**UNIT WISE PRACTICE QUESTION PAPER**

**Class-XII**
**Subject: Mathematics (041)**

---

**CHAPTER/UNITS: DIFFERENTIAL EQUATION, VECTOR AND 3D GEOMETRY**

---

**Time:3 Hours** **Maximum Marks:80**

---

**General Instructions:**
1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3**. Section B** has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. **Section C** has 6 Short Answer (SA)-type questions of 3 marks each.
5**. Section D** has 4 Long Answer (LA)-type questions of 5 marks each.
6. **Section E** has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.
7.Use of calculators is **not** allowed.

---

**SECTION – A**
***(Multiple Choice Questions)***
***Each question carries One Mark***

Q.1 If $|\vec{a}.\vec{b}|^2 + |\vec{a}\times\vec{b}|^2 = 144$ and $|\vec{a}| = 4$, then $|\vec{b}|$ is
(A) 9 (B) 16 (C) 3 (D) None of these

Q.2 If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\sqrt{3}\vec{a} - \vec{b}$ is also an unit vector, then the angle between $\vec{a}$ and $\vec{b}$ is
(A) $30^0$ (B) $45^0$ (C) $60^0$ (D) None of these

Q.3 If $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$ then, the angle between $\vec{a}$ and $\vec{b}$ is
(A)$90^0$ (B) $45^0$ (C) $60^0$ (D) None of these

Q.4 Let $\vec{a} = 2\,\hat{\imath} + 3\,\hat{\jmath} + \lambda\hat{k}$. The value of λ if $|-5\vec{a}| = 25$ is
(A) 0 (B) $\pm 2\sqrt{3}$ (C) ±1 (D) ±12

Q.5 The value of λ for which the two vectors $2\,\hat{\imath} - \hat{\jmath} + 2\,\hat{k}$ *and* $3\,\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$ are orthogonal (i.e.perpendicular) is
(A)2 (B) 4 (C) 6 (D) 8

Q.6 If $\overrightarrow{AB} = 2\,\hat{\imath} + \hat{\jmath} - 3\,\hat{k}$ and $A(1,2,-1)$ is the given point, then the coordinates of B are
(A) $(3,-3,4)$ (B)$(3,3,4)$ (C)$(-3,-3,4)$ (D) $(3,3,-4)$

Q.7 The integrating factor of the differential equation :

$$(1-y^2)\frac{dx}{dy}+yx=ay, (-1<y<1) \text{ is}$$

(A) $\frac{1}{\sqrt{1-y^2}}$ (B) $\frac{1}{1-y^2}$ (C) $\frac{1}{\sqrt{y^2-1}}$ (D) $\frac{1}{y^2-1}$

Q.8 Familyy $= Ax + A^3$ of curves will corresponds to a differential equation of order

(A) 3 (B) 2 (C) 1 (D) not defined

Q.9 If m and n are the order and the degree of the differential equation

$1+\left(\frac{d^3y}{dx^3}\right)^2=\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}}$, then the value of $4m-3n$ is

(A) 0 (B) 2 (C) 1 (D) not defined

Q.10 The number of solution of $\frac{dy}{dx}=\frac{y+1}{x-1}$wheny(1)=2 is

(A) none (B) one (C) two (D) infinite

Q.11 Which of the following is not a homogeneous function of x and y ?

(A) $x^2+2xy$ (B) $2x-y$ (C) $\cos^2\left(\frac{y}{x}\right)+\frac{y}{x}$ (D) $\sin x-\cos y$

Q.12 The solution of the differential equation is $\frac{dx}{x}+\frac{dy}{y}=0$is

(A) $\frac{1}{x}+\frac{1}{y}=C$ (B) $\log x-\log y=C$ (C) xy=C (D) x +y=C

Q.13 Direction cosines of the line $\frac{x-1}{2}=\frac{1-y}{3}=\frac{2z-1}{12}$ are

(A) $\frac{2}{7},\frac{3}{7},\frac{6}{7}$ (B) $\frac{2}{7},-\frac{3}{7},-\frac{6}{7}$ (C) $\frac{2}{\sqrt{157}},-\frac{3}{\sqrt{157}},\frac{12}{\sqrt{157}}$ (D) $\frac{2}{7},-\frac{3}{7},\frac{6}{7}$

Q.14 The coordinates of the foot of the perpendicular drawn from the point $(2,-3,4)$ on the y-axis is

(A) $(2,3,4)$ (B) $(-2,-3,-4)$ (C) $(0,-3,0)$ (D) $(2,0,4)$

Q.15 If a line makes angles $\alpha$,$\beta$ and $\gamma$ with the axes respectively,then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma=$

(A) -2 (B) -1 (C) 1 (D) 2

Q.16 The distance of the point P( a, b, c ) from y-axis is

(A) b (B) $|b|$ (C) $|b|+|c|$ (D) $\sqrt{a^2+c^2}$

Q.17 If the line through the points $(1,-1,2)$ ,$(3,4,-2)$ is perpendicular to the line through points $(\lambda,3,2)$,$(3,5,6)$, then the value of $\lambda$ is

