SUBJECT: MATHEMATICS (041)

UNIT WISE PRACTICE QUESTION PAPER
(UNITS: Continuity and differentiability, Application of derivatives)

Time: 3 Hours **Max. Marks: 80**

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains 38 questions. All questions are compulsory.
(ii) This Question paper is divided into five Sections - A, B, C, D and E.
(iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no.19 and 20 are Assertion-Reason based questions of 1 mark each.
(iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
(v) In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
(vi) In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
(vii) In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
(viii) Use of calculators is not allowed.

SECTION A **[1×20 = 20]**

(This section comprises of Multiple –choice questions (MCQ) of 1 mark each.)

1. If $f(x) = x^2 \sin\frac{1}{x}$ where $x \neq 0$, then the value of the function f at x = 0, so that the function f(x) is continuous at x = 0, is

a) 1 b) – 1 c) 0 d) 2

2. The function f(x) = [x] , where [x] denotes the greatest integer function, is continuous at

a) 4 b) 2 c) 1 d) 1.5

3. The relation between a and b so that the function f defined by

$$f(x) = \begin{cases} ax+1, & if \quad x \leq 3 \\ bx+3, & if \quad x > 3. \end{cases}$$ continuous at x = 3 is

a) $3a = 2 - 3b$ b) $3a - 3b = 2$ c) $3a + 3b = 2$ d) $3b - 3a = 2$.

4. The value of the constant k so that the function f defined by

$$f(x) = \begin{cases} \frac{1-\cos 4x}{8x^2}, & x \neq 0, \\ k, & x = 0 \end{cases}$$ continuous at x = 0 is

a) 0 b) 1 c) 2 d) 3

5. If $e^x + e^y = e^{x+y}$, then dy/dx

a) e^{y-x} b) $-e^{y-x}$ c) e^{x+y} d) e^{x-y}

6. Derivative of $\sin(\tan^{-1} e^x)$ is

a) $\cos(\tan^{-1} e^x) \cdot e^x$ b) $\frac{e^x \cos(\tan^{-1} e^x)}{1+e^{2x}}$ c) $-(\cos(\tan^{-1} e^x) \cdot e^x)/ 1+ x^2$

d) $- \frac{e^x \cos(\tan^{-1} e^x)}{1+e^{2x}}$

7. If y = A sinx + B cos x , then which of the following is correct ?

a) $D^2y + y = 0$ b) $D^2y - y = 0$ c) $D^2y + 2y = 0$ d) $D^2y - 2y = 0$.

8. Derivative of $\sin^2 x$ w r t $e^{\cos x}$ is

a) $-2 \cos x \cdot e^{-\cos x}$ b) $2\cos x\, e^{\cos x}$ c) $2 \sin x\, e^{\cos x}$ d) $-2\sin x\, e^{-\cos x}$

9. If $f(x) = (\sin x)^{\sin x}$, for all $0< x < \pi$, then $f'(x)$ is equal to

a) $(1 - \log(\sin x))(\sin x)^{\sin x} \cos x$ b) $(1 + \log(\sin x))(\sin x)^{\sin x} \cos x$

c) $(1 - \log(\cos x))(\sin x)^{\sin x} \cos x$ d) $(1 + \log(\cos x))(\sin x)^{\sin x} \cos x$

10 If x = a(cost +t sint) and y = a (sint – t cost), then $\frac{d^2y}{dx^2}$ is

a) $\cos^3 t/ at$ b) $\sec^3 t/ at$ c) $\sin^3 t / at$ d) $at \sec^3 t$.

11. If $y = \tan^{-1} \frac{\cos x}{1+\sin x}$ then dy/dx =

a) 1 b) 0 c) ½ d) – ½

12. The total revenue in Rupees from the sale of x units of a product is given by $R(x)= 3x^2 +36x +5$. Then the marginal revenue when x = 5 is

a) ₹44 b) ₹66 c)₹ 360 d) ₹88

13. The rate of change of the area of a circle with respect to its radius r at r = 6 cm is

a) 10π b) 12π c) 8π d) 11π

14. The function $f(x) = \sin 3x$, $x \in \left[0, \frac{\pi}{2}\right]$ is increasing in

a) $\left[0, \frac{\pi}{3}\right]$ b) $\left[0, \frac{\pi}{4}\right]$ c) $\left[0, \frac{\pi}{6}\right]$ d) $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

15. The interval on which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is strictly decreasing in

a) [-1 , ∝] b) (-2, -1) c) (- ∝ , −2) d) [- 1 ,1]

16. The absolute maximum and minimum values of a function f given by $f(x) = 2x^3 - 15x^2 + 36x + 1$ on the interval [1,5] respectively are

a) 56 and 28 b) 56 and 29 c) 29 and 24 d) 56 and 24

17. Let f have second derivative at c such that $f'(c) = 0$, $f''(c) \geq 0$, then c is a point of

a)local minima b) local maxima c) extreme value of f d) neither maxima nor minima.

18. The function $f(x) = 2x^3 - 3x^2 - 12x + 4$, has

a) two points of local maximum b) two points of local minimum

c) one maxima and one minima d) no maxima or minima.

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each.

Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.)

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

19. Assertion (A): If $3 \leq x \leq 10$ and $5 \leq y \leq 15$, then minimum value of (x/y) is 2.

Reason (R): If $3 \leq x \leq 10$ and $5 \leq y \leq 15$, then minimum value of (x/y) is 1/5 .

