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1. Relations and functions

(Previous years questions from 2008 to 2025)

Mcq's :

1.

The function $f: R \to [-1, 1]$ defined by $f(x) = \cos x$ is

(A) both one-one and onto

(B) not one-one, but onto

(C) one-one, but not onto

(D) neither one-one, nor onto

2.

Let R be the relation in the set N given by R = {(a, b) : a = b – 2, b > 6}, then:

a) $(2,4) \in R$	b) $(3,8) \in R$
c) $(6,8) \in R$	d) $(8,7) \in R$

3.

Let A = {1, 3, 5}. Then the number of equivalence relations in A containing (1, 3) is

(A) 1

(B) 2

(C) 3

(D) 4

4.

The relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1), (1, 1)} is

(A) symmetric and transitive, but not reflexive

(B) reflexive and symmetric, but not transitive

(C) symmetric, but neither reflexive nor transitive

(D) an equivalence relation

5.

What is the range of the function $f(x) = \frac{|x-1|}{(x-1)}$?

6.

If f and g are two functions from R to R defined as $f(x) = |x| + x$ and $g(x) = |x| - x$, then fog (x) for $x < 0$ is

(A) $4x$

(B) $2x$

(C) 0

(D) $-4x$

7.

The relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1), (1, 1)} is

(A) symmetric and transitive, but not reflexive

(B) reflexive and symmetric, but not transitive

(C) symmetric, but neither reflexive nor transitive

(D) an equivalence relation

8. 2023

Let A = {3, 5}. Then number of reflexive relations on A is

(a) 2 (b) 4

(c) 0 (d) 8

9.

Let R be a relation in the set N given by

$R = \{(a, b) : a = b - 2, b > 6\}$.

Then

(a) $(8, 7) \in R$ (b) $(6, 8) \in R$

(c) $(3, 8) \in R$ (d) $(2, 4) \in R$

10.

If $f(x) = |\cos x|$, then $f\left(\frac{3\pi}{4}\right)$ is

(a) 1 (b) -1

(c) $\frac{-1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}$

11.

A relation R in set A = {1,2,3} is defined as R = {(1, 1), (1, 2), (2, 2), (3, 3)}. Which of the following ordered pair in R shall be removed to make it an equivalence relation in A?

a) (1, 1)	b) (1, 2)
c) (2, 2)	d) (3, 3)

13.

Let the relation R in the set A = $\{x \in Z : 0 \le x \le 12\}$, given by R = {(a, b) : |a – b| is a multiple of 4}. Then [1], the equivalence class containing 1, is:

a) {1, 5, 9}	b) {0, 1, 2, 5}
c) ϕ	d) A

14.

State the reason for the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} not to be transitive.

2023 march :

1.

Assertion (A) : The number of onto functions from a set P containing 5 elements to a set Q containing 2 elements is 30.

Reason (R) : Number of onto functions from a set containing m elements to a set containing n elements is n^m.

2024 March :

1.

A function $f : R_+ \to R$ (where R_+ is the set of all non-negative real numbers) defined by $f(x) = 4x + 3$ is :

(A) one-one but not onto

(B) onto but not one-one

(C) both one-one and onto

(D) neither one-one nor onto

2.

A relation R defined on set $A = \{x : x \in Z \text{ and } 0 \leq x \leq 10\}$ as $R = \{(x, y) : x = y\}$ is given to be an equivalence relation. The number of equivalence classes is :

(A) 1 (B) 2

(C) 10 (D) 11

3.

A relation R defined on a set of human beings as

$R = \{(x, y) : x \text{ is 5 cm shorter than } y\}$

is :

(A) reflexive only

(B) reflexive and transitive

(C) symmetric and transitive

(D) neither transitive, nor symmetric, nor reflexive

4.

Let R_+ denote the set of all non-negative real numbers. Then the function $f : R_+ \to R_+$ defined as $f(x) = x^2 + 1$ is :

(A) one-one but not onto (B) onto but not one-one

(C) both one-one and onto (D) neither one-one nor onto

5.

Let $f : R_+ \to [-5, \infty)$ be defined as $f(x) = 9x^2 + 6x - 5$, where R_+ is the set of all non-negative real numbers. Then, f is :

(A) one-one

(B) onto

(C) bijective

(D) neither one-one nor onto

6.

A function $f : R \to A$ defined as $f(x) = x^2 + 1$ is onto, if A is :

(A) $(-\infty, \infty)$ (B) $(1, \infty)$

(C) $[1, \infty)$ (D) $[-1, \infty)$

7.

Let Z denote the set of integers, then function $f : Z \to Z$ defined as $f(x) = x^3 - 1$ is :

(A) both one-one and onto

(B) one-one but not onto

(C) onto but not one-one

(D) neither one-one nor onto

8.

Assertion (A) : The relation $R = \{(x, y) : (x + y)$ is a prime number and $x, y \in N\}$ is not a reflexive relation.

Reason (R) : The number '2n' is composite for all natural numbers n.

9.

A function $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = x^2 - 4x + 5$ is :

(A) injective but not surjective. (B) surjective but not injective.

(C) both injective and surjective. (D) neither injective nor surjective.

10.

Which of the following statements is ***not*** true about equivalence classes A_i (i = 1, 2, ... n) formed by an equivalence relation R defined on a set A ?

(A) $\bigcup_{i=1}^{n} A_i = A$

(B) $A_i \cap A_j \neq \phi, i \neq j$

(C) $x \in A_i$ and $x \in A_j \Rightarrow A_i = A_j$

(D) All elements of A_i are related to each other, for all i

2025 March :

1.

Assertion (A) : Let Z be the set of integers. A function $f : Z \to Z$ defined as $f(x) = 3x - 5, \forall x \in Z$ is a bijective.

Reason (R) : A function is a bijective if it is both surjective and injective.

2.

For real x, let $f(x) = x^3 + 5x + 1$. Then :

(A) f is one-one but not onto on R

(B) f is onto on R but not one-one

(C) f is one-one and onto on R

(D) f is neither one-one nor onto on R

3.

If $f : N \to W$ is defined as

$$f(n) = \begin{cases} \frac{n}{2}, & \text{if n is even} \\ 0, & \text{if n is odd} \end{cases},$$

then f is :

(A) injective only

(B) surjective only

(C) a bijection

(D) neither surjective nor injective

4.

Assertion (A) : Let $A = \{x \in R : -1 \le x \le 1\}$. If $f : A \to A$ be defined as $f(x) = x^2$, then f is not an onto function.

Reason (R) : If $y = -1 \in A$, then $x = \pm\sqrt{-1} \notin A$.

5.

If R be a relation defined as aRb iff $|a - b| > 0$ $a, b \in \mathbb{R}$ then R is :

(A) reflexive (B) symmetric

(C) transitive (D) symmetric and transitive

I. Relations

2 marks :

1.

Check if the relation R in the set $\mathbb{R}$ of real numbers defined as R = {(a, b) : a < b} is (i) symmetric, (ii) transitive

Sol.

$(i)\ 1,2 \in \mathbb{R}$ such that $1<2 \Rightarrow (1,2) \in R,$

but since 2 is not less than $1 \Rightarrow (2,1) \notin R.$

Hence R is not symmetric.

(ii) Let $(a,b) \in R$ and $(b,c) \in R, \therefore a<b$ and $b<c$

$\Rightarrow a<c \Rightarrow (a,c) \in R \therefore R$ is transitive.

2.

Check if the relation R on the set $A=\{1,2,3,4,5,6\}$ defined as $R=\{(x,y): y \text{ is divisible by } x\}$ is (i) symmetric (ii) transitive.

Sol.

(i) As $(2,4) \in R$ but $(4,2) \notin R \Rightarrow R$ is not symmetric.

(ii) Let $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow b=\lambda a$ and $c=\mu b$

Now, $c=\mu b=\mu(\lambda a) \Rightarrow (a,c) \in R$

$\Rightarrow R$ is transitive.

3.

Check whether the relation R defined on the set {1, 2, 3, 4} as R = {(a, b) : b = a + 1} is transitive. Justify your answer.

3.b

Check whether the relation R defined on the set $A = \{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive.

Sol.

$R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$

For $1 \in A$, $(1, 1) \notin R \Rightarrow$ R is not reflexive

For $1, 2 \in A$, $(1, 2) \in R$ but $(2, 1) \notin R \Rightarrow$ R is not symmetric

For $1, 2, 3 \in A$, $(1, 2), (2, 3) \in R$ but $(1, 3) \notin R \Rightarrow$ R is not transitive

4.

If the relation R on the set $A = \{x : 0 \le x \le 12\}$ given by $R = \{(a, b) : a = b\}$ is an equivalence relation, then find the set of all elements related to 1.

5.a

Let the relation R be given as

$R = \{(x, y) : x, y \in N \text{ and } x + 3y = 12\}$. Find the domain and range of R.

