SUBJECT: MATHEMATICS (041)

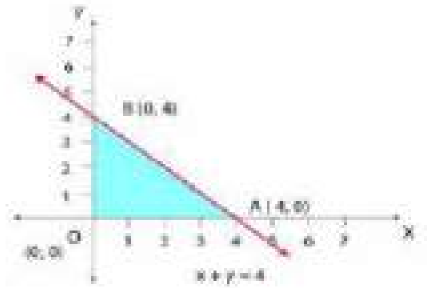
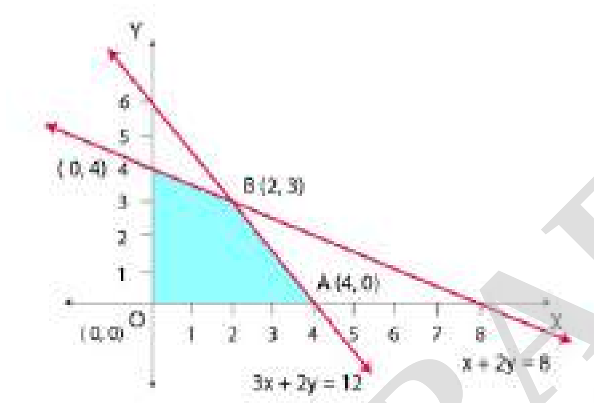
UNIT WISE PRACTICE QUESTION PAPER
(UNITS: Inverse Trigonometric functions, LPP and Probability)

Time: 3 Hours **Max. Marks: 80**

Marking Scheme

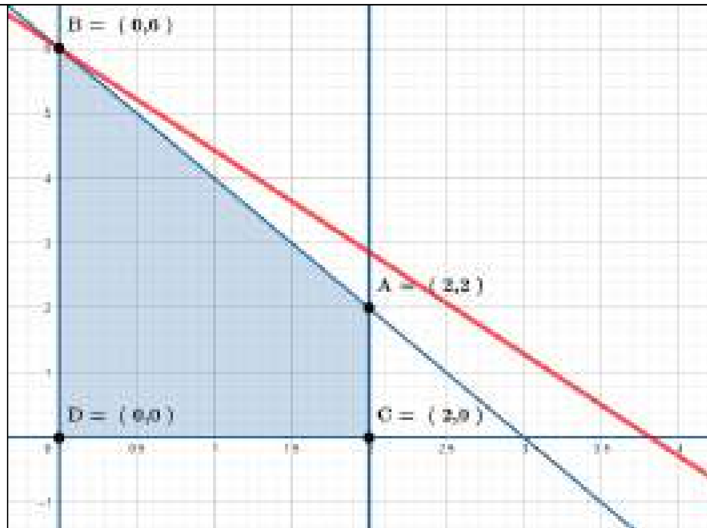
Q.NO	ANSWER	MARKS
1	(B) $\frac{5\pi}{6}$	1
2	(B) $\frac{-\pi}{2}$	1
3	**(D)** $\frac{\sqrt{1-x^2}}{x}$	1
4	(C) $\frac{5\pi}{6}$	1
5	(B) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$	1
6	(A) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	1
7	(A)[1,2]	1
8	**(C) Z is maximum at (40,15),minimum at (15,20)**	1
9	**(C) x + y ≤ 4, x ≥ 0, y ≥ 0**	1
10	**(A)All possible solutions satisfying all the constraints of the problems exist.**	1
11	(D) Infinite	1
12	(B) p=$\frac{q}{2}$	1
13	D) at every point of the line segment joining points (3,0) and (0.6,1.6)	1
14	**A) B) A and B are not independent events**	1
15	(C)$\frac{7}{8}$	1
16	(C)$\frac{1}{12}$	1
17	(A)$\frac{1}{36}$	1
18	D) $\frac{4}{7}$	1
19	**(A). Both (A) and (R) are true and (R) is the correct explanation**	1

