UNIT WISE PRACTICE QUESTIONS PAPER

(UNITS: Continuity and differentiability, Application of derivatives)

Time: 3 Hours **Max. Marks: 80**

Marking Scheme

1. c) 0 , since ,

   $\lim_{x\to0} f(x) = \lim_{x\to0} x^2 \sin\left(\frac{1}{x}\right) = 0, as \sin\left(\frac{1}{x}\right)$ $is\ \ always\ a\ definite\ value\ \ as$
   $-1 \le sinx \le 1$

2. d) 1.5, since greatest integer function [x] is discontinuous at all integral values of x . Hence at x = 1.5 is continuous.

3. b) Since the function is continuous at x = 3 , hence

$$\lim_{x\to3-} f(x) = \lim_{x\to3+} f(x)$$
$$\lim_{x\to3-} (ax+1) = \lim_{x\to3+} (bx+3)$$
$$3a + 1 = 3b + 3$$
$$3a – 3b = 2$$

4. b) 1 . Given , f is continuous at x = 0 , hence

$$\lim_{x\to0} f(x) = \quad f(0)$$
$$\lim_{x\to0} \frac{1-cos4x}{8x^2} = k$$
$$\lim_{x\to0} \frac{2sin^2 2x}{8x^2} = k$$
$$\lim_{x\to0} (\frac{sin2x}{2x})^2 = k$$
$$K=1$$

5. b) $– e^{y – x}$. Here, $e^x + e^y = e^{x + y}$, differentiating both sides w r t x , we have,

   $e^x + e^y \frac{dy}{dx} = e^{x+y} (1 + \frac{dy}{dx})$

   $(e^y - e^{x + y}) \frac{dy}{dx} = e^{x + y} - e^y$

   dy/dx = $\frac{e^{x+y}-e^y}{e^y-e^{x+y}} = \frac{(e^x + e^y-e^y}{e^y-e^x-e^y} = - e^{y-x}$

6. b) . By using chain rule , $\frac{d}{dx}(\sin (\tan^{-1}e^x)) = \frac{e^x\cos (tan^{-1}e^x)}{1+e^{2x}}$

7. a) , y = A sinx + Bcosx, Dy = A cosx – Bsinx ---------(i)

$D^2y = - A\sin x - B\cos x = - y$

$D^2y + y = 0.$

8. a) , Let, $u = \sin^2 x$, $v = e^{\cos x}$, $du/dx = 2 \sin x \cos x$, $dv/dx = - \sin x\, e^{\cos x}$

$du/dv = - 2 \cos x / e^{\cos x} = - 2\cos x\, e^{-\cos x}$.

9. b) . Here, $y = (\sin x)^{\sin x}$

$\log y = \sin x \log(\sin x)$

Differentiating w r t x : $1/y\, dy/dx = \sin x/\sin x \,.\, \cos x + \log(\sin x)(\cos x)$

$dy/dx = (1 + \log(\sin x)) \cos x\, (\sin x)^{\sin x}$

10. b). , Here, $dx/dt = a(- \sin t + t\cos t + \sin t) = at \cos t$

$dy/dt = a(\cos t - \cos t + t \sin t) = at \sin t.$

$dy/dx = dy/dt / dx/dt = \tan t$

Hence, $\frac{d^2y}{dx^2} = \sec^2 t\ dt/dx = \sec^2 t \,.\, 1/at\cos t = \sec^3 t/at.$

11. d) -1/2 , $y = \tan^{-1}\left(\frac{\cos x}{1+\sin}\right) = \tan^{-1}\left(\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{(\sin\frac{x}{2}+\cos\frac{x}{2})^2}\right.$

$= \tan^{-1}\left(\frac{\cos\frac{x}{2}-s\ \ \frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2})}\right. = \tan^{-1}\left(\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right)$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right) = \frac{\pi}{4}-\frac{x}{2}$