5.b 2025

Let A = {1, 2, 3} and B = {4, 5, 6}. A relation R from A to B is defined as R = {(x, y) : x + y = 6, x ∈ A, y ∈ B}.

(i) Write all elements of R.

(ii) Is R a function ? Justify.

(iii) Determine domain and range of R.

Sol.

$(i)\, R = \{(1,5),(2,4)\}$

(ii) R is not a function as $3 \in A$ do not have an image in co-domain.

(iii) Domain of $R = \{1,2\}$, Range of $R = \{4,5\}$

5.c 2025

A student wants to pair up natural numbers in such a way that they satisfy the equation 2x + y = 41, x, y ∈ N. Find the domain and range of the relation. Check if the relation thus formed is reflexive, symmetric and transitive. Hence, state whether it is an equivalence relation or not.

Sol.

R = {(1,39), (2, 37),_ _ _ ,(20, 1)}

Domain = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}

Range ={1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39}

(1, 1) does not belong to R hence not reflexive

(1, 39) belongs to R but (39, 1) does not belong to R hence not symmetric

(11, 19) and (19, 3) belong to R but (11, 3) does not belong to R hence not transitive.
Hence R is not an equivalence relation.

6.

How many equivalence relations on the set {1,2,3} containing (1,2) and (2,1) are there in all ? Justify your answer.

7.

Let $A = \{1,2,3,4\}$. Let R be the equivalence relation on $A \times A$ defined by $(a,b)R(c,d)$ iff $a+d=b+c$. Find the equivalence class $[(1,3)]$.

5 Marks :

a. Relations based on "divisible by" /

"sum or difference of two numbers is divisibly by"/

"sum or difference of two numbers is even or odd"/

"multiple of " / "factor of "/ "divisor of" /"even or odd" :

1.a

Check whether the relation R in the set N of natural numbers given by

$$R = \{(a, b) : a \text{ is divisor of } b\}$$

is reflexive, symmetric or transitive. Also determine whether R is an equivalence relation.

Sol.

For reflexive

Let $a \in N$ clearly a divides a $\therefore$ $(a, a) \in R$

$\therefore$ R is reflexive

For symmetric

$(1, 2) \in R$ but $(2, 1) \notin R$

$\therefore$ R is not symmetric

For transitive

Let (a, b) , $(b, c) \in R$

$\therefore$ a divides b and b divides c

$\Rightarrow$ a divides c $\therefore$ $(a, c) \in R$

R is transitive

As R is not symmetric $\therefore$ It is not an equivalence relation

prepared by : BALAJI KANCHI

1.b 2025

Let R be a relation defined over N, where N is set of natural numbers, defined as “mRn if and only if m is a multiple of n, m, n $\in$ N.” Find whether R is reflexive, symmetric and transitive or not.

Sol.

Let $x \in N$. Then we know that x is a multiple of itself.

$\Rightarrow xRx$

Hence, R is reflexive.

We have $2, 8 \in N$ such that 8 is a multiple of 2
$\Rightarrow 8R2$

But, 2 is not a multiple of 8. Hence, 2 is not R-related to 8.

Therefore, R is not symmetric.

Let $x, y, z \in N$ such that xRy, yRz

Then $x = my, y = nz$ for some $m, n \in N$

$\Rightarrow x = mnz \Rightarrow x = pz$, where $p = mn \in N$. Hence, xRz

Therefore, R is transitive.

prepared by : **BALAJI KANCHI**

2.

Show that the relation R on the set Z of all integers, given by $R = \{(a, b) : 2 \text{ divides } (a - b)\}$ is an equivalence relation.

Sol.

For $a \in Z, (a, a) \in R \because a - a = 0$ is divisible by 2

$\therefore$ R is reflexive ...(i)

Let $(a, b) \in R$ for $a, b \in Z$, then $a - b$ is divisible by 2

$\Rightarrow (b - a)$ is also divisible by 2

$\therefore (b, a) \in R \Rightarrow R$ is symmetric ...(ii)

For $a, b, c, \in Z$, Let $(a, b) \in R$ and $(b, c) \in R$

$\therefore a - b = 2p, p \in Z$, and $b - c = 2q, q \in Z$,

adding, $a - c = 2(p + q) \Rightarrow (a - c)$ is divisible by 2

$\Rightarrow (a, c) \in R$, so R is transitive ...(iii)

(i), (ii), and (iii) $\Rightarrow$ R is an equivalence relation.

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3.a

Show that the relation S in the set $A = \{x \in Z : 0 \le x \le 12\}$ given by $S = \{(a, b) : a, b \in Z, |a - b| \text{ is divisible by } 3\}$ is an equivalence relation.

Sol.

reflexive

symmetric

For transitive

Let $(a, b) \in S$ & $(b, c) \in S$

$|a - b| = 3m, |b - c| = 3n$

$a - b = \pm 3m \qquad b - c = \pm 3n$

$a - c = 3(\pm m \pm n) \Rightarrow a - c$ is divisible by 3

$\Rightarrow |a - c|$ is divisible by 3

$\Rightarrow (a, c) \in S$

S is transitive

As S is reflexive, symmetric & transitive

$\therefore$ S is an equivalence reation.

3.b

Let $A = \{x \in Z : 0 \le x \le 12\}$. Show that

$R = \{(a, b) : a, b \in A, |a - b| \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class [2].

Sol.

Reflexive: $|a - a| = 0$, which is divisible by 4, $\forall \ a \in A$

$\therefore \quad (a, a) \in R, \forall a \in A \quad \therefore \ R$ is reflexive

Symmetric: let $(a, b) \in R$

$\Rightarrow \quad |a - b|$ is divisible by 4

$\Rightarrow \quad |b - a|$ is divisible by 4 ($\because |a - b| = |b - a|$)

$\Rightarrow \quad (b, a) \in R \quad \therefore R$ is symmetric.

Transitive: let $(a, b), (b, c) \in R$

$\Rightarrow \quad |a - b|$ & $|b - c|$ are divisible by 4

$\Rightarrow \quad a - b = \pm 4m, \ b - c = \pm 4n, \ m, n \in Z$

Adding we get, $a - c = 4(\pm m \pm n)$

$\Rightarrow \quad (a - c)$ is divisible by 4

$\Rightarrow \quad |a - c|$ is divisible by 4 $\quad \therefore (a, c) \in R$

$\therefore R$ is transitive

Hence R is an equivalence relation in A

set of elements related to 1 is $\{1, 5, 9\}$

and $[2] = \{2, 6, 10\}$. prepared by : BALAJI KANCHI

3.c

A relation R on set $A = \{x : -10 \leq x \leq 10, x \in Z\}$ is defined as $R = \{(x, y) : (x - y)$ is divisible by $5\}$. Show that R is an equivalence relation. Also, write the equivalence class [5].

Sol.

For reflexive relation

To prove $(x, x) \in R$, $x - x = 0$ which is divisible by 5

$\therefore (x, x) \in R \Rightarrow R$ is reflexive

For symmetric relation

Let $(x, y) \in R \Rightarrow x - y$ is divisible by 5

$\Rightarrow x - y = 5m \Rightarrow y - x = 5(-m)$

$\Rightarrow y - x$ is divisible by 5

$\Rightarrow (y, x) \in R \therefore R$ is symmetric

For transitive relation

Let $(x, y) \in R$ and $(y, z) \in R$

$x - y$ is divisible by 5 $\quad \Rightarrow x - y = 5\,m$

$y - z$ is divisible by 5 $\quad \Rightarrow y - z = 5\,n$

$\Rightarrow x - y + y - z = 5(m - n) \Rightarrow x - z = 5(m - n)$

$\therefore x - z$ is divisible by 5

$\Rightarrow (x, z) \in R \therefore R$ is transitive.

R is an equivalence relation.

$[5] = \{-10, -5, 0, 5, 10\}$

prepared by : BALAJI KANCHI

3.d

Let $A = \{x \mid x \in Z,\ 0 \le x \le 12\}$. Show that the relation $R = \{(a, b) : a,\ b \in A,\ (a - b)$ is divisible by $4\}$ is an equivalence relation. Find the set of elements related to 2.

Sol.

$R = \{(a, b) : a, b \in A, (a - b)$ is divisible by $4\}$

(i) For every $a \in A$ we have $a - a = 0$ which is divisible by 4

$\Rightarrow (a, a) \in R$ for all $a \in A$ Hence R is reflexive

(ii) Let $(a, b) \in R$, $a, b \in A$

$\Rightarrow (a - b)$ is divisible by 4 $\Rightarrow (b - a)$ is also divisible by 4

$\therefore (b, a) \in R$ Hence R is symmetric

(iii) Let $(a, b) \in R$ and $(b, c) \in R$, $\because$ $a, b, c \in A$

Then$(a - b)$ and $(b - c)$ are divisible by 4

$\Rightarrow a - c = (a - b) + (b - c)$ which is divisible by 4

$\therefore (a, c) \in R$ Hence R is transitive

R is an equivalence relation

$[2] = \{2, 6, 10\}$

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3.d

Prove that the relation R on Z, defined by R {(x, y) : (x – y) is divisible by 5} is an equivalence relation.

