UNIT WISE PRACTICE QUESTION PAPER

MARKING SCHEME CLASS XII

MATHEMATICS(CODE-041)

CHAPTER/UNITS: DIFFERENTIAL EQUATION, VECTOR AND 3D GEOMETRY

SECTION:A

(Solution of MCQs of 1 Mark each)

Q.NO.	ANS	SOLUTION
1.	(C)	Given: $\left\vert\vec{a}.\vec{b}\right\vert^2 + \left\vert\vec{a}\times\vec{b}\right\vert^2 = 144 \,\&\, \vert\vec{a}\vert = 4$ By Langrange's identity, $\left\vert\vec{a}.\vec{b}\right\vert^2 + \left\vert\vec{a}\times\vec{b}\right\vert^2 = \vert\vec{a}\vert^2\left\vert\vec{b}\right\vert^2$ $\Rightarrow 144 = 4^2\left\vert\vec{b}\right\vert^2$ $\Rightarrow \left\vert\vec{b}\right\vert^2 = 9$ $\Rightarrow \left\vert\vec{b}\right\vert = 3$
2.	(A)	Given: $\sqrt{3}\vec{a} - \vec{b}$ is an unit vector $\Rightarrow \left\vert\sqrt{3}\vec{a} - \vec{b}\right\vert = 1$ Squaring on both sides, $\left\vert\sqrt{3}\vec{a} - \vec{b}\right\vert^2 = 1$ $\Rightarrow \left\vert\sqrt{3}\vec{a}\right\vert^2 - 2\sqrt{3}\vec{a}.\vec{b} + \left\vert\vec{b}\right\vert^2 = 1$ $\Rightarrow \left\vert\sqrt{3}\right\vert^2 \vert\vec{a}\vert^2 - 2\sqrt{3}\vert\vec{a}\vert\left\vert\vec{b}\right\vert\cos\Theta + \left\vert\vec{b}\right\vert^2 = 1$ $\Rightarrow 3 - 2\sqrt{3}\cos\Theta + 1 = 1 \;\left(\text{since } \vert\vec{a}\vert = 1 \,\&\, \left\vert\vec{b}\right\vert = 1\right)$ $\therefore \cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{6} = 30^0$
3.	(A)	Given: $\left\vert\vec{a} + \vec{b}\right\vert = \left\vert\vec{a} - \vec{b}\right\vert$ Squaring on both sides, $\left\vert\vec{a} + \vec{b}\right\vert^2 = \left\vert\vec{a} - \vec{b}\right\vert^2$ $\Rightarrow \vert\vec{a}\vert^2 + \left\vert\vec{b}\right\vert^2 + 2\,\vec{a}.\vec{b} = \vert\vec{a}\vert^2 + \left\vert\vec{b}\right\vert^2 - 2\,\vec{a}.\vec{b}$ $\Rightarrow 4\,\vec{a}.\vec{b} = 0$ $\Rightarrow 4\,\vert\vec{a}\vert\left\vert\vec{b}\right\vert\cos\Theta = 0$ $\therefore \cos\theta = 0 \Rightarrow \theta = 90^0$

4		Given: $\vec{a} = 2\,\hat{i} + 3\,\hat{j} + \lambda\hat{k}$ and $\lvert -5\vec{a}$ $\rvert = 25$ $\Rightarrow \lvert -5 \rvert \lvert \vec{a} \rvert = 25$ $\Rightarrow 5\sqrt{2^2 + 3^2 + \lambda^2} = 25$ $\Rightarrow \sqrt{13 + \lambda^2}$ $\Rightarrow 13 + \lambda^2 = 25 \Rightarrow \lambda^2 = 12 \Rightarrow \lambda = \pm 2\sqrt{3}$
5	(D)	Given: $2\,\hat{i} - \hat{j} + 2\,\hat{k}$ *and* $3\,\hat{i} + \lambda\hat{j} + \hat{k}$ are perpendicular $\Rightarrow (2\,\hat{i} - \hat{j} + 2\,\hat{k}).(3\,\hat{i} + \lambda\hat{j}$ $+ \hat{k})$ $\Rightarrow 2.3$ $\Rightarrow \lambda = 8$
6	(D)	Given: $\overrightarrow{AB} = 2\,\hat{i} + \hat{j} - 3\,\hat{k}$ and A $(1,2,-1)$ $\Rightarrow \text{p.v. of B} - \text{p.v. of A} = 2\,\hat{i} + \hat{j} - 3\,\hat{k}$ $\Rightarrow \text{p.v. of B} - (\hat{i} + 2\hat{j} - \hat{k}) = 2\,\hat{i} + \hat{j} - 3\,\hat{k}$ $\Rightarrow \text{p.v. of B} = 3\,\hat{i} + 3\,\hat{j} - 4\,\hat{k}$
7	(A)	Given: $(1 - y^2)\frac{dx}{dy} + yx = ay$ $\Rightarrow \frac{dx}{dy} + \frac{y}{(1 - y^2)}x = \frac{ay}{(1 - y^2)}$, which is in the form of $\frac{dx}{dy} + Px = Q$ $\therefore \text{I.F.} = e^{\int P dy} = e^{\int \frac{y}{1-y^2} dy}$ $= e^{\frac{-1}{2}\int \frac{-2y}{1-y^2} dy}$ $= e^{\frac{-1}{2}\log(1-y^2)}$ $= e^{\log(1-y^2)^{\frac{-1}{2}}}$ $= (1 - y^2)^{\frac{-1}{2}} = \frac{1}{\sqrt{1 - y^2}}$
8	(C)	$y = Ax + A^3$ is corresponds to the D.E. of order one as it is involved only one arbitrary constant i.e. A
9	(D)	$1 + \left(\frac{d^3y}{dx^3}\right)^2$ $= \left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}}$

