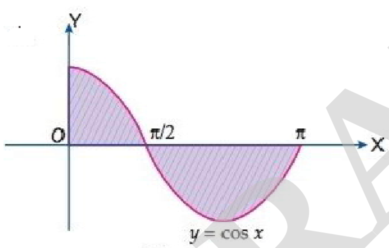
MARKINGSCHEME CLASS XII

MATHEMATICS (CODE-041)

CHAPTER/UNITS: INTEGRATION, APPLICATION OF INTEGRATION

SECTION:A

(Solution of MCQs of 1 Mark Each)

Q.NO.	ANS	SOLUTION
1.	(D)	Equation of circle is $x^2 + y^2 = 6^2 \Rightarrow y = \sqrt{6^2 - x^2}$. So, area of the shaded region =area of the region bounded by the circle between the y-axis i.e. x=0 and x=6(radius) upto x-axis. $=\int_0^6 \sqrt{6^2 - x^2}\, dx$
2.	(A)	 Area $=\int_0^{\frac{\pi}{2}} cosx\, dx + \left\lvert\int_{\frac{\pi}{2}}^{\pi} cosx\, dx\right\rvert$= 1+1 =2 sq units
3.	(B)	$f(x)\cos^3 x$ is an odd function since f(x) as odd
4.	(A)	$$\int x^2 e^{x^3} dx$$ Putting $z = x^3 \Rightarrow dz = 3x^2\, dx$ $$\int \frac{1}{3} e^z dz = \frac{1}{3} e^z$$
5.	(A)	$$\int \frac{e^x(1+x)}{\cos^2(xe^x)}\, dx$$ Putting $z = xe^x \Rightarrow dz=(xe^x + e^x)dx \Rightarrow dz=e^x(x+1)dx$ $$\int \frac{1}{\cos^2 z} dz = \int \sec^2 z dz = \tan z + C = \tan(xe^x) + C$$

6.	(D)	$$\int_{-1}^{1}\frac{\lvert x-2\rvert}{x-2}dx$$ $= \int_{-1}^{1}(-1)\,\text{dx}$, since $\lvert x-2\rvert = -(x-2)\text{in}[-1,1]$ $= -[x]_{-1}^{1}$ = -2
7.	(B)	Given: $f'(x) = x + \frac{1}{x} \Rightarrow f(x) = \int\left(x+\frac{1}{x}\right)dx = \frac{x^2}{2} + \log\lvert x\rvert + C$
8.	(C)	$\int \sin x^0 dx = \int \sin\frac{\pi x}{180}\,dx = -\frac{180}{\pi}\cos\frac{\pi x}{180} + C$
9.	(A)	$$\int \log_5 x\,dx = \int\frac{\log_e x}{\log_e 5}\,dx = \frac{1}{\log_e 5}\int \log_e x\,dx$$ $= \frac{1}{\log_e 5}[(\log_e x)x - \int\frac{1}{x}.x\,dx]$, (by parts) $=\frac{1}{\log_e 5}(x\log_e x - x) + C$
10.	(C)	$\int\frac{x}{x^2+1}\,dx = \frac{1}{2}\int\frac{2x}{x^2+1}\,dx = \frac{1}{2}\log(x^2+1) + C$, by $\frac{f'(x)}{f(x)}$ form.
11.	(A)	Area$= \int_2^3(x+1)dx = \left[\frac{(x+1)^2}{2}\right]_2^3 = \frac{7}{2}$
12.	(A)	$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^7 x\,dx = 0$, as $\sin^7 x$ an odd function
13	(B)	$\int\frac{\cos(\log x)}{x}dx = \sin(\log x) + C$, by putting $Z = \log x$
15	(C)	$\int e^{5\log x}\,dx$ $= \int e^{\log x^5}\,dx$ $= \int x^5\,dx$ $= \frac{x^6}{6} + C$
16	(A)	$\int e^x.\frac{(1+\sin x)}{(1+\cos x)}\,dx = \int e^x\left(\frac{1+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)dx = \int e^x\left\{\frac{1}{2}sec^2\frac{x}{2} + tan\frac{x}{2}\right\}dx$ $= e^x\tan\frac{x}{2} + C$, by $\int e^x\{f'(x)+f(x)\}\,dx$
17	(B)	$\int \cos x.\cos 2x\,dx = \frac{1}{2}\int(cos3x + cosx)\,dx$, by $\cos(A+B) + \cos(A-B)$

		$=\frac{1}{2}\left(\frac{Sin3x}{3}+\ sinx\right)+C$
17	(C)	$\int e^{x}(1-\cot x+\ \mathrm{cosec}^2x)dx$ $= e^{x}(1\text{-}\cot x)\ +C$, by $\int e^{x}\{f'(x)+f(x)\}\,dx$ where $f(x)=1-\cot x$
18	(A)	$\int \frac{\sec^2(\log x)}{x}\,dx=\tan(\log x)+C.\ by\ putting\ Z=\log x$
19	(C)	$\int_0^2 4x^3dx=16$, So (A) is true but (R) is false
20	(A)	Assertion (A)and Reason(R) both are correct, ***(R) is the correct explanation of (A)***

Section–B

[This section comprises of solution of very short answer type questions (VSA) of 2 marks each]