Sol.

For reflexive

$x - x = 0$, for every $x \in Z$ is divisible by 5 $\Rightarrow (x, x) \in R$

For symmetric

$(x, y) \in R \Rightarrow x - y$ is divisible by $5 \Rightarrow y - x$ is divisible by 5

$\Rightarrow (y, x) \in R \Rightarrow R$ is symmetric.

For transitive

Let $(x, y) \in R$ and $(y, z) \in R$

$(x, y) \in R \Rightarrow x - y = 5\lambda \quad ...(i)$

$(y, z) \in R \Rightarrow y - z = 5\mu \quad ...(ii)$

adding (i) and (ii), $x - z = 5(\lambda + \mu) = 5k$

$\Rightarrow (x, z) \in R \Rightarrow R$ is transitive.

Hence R is an equivalence relation.

3.e

Show that the relation R on the set Z of all integers defined by $(x, y) \in R \Leftrightarrow (x - y)$ is divisible by 3 is an equivalence relation.

Sol.

$(x - x) = 0$ is divisible by 3 for all $x \in z$. So, $(x, x) \in R$

$\therefore$ R is reflexive.

$(x - y)$ is divisible by 3 implies $(y - x)$ is divisible by 3.

So $(x, y) \in R$ implies $(y, x) \in R$, $x, y \in z$

$\Rightarrow$ R is symmetric.

$(x - y)$ is divisible by 3 and $(y - z)$ is divisble by 3.

So $(x - z) = (x - y) + (y - z)$ is divisible by 3.

Hence $(x, z) \in R \Rightarrow$ R is transitive

$\Rightarrow$ R is an equivalence relation

4.a

Show that the relation R in the set A = {1, 2, 3, 4, 5, 6} given by R = {(a, b) : | a – b | is divisible by 2} is an equivalence relation.

Sol.

Reflexive: $|a-a|=0$, which is divisible by 2 for all a ∈ A.

$\therefore$ (a, a) ∈ R ⇒ R is reflexive.

Symmetric: Let (a, b) ∈ R i.e., $|a-b|=2\lambda$, $\lambda \in \omega$

then $|b-a|=|-(a-b)|=|a-b|=2\lambda$

Transitive : Let (a, b), (b, c) ∈ R i.e., $|a-b|=2\lambda$, $|b-c|=2\mu$

$$a-c=(a-b)+(b-c)=\pm 2\lambda \pm 2\mu = \pm 2(\lambda+\mu)$$

$|a-c|=2|\lambda+\mu|$, which is divisible by 2

⇒ (a, c) ∈ R ⇒ R is transitive.

Hence R is an equivalence relation.

4.b

Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is divisible by 2} is an equivalence relation. Write all the equivalence classes of R.

4.c

A relation R in the set A = {5, 6, 7, 8, 9} is given by R = {(x, y) : |x – y| is divisible by 2}. Write R in roster form and prove that R is an equivalence relation. Also, find the elements related to element 7.

4.d

Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is divisible by 2 } is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other, but no element of {1, 3, 5} is related to any element of {2, 4}.

4.e

Check whether the relation R in the set Z of integers defined as R = $\{(a, b):\ a+b \text{ is "divisible by 2"}\}$ is reflexive, symmetric or transitive. Write the equivalence class containing 0 i.e. [0].

4.f

Prove that the relation R in the set of integers Z defined as

R = {(a, b) : 2 divides (a + b)} is an equivalence relation. Also, determine [3].

Sol.

Reflexive: $a+a=2a$, which is divisible by 2, $\forall a \in Z \Rightarrow (a,a) \in R, \forall a \in Z$,

$\therefore$ R is reflexive

Symmetric: Let $(a,b) \in R \Rightarrow 2$ divides $(a+b) \Rightarrow 2$ divides $(b+a) \Rightarrow (b,a) \in R$

$\therefore$ R is symmetric.

Transitive: Let $(a,b),(b,c) \in R \Rightarrow 2$ divides $(a+b)\ \&\ (b+c)$ both

$\Rightarrow a+b=2m, b+c=2n$

$\Rightarrow a+2b+c=2m+2n \therefore a+c=2(m+n-b)$

$\therefore 2$ divides $(a+c) \Rightarrow$ S isTransitive.

$\therefore$ S is an equivalance relation.

$[3]=\{x : x \text{ is an odd integer}\}$ or $[3]=\{\ldots,-1,0,1,3,5,7,\ldots\}$

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4.g

A relation R on set A = {– 4, – 3, – 2, – 1, 0, 1, 2, 3, 4} be defined as R = {(x, y) : x + y is an integer divisible by 2}. Show that R is an equivalence relation. Also, write the equivalence class [2].

Sol.

For reflexive: clearly $x + x$ i.e. $2x$ is integer divisible by 2.

$\Rightarrow (x,x) \in R \Rightarrow R$ is reflexive.

For symmetric : $(x,y) \in R \Rightarrow x + y$ is integer divisible by 2.

$\Rightarrow y + x$ is integer divisible by $2 \Rightarrow (y,x) \in R$

For transitive : $(x,y) \in R \Rightarrow x + y$ is integer divisible by 2.

and $(y,z) \in R \Rightarrow y + z$ is integer divisible by 2.

so, $(x+z) + 2y$ is integer divisible by 2.

$\Rightarrow x + z$ is integer divisible by $2 \Rightarrow (x,z) \in R$

Equivalence class $[2] = \{-4, -2, 0, 2, 4\}$

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5.a

Prove that the relation R in the set A = {1, 2, 3, 4, 5, 6, 7} given by R = {(a, b) : $|a - b|$ is even} is an equivalence relation.

Sol.

(i) **Reflexive:** $\forall\, a \in A, |a - a| = 0$ which is even

$\Rightarrow (a, a) \in R$, hence R is reflexive

(ii) **Symmetric:** Let $(a, b) \in R \Rightarrow |a - b|$ is even

$\Rightarrow |-(b - a)|$ is even $\Rightarrow |b - a|$ is even

so, $(b, a) \in R$

hence R is symmetric.

(iii) **Transitive:** Let $(a, b), (b, c) \in R$

so, $|a - b|$ is even and $|b - c|$ is even

$\Rightarrow a - b = 2\lambda,\ b - c = 2\mu$ where $\lambda, \mu \in Z$

Now, $a - c = (a - b) + (b - c) = 2(\lambda + \mu)$

$\Rightarrow |a - c|$ is even, so $(a, c) \in R$

hence R is transitive.

Since R is reflexive, symmetric and transitive therefore its an equivalence relation

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5.b

Prove that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b): $|a - b|$ is even}, is an equivalence relation.

6. 2023

Let R be the relation defined in the set A = {1, 2, 3, 4, 5, 6, 7} by R = {(a, b) : both a and b are either odd or even}. Show that R is an equivalence relation. Hence, find the elements of equivalence class [1].

Sol.

$R=\{(a,b):\text{both } a \text{ and } b \text{ are either odd or even}\}$

for reflexive : Let $a \in A$

clearly both a and a are either odd or even

$\therefore (a,a) \in R \Rightarrow R$ is reflexive.

for symmetric : Let $a, b \in A$. Let $(a,b) \in R$

$\Rightarrow$ both a and b are either odd or even

$\Rightarrow$ both b and a are either odd or even

so, $(a,\text{b}) \in R \Rightarrow (b,a) \in R \Rightarrow R$ is symmetric.

for transitive : Let $a,b,c \in A$. Let $(a,b) \in R, (b,c) \in R$

$\Rightarrow$ both a and b are either odd or even & both b and c are either odd or even

$\Rightarrow$ both a and c are either odd or even

so, $(a,\text{b}) \in R, (b,c) \in R \Rightarrow (\text{a},\text{c}) \in R \Rightarrow R$ is transitive.

equivalence class of $[1]=\{1,3,5,7\}$ prepared by : BALAJI KANCHI

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6.b

Let R be the relation defined in the set A = {1, 2, 3, 4, 5, 6, 7, 8, 9} by R = {(x, y) : x, y ∈ A, x and y are either both odd or both even}. Show that R is an equivalence relation. Write all the equivalence classes of set A.

Sol.

Reflexive

Clearly $(x, x) \in R$ $\quad \because x$ is either odd or even. So R is reflexive

Symmetric

Let $(x, y) \in R$

$\therefore$ x, y are both either odd or even

$\Rightarrow y, x$ are both either odd or even

$\Rightarrow (y, x) \in R$, So R is symmetric

Transitive

Let $(x, y) \in R$ and $(y, z) \in R$

Case (i) x and y are both odd so y and z are both odd

$\therefore$ x and z are both odd

$\therefore (x, z) \in R$

Case (ii) x and y are both even, so y and z are both even

$\therefore$ x and z are both even

$\therefore (x, z) \in R$

Thus (x, y) and $(y, z) \in R \Rightarrow (x, z) \in R$

R is transitive

As R is reflexive, symmetric and transitive so R is an equivalence relation.

$$\left.\begin{array}{l}[1] = \{1, 3, 5, 7, 9\} \\ [2] = \{2, 4, 6, 8\}\end{array}\right\} \text{ are required equivalence classes}$$

b. Number system properties relations:

1. 2023

A relation R is defined on a set of real numbers $\mathbb{R}$ as

$$R = \{(x, y) : x \cdot y \text{ is an irrational number}\}.$$

Check whether R is reflexive, symmetric and transitive or not.