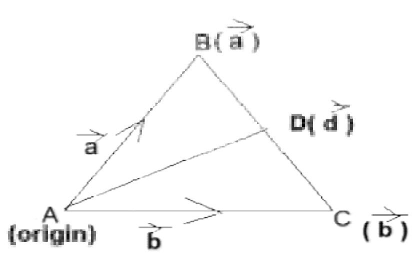
10	(B)	Concept: Solution of a D.E. under an initial given condition is a particular solution. And, a particular solution of a D.E. of order one and degree one have only one solution. Explanation: Given: $\frac{dy}{dx} = \frac{y+1}{x-1}$wheny(1)=2 (initial condition) $\Rightarrow \frac{1}{y+1}dy = \frac{1}{x-1}dx \Rightarrow \int \frac{1}{y+1}dy = \int \frac{1}{x-1}dx \Rightarrow \log(y+1) = \log(x-1) + C$ $\Rightarrow \log(y+1) - \log(x-1) = C$ $\Rightarrow \log\left(\frac{y+1}{x-1}\right) = \log A$ $\Rightarrow \frac{y+1}{x-1} = A$ $\Rightarrow y + 1 = A(x-1)$………(i) But, when x=1, y=2 ; (i) $\Rightarrow 2 + 1 = A(0-1) \Rightarrow A = -3$ Hence ,from(i), the P.S. is $y + 1 = -3(x-1)$ $\Rightarrow\ y + 3x = 2$
11	(D)	Here,in option (D) the degree of x and y not defined. A homogeneous function is a function that has the same degree of the polynomial in each variablesHomogeneous function of x and y is a function that can be expressed in the form of either $f\left(\frac{x}{y}\right)$ or $f\left(\frac{y}{x}\right)$.
12	(C)	Given:$\frac{dx}{x} + \frac{dy}{y} = 0$ $\Rightarrow \frac{1}{x}dx$ $\Rightarrow \int \frac{1}{x}dx$ $+ \int \frac{1}{y}dy$ $\Rightarrow \log x$ $\Rightarrow \log xy$ $\Rightarrow xy = C$

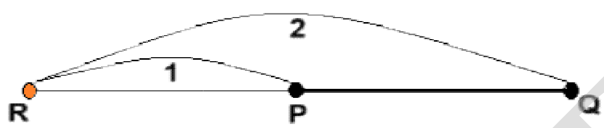
13	(D)	Given: $\frac{x-1}{2}=\frac{1-y}{3}=\frac{2z-1}{12}$ Standard form: $\frac{x-1}{2}=\frac{y-1}{-3}=\frac{z-\frac{1}{2}}{6}$ Direction ratios are: $\langle 2,-3,6 \rangle$ Direction cosines are: $\langle \frac{2}{\sqrt{2^2+(-3)^2+6^2}}, \frac{-3}{\sqrt{2^2+(-3)^2+6^2}}, \frac{6}{\sqrt{2^2+(-3)^2+6^2}} \rangle$ i.e. $\langle \frac{2}{7}, \frac{-3}{7}, \frac{6}{7} \rangle$
14	(C)	The x and z co-ordinates of a point on y-axis are 0. Therefore, required point on the y-axis = (0,-3,0)
15	(B)	We know that $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$ Now, $\cos2\alpha+\cos2\beta+\cos2\gamma=(2\cos^2\alpha-1)+(2\cos^2\beta-1)+(2\cos^2\gamma-1)$ $=2(\cos^2\alpha+\cos^2\beta+\cos^2\gamma)-3$ $=-1$
16	(D)	The required distance is the distance of P(a,b,c) from Q(0,b,0) $=\sqrt{a^2+c^2}$
17	(A)	Direction ratios of the through $(1,-1,2)$&$(3,4,-2)$ are $\langle 2,5,-4 \rangle$ Direction ratios of the through $(\lambda,3,2)$&$(3,5,6)$ are $\langle 3-\lambda,2,4 \rangle$ Since, the lines are perpendicular we have $a_1a_2+b_1b_2+c_1c_2=0$ $\Rightarrow 2(3-\lambda)+5.2+(-4).4=0$ $\Rightarrow 6-2\lambda+10-16=0$ $\Rightarrow \lambda=0$
18	(B)	Given lines: $\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$ and $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ Standard forms of the lines: $\frac{x-1}{-3}=\frac{y-2}{\frac{2p}{7}}=\frac{z-3}{2}$ and $\frac{x-1}{-\frac{3p}{7}}=\frac{y-5}{1}=\frac{z-6}{-5}$ Since, the lines are perpendicular we have $a_1a_2+b_1b_2+c_1c_2=0$ $\Rightarrow -3.(-\frac{3p}{7})+\frac{2p}{7}.1+2.(-5)=0$ $\Rightarrow \frac{11p}{7}=10$ $\Rightarrow p=\frac{70}{11}$
19	(C)	The assertion(A) is true as, for the Cartesian equation $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-y_1}{c}$, the vector equation is $\vec{r}=x_1\hat{i}+y_1\hat{j}+y_1\hat{k}+\lambda(a\hat{i}+b\hat{j}+c\hat{k})$. The reason (R) is not true because the correct equation of the line

