CLASS: XII
SUBJECT: MATHEMATICS (041)

UNIT WISE PRACTICE QUESTION PAPER
(UNITS: INTEGRATION, APPLICATION OF INTEGRATION)

Time: 3 Hours **Max. Marks: 80**

General Instructions:

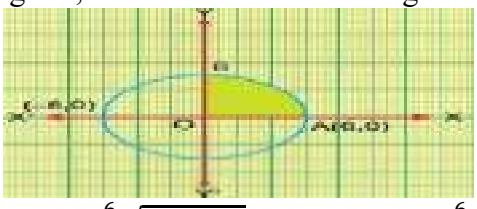
Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains 38 questions. All questions are compulsory.
(ii) This Question paper is divided into five Sections - A, B, C, D and E.
(iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no.19 and 20 are Assertion-Reason based questions of 1 mark each.
(iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
(v) In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
(vi) In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
(vii) In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
(viii) Use of calculators is not allowed.

SECTION – A

(Multiple Choice Questions)
Each question carries One Mark

Q.1 For the below given diagram, the area of the shaded region is



(A) $\int_{-6}^{6} \sqrt{x^2 - 6^2}\, dx$ (B) $\int_{0}^{6} \sqrt{x^2 - 6^2}\, dx$ (C) $4\int_{0}^{6} \sqrt{x^2 - 6^2}\, dx$ (D) None of these

Q.2 The area of the region bounded by the curvey $=$ cosx between x $= 0$ and x $= \pi$, x $-$ axis is

(A) 2 sq.units (B) 4sq. units (C) 6sq. units (D) 8sq.units

Q.3 If f(x) is an odd function, then $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)\cos^3 x\, dx$ equals:

(A) $2\int_0^{\frac{\pi}{2}} f(x)\cos^3 x\, dx$ (B) 0 (C) $2\int_0^{\frac{\pi}{2}} f(x)\, dx$ (C) $2\int_0^{\frac{\pi}{2}} \cos^3 x\, dx$.

Q.4 $\int x^2 e^{x^3} dx$ is equal to

(A) $\frac{1}{3}e^{x^3} + C$ (B) $\frac{1}{3}e^{x^4} + C$ (C) $\frac{1}{2}e^{x^3} + C$ (D) $\frac{1}{2}e^{x^2} + C$

Q.5 $\int \frac{e^x(1+x)}{\cos^2(xe^x)}$ dxis equal to

(A) $\tan(xe^x) + C$ (B) $\cot(xe^x) + C$ (C) $\cot(e^x) + C$

(D) $\tan[e^x(1+x)] + C$

Q.6 $\int_{-1}^{1} \frac{|x-2|}{x-2}$ dx, x≠0 is equal to

(A) 1 (B) -1 (C) 2 (D) -2

Q.7 If $f'(x) = x+\frac{1}{x}$, then f(x) is

A. $\log(x^2) + C$ B. $\frac{x^2}{2} + \log|x| + C$. C. $\frac{1}{x} + \log x + C$ D. None of these.

Q.8 Value of $\int \sin x^0 dx$ is

A. $-\cos x^0 + C$ B. $\cos x + C$ C. $-\frac{180}{\pi}\cos\frac{\pi x}{180} + C$ D. None of these

Q.9 Value of $\int \log_5 x\, dx$ is

A. $\frac{1}{\log_e 5}(x\log_e x - x) + C$ B. $x\log_e x - x + C$ C. $\frac{1}{x} + C$ D. None of these

Q.10 $\int \frac{x}{x^2+1} dx$ is

A. $\log(x^2+1) + C$ B. $2\log(x^2+1) + C$ C. $\frac{1}{2}\log(x^2+1) + C$

D. None of these

Q.11 Area of region bounded by curve y = x+1 and the lines x=2 and x= 3 is

A. $\frac{7}{2}$sq. unit B. $\frac{13}{2}$ sq. unit C. $\frac{9}{2}$ sq. unit D. $\frac{11}{2}$

Q.12 $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx$ is

A. 0 B. -2 C. 2 D. 1

Q.13 $\int \frac{\cos(\log x)}{x} dx$ is

A. logx +C B. sin(logx) + C C. log(sinx) + C D. log(cosx) + C

Q.14 $\int e^{5\log x} dx$ is

A. x + C B. $e^{\log x}$ + C C. $\frac{x^6}{6}$ + C D. None of these

Q.15 $\int e^x . \frac{(1+\sin x)}{(1+\cos x)} dx$ is

A. e^x tanx +C B. tan(sinx) + C C. $e^x \tan\frac{x}{2}$ + C D. None of these

Q.16 $\int \cos x . \cos 2x \, dx$ is

A. $\frac{1}{2}(\frac{\sin 3x}{3} + \sin x) + C$ B. $2(\frac{\sin}{3} + \sin x)$ C. sinx.sin2x + C D. None of these

Q.17 $\int e^x (1 - \cot x + \text{Cosec}^2 x) dx$ is

A. e^xcotx + C B. e^x(1-cotx) + C C. e^x(1+cosecx) + c D. None of these

Q.18 $\int \frac{\sec^2(\log x)}{x} dx$ is

A. tan(logx) + C B. cot(logx) + C C. log(tanx) + C D. None of these

ASSERTION-REASON BASED QUSETIONS

Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

A) Both A and R are true and R is the correct explanation of A.