	of (A).	
20	**(A). Both (A) and (R) are true and (R) is the correct explanation of (A)**	1
21	$\tan^{-1}\frac{1-sin\theta}{cos\theta} = \tan^{-1}\left(\frac{1-\cos\left(\frac{\pi}{2}-\theta\right)}{\sin\left(\frac{\pi}{2}-\theta\right)}\right)$ $= \tan^{-1}\left(\frac{2\sin^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}{2\sin\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}\right)$ $= \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\right) = \frac{\pi}{4}-\frac{\theta}{2}$	1 1
22	$\mathbf{\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)}$ $=\frac{\pi}{4} + (\pi - \frac{\pi}{3}) + \left(-\frac{\pi}{6}\right)$ $=\frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$ $=\frac{3\pi + 8\pi - 2\pi}{12}$ $=\frac{9\pi}{12}$ $=\frac{3\pi}{4}$	1 1
23	**Given: y=** $\mathbf{\sin^{-1}(x^2-4)}$ $\Rightarrow -1 \leq x^2 - 4 \leq 1$ $\Rightarrow 3 \leq x^2 \leq 5$ $\Rightarrow x^2 \geq 3 \; and \; x^2 \leq 5$ $\Rightarrow$ **x** $\leq -\sqrt{3}$ **, x** $\geq \sqrt{3}$ **and** $-\sqrt{5} \leq x \leq \sqrt{5}$ **The domain of y is** $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$	1 $\frac{1}{2}$ $\frac{1}{2}$
23(OR)	**Given: f(x) = 2** $\mathbf{\sin^{-1} x + \frac{3\pi}{2}}$ $-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$ $\Rightarrow -\pi \leq 2\sin^{-1} x \leq \pi$ $\Rightarrow -\pi + \frac{3\pi}{2} \leq 2\sin^{-1} x + \frac{3\pi}{2} \leq \pi + \frac{3\pi}{2}$ $\Rightarrow -\frac{\pi}{2} \leq 2\sin^{-1} x + \frac{3\pi}{2} \leq \frac{5\pi}{2}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

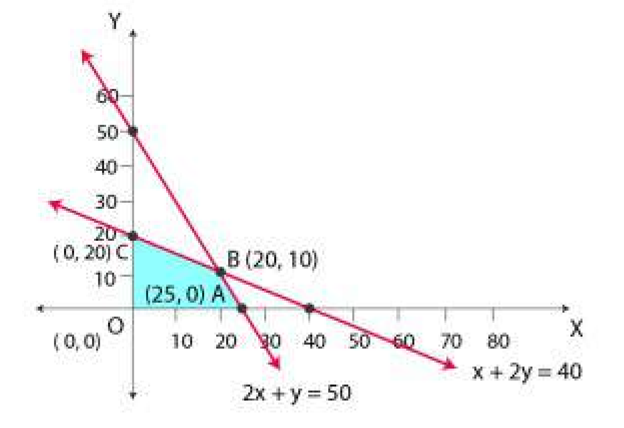
	The range of f(x) is $\left[-\frac{\pi}{2},\frac{5\pi}{2}\right]$	$\frac{1}{2}$
24	 **The corner points are O(0,0),A(4,0) and B(0,4)** **Maximum Value of Z is 16 attained at B(0,4)**	For figure-1 1
24 (OR)	 **The corner points are O(0,0),A(4,0) ,B(2,3) and C(0,4)** **Minimum Value of Z is -12 attained at A(4,0)**	For figure-1 1
25	We have P(A)=$\frac{18}{36}=\frac{1}{2}$ P(B)=$\frac{18}{36}=\frac{1}{2}$ Also P($A\cap B$)=P(Odd number on both throws) $=\frac{9}{36}=\frac{1}{4}$ Now P(A)P(B)=$\frac{1}{2}.\frac{1}{2}=\frac{1}{4}$ P($A\cap B$)= P(A)P(B) A and B are independent events	$\frac{1}{2}$ $\frac{1}{2}$ 1
26	**Given**: $\sin^{-1}(1-x)$-2$\sin^{-1}x=\frac{\pi}{2}$ $\Rightarrow \sin^{-1}(1-x)=\frac{\pi}{2}+2\sin^{-1}x$ ⇒1-x=$\sin\left(\frac{\pi}{2}+2\sin^{-1}x\right)$ ⇒1-x=$\cos(2\sin^{-1}x)$………(i)	1