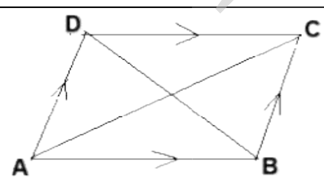
		passing through (–2, 4, –5) and parallel to the line given by $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$ is $\frac{x+2}{3}=\frac{y-4}{5}=\frac{z+5}{6}$
20	(A)	The assertion(A) that a line in space cannot be drawn perpendicular to x , y and z-axes simultaneously is true, as a line can only be perpendicular to a single axis or lie in a plane that is perpendicular to two axes,but not all three simultaneously. The reason (R) is also true as well as (R) is the correct explaination of (A) as, suppose the line is perpendicular to the 3 axes simultaneously then $\cos^2 90^0+\cos^2 90^0+\cos^2 90^0=0\neq 1$

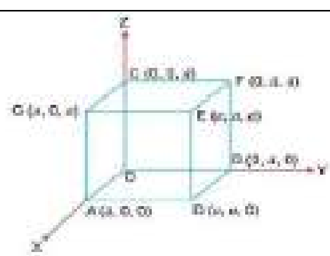
<u>Section–B</u>

[This section comprises of solution of very short answer type questions **(VSA)of** 2 marks **ea**ch]

21.	Given: $(\vec{a}+\vec{b})\perp\vec{a}$ and $(2\vec{a}+\vec{b})\perp\vec{b}$ $\Rightarrow(\vec{a}+\vec{b}).\vec{a}=0$ and $(2\vec{a}+\vec{b}).\vec{b}=0$ $\Rightarrow\|\vec{a}\|^2+\vec{a}.\vec{b}=0$ …. (i)and $2\vec{a}.\vec{b}+\|\vec{b}\|^2$	$\frac{1}{2}$ 1 $\frac{1}{2}$
21 (OR		$\frac{1}{2}$

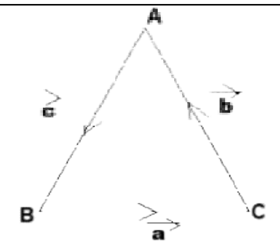
	Given .$\vec{a}$=p.v. of B=$\overrightarrow{AB}$ =$\hat{j} + \hat{k}$ and $\vec{b}$ = p.v. of C = $\overrightarrow{AC} = 3\,\hat{\imath} - \hat{j} + 4\,\hat{k}$ $\therefore$ Median $\overrightarrow{AD} = \vec{d}$ i.e. by mid point formula, $\vec{d} = \dfrac{\vec{a}+\vec{b}}{2}$ $= \dfrac{3}{2}\hat{\imath} + 0\hat{j} + \dfrac{5}{2}\hat{k}$ $\therefore$ Length of the median $\overrightarrow{AD} = \lvert\vec{\square}\rvert = \sqrt{\left(\frac{3}{2}\right)^2 + 0^2 + \left(\frac{5}{2}\right)^2} = \dfrac{\sqrt{34}}{2}$	1 $\frac{1}{2}$
22.	 $(2\vec{a} + \vec{b})$ $\quad\quad$ $(\vec{a} - 3\vec{b})$ By section formula ,position vector of R= $\frac{2(2\vec{a}+\vec{b})-1(\vec{a}-3\vec{b})}{2-1} = 3\vec{a} + 5\vec{b}$ Again,mid-point of RQ= $\frac{(3\vec{a}+5\vec{b})+(\vec{a}-3\vec{b})}{2} = 2\vec{a} + \vec{b}$ = Position vector of P.Hence,P is the mid-point of RQ	1 1
22. (OR)	The vector of magnitude 9 unit perpendicular to both the vectors $\hat{\imath} - \hat{j}$ and $\hat{\imath} + \hat{j}$ $= 9.\left[\dfrac{(\hat{\imath} - \hat{j}) \times (\hat{\imath} + \hat{j})}{\lvert(\hat{\imath} - \hat{j}) \times (\hat{\imath} + \hat{j})\rvert}\right]$ $= 9.\left[\dfrac{\hat{\imath}\times\hat{\imath} + \hat{\imath}\times\hat{j} - \hat{j}\times\hat{\imath} - \hat{j}\times\hat{j}}{\lvert\hat{\imath}\times\hat{\imath} + \hat{\imath}\times\hat{j} - \hat{j}\times\hat{\imath} - \hat{j}\times\hat{j}\rvert}\right]$ $= 9.\left[\dfrac{0 + \hat{k} - (-\hat{k}) - 0}{\lvert 0 + \hat{k} - (-\hat{k}) - 0\rvert}\right]$ by right handed system $= 9.\left[\frac{2\hat{k}}{\lvert 2\hat{k}\rvert}\right]$=9$\hat{k}$ (since $\lvert 2\hat{k}\rvert = 2\lvert\hat{k}\rvert = 2$)	1 $\frac{1}{2}$ $\frac{1}{2}$