(A) 0 (B) 2 (C) 1 (D) not defined

Q.18 The value of p for which the lines $\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$ and $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles is

(A) $-\frac{11}{70}$ (B) $\frac{70}{11}$ (C) $\frac{70}{\sqrt{11}}$ (D) $-\frac{70}{\sqrt{11}}$

## ASSERTION-REASON BASED QUSETIONS

***Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.***
***A) Both A and R are true and R is the correct explanation of A.***
***B) Both A and R are true but R is not the correct explanation of A.***
***C) A is true but R is false.***
***D) A is false but R is true***

Q.19 **Assertion(A)**: If the cartesian equation of a line is $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$ then its vector form is

$$\vec{r}=5\hat{i}-4\hat{j}+6\hat{k}+\lambda(3\hat{i}+7\hat{j}+2\hat{k})$$

**Reason(R)**: The cartesian equation of the line which passes through the point (–2, 4, –5) and parallel to the line given by $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$ is $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$

Q.20 **Assertion(A)**: A line in space cannot be drawn perpendicular to x,y and z axes simultaneously.

**Reason(R)**: For any line making angles α,β and γ with the positive directions of x,y and z- axes respectively, $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$



## SECTION B

***(Each question carries 2 marks)***

Q.21 If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $(\vec{a}+\vec{b})\perp\vec{a}$ and $(2\vec{a}+\vec{b})\perp\vec{b}$ , Then prove that $|\vec{b}|=\sqrt{2}|\vec{a}|$

**(OR)**

The two vectors $\hat{j}+\hat{k}$ and $3\hat{i}-\hat{j}+4\hat{k}$ represents the two sides AB and AC, respectively of a triangle ABC.Find the length of the median through A.

Q.22 Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $2\vec{a}+\vec{b}$ and $\vec{a}-3\vec{b}$, externally in the ratio 1:2 .Also, show that P is the mid point the line segment RQ.

**(OR)**

Find a vector of magnitude 9 unit perpendicular to both the vectors $\hat{i}-\hat{j}$ and $\hat{i}+\hat{j}$

Q.23 Find the general solution of $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

Q.24 Given that $\frac{dy}{dx} = ye^x$ and x=0, y=e. Find the value of y when x=1

Q.25 Find the angle between the lines
$\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k} + \lambda(2\hat{i} + \hat{j} + 2\hat{k})$ and $\vec{r} = 2\hat{j} - 5\hat{k} + \mu(6\hat{i} + 3\hat{j} + 2\hat{k})$

## SECTION C

***(Each question carries 3 marks)***

Q.26 The two adjacent sides of a parallelogram are $2\,\hat{i} - 4\,\hat{j} - 5\,\hat{k}$ *and* $2\,\hat{i} + 2\,\hat{j} + 3\,\hat{k}$ .
Find the two-unit vectors parallel to its diagonals. Using the diagonals vectors, find the area of the parallelogram.

**(OR)**

If A,B,C,D are the points with position vectors $\hat{i} + \hat{j} - \hat{k}$, $2\,\hat{i} - \hat{j} + 3\,\hat{k}$, $2\,\hat{i} - 3\,\hat{k}$ and $3\,\hat{i} - 2\,\hat{j} + \hat{k}$ respectively, find the projection of $\overrightarrow{AB}$ along $\overrightarrow{CD}$.

Q.27 Find the position vector of a point P in space such that $\overrightarrow{OP}$ is inclined at $60^0$ to OX and at $45^0$ to OY and $|\overrightarrow{OP}|$=10 units.

Q.28 Find the angle between any two diagonals of a cube.

Q.29 Solve: $\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + xdy = 0; y = \frac{\pi}{4}$ when $x = 1$

Q.30 Find the particular solution of $(1 + x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$; $y = 0$ when $x = 1$

Q.31 let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = 1, |\vec{b}| = 2, |\vec{c}| = 3$. If the projection of $\vec{b}$ along$\vec{a}$ is equal tothe projection of $\vec{c}$ along$\vec{a}$ and $\vec{b}$,$\vec{c}$ are perpendicular to each other, then find$|3\vec{a} - 2\vec{b} + 2\vec{c}|$.

OR The magnitude of the vector product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$and $\lambda\hat{i} + 2\hat{j} + 3\,\hat{k}$ is equal to $\sqrt{2}$ .Find the value of λ.

## SECTION D

***(Each question carries 5 marks)***

Q.32 By using vectors, in a $\Delta$ABC, prove that, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
where a,b.c are represent the magnitude of the sides opposite to the vertices A,B,C respectively.

**(OR)**

If $\vec{a}$ and $\vec{b}$are two unit vectors inclined an angle Θ, then prove that:

(i) $\cos\frac{\theta}{2} = \frac{1}{2}\left|\vec{a} + \vec{b}\right|$

(ii) $\sin\frac{\theta}{2} = \frac{1}{2}\left|\vec{a} - \vec{b}\right|$

(iii) $\tan\frac{\theta}{2} = \frac{|\vec{a} - \vec{b}|}{|\vec{a} + \vec{b}|}$.