	Let , $\sin^{-1} x = \theta$ $\Rightarrow$x=sinθ From(i),we get 1-x=cos2θ $\Rightarrow$1-x=1-2$sin^2\theta$ $\Rightarrow$1-x=1-2x^2 $\Rightarrow$ $2x^2$-x= 0$\Rightarrow$ $x(2x-1)=0$ $\Rightarrow$ x=0 or x=$\frac{1}{2}$ Putting x = $\frac{1}{2}$ in the given equation ,we get $\sin^{-1}\left(\frac{1}{2}\right)-2\sin^{-1}\frac{1}{2}$ =$\frac{\pi}{6}$ - 2$\times\frac{\pi}{6} \neq \frac{\pi}{2}$ So,$x \neq \frac{1}{2}$ $\therefore$x=0	$\frac{1}{2}$ 1 $\frac{1}{2}$
27	To show that: $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)= \frac{4-\sqrt{7}}{3}$ Let $\sin^{-1}\frac{3}{4}=\theta$ $\Rightarrow$ sinθ $=\frac{3}{4}$ $\Rightarrow \frac{2\tan\frac{\theta}{2}}{1+tan^2\frac{\theta}{2}}= \frac{3}{4}$ $\Rightarrow$3+3$tan^2\frac{\theta}{2}$=8$\tan\frac{\theta}{2}$ $\Rightarrow$3$tan^2\frac{\theta}{2}$-8$\tan\frac{\theta}{2}$ + 3=0 $\Rightarrow \tan\frac{\theta}{2} = \frac{8\pm\sqrt{64-36}}{6}$ $\Rightarrow \tan\frac{\theta}{2} = \frac{8\pm2\sqrt{7}}{6}$ $\Rightarrow \tan\frac{\theta}{2} = \frac{4\pm\sqrt{7}}{3}$ $\therefore$ $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)= \frac{4-\sqrt{7}}{3}$	1 1 1
28	Total number of questions=1400 n(S)=1400 Let E=Selected questions is easy F= Selected questions is M.C.Q E∩F= Selected questions is Easy and M.C.Q n($\mathbf{E} \cap \mathbf{F}$)=500+400=900	1

	$\therefore P(\mathbf{E} \cap \mathbf{F}) = \frac{n(E\cap F)}{n(S)} = \frac{500}{1400}$ $\therefore P(\mathbf{F}) = \frac{n(F)}{n(S)} = \frac{900}{1400}$ Required Probability=P(E/F) = $\frac{P(E\cap F)}{P(S)} = \frac{5}{9}$	1 1
28 (OR)	P(A) = $\frac{2}{7}$ i.e A coming on time, P(B) = $\frac{4}{7}$ i.e B coming on time, $P(\bar{A}) = 1- \frac{2}{7} = \frac{5}{7}$ $P(\bar{B}) = 1- \frac{4}{7} = \frac{3}{7}$ ∴Probability of only one of them coming to school on time $=P(A)\,P(\bar{B}) + P(\bar{A})\,P(B)$ $=\frac{2}{7}\times\frac{3}{7} + \frac{5}{7}\times\frac{4}{7}$ $=\frac{26}{49}$	1 1 1
29	We know that ,A∪ B denotes the occurrence of atleast one of A and B and A∩B denotes the occurrence of both A and B simultaneously Thus,$P(A \cup B) = 0.6$ and $P(A \cap B) = 0.3$ $\therefore P(A \cup B) = 0.6$ $\Rightarrow P(A) + P(B) - P(A \cap B)=0.6$ $\Rightarrow P(A) + P(B) =0.6 + 0.3 =0.9$ $\Rightarrow [1 - P(\bar{A})] + [1 - P(\bar{B})] = 0.9$ $\Rightarrow P(\bar{A}) + P(\bar{B}) = 2-0.9=1.1$	$1\frac{1}{2}$ $1\frac{1}{2}$
29 (OR)	We have P(atleast one of A and B) = P(A∪B) $= P(A) + P(B) - P(A \cap B)$ $=P(A) + P(B) - P(A)P(B)$ $=P(A) + P(B)[1 - P(A)]$ $=P(A) + P(B)P(\bar{A})$ $=1-P(\bar{A}) + P(B)P(\bar{A})$ $=1-P(\bar{A})[1 - P(B)]$ $=1- P(\bar{A})P(\bar{B})$	1 1 1