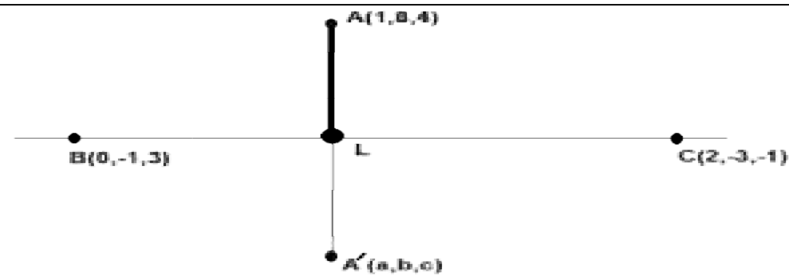
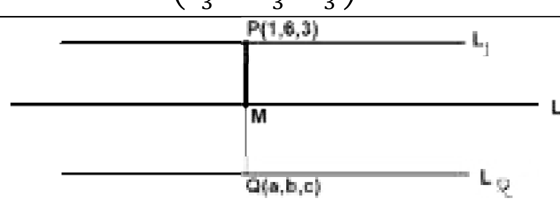
23.	Given: $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ $\Rightarrow \frac{dy}{dx} = \frac{e^x}{e^y} + \frac{x^2}{e^y}$ $\Rightarrow e^y \; dy = (e^x + x^2)\, dx$ $\Rightarrow \int e^y \; dy = \int (e^x + x^2)\, dx$ $\Rightarrow e^y = e^x + \frac{x^3}{3} + C$	$\frac{1}{2}$ 1 $\frac{1}{2}$
24.	Given that: $\frac{dy}{dx} = ye^x$ and x=0, y=e $\Rightarrow \frac{1}{y} \; dy = e^x \; dx$ (variables separation) $\Rightarrow \int \frac{1}{y} \; dy = \int e^x \; dx$ $\Rightarrow \log y = e^x + C$……….(i) But $y = e$ when $x = 0$, (i) gives $\log e = e^0 + C \Rightarrow 1 = 1 + C \Rightarrow C = 0$ From (i), now $\log y = e^x$ When $x = 1$, $\log y = e^1 = e \Rightarrow y = e^e$	$\frac{1}{2}$ $\frac{1}{2}$ 1
25	Given lines: $\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k} + \lambda(2\hat{i} + \hat{j} + 2\hat{k})$ and $\vec{r} = 2\hat{j} - 5\hat{k} + \mu(6\hat{i} + 3\hat{j} + 2\hat{k})$ Direction ratios of the line are $\langle 2,1,2 \rangle$ and $\langle 6,3,2 \rangle$ If θ ia an angle between the lines, then $\cos\theta = \frac{2.6 + 1.3 + 2.2}{\sqrt{2^2 + 1^2 + 2^2}\sqrt{6^2 + 3^2 + 2^2}} = \frac{19}{\sqrt{9}\sqrt{49}} = \frac{19}{21}$ $\therefore \theta = \cos^{-1}\left(\frac{19}{21}\right)$	1 1
	SECTION C **(Each question carries 3 marks)**	
26	 Let the parallelogram be ABCD with $\overrightarrow{AB} = \overrightarrow{DC} = 2\,\hat{i} - 4\,\hat{j} - 5\,\hat{k}$ and $\overrightarrow{AB} = \overrightarrow{BC} = 2\,\hat{i} + 2\,\hat{j} + 3\,\hat{k}$ Now, $\overrightarrow{d_1} = \overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = (2\,\hat{i} - 4\,\hat{j} - 5\,\hat{k}) + (2\,\hat{i} + 2\,\hat{j} + 3\,\hat{k}) = 4\,\hat{i} - 2\,\hat{j} - 2\,\hat{k}$ $\overrightarrow{d_2} = \overrightarrow{BD} = -\overrightarrow{AB} + \overrightarrow{AD} = -(2\,\hat{i} - 4\,\hat{j} - 5\,\hat{k}) + (2\,\hat{i} + 2\,\hat{j} + 3\,\hat{k}) = 0\,\hat{i} + 6\,\hat{j} + 8\,\hat{k}$ Also, $\lvert\overrightarrow{d_1}\rvert = \sqrt{4^2 + 2^2 + (-2)^2} = \sqrt{24}$ and $\lvert\overrightarrow{d_2}\rvert = \sqrt{0^2 + 6^2 + 8^2} = \sqrt{100} = 10$	1 1 1

	Thus, $\widehat{d_1} = \frac{\overrightarrow{d_1}}{\lvert\overrightarrow{d_1}\rvert} = \frac{4\,\hat{i}-2\,\hat{j}-2\,\hat{k}}{\sqrt{24}}$ and $\widehat{d_2} = \frac{\overrightarrow{d_2}}{\lvert\overrightarrow{d_2}\rvert} = \frac{6\,\hat{j}+8\,\hat{k}}{10}$ Area of parallelogram= $\frac{1}{2}\lvert\overrightarrow{d_1} \times \overrightarrow{d_2}\rvert = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & -2 \\ 0 & 6 & 8 \end{vmatrix} =$ $\frac{1}{2}\lvert -4\,\hat{i} - 32\,\hat{j} + 24\,\hat{k}\rvert = \frac{1}{2}\sqrt{1616}$	
26. (OR)	Given : A, B, C, D are the points with position vectors $\hat{i} + \hat{j} - \hat{k}$, $2\,\hat{i} - \hat{j} + 3\,\hat{k}$, $2\,\hat{i} - 3\,\hat{k}$ and $3\,\hat{i} - 2\,\hat{j} + \hat{k}$ respectively. Now, $\overrightarrow{AB} = p.v.of\ B - p.v.of\ A = (2\,\hat{i} - \hat{j} + 3\,\hat{k}) - (\hat{i} + \hat{j} - \hat{k}) = \hat{i} - 2\,\hat{j} + 4\,\hat{k}$ $\overrightarrow{CD} = p.v.of\ D - p.v.of\ C = (3\,\hat{i} - 2\,\hat{j} + \hat{k}) - (2\,\hat{i} - 3\,\hat{k}) = \hat{i} - 2\,\hat{j} + 4\,\hat{k}$ Hence, projection of $\overrightarrow{AB}$ along $\overrightarrow{CD} = \frac{\overrightarrow{AB}\,.\,\overrightarrow{CD}}{\lvert\overrightarrow{CD}\rvert} = \frac{1.1+(-2).(-2)+4.4}{\sqrt{1^2+(-2)^2+4^2}} = \frac{21}{\sqrt{21}} = \sqrt{21}$ sq.units	1 2
27.	Given : $\alpha = 60^0, \beta = 45^0$ We know, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ $\Rightarrow \cos^2 60^0 + \cos^2 45^0 + \cos^2\gamma = 1$ $\Rightarrow \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2\gamma = 1$ $\Rightarrow \cos^2\gamma = 1 - \frac{1}{4} - \frac{1}{2} = \frac{1}{4}$ $\therefore\ \cos\gamma = \frac{1}{2} \Rightarrow\ \gamma = 60^0$ $\therefore\ l = \cos 60^0 = \frac{1}{2}, m = \cos 45^0 = \frac{1}{\sqrt{2}}, n = \cos 45^0 = \frac{1}{2}$ $\therefore\ \overrightarrow{OA} = \lvert\overrightarrow{OA}\rvert(l\hat{i} + m\hat{j} + n\hat{k})$ $= 10\left(\frac{1}{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{2}\hat{k}\right) = 5\hat{i} + 5\sqrt{2}\hat{j} + 5\hat{k}$	1 1 1
28	 Diagonals are OE,AF,BG,CD. Direction ratios of OE are $\langle a-0, a-0, a-0\rangle$ $i.e.\langle a, a, a\rangle$ $\therefore$ direction cosines of OE are	1 1