Sol.

For reflexive

$(1, 1) \notin R$ as 1^2 is rational (or any other counter example)

R is not reflexive

For symmetric

Let $(x, y) \in R$ $\therefore$ x.y is an irrational number

$\therefore$ (y.x) is an irrational number

$\therefore$ $(y, x) \in R$

$\therefore$ R is symmetric

For Transitive

$(1, \sqrt{2}) \in R$, $(\sqrt{2}, 2) \in R$ but $(1, 2) \notin R$ (or any other counter example)

$\therefore$ R is not transitive

prepared by : BALAJI KANCHI

2. a 2024

Check whether the relation S in the set of real numbers R defined by S = {(a, b) : where $a - b + \sqrt{2}$ is an irrational number} is reflexive, symmetric or transitive.

Sol.

Reflexive: For $a \in S$

$\Rightarrow a - a + \sqrt{2}$ is irrational number

$\Rightarrow \sqrt{2}$ is irrational number

$\Rightarrow (a, a) \in S$

Thus, S is Reflexive Relation.

Symmetric: Let $(a,b) \in S \Rightarrow a - b + \sqrt{2}$ is irrational number

but $b - a + \sqrt{2}$ may not be irrational number

For example, $(\sqrt{2}, 1) \in S \Rightarrow \sqrt{2} - 1 + \sqrt{2} = 2\sqrt{2} - 1$ is irrational number

$(1, \sqrt{2}) \notin S$ as $1 - \sqrt{2} + \sqrt{2} = 1$ is not irrational number

$\therefore (b, a) \notin S$, So S is NOT Symmetric Relation.

prepared by : BALAJI KANCHI

2.b 2025

Let R be a relation on set of real numbers ℝ defined as $\{(x, y) : x - y + \sqrt{3}$ is an irrational number, $x, y \in \mathbb{R}\}$ Verify R for reflexivity, symmetry and transitivity.

Sol.

Let $x \in \mathbb{R}$. Then we know that $x - x + \sqrt{3} = \sqrt{3}$, which is an irrational number.

$\Rightarrow (x, x) \in R$

Hence, R is reflexive.

We have $\sqrt{3}, 2 \in \mathbb{R}$ such that $\sqrt{3} - 2 + \sqrt{3} = 2(\sqrt{3} - 1)$, which is an irrational number

$\Rightarrow (\sqrt{3}, 2) \in R$.

But, $2 - \sqrt{3} + \sqrt{3} = 2$, which is a rational number.

Hence, $\Rightarrow (2, \sqrt{3}) \notin R$.

Therefore, R is not symmetric.

Let $-\sqrt{3}, \sqrt{3}, 2 \in \mathbb{R}$ such that $\left(-\sqrt{3}, \sqrt{3}\right), \left(\sqrt{3}, 2\right) \in R$.

But, $\left(-\sqrt{3}, 2\right) \notin R$

Therefore, R is not transitive. prepared by : BALAJI KANCHI

3. 2025

Let R be a relation defined on a set N of natural numbers such that R = {(x, y) : xy is a square of a natural number, x, y ∈ N}. Determine if the relation R is an equivalence relation.

Sol.

Reflexive: For any $x \in N$, $x \cdot x = x^2$, which is square of the natural number 'x'.

$$\Rightarrow (x,x) \in R$$

∴ 'R' is a Reflexive relation.

Symmetric: Let $(x,y) \in R \Rightarrow xy$ is a square of a natural number

$\Rightarrow yx$ is a square of a natural number, $\because xy = yx$.

$\Rightarrow (y,x) \in R$

∴ 'R' is a Symmetric relation.

Transitive: Let $(x,y),(y,z) \in R \Rightarrow xy = a^2, yz = b^2$ for some $a,b \in N$,

$$\therefore \frac{a^2}{y} = x, \frac{b^2}{y} = z \in N$$

$$\Rightarrow xz = \frac{a^2}{y} \cdot \frac{b^2}{y} = \left(\frac{ab}{y}\right)^2, \frac{ab}{y} \in N$$

$$\Rightarrow (x,z) \in R$$

∴ 'R' is a Transitive relation.

Hence, R is an Equivalence relation

c. Inequality relations:

1.

Show that the relation R on $\mathbb{R}$ defined as $R = \{(a, b) : a \leq b\}$, is reflexive, and transitive but not symmetric.

Sol.

Clearly $a \leq a \ \forall \ a \in \mathbb{R} \Rightarrow (a, a) \in R \Rightarrow R$ is reflexive.

For transitive:

Let $(a, b) \in R$ and $(b, c) \in R$, $a, b, c \in \mathbb{R}$

$\Rightarrow a \leq b$ and $b \leq c \Rightarrow a \leq c \Rightarrow (a, c) \in R$

$\Rightarrow R$ is transitive.

For non-symmetric:

Let $a = 1$, $b = 2$. As $1 \leq 2 \Rightarrow (1, 2) \in R$ but $2 \not\leq 1 \Rightarrow (2, 1) \notin R$

$\Rightarrow R$ is non-symmetric.

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2. **2023**

Show that the relation S in set $\mathbb{R}$ of real numbers defined by

$$S = \{(a, b) : a \le b^3,\ a \in \mathbb{R},\ b \in \mathbb{R}\}$$

is neither reflexive, nor symmetric, nor transitive.

Sol.

Reflexive: Let $\frac{1}{2} \not\le \left(\frac{1}{2}\right)^3$, $\therefore$ S is not reflexive.

Symmetric: $1 \le 2^3 \Rightarrow (1,2) \in S$ but $2 \not\le 1^3 \Rightarrow (2,1) \notin S$. $\therefore$ S is not symmetric.

Transitive: $10 \le 6^3 \Rightarrow (10,6) \in S$ & $6 \le 2^3 \Rightarrow (6,2) \in S$ but $10 \not\le 2^3 \Rightarrow (10,2) \notin S$
$\therefore$ S is not Transitive.

prepared by : **BALAJI KANCHI**

OR

We have $S = \{(a, b : a \le b^3\}$ where $a, b \in R$.

(i) Reflexive: we observe that, $\frac{1}{2} \le \left(\frac{1}{2}\right)^3$ is not true.

$\therefore \left(\frac{1}{2}, \frac{1}{2}\right) \notin S$. So, S is not reflexive.

(ii) Symmetric: We observe that $1 \le 3^3$ but $3 \not\le 1^3$ i.e., $(1, 3) \in S$ but $(3, 1) \notin S$.
So, S is not symmetric.

(iii) Transitive: We observe that, $10 \le 3^3$ and $3 \le 2^3$ but $10 \not\le 2^3$.
i.e., $(10, 3) \in S$ and $(3, 2) \in S$ but $(10, 2) \notin S$.
So, S is not transitive.
$\therefore$ S is neither reflexive nor symmetric, not transitive.

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1.c

Check whether the relation S in the set $\mathbb{R}$ of real numbers, defined as $S = \{(a, b) : a \leq b^2\}$ is reflexive, symmetric or transitive. Also, determine all R such that $(x, x) \in S$.

Sol.

Reflexive: Let $\frac{1}{2} \not\leq \left(\frac{1}{2}\right)^2$, $\therefore$ S is not reflexive.

Symmetric: $1 \leq 2^2 \Rightarrow (1,2) \in S$ but $2 \not\leq 1^2 \Rightarrow (2,1) \notin S$.

$\therefore$ S is not symmetric.

Transitive: $10 \leq 6^2 \Rightarrow (10,6) \in S$ & $6 \leq 3^2$

$\therefore$ S is not Transitive.

$\Rightarrow (6,3) \in S$ but $10 \not\leq 3^2 \Rightarrow (10,3) \notin S$

Let $(x,x) \in S \Leftrightarrow x \leq x^2 \Leftrightarrow x \in R - (0,1)$

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3.

Check whether the relation R in the set **R** of real numbers, defined by $R = \{(a, b) : 1 + ab > 0\}$, is reflexive, symmetric or transitive.

4. 2024

A relation R on set A = {1, 2, 3, 4, 5} is defined as $R = \{(x, y) : |x^2 - y^2| < 8\}$. Check whether the relation R is reflexive, symmetric and transitive.

Sol.

(a) Reflexive:

$\because |x^2 - x^2| < 8 \; \forall \; x \in A \Rightarrow (x, x) \in R \;\; \therefore R$ is reflexive .