30	 The corner points are O(0,0),C(2,0),a(2,2) and B(0,6) The maximum value of Z is 42 and which attained at B(0,6)	For graph 2 1
31	When two dice are thrown together, the number of possible outcomes=36 Given E:event of outcomes whose total score is 4 ={(1,3)(2,2)(3,1)} F: event of outcomes whose total score is 9 or more {(3,6)(4,5),(4,6)(5,4),(5,5),(5,6),(6,3),(6,4),(6,5)} G: event of outcomes whose total score is divisible by 5 {(1,4)(2,3),(3,2),(4,1),(2,3),(3,2),(4,1),(4,6),(5,5),(6,4)} : P(E) $= \frac{1}{12}$, P(F) = $\frac{1}{4}$, P(G) = $\frac{5}{18}$ E∩ $\boldsymbol{F}$={}, E∩ $\boldsymbol{G}$={},F∩G={(4,6),(5,5),(6,4)} P(E∩ F) = 0 P(E)XP(F)=$\frac{1}{48}$ P(E∩ F) ≠P(E)XP(F) E and F are not independent P(E∩ G) = 0 P(E)×P(G)=$\frac{1}{12} \times \frac{5}{18} = \frac{5}{216}$ ∴ P(E∩ G) ≠P(E)×P(G) E and G are not independent	1 1 1

	$P(F\cap G)=\frac{1}{12}$ $P(F)\times P(G)=\frac{1}{4}\times\frac{5}{18}=\frac{5}{72}$ $\therefore P(F\cap G)\neq P(F)\times P(G)$ F and G are not independent No pairs are independent	1
31(or)	Probability that A hits the target,P(A)= $\frac{1}{3}$ Probability that B hits the target,P(B)= $\frac{2}{5}$ Probability that A does not hit the target,$P(\bar{A})=1-\frac{1}{3}=\frac{2}{3}$ Probability that B does not hit the target,$P(\bar{B})=1-\frac{2}{5}=\frac{3}{5}$ Probability that the target is hit=At least one of them hit the target $=1-P(\bar{A})\,P(\bar{B})$ $=1-\frac{2}{3}\times\frac{3}{5}$ $=\frac{3}{5}$	$\frac{1}{2}$ 1 $\frac{1}{2}$ 1
32	Let the first kind of cake be x and second kind of cakes be y. Hence, $x\geq 0$ and $y\geq 0$ The total number of cakes z =x+y The mathematical formulation of the given problem can be written as Maximise, z = x + y subject to the constraints,$2x + y \leq 50$,$x + 2y \leq 40$, $x, y \geq 0$	1



Graph-2

OABC represent the feasible region.

