times \vec{b_2}| = 3\sqrt{2}$$

$$(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = -9.$$

$$d = \left| \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

$$= \left| \frac{-9}{3\sqrt{2}} \right| = \frac{3\sqrt{2}}{2} \text{ units.}$$

34(b) Let the equation of line passing through $(1, 2, -4)$ and perpendicular to the lines.

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \text{ and}$$

$$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5} \quad \text{be}$$

$$\frac{x-1}{l} = \frac{y-2}{m} = \frac{z+4}{n}$$

$\therefore\ l(3) + m(-16) + n(7) = 0$ and

$$3l + 8m - 5n = 0$$

$$\Rightarrow \frac{l}{80-56} = \frac{m}{21+15} = \frac{n}{24+48}$$

$$\Rightarrow \frac{l}{24} = \frac{m}{36} = \frac{n}{72}$$

$$\Rightarrow \frac{l}{2} = \frac{m}{3} = \frac{n}{6}$$

Equation of the required line is,

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}.$$

Vector equation is.

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

35(a) For reflexive: Let $(a,b) \in A \times A$

$(a,b)\ R\ (a,b) \Rightarrow a+b = b+a$, true.

For symmetric: Let for $(a,b), (c,d) \in A \times A$

$(a,b)\ R\ (c,d) \Rightarrow a+d = b+c$

$\Rightarrow c+b = d+a$

$\Rightarrow (c,d)\ R\ (a,b)$

For transitive:

$(a,b), (c,d), (e,f), \in A \times A$

$(a,b)\ R\ (c,d)$ and $(c,d)\ R\ (e,f)$

$a+d = b+c$ and $c+f = d+e$

$a+d+c+f = b+c+d+e$

$a+f = b+e \Rightarrow (a,b)\ R\ (e,f)$

Hence R is an equivalence relation.

$[(2,5)] = \{(1,4), (2,5), (3,6), (4,7), (5,8), (6,9)\}$.

OR

35(b) For reflexive: For any $(a,b) \in N \times N$

$(a,b)\ R\ (a,b) \Rightarrow ab = ba$, true

For symmetric :

$$\text{For } (a,b),(c,d) \in N \times N$$

$$(a,b) R (c,d) \Rightarrow ad = bc$$

$$\Rightarrow bc = da$$

$$\Rightarrow (c,d) R (a,b)$$

For transitive :

$$\text{For any } (a,b), (c,d), (e,f) \in N \times N$$

$$(a,b) \in (c,d) \text{ and } (c,d) R (e,f)$$

$$ad = bc \text{ and } cf = de$$

$$ad \cdot cf = bc \cdot de$$

$$af = be$$

$(a,b) R (e,f)$, R is transitive.

R is an equivalence relation.

Section E

36. i) $y = 4x - \frac{x^2}{2}$ (1 MARK)

$$\frac{dy}{dx} = 4 - x$$

Rate of growth of plant w.r.t. Sunlight is $(4-x)$ cm/day

ii) $y = 4x - \frac{x^2}{2}$ (1 MARK)

$\frac{dy}{dx} = 4 - x.$

For maximum height,

$\frac{dy}{dx} = 0 \Rightarrow 4 - x = 0 \Rightarrow x = 4.$

$\frac{d^2y}{dx^2} = -1 \Rightarrow \left.\frac{d^2y}{dx^2}\right|_{x=4} = -1 < 0$

y is maximum at $x = 4$

iii) y is maximum at $x = 4$. (2 Mark)

So, max. height of plant $= [y]_{x=4}$

$= 4 \times 4 - \frac{16}{2} = 8$cm

OR

iii) $y = 4x - \frac{x^2}{2}$

$x = 2,\ y = 4 \times 2 - \frac{4}{2} = 6$cm.

Height of plant after 2 days is 6cm.

37. i) Order 1 (1 M)

ii) Variable separable method. (1 M)

iii) We have $\frac{dy}{dx} = k(50-y)$

$$\frac{dy}{50-y} = k\,dx$$

$$\int \frac{dy}{50-y} = k\int dx$$

$$-\log(50-y) = kx + C. \quad (2\text{ M})$$

OR

iii) $-\log(50-y) = kx + c.$

putting $x = y = 0$ in the above eqn,

we get.

$$-\log 50 = c$$

$$\Rightarrow \quad c = \log\frac{1}{50}.$$

(2 M)

38. i) E = A fails   F = B fails

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$= 0.2 + 0.3 - 0.15$$

$$= \underline{\underline{0.35}}.$$ (2M)

ii) E = A fails F = B fails

$P(E) = 0.2$ $P(E \cap F) = 0.15$

$$P(F/E) = \frac{P(F \cap E)}{P(E)}$$

$$= \frac{0.15}{0.2} = \underline{\underline{\frac{3}{4}}}$$ (2M)