	$\langle \frac{a}{\sqrt{a^2+a^2+a^2}}, \frac{a}{\sqrt{a^2+a^2+a^2}}, \frac{a}{\sqrt{a^2+a^2+a^2}} \rangle$ i.e. $\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$ Similarly, direction cosines of AF,BG,CD are $\langle \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle, \langle \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle, \langle \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$ respectively Let α be the angle between the two diagonals OE& AF. We have, $cos\alpha = l_1 l_2 + m_1 m_2 + n_1 n_2$ $= \frac{1}{\sqrt{3}}.\frac{-1}{\sqrt{3}} + \frac{1}{\sqrt{3}}.\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}.\frac{1}{\sqrt{3}} = \frac{-1}{3} + \frac{1}{3} + \frac{1}{3}$ $= \frac{1}{3}$ $\therefore \alpha = \cos^{-1}\left(\frac{1}{3}\right)$ Similarly ,we can prove that angle between any two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$	1
29.	Given, $\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + xdy = 0; y = \frac{\pi}{4}$ when $x = 1$ $\Rightarrow \frac{\left[x\sin^2\left(\frac{y}{x}\right) - y\right]}{x} + \frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \sin^2\left(\frac{y}{x}\right)$(It is homogeneous differential equation) Putting $y = vx$ *and* $\frac{dy}{dx} = v + x\frac{dv}{dx}$ $\Rightarrow v + x\frac{dv}{dx} = v - \sin^2 v$ $\Rightarrow x\frac{dv}{dx} = -\sin^2 v$ $\Rightarrow \frac{dv}{\sin^2 v} = -\frac{dx}{x}$ Integrate on both sides $\Rightarrow \int \text{cosec}^2 v\, dv = -\int \frac{1}{x}\, dx$ $\Rightarrow -\cot v = -\log x + C$ $\Rightarrow \log x - \cot\left(\frac{y}{x}\right) = C \ldots\ldots\ldots (i)$ $y = \frac{\pi}{4}$ when $x = 1$, (i)$\Rightarrow \log 1 - \cot\left(\frac{\frac{\pi}{4}}{1}\right) = C \Rightarrow 0 - 1 = C \Rightarrow C = -1$ Hence, the reqd. particular solution is,$\log x - \cot\left(\frac{y}{x}\right) = -1$	1 1 1
30.	Given:$(1 + x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$; $y = 0$ when $x = 1$ $\Rightarrow \frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{1}{(1+x^2)^2}$,which is in the form of $\frac{dy}{dx} + Py = Q$ i.e. linear D.E. in y. Now,$I.F = e^{\int P\,dx} = e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = (1 + x^2)$ Hence,the solution of the D.E.:$y.(I.F) = \int Q(I.F)dx + C$	1 1

	$\therefore y.(1+x^2) = \int \frac{1}{(1+x^2)^2}.(1+x^2)dx + C$ $= \int \frac{1}{(1+x^2)}dx + C$ $\therefore y.(1+x^2) = \tan^{-1}x + C$ ……(i) By question $y = 0$ when $x = 1$,(i) $\Rightarrow 0.(1+1^2) = \tan^{-1}1 + C \Rightarrow 0 = \frac{\pi}{4} + C \Rightarrow C = -\frac{\pi}{4}$ Hence, the reqd. particular solution is, $y.(1+x^2) = \tan^{-1}x - \frac{\pi}{4}$	1
31.	Given, projection of $\vec{b}$ along $\vec{a}$ = the projection of $\vec{c}$ along $\vec{a}$ $\Rightarrow \frac{\vec{b}.\vec{a}}{\lvert\vec{b}\rvert} = \frac{\vec{c}.\vec{a}}{\lvert\vec{b}\rvert}$ $\Rightarrow \vec{b}.\vec{a} = \vec{c}.\vec{a}$ ....(i) Also, given $\vec{b} \perp \vec{c} \Rightarrow \vec{b}.\vec{c} = 0$ ……(ii) Then, $\lvert 3\vec{a} - 2\vec{b} + 2\vec{c}\rvert^2 = 9\lvert\vec{a}\rvert^2 + 4\lvert\vec{b}\rvert^2 + 4\lvert\vec{c}\rvert^2 - 12\,\vec{a}.\vec{b} - 8\vec{b}.\vec{c} + 12\vec{c}.\vec{a}$ $= 9.1^2 + 4.2^2 + 4.3^2 - 12\,\vec{a}.\vec{b} - 0 + 12\,\vec{a}.\vec{b}$, (by(i) & (ii)) $= 9 + 16 + 36 = 31$	1 1 1
31. (OR)	Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$. According to question, $\lvert\vec{a} \times \hat{p}\rvert = \sqrt{2}$, where $\hat{p} = \frac{\vec{b}+\vec{c}}{\lvert\vec{b}+\vec{c}\rvert} = \frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+6^2+(-2)^2}} = \frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}} =$ $\therefore \frac{1}{\sqrt{\lambda^2+4\lambda+44}} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2+\lambda & 6 & -2 \end{vmatrix} = \sqrt{2}$ $\Rightarrow \lvert(-2-6)\hat{i} - \{-2-(2+\lambda)\}\hat{j} + \{6-(2+\lambda)\}\hat{k}\rvert$ $= \sqrt{2}\sqrt{\lambda^2+4\lambda+44}$ $\Rightarrow \lvert -8\hat{i} + (4+\lambda)\hat{j} + (4-\lambda)\hat{k}\rvert = \sqrt{2}\sqrt{\lambda^2+4\lambda+44}$ $\Rightarrow \sqrt{(-8)^2+(4+\lambda)^2+(4-\lambda)^2} = \sqrt{2}\sqrt{\lambda^2+4\lambda+44}$ $\Rightarrow \lambda = 1$ (after squaring on both sides)	1 1 1
	SECTION D (Each question carries 5 marks)	