Q.33 Find the coordinates of the foot of perpendicular drawn from the point A(1, 8, 4) to the line

joining the points B(0,-1,3) and C(2,-3,-1) . Also find
(i) length of the perpendicular
(ii) image of A in the line through B and C

**(OR)**

Find the equation of a line $l_2$ which is the mirror image of the line$l_1$ with repect to the line $l$: $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$, given that the line $l_1$ passes through the point P(1, 6, 3) and parallel to line $l$. **(CBSE 2024 65/1/3)**

Q.34 Find the shortest distance between the lines whose vector equations are:
$\vec{r} = (1-t)\hat{\imath} + (t-2)\hat{\jmath} + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{\imath} + (2s-1)\hat{\jmath} - (2s+1)\hat{k}$).

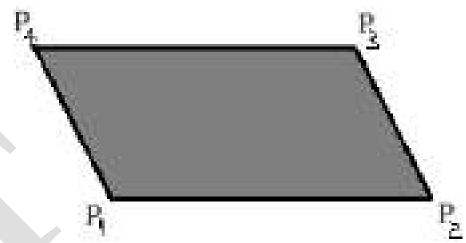
Q.35 Solve:$y + \frac{d}{dx}(xy) = x(\sin x + \log x)$

## SECTION- E [4x3=12]

(***This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)***

**Case study-I**

Q.36 Solar Panels have to be installed carefully so that the tilt of the roof, and the direction to the sun, produce the largest possible electrical power in the solar panels. A surveyor uses his instrument to determine the coordinates of the four corners of a roof where solar panels are to be mounted. In the picture, suppose the points are labelled counter clockwise from the roof corner nearest to the camera in units of meters $P_1$ (6,8,4) , $P_2$ (21,8,4), $P_3$ (21,16,10) and $P_4$ (6,16,10).






Based on the above information, answer the following questions

**(i)** What are the components to the two edge vectors defined by $\vec{\mathbf{A}}$ = PV of $P_2$ – PV of $P_1$ and $\vec{\mathbf{B}}$ = PV of $P_4$ – PV of $P_1$ ? (where PV stands for position vector). 1

**(ii)** Find the vector $\vec{\mathbf{N}}$, perpendicular to $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ and the surface of the roof? 1

**(iii)** (a) If the flow of solar energy is given by the vector $\vec{\mathbf{F}} = \mathbf{6\hat{i} - 2\hat{j} + 3\hat{k}}$ what is the dot product of vectors $\vec{\mathbf{F}}$ with $\vec{\mathbf{N}}$. 2

OR

(b) What is the angle between vectors $\vec{\mathbf{N}}$ and $\vec{\mathbf{F}}$ ?

**Case study-II**

Q.37 Two motorcycles A and B are running at the speed more than the allowed speed on the roads represented by the lines $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k})$ respectively.

Base on the above information, answer the following

(i) Write the Cartesian equation of the line along which the motorcycle A is running. 1

(ii) Find the direction cosines of the line along which motorcycle B is running 2

(iii) (a) Find shortest distance between the given lines 2

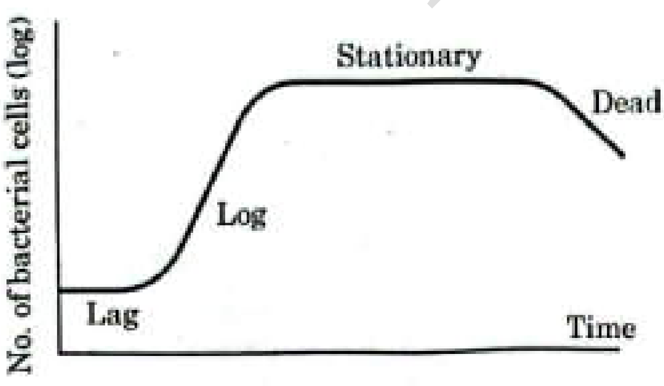
OR

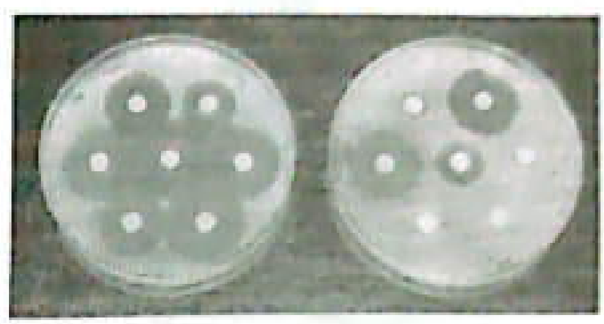
(b) Find the angle between the given lines



**Case study-III**

Q.38 A bacteria sample of certain number of bacteria is observed to grow exponentially in a given amount of time. Using exponential growth, the rate of growth of this sample bacteria are calculated.



The differential equation representing the growth of bacteria is given as:

$\frac{dP}{dt} = \mathbf{kP}$ ,where P is the population of bacteria at any time ‘t’.