$dy/dx = - ½$

12. b) 66. Here, Marginal Revenue, $dR/dx = 6x + 36$ at $x = 5$,

$dR/dx = 30 + 36 = 66$

13. b) 12π Area of a circle , $A = \pi r^2$,

$dA/dr = 2\pi r$

dA/dr at $r = 6$ is 12π

14. c) $\left[0,\frac{\pi}{6}\right]$

Here, $f(x) = \sin 3x$

$f'(x) = 3 \cos 3x.$

Now , $f'(x) = 0$ gives, $\cos 3x = 0$, => $3x = \frac{\pi}{2}$, $\frac{3\pi}{2}$ => $x = \frac{\pi}{6}, \frac{\pi}{2}$

Hence the intervals are, $[0,\frac{\pi}{6}]$ and $[\frac{\pi}{6},\frac{\pi}{2}]$

Now, $f'(x) > 0$ in $[0,\frac{\pi}{6}]$. Hence f(x) is increasing in $[0,\frac{\pi}{6}]$.

15. b) . $f(x) = 2x^3 + 9x^2 + 12x - 1$

$f'(x) = 6x^2 + 18x + 12$

$f'(x) = 6(x^2 + 3x + 2) = 6(x+2)(x+1)$

$f'(x) = 0$ => $x = - 2 , - 1$.

Hence the intervals are , $(-\propto, -2)$, $(-2, -1)$, $(-1, \propto)$.
In (-2,-1) , $f'(x)$ = (+ve)(-ve) = - ve, hence decreasing in (-2 , -1)

16. d) . We have $f(x) = 2x^3 - 15x^2 + 36x + 1$
$f'(x) = 6x^2 - 30x + 36 = 6(x^2 - 5x + 6)$
$= 6(x-3)(x-2)$.
$f'(x) = 0 \Rightarrow x = 2, 3$
Hence, f(2)= 29 , f(3) = 28 , f(1)= 24 , f(5) = 56.
Thus absolute maximum value is 56 and absolute minimum value is 24.

17. a) local minima.
18. c) one maxima and one minima.
19. b) Both A and R correct but R is not the correct explanation of A.
20. a) (a) Both A and R are true and R is the correct explanation of A. Explanation: We have,
$(x-5)(x-7) = x^2 - 12x + 35$ We know that, $ax^2 + bx + c$ has minimum value . Here, a = 1, b = -12 and c = 35 Minimum value of (x - 5)(x - 7) is -1 .
21. Here, f(0) = a,
Left hand limit at x = 0, $\lim_{x\to o-} f(x) = \lim_{x\to o-} \frac{1-c0s4x}{x^2} = \lim_{x\to o-} \frac{2sin^2 2x}{x^2}$
$= \lim_{x\to o-} 8\left(\frac{sin2x}{2x}\right)^2 = 8 \times 1 = 8$.
Thus , a = 8 .
22. Now , $xy = e^{x-y}$
Log xy = x – y,
Logx + logy = x – y
Differentiating w r t x we get, 1/x + 1/y dy/dx = 1 - dy/dx
(1/y + 1) dy/dx = 1 – 1/x
Dy/dx = $\frac{x-1}{y+1}\left(\frac{y}{x}\right)$
23. $f(x) = x^2 - 4x + 6$, $f'(x) = 2x - 4$
$f'(x) = 0 \Rightarrow x = 2$. Thus the intervals are, $(-\propto, 2)$ *and* $(2, \propto)$

$f'(x) > 0$ in $(2, \propto)$ and $f'(x) < 0$ in $(-\propto, 2)$.
Hence f(x) is increasing in $(2, \propto)$ and f(x) is decreasing in $(-\propto, 2)$.

24. Here, $f(x) = x^3 - 3x^2 + 6x - 100$. $f'(x) = 3x^2 - 6x + 6 = 3(x^2 - 2x + 2)$
$= 3\{(x-1)^2 + 1\} > 0$ for all $x \in R$
Thus f(x) is increasing function.