21.	$\int \frac{dx}{x^2-6x+13}=\int \frac{dx}{(x-3)^2+4}\ \ =\ \frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right)+C$
22.	$\int \frac{cos2x+2sin^2x}{cos^2x}\ dx$ Using relation, $\cos2x = 1-\sin^2x$ $\int \frac{1-2sin^2x+2sin^2x}{cos^2x}\ dx = \tan x + C$
23	$=\int sinx.Logcosx\ dx$ putting z =cosx $I=-\int 1.logzdz$ $=\cos x\{1-\log(\cos x)\} + C$
24	$I=\int \frac{sin^{-1}x}{(1-x^2)^{\frac{3}{2}}}\,dx$ Put $z= \sin^{-1}x$ $dz=\frac{1}{\sqrt{1-x^2}}dx$ $I=\int zsec^2zdz$ $=\frac{x}{\sqrt{1-x^2}}\sin^{-1}x+\log\left\|\sqrt{1-x^2}\right\|+C$
25	$\int \frac{sinx+cosx}{\sqrt{1+sin2x}}\,dx=\int \frac{sinx+cosx}{sinx+cosx}dx=\int dx= x+C$

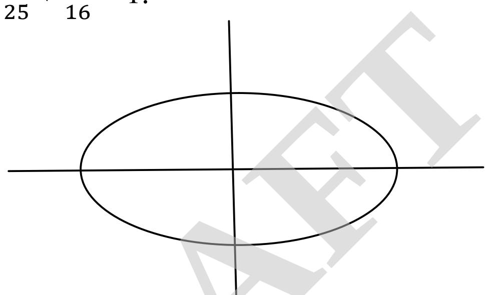
SECTION C (Each question carries 3 marks)

26	$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{tanx}} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{cosx}}{\sqrt{cosx}+\sqrt{sinx}}$ dx ....1 Using Property, $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$ $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{sinx}}{\sqrt{sinx}+\sqrt{cosx}}$ dx ....2 (1) + (2) $2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} dx$ $I = \frac{\pi}{12}$
27	$I = \int_0^{\pi} \frac{x}{a^2cos^2x + b^2sin^2x}$ dx ....(1) Use prop, $\int_0^a f(x) = \int_0^a f(a-x)dx$ $I = \int_0^{\pi} \frac{\pi - x}{a^2cos^2x + b^2sin^2x}$ dx ....(2) (1) + (2) $2I = \int_0^{\pi} \frac{\pi}{a^2cos^2x + b^2sin^2x}$ dx $I = \frac{\pi}{2}\int_0^{\pi} \frac{1}{a^2cos^2x + b^2sin^2x}$ dx $= \frac{\pi}{2}\int_0^{\pi} \frac{sec^2x}{a^2 + b^2tan^2x}$ dx
28	Use Linear = A$\frac{d}{dx}$Quadratic + B Solution is $6\sqrt{x^2 - 9x + 20}$ $+34\log\left\lvert x - \frac{9}{2} + \sqrt{x^2 - 9x + 20}\right\rvert + C$
29.	$\int \frac{x^4}{(x-1)(x^2+1)}$ dx $= \int \left\{x + 1 + \frac{1}{(x-1)(x^2+1)}\right\}$ dx Apply partial Fraction in the above $I = \int (x+1)dx + \frac{1}{2}\int \frac{1}{x-1}dx - \frac{1}{2}\int \frac{x+1}{x^2+1}dx$ $= \frac{x^2}{2} + x + \frac{1}{2}\log\lvert x-1\rvert - \frac{1}{4}\log\lvert x^2+1\rvert - \frac{1}{2}\tan^{-1}x + C$
30	$I = \int_0^{2\pi} \frac{1}{1+e^{Sinx}}$ dx ....(1) Use property, $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

	I=$\int \int_0^{2\pi} \frac{1}{1+e^{-sinx}}$ dx $=\int_0^{2\pi} \frac{e^{sinx}}{1+e^{sinx}}$ dx ....(2) (1)+(2) 2I = $\int_0^{2\pi} \frac{1+e^{sinx}}{1+e^{sinx}}$ dx $= 2\pi$ I = π
31	I = $\int \frac{2x^2+1}{x^2(x^2+1)}$ dx Replace $x^2 = z$ Apply Partial Fraction Soln is $\frac{-1}{4x}+\frac{7}{8}\tan^{-1}\frac{x}{2}+C$

SECTION D

(Each question carries 5 marks)

32	Equation of ellipse is $\frac{x^2}{25}+\frac{y^2}{16}=1$. Find y = $\frac{4}{5}\sqrt{5^2-x^2}$  Area of ellipse = 4 x area of ellipse in 1[st] Quadrant $= 20\pi$ sq. unit
33	$\int_{-1}^{\frac{3}{2}} \lvert x\sin\pi x\rvert$dx When xsinx =0 x=n$\pi$ x= 0,1$\in (-1,\frac{3}{2})$ $\int_{-1}^{\frac{3}{2}} \lvert x\sin\pi x\rvert$dx = $\int_{-1}^{1} x\sin\pi x\,dx + \int_{1}^{\frac{3}{2}} -x\sin\pi x\,dx$ $= \frac{1+3\pi}{\pi^2}$
33.(OR)	OR $\int_{0}^{\frac{3}{2}} \lvert x\cos\pi x\rvert$dx When $x\cos\pi x = 0$