32.	 Here, by triagle law of vector addition $\vec{a}+\vec{b}=-\vec{c}$ ……….(i) By pre cross multiplication of (i) by $\vec{a}$, we get $\vec{a}\times\vec{a}+\vec{a}\times\vec{b}=-\vec{a}\times\vec{c}$ $\Rightarrow\vec{a}\times\vec{b}=\vec{c}\times\vec{a}$………(ii) By post cross multiplication of (i) by $\vec{b}$, we get $\vec{a}\times\vec{b}+\vec{b}\times\vec{b}=-\vec{c}\times\vec{b}$ $\Rightarrow\vec{a}\times\vec{b}=\vec{b}\times\vec{c}$………(iii) From (ii) and (iii) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$ $\Rightarrow\lvert\vec{a}\times\vec{b}\rvert=\lvert\vec{b}\times\vec{c}\rvert=\lvert\vec{c}\times\vec{a}\rvert$ $\Rightarrow\lvert\vec{a}\rvert\lvert\vec{b}\rvert\sin(\pi-C)=\lvert\vec{b}\rvert\lvert\vec{c}\rvert\sin(\pi-A)=\lvert\vec{c}\rvert\lvert\vec{a}\rvert\sin(\pi-B)$ $\Rightarrow ab\sin C=bc\sin A=ca\sin B$ Dividing by abc, we get $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{si}{c}$	0.5 0.5 1 1 2
32.(OR)	Here, $\lvert\vec{a}+\vec{b}\rvert^2=\lvert\vec{a}\rvert^2+\lvert\vec{b}\rvert^2+2\,\vec{a}.\vec{b}$ $=1^2+1^2+2\,\lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\Theta$ $=1^2+1^2+2.1.1.\cos\Theta$ $=2+2\cos\Theta=2(1+\cos\Theta)=2.2\cos^2\frac{\theta}{2}=4\cos^2\frac{\theta}{2}$ $\therefore\ \lvert\vec{a}+\vec{b}\rvert^2=4\cos^2\frac{\theta}{2}$ $\Rightarrow\lvert\vec{a}+\vec{b}\rvert=2\cos\frac{\theta}{2}$…….(i) And , $\lvert\vec{a}-\vec{b}\rvert^2=\lvert\vec{a}\rvert^2+\lvert\vec{b}\rvert^2-2\,\vec{a}.\vec{b}$ $=1^2+1^2-2\,\lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\Theta$ $=1^2+1^2-2.1.1.\cos\Theta$ $=2-2\cos\Theta=2(1-\cos\Theta)=2.2\sin^2\frac{\theta}{2}=4\sin^2\frac{\theta}{2}$ $\therefore\ \lvert\vec{a}-\vec{b}\rvert^2=4\sin^2\frac{\theta}{2}$ $\Rightarrow\lvert\vec{a}-\vec{b}\rvert=2\sin\frac{\theta}{2}$…….(ii) Dividing (ii) by (i)	2 2 1

	$\tan\frac{\theta}{2} = \frac{\lvert\vec{a} - \vec{b}\rvert}{\lvert\vec{a} + \vec{b}\rvert}$	
33.	 Equation of BC: $\frac{x-0}{2-0} = \frac{y+1}{-3+1} = \frac{z-3}{-1-3}$ i.e. $\frac{x-0}{2} = \frac{y+1}{-2} = \frac{z-3}{-4} = \lambda$, with dirction ratios $\langle 2, -2, -4\rangle$ Now , coordinates of L point on the line BC are $(2\lambda,\ -2\lambda - 1,\ -4\lambda + 3)$. Thus, direction ratios of AL are $\langle 2\lambda - 1, -2\lambda - 1 - 8,\ -4\lambda + 3 - 4\rangle$ i.e. $\langle 2\lambda - 1, -2\lambda - 9,\ -4\lambda - 1\rangle$ L is the foot of the perpendicular drawn from A on BC. Therefore, AL is perpendicular to BC. So, we have $2(2\lambda - 1) + (-2)(-2\lambda - 9) + (-4)(-4\lambda - 1) = 0$. $\Rightarrow 4\lambda - 2 + 4\lambda + 18 + 16\lambda + 4 = 0$ $\Rightarrow 24\lambda + 20 = 0 \Rightarrow \lambda = \frac{-5}{6}$ Hence , $L = (2\lambda, -2\lambda - 1, -4\lambda + 3) = \left(2\left(\frac{-5}{6}\right),\ -2\left(\frac{-5}{6}\right) - 1, -4\left(\frac{-5}{6}\right) + 3\right) = \left(\frac{-5}{3}, \frac{2}{3}, \frac{19}{3}\right)$ The length of the perpendicular $AL = \sqrt{\left(1 + \frac{5}{3}\right)^2 + \left(8 - \frac{2}{3}\right)^2 + \left(4 - \frac{19}{3}\right)^2} = \frac{\sqrt{597}}{3}$ If $A'(a, b, c)$ be the image of A(1,8,4) in the line through B and C, then L is the mid pint of AA' Therefore, $\frac{-5}{3} = \frac{1+a}{2}, \frac{2}{3} = \frac{8+b}{2}, \frac{19}{3} = \frac{4+c}{2}$ $\Rightarrow a = \frac{-13}{3},\ b = \frac{-20}{3},\ c = \frac{26}{3}$ Hence, $A' = \left(\frac{-13}{3}, \frac{-20}{3}, \frac{26}{3}\right)$	1 1 1 1 1
33.(OR)		1