25. Let r be the radius of the given disc and A be its area. Then $A = \pi r^2$
dA/dt = $2\pi r$ dr/dt.
Given, dr/dt = 0.05 cm/s,

Thus rate of change of area when r = 3.2 cm is $dA/dt = 2\pi . 3.2(0.05)$
$= 0.320\pi\ cm^{2}/s$.

26. $X = a\left(\cos t + \log \tan\frac{t}{2}\right)$ => $dx/dt = a\left(-\sin t + \frac{1}{2}\,\frac{\sec^2\frac{t}{2}}{\tan\frac{t}{2}}\right) = a\left(-\sin t + 1/\sin t\right) = a \cot t . \cos t$

$Y = a \sin t$ => $dy/dt = a \cos t$

$dy/dx = (dy/dt)/(dx/dt) = a \cos t/(a \cot t . \cos t) = \tan t$

$d^2y/dx^2 = \sec^2 t\, dt/dx. = \sec^2 t . \frac{1}{a \cot t . \cos t} = 1/a\ . \sec^4 t . \sin t$

When, $t = \frac{\pi}{3}$, $d^2y/dx^2 = 1/a\ . 2^4 . \sqrt{3}/2 = \frac{8\sqrt{3}}{a}$

27. Here , $x\sqrt{1+y} + y\sqrt{1+x} = 0$

$x\sqrt{1+y} = - y\sqrt{1+x}$

squaring both sides, $x^2(1+y) = y^2(1+x)$

- ⇨ $X^2 - y^2 + x^2 y - y^2 x = 0$
- ⇨ $(x-y)(x+y+xy) = 0$
- ⇨ $X=y$ or $x + y + xy = 0$, here $x \neq y$.
- ⇨ $X+y+xy = 0$
- ⇨ $Y(1+x) = - x$
- ⇨ $Y = \frac{-x}{1+x}$
- ⇨ Differentiating w r t x , $dy/dx = -\frac{1}{(1+x)^2}$
- ⇨

28. Here, $(\cos x)^y = (\cos y)^x$

- ⇨ Taking log on both sides, $y \log \cos x = x \log \cos y$
- ⇨ Differentiating both sides wrt x , $y.\frac{-\sin x}{\cos x} + \log \cos x\, dy/dx = x\frac{-\sin y}{\cos y}\frac{dy}{dx} + \log \cos y$
- ⇨ $dy/dx(\log \cos x + x \tan y) = y \tan x + \log \cos y$
- ⇨ $dy/dx = \frac{y\tan x + \log co}{\log\cos x + x \tan y}$

29. Here ,profit function is $p(x) = 41 - 72x - 18x^2$

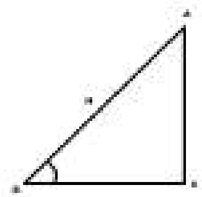
- ⇨ $p^{/}(x) = - 72 - 36x$,
- ⇨ $p^{//}(x) = - 36$

For extreme values of P(x) , $p^{/}(x) = 0$, => $- 72 - 36x = 0$

$x = - 2$

At $x = -2$, $p^{//}(2) = - 36 < 0$, hence p(x) is maximum at x = - 2 , Hence max profit is P(-2) = 41 + 144 – 72 = 113 .

30.



Let H be the hypotenuse AC and θ be the angle between the hypotenuse and the base BC of the right angles triangle ABC .

Then BC= H cos θ , AC = H sin θ

⇨ P= Perimeter of the right triangle

⇨ P = H + H cos θ + H sin θ

For extreme value of P , dP/d θ = 0 => H(- sin θ + cos θ) = 0

⇨ $\sin\theta = cos\theta$

⇨ tan θ = 1

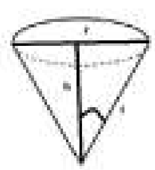
⇨ $\theta = \frac{\pi}{4}$

Now, $d^2P/d\theta^2$ = - H cos θ - H sin θ and is – ve for $\theta = \frac{\pi}{4}$.

Hence, P is maximum at $\theta = \frac{\pi}{4}$.

At , $\theta = \frac{\pi}{4}$, BC = AC = H/$\sqrt{2}$, Thus triangle ABC is isosceles triangle.