20. Assertion (A): Minimum value of (x - 5)(x -7) is -1.

Reason (R): Minimum value of $ax^2 + bx + c$ is $\frac{4ac- \ ^2}{4a}$

SECTION B **[2× 5 = 10]**

(This section comprises of 5 very short answer (VSA) type-questions of 2 marks each.)

21. If for $f(x) = \begin{cases} \frac{1-cos4x}{x^2}, if \quad x < 0 \\ a, \quad if\ x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, \quad if\ x > 0 \end{cases}$, f is continuous at x = 0 , find a.

22. Find dy/dx. Where, $xy = e^{x-y}$.

23. Find the intervals in which the function f given by $f(x) = x^2 - 4x + 6$ is

a. increasing b. Decreasing

24. Show that the function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on R.

25. A circular disc of radius 3 cm is being heated. Due to expansion, its radius increases at the rate 0.05 cm/s. Find the rate at which its area increasing when radius is 3.2 cm .

SECTION – C **[3× 6 = 18]**

(This section comprises of 6 short answer (SA) type questions of 3 marks each)

26. If x = a(cost + log tan$\frac{t}{2}$) , , y = a sin t, then evaluate d^2y/dx^2 at $t = \frac{\pi}{3}$.

27 .If $x\sqrt{1+y}$ + $y\sqrt{1+x}$ = 0 , for – 1<x<1 , prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$

28. Find *dy/dx* of the function $(\cos x)^y = (\cos y)^x$.

29. Find the maximum profit that a company can make , if the profit function is given by $p(x) = 41 - 72x - 18x^2$.

30. Prove that the perimeter of a right triangle of given hypotenuse is maximum when the triangle is isosceles.

SECTION – D **[5× 4 = 20]**

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

31. Water is dripping out at a steady rate of 1 cu cm / sec through a tiny hole at the vertex of the conical vessel, whose axis is vertical . When the slant height of water in the vessel is 4 cm, find the rate of decrease of slant height , where the vertical angle of the conical vessel is $\frac{\pi}{6}$.

32. Find the area greatest rectangle that can be inscribed in an ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

33 . Find dy/dx, if y = $e^{sin^2x}(2tan^{-1}\sqrt{\frac{1-x}{1+x}}$) . + $cot^{-1}\{\frac{\sqrt{1+si} \quad +\sqrt{1-sin}}{\sqrt{1+sin} \quad -\sqrt{1-sin}}\}$

34. .Find the intervals in which the function f(x) = sinx + cosx , $0 \leq x \leq 2\pi$, is strictly increasing or strictly decreasing.

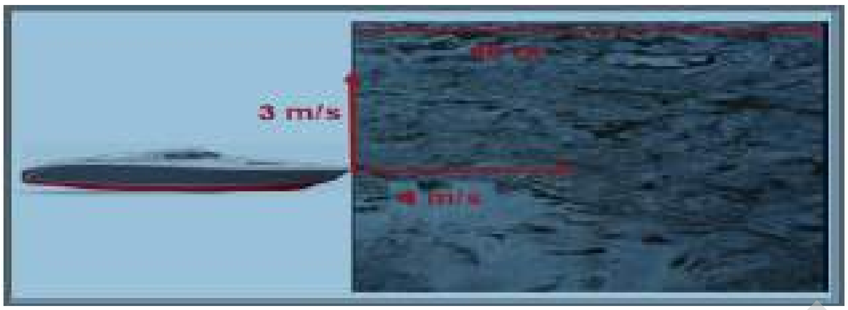
35. An open box with a square base is to be made out of a given quantity of cardboard of area c^2square unit. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic unit.

SECTION – E **[4× 3 = 12]**

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts)

36. **Read the following text carefully and answer the questions that follow:**

Once Ramesh was going to his native place at a village near Agra. From Delhi and Agra he went by flight, In the way, there was a river. Ramesh reached the river by taxi. Then Ramesh used a boat for crossing the river. The boat heads directly across the river 40 m wide at 4 m/s. The current was flowing downstream at 3 m/s.



i. What is the resultant velocity of the boat? (1)

ii. How much time does it take the boat to cross the river? (1)

iii. How far downstream is the boat when it reaches the other side? (2)

OR

If speeds of boat and current were 1.5 m/s and 2.0 m/s then what will be resultant velocity?

(2)

37 **Read the following text carefully and answer the questions that follow:**

Naina is creative she wants to prepare a sweet box for Diwali at home. She took a square piece of cardboard of side 18 cm which is to be made into an open box, by cutting a square from each corner and folding up the flaps to form the box. She wants to cover the top of the box with some decorative paper. Naina is interested in maximizing the volume of the box.

i. Find the volume of the open box formed by folding up the cutting each corner with x cm. (1)

ii. Naina is interested in maximizing the volume of the box. So, what should be the side of the square to be cut off so that the volume of the box is maximum? (1)

iii. Verify that volume of the box is maximum at $x = 3$ cm by second derivative test? (2)

OR

Find the maximum volume of the box. (2)

38. A mason wants to put a ladder on a wall. It is 5 m long and leaning against a wall. The bottom of the ladder is pulled by the man along the ground away from the wall at the rate of 2m/s.

If x is the distance of the bottom and top of the ladder then

i) write a relation between x and y. (1)

ii) How fast is its height on the wall decreasing ? (1)

iii) Find the rate of decreasing the height on the wall decreases when the foot of the ladder is 4 m away from the wall ? (2)