(b) Symmetric:

Let (x,y) $\in R$ for some x,y $\in$ A

$\therefore \; |x^2 - y^2| < 8 \Rightarrow |y^2 - x^2| < 8 \Rightarrow (y, x) \in R$

Hence R is symmetric.

(c) Transitive:

(1,2) , (2,3) $\in R$ as $|1^2 - 2^2| < 8$, $|2^2 - 3^2| < 8$ respectively

But $|1^2 - 3^2| \not< 8 \Rightarrow (1, 3) \notin R$

Hence R is not transitive. prepared by : BALAJI KANCHI

d. Based on Order Pairs :

1.a

Let set A = {1, 2, 3, ..., 10} and R be a relation in $A \times A$, defined by (a, b) R (c, d) $\Leftrightarrow$ a + d = b + c for all (a, b) and (c, d) $\in A \times A$. Prove that R is an equivalence relation.

1.b

Show that the relation R defind by (a, b) R (c, d) $\Rightarrow a+d=b+c$ on the $A \times A$, where A = {1, 2, 3, ..., 10} is an equivalence relation. Hence write the equivalence class [(3, 4)]; a, b, c, d $\in$ A.

Sol.

reflexive :

$$a+b=b+a \Rightarrow (a,b)R(a,b), \forall (a,b) \in R,$$

$\therefore$ **R is**

symmetric :

$$(a,b)R(c,d) \Rightarrow a+d=b+c$$

$$\Rightarrow c+b=d+a \Rightarrow (c,d)R(a,b),$$

$\therefore$ **R is symmetric**

transitive :

$$(a,b)R(c,d) \,\&\, (c,d)R(e,f)$$

$$\Rightarrow a+d=b+c \,;\, c+f=d+e,$$

Adding the two equations

$$\Rightarrow a+f=b+e, \therefore (a,b)R(e,f)$$

$\Rightarrow$ **R is a transitive relation**

$\therefore$ **R is an equivalence relation**

prepared by : **BALAJI KANCHI**

1.c 2024

A relation R is defined on N × N (where N is the set of natural numbers) as :

$$(a, b)\ R\ (c, d) \Leftrightarrow a - c = b - d$$

Show that R is an equivalence relation.

Sol.

Let $(a, b) \in N \times N$

We have
$a - a = b - b$
This implies that (a, b) R (a, b) $\forall (a, b) \in N \times N$
Hence R is reflexive

Let (a, b) R (c, d) for some $(a, b), (c, d) \in N \times N$
Then $a - c = b - d$
$\Rightarrow c - a = d - b$
$\Rightarrow$(c, d) R (a, b)
Hence, R is symmetric.

Let (a, b) R (c, d), (c, d) R (e, f) for some $(a, b), (c, d), (e, f) \in N \times N$
Then $a - c = b - d, c - e = d - f$
$\Rightarrow a - c + c - e = b - d + d - f$
$\Rightarrow a - e = b - f$
$\Rightarrow$(a, b) R (e, f)
Hence, R is transitive

Thus, R is an equivalence relation.

2.a

Let N be the set of natural numbers and R be the relation on $N \times N$ defined by (a, b) R (c, d) iff ad = bc for all a, b, c, d $\in$ N. Show that R is an equivalence relation.

Sol.

For reflexive:

As $ab = ba$

$\Rightarrow \quad (a, b) R (a, b) \qquad \therefore$ R is reflexive

For symmetric:

Let (a, b,) R (c, d)

$\Rightarrow \quad ad = bc$

$\Rightarrow \quad cb = da$

$\Rightarrow \quad (c, d) R (a, b) \qquad \therefore$ R is symmetric

For transitive:

Let a, b, c, d, e, f $\in$ N

Let (a, b) R(c, d) and (c, d) R(e, f)

$\Rightarrow \quad ad = bc$ and $cf = de$

$$\Rightarrow \qquad d = \frac{cf}{e}$$

$$\therefore \qquad a\left(\frac{cf}{e}\right) = bc$$

$\Rightarrow \quad acf = bce \Rightarrow af = be$

$\Rightarrow \quad (a, b) R (e, f) \qquad \therefore$ R is transitive

Since R is reflexive, symmetric and transitive $\therefore$ R is an equivalence relation.

OR

Reflexive: For any $(a, b) \in N \times N$

$a \cdot b = b \cdot a$

$\therefore$ $(a, b)\ R\ (a, b)$ thus R is reflexive

Symmetric: For $(a, b), (c, d) \in N \times N$

$(a, b)\ R\ (c, d) \Rightarrow a \cdot d = b \cdot c$

$\Rightarrow c \cdot b = d \cdot a$

$\Rightarrow (c, d)\ R\ (a, b)$ $\therefore$ R is symmetric

Transitive : For any $(a, b), (c, d), (e, f), \in N \times N$

$(a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f)$

$\Rightarrow a \cdot d = b \cdot c$ and $c \cdot f = d \cdot e$

$\Rightarrow a \cdot d \cdot c \cdot f = b \cdot c \cdot d \cdot e \Rightarrow a \cdot f = b \cdot e$

$\therefore (a, b)\ R\ (e, f)$, $\therefore$ R is transitive

$\therefore$ R is an equivalance Relation

prepared by : BALAJI KANCHI

2.b

A relation R is defined on N × N (where N is the set of natural numbers) as (a, b) R (c, d) ⇔ $\frac{a}{c} = \frac{b}{d}$. Show that R is an equivalence relation.

Sol.

3.

If N denotes the set of all natural numbers and R is the relation on N × N defined by (a, b) R (c, d), if ad(b + c) = bc(a + d). Show that R is an equivalence relation.

Sol.

Reflexive :Here, (a, b) R (a, b) ∀ (a, b) ∈ N X N

since ab(b + a) = ba(a + b) is always true.

Symmetric: Let (a, b) R (c, d) ∀ (a, b), (c, d) ∈ N X N. Then,

$$ad(b + c) = bc(a + d)$$

$$\Rightarrow bc(a + d) = ad(b + c)$$

$$\Rightarrow (c, d)\ R\ (a, b)$$

Transitive: Let (a, b) R (c, d) and (c, d) R (e, f) ∀ (a, b), (c, d), (e, f) ∈ N X N. Then

$$ad\ (b + c) = bc\ (a + d) \text{ and } cf(d + e) = de(c + f)$$

$$\Rightarrow \frac{b + c}{bc} = \frac{a + d}{ad} \text{ and } \frac{d + e}{de} = \frac{c + f}{cf}$$

$$\frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a} \Rightarrow \frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c}$$

Adding, we get

$$\frac{1}{c} + \frac{1}{b} + \frac{1}{e} + \frac{1}{d} = \frac{1}{d} + \frac{1}{a} + \frac{1}{f} + \frac{1}{c}$$

$$\Rightarrow \frac{1}{b} + \frac{1}{e} = \frac{1}{a} + \frac{1}{f}$$

$$\Rightarrow \frac{e+b}{be} = \frac{f+a}{af}$$

$$\Rightarrow af(b+e) = be(a+f)$$

$$\Rightarrow (a, b)\ R\ (e, f)$$

Hence, R is equivalence relation.

4.

Show that the relation R in the set $N \times N$ defined by

$(a, b)R(c, d)$ iff $a^2 + d^2 = b^2 + c^2 \forall a, b, c, d \in N$, is an equivalence relation.

5.

Given a non-empty set X, define the relation R in P(X) as follows:
For A, B $\in P(X)$, $(A, B) \in R$ iff $A \subset B$. Prove that R is reflexive, transitive and not symmetric.

e. Relations based on objects - places/humans:

1.

Let W denote the set of words in the English dictionary. Define the relation R by

R = {(x, y) ∈ W × W such that x and y have at least one letter in common}.

Show that this relation R is reflexive and symmetric, but not transitive.

Sol.

For any word $x \in W$

x and x have atleast one (all) letter in common

$\therefore (x, x) \in R, \forall x \in W \;\; \therefore$ R is reflexive

Symmetric : Let $(x, y) \in R$, $x, y \in W$

$\Rightarrow$ x and y have atleast one letter in common

$\Rightarrow$ y and x have atleast one letter in common

$\Rightarrow (y, x) \in R \;\therefore$ R is symmetric

Transitive : Taking example of three English dictionary words $x, y, z, \in W$ such that $(x, y), (y, z) \in R$ but $(x, z) \notin R$

$\therefore$ R is not transitive

f. Relations based on Geometry :

1. 2024

Let S be the relation defined by $S = \{(l_1, l_2) : l_1 \text{ is perpendicular to } l_2\}$

check whether the relation S is symmetric and transitive.

II. Functions :

2 marks :

1.

If $f = \{(1, 2), (2, 4), (3, 1), (4, k)\}$ is a one-one function from set A to A, where $A = \{1, 2, 3, 4\}$, then find the value of k.

2

5Marks :

a.Linear polynomial :

1.