B) Both A and R are true but R is not the correct explanation of A.

C) A is true but R is false.

D) A is false but R is true

Q.19 **Assertion(A)**: $\int_0^3 (x^2 + x + 1) dx = \frac{33}{2}$

Reason(R): $\int_0^2 4x^3 dx = \frac{81}{4}$

Q.20 **Assertion(A)**: $\int 3x^2 (cosx^3 + 8) dx = \sin x^3 + 8x^3 + C$

Reason(R): The above integration is solved using substitution method.

SECTION B

(Each question carries 2 marks)

Q.21 Evaluate: $\int \frac{dx}{x^2-6x+13}$

Q.22 $Evaluate$: $\int \frac{cos2x+2si\ ^2x}{cos^2x}$ dx

Q.23 $Evaluate$: $\int sinx.logcosx\ dx$

Q.24 Evaluate : $\int \frac{sin^{-1}x}{(1-x^2)^{\frac{3}{2}}}$ dx

Q.25 Evaluate : $\int \frac{sinx+cosx}{\sqrt{1+sin}}$ dx

SECTION C

(Each question carries 3 marks)

Q.26 Evaluate : $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{tanx}}$

Q.27 Evaluate : $\int_0^{\pi} \frac{x}{a^2cos^2x+ b^2sin^2x}$ dx

Q.28 Evaluate : $\int \frac{6x+7}{(6x-5)(x-4)}$ dx

Q.29 Evaluate : $\int \frac{x^4}{(x-1)(x^2+1)}$ dx

Q.30 Evaluate : $\int_0^{2\pi} \frac{1}{1+ e^{sinx}}$ dx

Q.31 Evaluate : $\int \frac{2x^2+1}{x^2(x^2+1)}$ dx

SECTION D

(Each question carries 5 marks)

Q.32 Find the area of region bounded by ellipse, $\frac{x^2}{25}+\frac{y^2}{16}=1$.

Q.33 Evaluate : $\int_{-1}^{\frac{3}{2}} |xsin\pi x|$dx

OR

$\int_0^{\frac{3}{2}} |x\cos\pi x| dx$

Q.34 Find the area of region bounded by parabola, $y^2 = 8x$ and line, x =2

Q.35 Evaluate: $\int e^x . sin2x dx$

SECTION- E [4x3=12]

(*This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)*

Case study-I

Q.36 Read the following text and answers the following questions on the basis of the same:
Reena and Sapna practice the problems based on integrals. They will try to evaluate the integrals based upon $\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + C$.
Reena first explains the steps to solve this type of integrals.
Step 1 : Obtain the integral, I = $\int \frac{f'(x)}{f(x)} dx$
Step 2 : Put f(x) = t and replace $f'(x)dx$ by dt to obtain I = $\int \frac{1}{t} dt$
Step 3 : Evaluate integral obtained in step II to obtain I = $\log|t| + C$
Step 4 : Replace t by f(x) in step III to get I = $\log|f(x)| + C$

i. Evaluate : $\int \frac{2x+5}{x^2+5x-7} dx$

ii. Evaluate : $\int \frac{1}{x(3+logx)} dx$

iii. Evaluate: $\int \frac{1}{1+e^{-x}} dx$

Or

Evaluate : $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$

Case study-II

Q.37 Read the following text and answer the following questions on the basis of the same:
Ram & Lakshman were discussing integration, in with following points:

a. In question like $\int e^x (f(x) + f'(x)) dx$, the result is $e^x f(x) = C$
b. If we have irrational terms in integration question, then we should try doing rationalizing these terms.

On the basis of the above information answer the following:

i. Evaluate : $\int \frac{1}{\sqrt{3x+4} - \sqrt{3x+1}} dx$

ii. Evaluate : $\int e^x(tanx + logsecx)$ dx

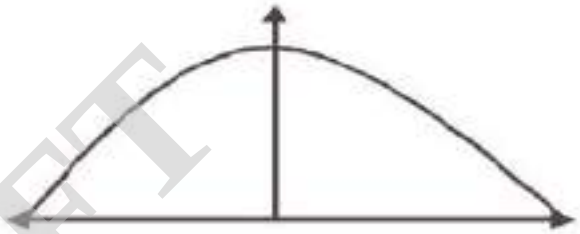
iii. Evaluate : $\int \frac{1}{\sqrt{1-2x}+\sqrt{3-2x}}$dx

Or

Evaluate : $\int e^x \left(\frac{1-sinx}{1-cosx}\right) dx$ dx

Case study-III

Q.38 The bridge connects 2 hills 100ft apart. The arch on the bridge is in a parabolic form. The highest point on the bridge is 10ft above the road at the middle of the bridge as seen in the figure :



i. The equation of the parabola:

A. $x^2 = 250y$ B. $x^2 = -250y$ C. $y^2 = 250x$ D. $y^2 = -250x$

ii. The value of integral $\int_{-50}^{50} \frac{x^2}{250}$dx is

A. $\frac{1000}{3}$ B. $\frac{250}{3}$ C. 1200 D. 0