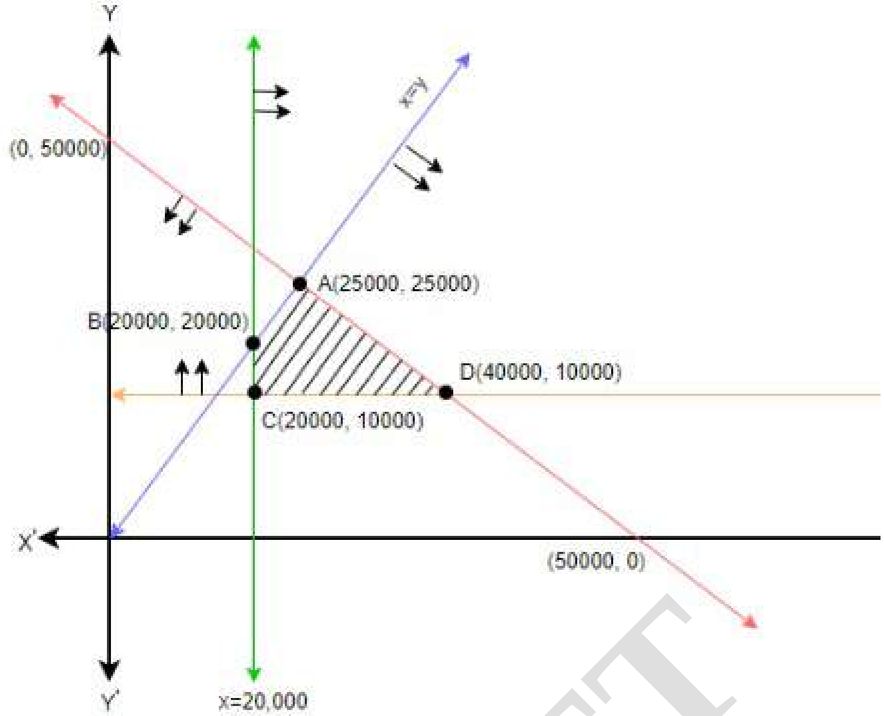
The corner points are A (25, 0), B (20, 10), O (0, 0) and C (0, 20).
We now find the value of Z at the corner points A (25, 0), B (20, 10), O (0, 0) and C (0, 20). $\frac{1}{2}$

Corner Points	Z=x+y	Conclusion
A(25,0)	25	
B(20,10)	30	
O(0,0)	0	
C(0,20)	20	

1

Hence, the maximum numbers of cakes that can be made are 30 (20 cakes of one kind and 10 cakes of other kind). $\frac{1}{2}$

33 Let the amounts invested by the person in bonds A and B are respectively Rsx and Rsy.
We now have the following mathematical model for the given problem.
Maximise, Z = 10% of x + 9% of y =0.1x+0.09y
Subject to the constraints x+y≤50,000
x≥ y ⇒ x - y≥ 0 and x≥ 20,000 , y≥ 10,000 1



Graph-2

$\frac{1}{2}$

ABCD is the feasible region ,which is bounded.

The corner points are A(25000,25000), B(20000,20000) ,C(20000,10000)and D(40000,10000)

We now find the value of Z at the corner points A(25000,25000), B(20000,20000) ,C(20000,10000)and D(40000,10000)

1

Corner Points	Z=0.1x+0.09y	Conclusion
A(25000,25000)	2500+2250=4750	
B(20000,20000)	2000+1800=3800	
C(20000,10000)	2000+900=2900	
D(40000,10000)	4000+900=4900	Maximum

$\frac{1}{2}$

So, in order to get the maximum return the man has to invest Rs. 40000 in bond A and Rs. 10000 in bond B and the maximum return will be Rs. 4900.

34 Let S denote the success(getting a 6) and F denote the failure (not getting a 6)

Thus P $(\boldsymbol{S}) = \frac{1}{6}$, P(F)= $\frac{5}{6}$ 1

P(A wins the first throw) = P $(\boldsymbol{S}) = \frac{1}{6}$

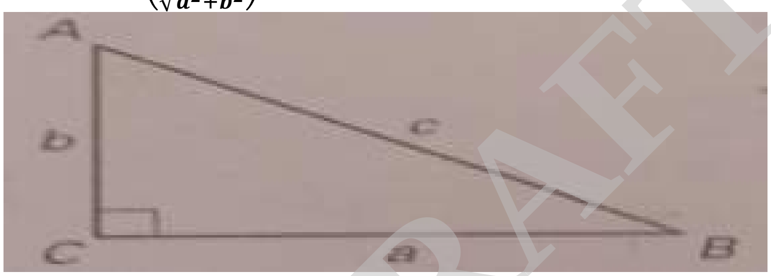
A gets the third throw ,when the first throw by A and second throw by B result into failures.