	L: $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$, with direction ratios $\langle 1,2,3 \rangle$ Coordinates of any point M on the L i.e. M= $(\lambda, 2\lambda+1, 3\lambda+2)$. Now, direction ratios of line PM are $\langle \lambda-1, 2\lambda+1-6, 3\lambda+2-3 \rangle$ i.e.$\langle \lambda-1, 2\lambda-5, 3\lambda-1 \rangle$ If M is the foot of the perpendicular drawn from P on the line L. Then PM is perpendicular to L $\therefore 1(\lambda-1)+2(2\lambda-5)+3(3\lambda-1)=0$ $\Rightarrow 14\lambda - 14 = 0 \Rightarrow \lambda = 1$ Hence, M= $(\lambda, 2\lambda+1, 3\lambda+2) = (1, 3, 5)$ Let Q(a, b, c) be the image of P(1, 6, 3) in the line L (but on the line L_2). Then, M is the mid-point of PQ (as object distance from the mirror is equal to the image distance from the mirror) Therefore, $1 = \frac{1+a}{2}, 3 = \frac{6+b}{2}, 5 = \frac{3+c}{2}$ $\Rightarrow$ a= 1, $b = 0$, $c = 7$ Thus , a point on the line L_2 is Q(1,0,7) Hence the equation of the line L_2 is $\frac{x-1}{1} = \frac{y}{2} = \frac{z-7}{3}$(since the lines are parallel , directions ratios are remain same)	1 1 1 1
34.	Given lines are L_1: $\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ $= \hat{i} - t\hat{i} + t\hat{j} - 2\hat{j} + 3\hat{k} - 2t\hat{k}$ $= (\hat{i} - 2\hat{j} + 3\hat{k}) - t(\hat{i} - \hat{j} + 2\hat{k})$. L_2: $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$. $= s\hat{i} + \hat{i} + 2s\hat{j} - \hat{j} - 2s\hat{k} - \hat{k}$. $= (\hat{i} - \hat{j} - \hat{k}) + s(\hat{i} + 2\hat{j} - 2\hat{k})$ Now, $\overrightarrow{a_2} - \overrightarrow{a_1} = (\hat{i} - \hat{j} - \hat{k}) - (\hat{i} - 2\hat{j} + 3\hat{k}) = 0\hat{i} + \hat{j} - 4\hat{k}$ Also, $\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 1 & 2 & -2 \end{vmatrix} = \hat{i}(2-4) - \hat{j}(-2-2) + \hat{k}(2+1)$ $= -2\hat{i} + 4\hat{j} + 3\hat{k}$. Hence, shortest distance between two lines L_1 and L_2= $\left\| \frac{(\overrightarrow{a_2} - \overrightarrow{a_1}).(\overrightarrow{b_1} \times \overrightarrow{b_2})}{\|\overrightarrow{b_1} \times \overrightarrow{b_2}\|} \right\|$ $= \left\| \frac{0.(-2)+1.4+(-4).3}{\sqrt{(-2)^2+4^2+3^2}} \right\| = \frac{8}{\sqrt{29}}$ units.	1 1 1 2

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Given: $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$

$$\Rightarrow y + x\frac{dy}{dx} + y = x(\sin x + \log x)$$

$$\Rightarrow x\frac{dy}{dx} + 2y = x(\sin x + \log x)$$

$\Rightarrow \frac{dy}{dx} + \frac{2}{x}y = \sin x + \log x$ which is a linear D.E. in the form $\frac{dy}{dx} + Py = Q$

Now, $I.F = e^{\int P\,dx} = e^{\int \frac{2}{x}dx} = e^{2lo} \quad = e^{\log x^2} = x^2$.

Hence, the solution of the D.E.: $y.(I.F) = \int Q(I.F)dx + C$

$\therefore y.x^2 = \int x^2(\sin x + \log x)dx + C$.

$$\therefore y.x^2 = \int x^2 \sin x\, dx + \int x^2 \log x\, dx + C$$

$$= x^2(-\cos x) - \int \{2x\,(-\cos x)\}\,dx + \log x\,.\frac{x^3}{3} - \int \left\{\frac{1}{x}.\frac{x^3}{3}\right\}dx + C$$

$$= -x^2\cos x + 2\int x\,.\cos x\,dx + \frac{x^3}{3}.\log x - \frac{1}{3}\int x^2 dx + C$$

$$= -x^2\cos x + 2\left[x.\sin x - \int 1.\sin x\,dx\right] + \frac{x^3}{3}.\log x - \frac{1}{3}.\frac{x^3}{3} + C$$

$$= -x^2\cos x + 2[x.\sin x - (-\cos x)] + \frac{x^3}{3}.\log x - \frac{x^3}{9} + C$$

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<u>Section –E</u>

(This section comprises solution of 3 case- study/passage-based questions of 4 marks each with two sub parts. Solution of the first two case study questions have three sub parts (i),(ii),(iii) of marks 1,1,2 respectively. Solution of the third case study question has two sub parts of 2 marks each.)