A function $f : A \rightarrow B$ defined as $f(x) = 2x$ is both one-one and onto. If $A = \{1, 2, 3, 4\}$, then find the set B.

Sol.

$f(1) = 2, f(2) = 4, f(3) = 6, f(4) = 8$

$\therefore B = \{2, 4, 6, 8\}$

2. 2023

Show that a function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = \frac{5x - 3}{4}$ is both one-one and onto.

Sol.

Let $x_1, x_2 \in R$ (Domain) such that $f(x_1) = f(x_2)$

$\Rightarrow \frac{5x_1 - 3}{4} = \frac{5x_2 - 3}{4}$

$\Rightarrow 5x_1 = 5x_2 \Rightarrow x_1 = x_2$

$\therefore$ f is one-one.

Let $y \in R$ (co-domain). Then f(x) = y for some x.

if, $y = \frac{5x-3}{4}, i.e., if, \quad x = \frac{4y+3}{5}$, which $\in R$(Domain)

Thus, for every y$\in R$(co − domain), there exists $\frac{4y+3}{5} \in R$(domain) such that $f\left(\frac{4y+3}{5}\right) = y$

∴ Range of f = R = codomain of f. Hence, f is onto.

3.a 2024

A function f is defined from R → R as f(x) = ax + b, such that f(1) = 1 and f(2) = 3. Find function f(x). Hence, check whether function f(x) is one-one and onto or not.

Sol.

f(x) = ax + b

Solving a+ b = 1 and 2a + b = 3 to get a=2 , b = −1

f(x) = 2 x − 1

Let f (x_1) = f (x_2) for some $x_1, x_2 \in R$

$2x_1 - 1 = 2x_2 - 1 \Rightarrow x_1 = x_2$

Hence f is one – one.

Let y = 2x -1, y ∈ R (Codomain)

$\Rightarrow x = \frac{y+1}{2} \in R$ (domain)

Also, f(x) = f $\left(\frac{y+1}{2}\right)$ = y

∴ f is onto.

3.b 2025

Prove that $f : N \to N$ defined as $f(x) = ax + b$ $(a, b \in N)$ is one-one but not onto.

Sol.

Let $x_1, x_2 \in N$ (Domain) such that $f(x_1) = f(x_2)$

$\Rightarrow ax_1 + b = ax_2 + b$

$\Rightarrow x_1 = x_2$

Therefore, f is one-one.

Let $y \in N$ (codomain). Then $f(x) = y$

if, $ax + b = y$

i.e., if, $x = \frac{y-b}{a}$, which may not belong to N (domain)

Therefore, f is not onto.

b. Rational function:

1.a 2025

Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$ be two sets. Prove that the function $f : A \to B$ given by $f(x) = \left(\frac{x-2}{x-3}\right)$ is onto. Is the function f one-one ? Justify your answer.

Sol.

Let $x_1, x_2 \in A$ such that $f(x_1) = f(x_2) \Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3} \Rightarrow x_1 = x_2$, $\therefore$ 'f' is one-one.

For each $y \in B$, there exists $x = \frac{3y-2}{y-1} \in R - \{3\}$, such that $f(x) = y$, $\therefore$ 'f' is onto

$\Rightarrow$ 'f' is one-one & onto, or 'f' is a bijective function.

1.b

Let $A = R - \{2\}$ and $B = R - \{1\}$. If $f : A \to B$ is a function defined by $f(x) = \dfrac{x-1}{x-2}$, show that f is one-one and onto.

Sol.

Let $x_1, x_2 \in R - \{2\}$ such that $f(x_1) = f(x_2)$

$$\Rightarrow \frac{x_1 - 1}{x_1 - 2} = \frac{x_2 - 1}{x_2 - 2}$$

$$\Rightarrow x_1x_2 - 2x_1 - x_2 + 2 = x_1x_2 - 2x_2 - x_1 + 2$$

$$\Rightarrow x_1 = x_2$$

So f is one – one

$$\Rightarrow x_1x_2 - 2x_1 - x_2 + 2 = x_1x_2 - 2x_2 - x_1 + 2$$

$$\Rightarrow x_1 = x_2$$

So f is one – one

For range let $f(x) = y$

$$\frac{x-1}{x-2} = y$$

$$x = \frac{2y-1}{y-1}$$

Range of $f = R - \{1\}$ = co domain B

So f is onto. prepared by : BALAJI KANCHI

1.c 2024

Let A = R − {5} and B = R − {1}. Consider the function f : A → B, defined by $f(x) = \frac{x-3}{x-5}$. Show that f is one-one and onto.

Sol.

Let $f(x_1) = f(x_2)$, for some $x_1, x_2 \in A$

$\Rightarrow \frac{x_1 - 3}{x_1 - 5} = \frac{x_2 - 3}{x_2 - 5}$

$\Rightarrow (x_1 - 3)(x_2 - 5) = (x_2 - 3)(x_1 - 5)$

$\Rightarrow x_1 = x_2$, So $\underline{f \text{ is one} - \text{one Function.}}$

Let $y = f(x) = \frac{x-3}{x-5} \Rightarrow y(x-5) = x-3$

$\Rightarrow yx - 5y = x - 3$

$\Rightarrow x = \frac{5y-3}{y-1}$, We observe that x is defined for all values of y except $y = 1$,

So, Range $= R - \{1\}$ and Co-domain is Given $R - \{1\}$ [As, $f : A \to B$]

Since, Range = Co-domain, f is onto Function.

Thus, f is one-one & onto function.

prepared by : BALAJI KANCHI

1.d

Let A = R – {3} and B = R – {a}. Find the value of 'a' such that the function $f : A \rightarrow B$ defined by $f(x) = \frac{x-2}{x-3}$ is onto. Also, check whether the given function is one-one or not.

Sol.

For onto, let f(x) = y

$\frac{x-2}{x-3} = y \quad \Rightarrow x - 2 = xy - 3y$

$\Rightarrow x(1 - y) = 2 - 3y \Rightarrow x = \frac{2 - 3y}{1 - y}$

For $y = 1, x \in A \therefore \text{Range} = R - \{1\} \therefore a = 1$

For one– one

Let $f(x_1) = f(x_2)$ where $x_1, x_2 \in A$

$\Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$

$\Rightarrow x_1 x_2 - 3x_1 - 2x_2 + 6 = x_2 x_1 - 2x_1 - 3x_2 + 6$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one – one.

1.f

Let $f: A \rightarrow B$ be a function defined as $f(x) = \frac{2x+3}{x-3}$, where $A = R - \{3\}$ and $B = R - \{2\}$. Is the function f one –one and onto?

1.g

Show that the function f in $A = R - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto.

Sol.

Let for $x_1, x_2 \in A$, $f(x_1) = f(x_2)$

$$\frac{4x_1+3}{6x_1-4} = \frac{4x_2+3}{6x_2-4}$$

$$\Rightarrow (4x_1+3)(6x_2-4) = (6x_1-4)(4x_2+3)$$

$\Rightarrow 34x_1 = 34x_2 \Rightarrow x_1 = x_2$, hence f is one-one.

For any $y \in A$ such that $y = \frac{4x+3}{6x-4}$ there exists x such that

$$6xy - 4y = 4x + 3 \Rightarrow (6y-4)x = 4y + 3$$

$$\Rightarrow x = \frac{4y+3}{6y-4}, y \in A, \; x = \frac{4y+3}{6y-4} \in A$$

$\Rightarrow$ f is onto. prepared by : **BALAJI KANCHI**

1.h

Consider $f : \mathbb{R} - \left\{-\frac{4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ given by $f(x) = \frac{4x+3}{3x+4}$.

Show that f is bijective.

Sol.

Let $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$ and $f(x_1) = f(x_2)$

$\Rightarrow \quad \frac{4x_1+3}{3x_1+4} = \frac{4x_2+3}{3x_2+4} \Rightarrow (4x_1+3)(3x_2+4) = (3x_1+4)(4x_2+3)$

$\Rightarrow \quad 12x_1x_2 + 16x_1 + 9x_2 + 12 = 12_1x_2 + 16x_2 + 9x_1 + 12$

$\Rightarrow \quad 16(x_1 - x_2) - 9(x_1 - x_2) = 0 \Rightarrow x_1 - x_2 = 0 \Rightarrow x_1 = x_2$

Hence f is a 1–1 function

Let $y = \frac{4x+3}{3x+4}$, for $y \in R - \left\{\frac{4}{3}\right\}$

$3xy + 4y = 4x + 3 \Rightarrow 4x - 3xy = 4y - 3$

$\Rightarrow \quad x = \frac{4y-3}{4-3y} \quad \therefore \ \forall\, y \in R - \left\{\frac{4}{3}\right\}, x \in R - \left\{-\frac{4}{3}\right\}$

Hence f is ONTO and so bijective

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1.i

Let A = $\mathbb{R} - \{4\}$ and B = $\mathbb{R} - \{1\}$ and let function $f : A \to B$ be defined as $f(x) = \frac{x-3}{x-4}$ for $\forall\, x \in A$. Show that f is one-one and onto.