Therefore, P(A wins the third throw) =P(FFS) = P(F)P(F)P(S)= $\frac{5}{6} \times \frac{5}{6}$ 1

	$\times \frac{1}{6}$ $=\left(\frac{5}{6}\right)^2 \times \frac{1}{6}$ P(A wins the 5th throw) =P(FFFFS) = P(F)P(F)P(F)P(F)P(S) $=\left(\frac{5}{6}\right)^4 \times \frac{1}{6}$ and so on Hence P(A wins) = $\frac{1}{6} + \left(\frac{5}{6}\right)^2 \times \frac{1}{6} + \left(\frac{5}{6}\right)^4 \times \frac{1}{6} + \ldots$ $= \frac{\frac{1}{6}}{1-\frac{25}{36}}$ $= \frac{6}{11}$ P(B wins) = 1- $\frac{6}{11} = \frac{5}{11}$	1 1 1
34 (OR)	Let A, E_1, E_2, E_3 and E_4 be the events as defined below: A:a Green bulb is selected E_1:Box I is selected E_2: Box II is selected E_3: Box III is selected E_4: Box IV is selected Since the boxes are chosen at random P(E_1) = P(E_2) = P(E_3) = P(E_4) $= \frac{1}{4}$ Also, $P(A/E_1) = \frac{3}{18}$, $P(A/E_2) = \frac{2}{8}$, $P(A/E_3) = \frac{1}{7}$ and $P(A/E_4) = \frac{4}{13}$ P(box III is selected given that the drawn bulb is green)=P(E_3/A) By the Bays' Theorem $P(E_3/A) = \frac{P(E_3).P(A/E_3)}{P(E_1).P(A/E_1)+P(E_2).P(A/E_2)+P(E_3).P(A/E_3)+P(E_4).P(A/E_4)}$	1 1 1 1 1

	$= \dfrac{\frac{1}{4}\times\frac{1}{7}}{\frac{1}{4}\times\frac{3}{18}+\frac{1}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{7}+\frac{1}{4}\times\frac{4}{13}}$ = 0.165	
35	Let A, E_1 and E_2 be the events as defined below: E_1:The lost card is a spade card E_2: The lost card is not a spade card A:drawing three spade cards from the remaining cards. P(E_1) = $\frac{13}{52} = \frac{1}{4}$, P(E_2) = $\frac{39}{52} = \frac{3}{4}$ $P(A/E_1) = \frac{12_{C3}}{51_{C3}} = \frac{12\times11\times10}{51\times50\times49}$, $P(A/E_2) = \frac{13_{C3}}{51_{C3}} = \frac{13\times12\times11}{51\times50\times49}$ By the Bays' Theorem $P(E_1/A) = \frac{P(E_1).P(A/E_1)}{P(E_1).P(A/E_1)+P(E_2).P(A/E_2)}$ $= \dfrac{\frac{1}{4}\times\frac{12\times11\times10}{51\times50\times49}}{\frac{1}{4}\times\frac{12\times11\times10}{51\times50\times49}+\frac{3}{4}\times\frac{13\times12\times11}{51\times50\times49}}$ $=\frac{10}{49}$	1 1 1 1 1
35 (OR)	Let A, E_1 and E_2 be the events as defined below: E_1:Event that 6 occurs . E_2: Event that 6 does not occurs A:The man reports that 6 occurs. P(E_1) = $\frac{1}{6}$, P(E_2) $= \frac{5}{6}$ Also, $P(A/E_1) = \frac{3}{4}$, $P(A/E_2) = 1- \frac{3}{4} = \frac{1}{4}$ By the Bays' Theorem	1 1 1