31.



Given that dv/dt = 1 cm^3/s, where v is the volume of water in the conical vessel..

From the fig, l = 4cm , h = lcos $\frac{\pi}{6} = \frac{\sqrt{3}}{2}l$, and r = $lsin\frac{\pi}{6} = \frac{l}{2}$

Now, v = $\frac{1}{3}\pi r^2 h = \frac{\pi}{3}\frac{l^2}{4}\frac{\sqrt{3}}{2}l = \frac{\sqrt{3}\pi}{24}l^3$

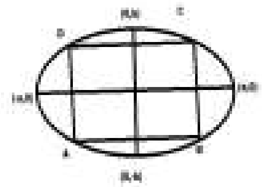
⇨ Dv/dl = $\frac{3\frac{\sqrt{3}\pi}{24}l^2 dl}{dt} = \frac{\sqrt{3}\pi}{8}l^2\frac{dl}{dt}$

But, dv/dt = 1 cm^3/s when l = 4 cm,

Hence, 1 = $\frac{\sqrt{3}\pi}{8}4^2\frac{dl}{dt}$ => $\frac{dl}{dt} = \frac{1}{2\sqrt{3}\pi}cm/s$

Thus rate of decrease of slant height , $\frac{dl}{dt} = \frac{1}{2\sqrt{3}\pi} cm/s$

32.



Let ABCD be the rectangle of maximum area of sides AB = 2x and BC = 2y, where C(x,y) be a point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Now area of the rectangle A = 4xy. Or, $A^2 = 16\, x^2\, y^2$ = S(say)

$$S = 16\, x^2\, y^2 = 16x^2 \left(1 - \frac{x^2}{a^2}\right)b^2 = 16\, \frac{b^2}{a^2}(a^2x^2 - x^4)$$

$$ds/dx = 16\, \frac{b^2}{a^2}(2a^2x \quad - 4x^3)$$

Now , ds/dx = 0 => $x = \frac{a}{\sqrt{2}} \quad and \quad y = \frac{b}{\sqrt{2}}$

$d^2s/dx^2 = 16\, \frac{b^2}{a^2}(2a^2 - 12x^2)$. For , $x = \frac{a}{\sqrt{2}}$, $d^2s/dx^2 = 16\, \frac{b^2}{a^2}(2a^2 - 6a^2) = 16\, \frac{b^2}{a^2}(-4a^2) < 0$

Thus area is maximum at $x = \frac{a}{\sqrt{2}} \quad and \quad y = \frac{b}{\sqrt{2}}$

Maximum area is , $A = 4xy = 4\, \frac{a}{\sqrt{2}}.\frac{b}{\sqrt{2}}$ =2ab.

33. Here, $y = e^{sin^2x}(2tan^{-1}\sqrt{\frac{1-x}{1+x}}) + cot^{-1}\{\frac{\sqrt{1+sinx}+\sqrt{1-sin}}{\sqrt{1+sinx}-\sqrt{1-sinx}}\}$

. For, $x = \cos\theta$, $tan^{-1}\sqrt{\frac{1-x}{1+x}} = \frac{1}{2}\, cos^{-1}x$, and

$cot^{-1}\{\frac{\sqrt{1+sinx}+\sqrt{1-sinx}}{\sqrt{1+sinx}-\sqrt{1-sinx}}\} = \cot^{-1} \{\frac{\sin\frac{x}{2}+\cos\frac{x}{2}+\cos\frac{x}{2}-\sin\frac{x}{2}}{\sin\frac{x}{2}+\cos\frac{x}{2}-\cos\frac{x}{2}+\sin\frac{x}{2}}\} = \cot^{-1}(\cot x/2) = x/2$

Thus, $y = e^{sin^2x}\, cos^{-1}x \; + x/2$

$=> dy/dx = e^{sin^2x}.\frac{-1}{\sqrt{1+x^2}} + cos^{-1}x\; e^{sin^2x}.2sinx.cosx. + ½$