Sol.

$f(x) = \frac{x - 3}{x - 4}$

Let $x_1, x_2 \in A$ such that $f(x_1) = f(x_2)$

$\therefore \frac{x_1-3}{x_1-4} = \frac{x_2-3}{x_2-4}$

$\Longrightarrow x_1 = x_2$

$\Longrightarrow$ f is one-one

Let $y = \frac{x-3}{x-4} \in \mathbb{R} - \{4\}$

Then $xy - 4y = x - 3$

$\Longrightarrow x = \frac{4y-3}{y-1}$

Range f = B = Codomain f

$\Longrightarrow$ f is onto

2.a

Show that the function $f : R - \{-1\} \to R - \{1\}$ given by $f(x) = \frac{x}{x+1}$ is bijective.

2.b

Show that $f : \mathbf{R} - \{2\} \rightarrow \mathbf{R} - \{1\}$ defined by $f(x) = \dfrac{x}{x-2}$ is one-one.

Sol.

Let $x_1, x_2 \in R - \{2\}$

Let $f(x_1) = f(x_2)$

$$\Rightarrow \quad \frac{x_1}{x_1 - 2} = \frac{x_2}{x_2 - 2} \Rightarrow x_1(x_2 - 2) = x_2(x_1 - 2)$$

$$\Rightarrow x_1 = x_2$$

$$\Rightarrow \text{f is one-one.}$$

2.c check one-one and onto.

$g : \mathbf{R} - \{1\} \rightarrow \mathbf{R} - \{2\}$ is defined as $g(x) = \dfrac{2x}{x-1}$

2.d

Let $f : \mathbb{R} - \left\{-\frac{4}{3}\right\} \rightarrow \mathbb{R}$ be a function defined as $f(x) = \dfrac{4x}{3x+4}$. Show that, in $f : \mathbb{R} - \left\{-\frac{4}{3}\right\} \rightarrow$ Range of f, f is one-one and onto. Hence find Range $f \rightarrow \mathbb{R} - \left\{-\frac{4}{3}\right\}$.

2.e

Let $f : \mathbb{R} - \left\{-\frac{4}{3}\right\} \to \mathbb{R}$ be a function defined as $f(x) = \frac{4x}{3x+4}$. Show that f is a one-one function. Also, check whether f is an onto function or not.

sol.

For one-one

Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$

$\Rightarrow \frac{4x_1}{3x_1 + 4} = \frac{4x_2}{3x_2 + 4}$

$\Rightarrow 12x_1x_2 + 16x_1 = 12x_1x_2 + 16x_2$

$\Rightarrow 16x_1 = 16x_2$

$\Rightarrow x_1 = x_2$

$\Rightarrow$ f is one-one

For onto let $y \in R$, and for some x.

Let $y = \frac{4x}{3x + 4}$

$\Rightarrow 3xy + 4y = 4x$

$\Rightarrow x(3y - 4) = -4y$

$\Rightarrow x = -\frac{4y}{3y - 4}$ or $x = \frac{4y}{4 - 3y}$

x is real if $y \neq \frac{4}{3}$. So $R_f = R - \left\{\frac{4}{3}\right\} \neq Codomain\,(f)$

So, f is not onto. prepared by : BALAJI KANCHI

3.

Show that the function $f : (-\infty, 0) \rightarrow (-1, 0)$ defined by $f(x) = \dfrac{x}{1+|x|}$, $x \in (-\infty, 0)$ is one-one and onto.

Sol.

Let $x_1, x_2 \in (-\infty, 0)$ such that $f(x_1) = f(x_2)$

i.e., $\dfrac{x_1}{1+|x_1|} = \dfrac{x_2}{1+|x_2|}$

$\Rightarrow \quad \dfrac{x_1}{1-x_1} = \dfrac{x_2}{1-x_2}$

$\Rightarrow \quad x_1 - x_1x_2 = x_2 - x_1x_2$

$\Rightarrow \quad x_1 = x_2$

$\therefore$ f is one-one.

Let $y \in (-1, 0)$ such that $y = \dfrac{x}{1+|x|}$

$\Rightarrow \quad y = \dfrac{x}{1-x}$

$\Rightarrow \quad x = \dfrac{y}{1+y}$

For each $y \in (-1, 0)$, there exists $x \in (-\infty, 0)$,

such that $f(x) = f\left(\dfrac{y}{1+y}\right) = \dfrac{\frac{y}{1+y}}{1+\left|\frac{y}{1+y}\right|}$

$= \dfrac{\frac{y}{1+y}}{1-\frac{y}{1+y}} = y$ Hence f is onto.

c. Conditional function :

1.a check one-one and onto**.**

Let $f : W \to W$ be defined as

$$f(n) = \begin{cases} n - 1, & \text{if } n \text{ is odd} \\ n + 1, & \text{if } n \text{ is even} \end{cases}$$

1.b 2025

Show that the function $f : N \to N$, where N is a set of natural numbers, given by $f(n) = \begin{cases} n - 1, \text{ if } n \text{ is even} \\ n + 1, \text{ if } n \text{ is odd} \end{cases}$ is a bijection.

Sol.

Let f(x) = f(y)
Let x and y are both odd or both even
Then either x+1 = y + 1 or x-1 = y-1 gives
x = y
x odd and y even is rejected as
x + 1 = y -1 gives x – y = -2 not possible as odd number and even number cannot differ by 2
Hence f is one-one

For onto: Let f(x) = y gives x = y + 1 or x = y - 1
If y is odd, x is even and if y is even, x is odd.
Range = N = co-domain, hence onto
As f is both one-one and onto hence bijective

2.

Prove that the function f is surjective, where $f: N \to N$ such that

$$f(n) = \begin{cases} \frac{n+1}{2}, & if\ n\ is\ odd \\ \frac{n}{2}, & if\ n\ is\ even \end{cases}$$

Is the function injective? Justify your answer.

3.

Let A and B be two sets. Show that $f : A \times B \to B \times A$ such that $f(a, b) = (b, a)$ is a bijective function.

d. Quadratic function :

1.a

Show that a function $f : R \to R$ defined as $f(x) = x^2 + x + 1$ is neither one-one nor onto. Also, find all the values of x for which $f(x) = 3$.

Sol.

Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in R$
Then $x_1{}^2 + x_1 + 1 = x_2{}^2 + x_2 + 1$
$\Rightarrow (x_1 - x_2)(1 + x_1 + x_2) = 0$
$\Rightarrow x_1 - x_2 = 0$ or $x_1 + x_2 = -1$
$\Rightarrow x_1 = x_2$ or $x_1 + x_2 = -1$ so if $x_1 + x_2 = -1, x_1 \neq x_2$

Hence f is not one -one

Let y = f(x) where $x \in R$
Then $y = x^2 + x + 1$.

$\Rightarrow x^2 + x + 1 - y = 0$

$$\Rightarrow x = \frac{-1 \pm \sqrt{4y - 3}}{2}$$

For x to be real, $4y - 3 \geq 0$

$$\Rightarrow y \geq \frac{3}{4}$$

Hence, range = $[\frac{3}{4}, \infty) \neq codomain$
Hence, f is not onto.

$f(x) = 3 \Rightarrow x^2 + x + 1 = 3 \Rightarrow x^2 + x - 2 = 0$

$$\Rightarrow x = \frac{-1 \pm \sqrt{9}}{2} = -2, 1$$

prepared by : BALAJI KANCHI

1.b

Prove that the function $f : N \to N$, defined by $f(x) = x^2 + x + 1$ is one-one but not onto.

Sol.

For one-one. Let $x_1, x_2 \in N$.

$$f(x_1) = f(x_2) \Rightarrow x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$$

$$\Rightarrow (x_1 - x_2)(x_1 + x_2 + 1) = 0$$

$$\Rightarrow x_1 = x_2 \quad \text{as} \quad x_1 + x_2 + 1 \neq 0 \quad (\because x_1, x_2 \in N)$$

$$\Rightarrow f \text{ is one-one.}$$

For not onto.

$$y = f(x) \Rightarrow y = x^2 + x + 1 \Rightarrow y = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$\Rightarrow x = \frac{\sqrt{4y-3} - 1}{2}$$

for $y = 1 \in N$, there is no $x \in N$ for which $f(x) = 1$

prepared by : **BALAJI KANCHI**

1.c

Consider $f : R_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$ where R_+ is the set of all non-negative real numbers. Prove that f is one- one and onto function.

1.d

check one-one and onto

Consider $f : \mathbb{R}_+ \to [-9, \infty]$ given by $f(x) = 5x^2 + 6x - 9$.

1.e

Let $f : N \to \mathbb{R}$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : N \to S$, where S is the range of f,

1.f 2023

Prove that a function $f : [0, \infty) \to [-5, \infty)$ defined as $f(x) = 4x^2 + 4x - 5$ is both one-one and onto.

sol.