	$P(\boldsymbol{E_1/A}) = \frac{P(E_1).P(A/E_1)}{P(E_1).P(A/E_1)+P(E_2).P(A/E_2)}$ $= \frac{\frac{1}{6}\times\frac{3}{4}}{\frac{1}{6}\times\frac{3}{4}+\frac{5}{6}\times\frac{1}{4}}$ $= \frac{1}{8}\times\frac{24}{8}$ $=\frac{3}{8}$	1 1
36	By the Pythagoras theorem $C=\sqrt{a^2+b^2}$ (A)Sin$\angle ABC = \frac{AC}{AB} = \frac{b}{\sqrt{a^2+b^2}}$ $\Rightarrow \angle ABC = \sin^{-1}\left(\frac{b}{\sqrt{a^2+b^2}}\right)$ $\theta = \sin^{-1}\left(\frac{b}{\sqrt{a^2+b^2}}\right)$	1



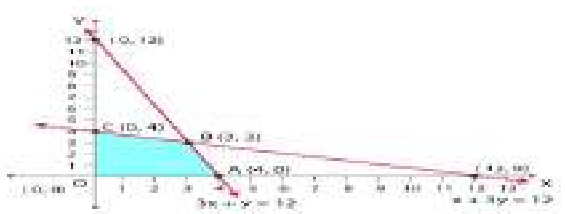
	(B) Cos$\angle ABC = \frac{BC}{AB} = \frac{a}{\sqrt{a^2+b^2}}$ $\Rightarrow \angle ABC = \cos^{-1}\left(\frac{a}{\sqrt{a^2+b^2}}\right)$ $\therefore \theta = \sin^{-1}\left(\frac{b}{\sqrt{a^2+b^2}}\right)$ (C)Given, A=5m ,b=2m $c=\sqrt{5^2+2^2}=\sqrt{29}$ $\therefore \sin\angle CAB = \frac{BC}{AB}$ $\Rightarrow \angle CAB = \sin^{-1}\frac{5}{\sqrt{29}}$ **OR** (iii)Given, a=5m ,b=2m $C=\sqrt{5^2+2^2}=\sqrt{29}$	1 2 2

	$\cos\angle CAB = \frac{AC}{AB}$ $\Rightarrow \theta = \cos^{-1}\left(\frac{2}{\sqrt{29}}\right)$	
37	(A) No. of total oranges=15 3 oranges out of 15 oranges can be arranged in 15_{C3} ways The total number of ways that can be arranged taking 3 oranges at once out of 15 oranges=15_{C3} $= \frac{15\times14\times13}{3!}$ =455 (B) No. of good oranges=12 The number of arrangement that contained only god oranges =The number of ways that can be arranged taking 3 oranges at once out of 12 oranges=12_{C3} $= \frac{12\times11\times10}{3!}$ =220 (C) The probability that the box is approved for sale=Probability that all the three oranges drawn are good. $=\frac{220}{455}$ $=\frac{44}{91}$ OR, The probability that the box is not approved for sale=Probability that all the three oranges drawn are bad one. $= 1- \frac{44}{91}$ $=\frac{47}{91}$	1 1 2 2
38	(A) The constraints are x + 3y ≤ 12(constraint related to machine-A) , 3x + y ≤ 12(constraint related to machine B) x≥0 and y≥0	1

(B) The total profit is Z = 17.5x + 7y

1

(C)



OABC Is the feasible region

The corner points are O(0,0) ,A (4, 0), B (3, 3) and C (0, 4)

We now find the value of Z at the corner points O(0,0), A (4, 0), B (3, 3),
and C (0, 4).

Graph($1\frac{1}{2}$)

Corner Points	Z=17.5x+7y	Conclusion
O(0,0)	0	
A(4,0)	70	
B(3,3)	73.5	
C(0,4)	28	

$\frac{1}{2}$

Hence, 3 packages of nuts and 3 packages of bolts should be produced each day to get the maximum profit

Graph($1\frac{1}{2}$) The graph is same as above

Or

The maximum profit Rs 73.50.

$\frac{1}{2}$