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Corners of the roof are $P_1(6,8,4)$, $P_2(21,8,4)$, $P_3(21,16,10)$ and $P_4(6,16,10)$.

(i) Here, $\vec{A}$ = PV of P_2 − PV of P_1 = $(21\hat{i} + 8\hat{j} + 4\hat{k}) - (6\hat{i} + 8\hat{j} + 4\hat{k}) = 15\hat{i} + 0\hat{j} + 0\hat{k}$

and $\vec{B}$ = PV of P_4 − PV of P_1 = $(6\hat{i} + 16\hat{j} + 10\hat{k}) - (6\hat{i} + 8\hat{j} + 4\hat{k}) = 0\hat{i} + 8\hat{j} + 6\hat{k}$

Therefore, components of $\vec{A}$ and $\vec{B}$ are 15 , 0 , 0 and 0 , 8 , 6

(ii) $\vec{N} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 15 & 0 & 0 \\ 0 & 8 & 6 \end{vmatrix} = \hat{i}(0-0) - \hat{j}(90-0) + \hat{k}(120-0) = 0\hat{i} - 90\hat{j} + 120\hat{k}$

(iii) Given: $\vec{F} = 6\hat{i} - 2\hat{j} + 3\hat{k}$

Now, $\vec{F}.\vec{N} = 6 \times 0 + 2 \times 90 + 3 \times 120 = 540$

OR

$$\cos\theta = \frac{\vec{F}\,.\,\vec{N}}{|\vec{F}||\vec{N}|} = \frac{540}{\sqrt{(6)^2 + (-2)^2 + (3)^2}\sqrt{(0)^2 + (-90)^2 + (120)^2}} = \frac{540}{7 \times 150} = \frac{18}{35}$$

$$\therefore \theta = \cos^{-1}\left(\frac{18}{35}\right)$$

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37.	Given, line for motorcycle A, $L_1: \vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ And, line for motorcycle B, $L_2: \vec{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k})$ (i) $L_1: \vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ $\Rightarrow x\hat{i} + y\hat{j} + z\hat{k} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ $\therefore x=\lambda$, $y= 2\lambda$, $z= -\lambda$ $\therefore \frac{x}{1} = \lambda, \quad \frac{y}{2} = \lambda, \quad \frac{z}{-1} = \lambda,$ $\therefore \frac{x}{1} = \frac{y}{2} = \frac{z}{-1} = \lambda$, which is the reqd. cartesian equation. (ii) $L_2: \vec{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k})$ Direction ratios are $\langle 2, 1, 1 \rangle$ $\therefore$ direction cosines of OE are $\langle \frac{2}{\sqrt{2^2+1^2+1^2}}, \frac{1}{\sqrt{2^2+1^2+1^2}}, \frac{1}{\sqrt{2^2+1^2+1^2}} \rangle$ $i.e. \langle \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \rangle$ (iii) Now, $\vec{a_2} - \vec{a_1} = (3\hat{i} + 3\hat{j}) - (0\hat{i} + 0\hat{j} + 0\hat{k}) = 3\hat{i} + 3\hat{j}$ Also, $\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{vmatrix} = \hat{i}(2+1) - \hat{j}(1+2) + \hat{k}(1-4) = 3\hat{i} - 3\hat{j} - 3\hat{k}.$ Hence, shortest distance between two lines L_1 and $L_2 = \left\| \frac{(\vec{a_2}-\vec{a_1}).(\vec{b_1}\times\vec{b_2})}{\|\vec{b_1}\times\vec{b_2}\|} \right\|$ $= \left\| \frac{3.(3)+3.(-3)+0.(-3)}{\sqrt{3^2+(-3)^2+3^2}} \right\| = 0$ units OR $\cos\theta = \frac{\vec{b_1} \cdot \vec{b_2}}{\|\vec{b_1}\|\|\vec{b_2}\|} = \frac{1.2 + 2.1 + (-1).1}{\sqrt{(1)^2 + (2)^2 + (-1)^2}\sqrt{(2)^2 + (1)^2 + (1)^2}} = \frac{3}{6} = \frac{1}{2}$ $\therefore \theta = 60^0$	1 1 2
38.	(i) Given : $\frac{dP}{dt} = \mathbf{kP}$ $\Rightarrow \frac{dP}{P} = k\,dt$ Integrate on both sides $\Rightarrow \int \frac{1}{P}\,dP = \int k\,dt$ $\Rightarrow \log P = kt + C$ $\Rightarrow \log P = kt + \log C$ $\Rightarrow \log P - \log C = kt$ $\Rightarrow \log\frac{P}{C} = kt$ $\therefore \frac{P}{C} = e^{kt}$ $\therefore P = C.e^{kt}$…………(i) (ii) According to question, at $t=0, P=1000$ In this case, (i) $\Rightarrow 1000 = C.e^0$ $\Rightarrow C = 1000$	2 2

	Hence, (i) $\Rightarrow P = 1000\ e^{kt}$……….(ii) Again, at t=1,P=2000 In this case,(ii)) $\Rightarrow 2000 = 1000\ e^{k}$ $\Rightarrow 2 = e^{k} \therefore k = \log 2$	