Let $x_1, x_2 \in [0, \infty)$ such that $f(x_1) = f(x_2)$

Then this $\Rightarrow 4x_1^2 + 4x_1 - 5 = 4x_2^2 + 4x_2 - 5$

$\Rightarrow (x_1 + x_2)(x_1 - x_2) + (x_1 - x_2) = 0$

$\Rightarrow (x_1 - x_2)[(x_1 + x_2) + 1] = 0$

$\Rightarrow (x_1 - x_2) = 0$ or $x_1 + x_2 = -1$, which is rejected as $x_1, x_2 \geq 0$

$\Rightarrow x_1 = x_2$ $\quad \therefore$ f is one-one.

Let $f(x) = y \Rightarrow y = 4x^2 + 4x - 5$ for $x \in [0, \infty)$

$\Rightarrow 4x^2 + 4x - 5 - y = 0$

$$\Rightarrow x = \frac{-4 \pm \sqrt{16 - 16(-5 - y)}}{8} \Rightarrow x = \frac{-4 + 4\sqrt{6 + y}}{8} = \frac{-1 + \sqrt{6 + y}}{2}$$

Since, $x \geq 0$, we have $y + 6 \geq 1 \Rightarrow y \in [-5, \infty)$

$\therefore$ Range = Codomain $= [-5, \infty)$

Hence f is onto.

prepared by : BALAJI KANCHI

1.g

Let $f : N \to N$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that

1.h

Prove that function $f : R_+ \to (7, \infty)$ given by $f(x) = 16x^2 + 24x + 7$

One and onto.

2.a

Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \frac{x}{x^2+1}$, $\forall\, x \in \mathbb{R}$ is neither one-one nor onto.

Sol.

Checking for one-one:

here $f(x) = f\left(\frac{1}{x}\right)$. For example $f(2) = f\left(\frac{1}{2}\right)$

$\therefore f$ is not one-one.

Checking for onto:

Let $y = 1 \in R$ (co-domain). Then

$y = f(x) \Rightarrow \frac{x}{x^2+1} = 1$

$\Rightarrow x^2 - x + 1 = 0$, which has no real roots.

$\therefore R_f \neq$ co-domain $\Rightarrow f$ is not onto.

OR

Here $f(2) = f\left(\frac{1}{2}\right) = \frac{2}{5}$ but $2 \neq \frac{1}{2}$

$\therefore \quad f$ is not 1-1

for $y = \frac{1}{\sqrt{2}}$ let $f(x) = \frac{1}{\sqrt{2}} \Rightarrow x^2 - \sqrt{2}x + 1 = 0$

As $D = (-\sqrt{2})^2 - 4(1)(1) < 0$, $\therefore$ No real solution

$\therefore \quad f(x) \neq \frac{1}{\sqrt{2}}$, for any $x \in R(D_f)$ $\therefore f$ is not onto

2.b 2023

Check whether a function $f : \mathbb{R} \rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$ is one-one and onto or not.

Sol.

Let $f(x_1) = f(x_2) \Rightarrow \frac{x_1}{1+x_1^2} = \frac{x_2}{1+x_2^2}$

$\Rightarrow x_1 + x_1 x_2^2 = x_2 + x_1^2 x_2$

$\Rightarrow (x_1 - x_2)(1 - x_1 x_2) = 0$

for $x_1 = 2, x_2 = \frac{1}{2}$

we have $(x_1 - x_2)(1 - x_1 x_2) = 0$ but $x_1 \neq x_2$

$\Rightarrow f$ is not one-one.

Let $x \in R$ such that $f(x) = y \Rightarrow y = \frac{x}{1+x^2}$

$x^2 y - x + y = 0$

$\Rightarrow x = \frac{1 \pm \sqrt{1-4y^2}}{2y} \, (y \neq 0).$

[For $y = 0 \in [-\frac{1}{2}, \frac{1}{2}]$, we have $0 \in R$ such that $f(0) = 0$]

$x \neq 0, x \in R \Rightarrow 1 - 4y^2 \geq 0, y \neq 0$

$\Rightarrow y \in \left[-\frac{1}{2}, \frac{1}{2}\right] - \{0\}$. Also, $y = 0$ when $x = 0$

$\therefore$ Range $= \left[-\frac{1}{2}, \frac{1}{2}\right] =$ Codomain

$\therefore$ f is onto.

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2.c

Show that a function $f : R \to R$ defined by $f(x) = \dfrac{2x}{1+x^2}$ is neither one-one nor onto. Further, find set A so that the given function $f : R \to A$ becomes an onto function.

Sol.

Let $f(x_1) = f(x_2)\ for\ some\ x_1, x_2 \in R$
Then $\frac{2x_1}{1+x_1{}^2} = \frac{2x_2}{1+x_2{}^2}$

$\Rightarrow x_1 + x_1x_2{}^2 = x_2 + x_1{}^2x_2$
$\Rightarrow (x_1 - x_2) - x_1x_2(x_1 - x_2) = 0$
$\Rightarrow (x_1 - x_2)(1 - x_1x_2) = 0$
$\Rightarrow x_1 - x_2 = 0$ or $1 - x_1x_2 = 0$
$\Rightarrow x_1 = x_2$ or $x_1x_2 = 1$, so if $x_1x_2 = 1, x_1 \neq x_2$
Hence f is not one -one

Let y = f(x) where $x \in R$
Then $y = \frac{2x}{1+x^2}$. Here, for x = 0, y = 0

If $y \neq 0$, then $y = \frac{2x}{1+x^2}$
$\Rightarrow yx^2 - 2x + y = 0$
$\Rightarrow x = \dfrac{2 \pm \sqrt{4 - 4y^2}}{2y}$
For x to be real, $4 - 4y^2 \geq 0$
$\Rightarrow y^2 \leq 1$
$\Rightarrow -1 \leq y \leq 1$
Hence, range = $[-1,1] \neq codomain$
Hence, f is not onto.

For the given function to become onto, $A = [-1,1]$

3.

A function f: [– 4, 4] → [0, 4] is given by $f(x) = \sqrt{16 - x^2}$. Show that f is an onto function but not a one-one function. Further, find all possible values of 'a' for which $f(a) = \sqrt{7}$.

Sol.

Onto: Let $y = \sqrt{16 - x^2} \Rightarrow y \geq 0$

Squaring we get, $x^2 = 16 - y^2 \Rightarrow x = \pm\sqrt{16 - y^2}$

For each $y \in [-4, 4]$**, 'x' is a real number,**

$\therefore 0 \leq y \leq 4 \Rightarrow R_f = [0, 4] = \text{Co} - \text{domain}$

$\therefore$ **'f' is an onto function.**

One-One: $f(-1) = f(1) = \sqrt{15}$ **but** $-1 \neq 1$,

$\therefore$ **'f' is not a one-one function.**

$f(a) = \sqrt{7} \Rightarrow \sqrt{16 - a^2} = \sqrt{7} \Rightarrow a = \pm 3$

prepared by : BALAJI KANCHI

e. Special functions : modulus/greatest integer/even/odd function :

1.

Prove that the greatest integer function $f : \mathbb{R} \to \mathbb{R}$, given by $f(x) = [x]$, is neither one-one nor onto.

Sol.

For not one-one:

$1.1, 1.2 \in R$ (domain)

now, $1.1 \neq 1.2$ but $f(1.1) = f(1.2) = 1 \Rightarrow f$ is not one-one.

For not onto :

Let $\frac{1}{2} \in R$ (co-domain), but $[x] = \frac{1}{2}$ is not possible for x in domain.

so, f is not onto.

2.

Show that the function $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^4$ is neither one-one nor onto.

3. a

Check the injectivity and surjectivity of the function $f : N \to N$, given by $f(x) = x^3$.

3.b 2025

Show that the function $f : R \to R$ defined by $f(x) = 4x^3 - 5$, $\forall\, x \in R$ is one-one and onto.

Sol.

One-One: Let $x_1, x_2 \in R$ such that

$f(x_1) = f(x_2) \Rightarrow 4x_1^3 - 5 = 4x_2^3 - 5 \Rightarrow x_1^3 = x_2^3 \Rightarrow x_1 = x_2$, $\therefore$ 'f' is one-one

Onto: $x \in R\ (D_f) \Rightarrow x^3 \in R \Rightarrow 4x^3 - 5 \in R \Rightarrow f(x) \in R$, $\therefore\ R_f = \text{Co-domain}(f)$

$\therefore$ 'f' is an onto function

$\Rightarrow$ 'f' is one-one & onto both

4. 2025

If $f : R^+ \to R$ is defined as $f(x) = \log_a x$ $(a > 0$ and $a \neq 1)$, prove that f is a bijection.

(R^+ is a set of all positive real numbers.)

Sol.

$f(x) = \log_a x \quad (a > 0, a \neq 1)$

Let $x_1, x_2 \in R^+$ such that $f(x_1) = f(x_2)$

$\Rightarrow \log_a x_1 = \log_a x_2$

$\Rightarrow x_1 = x_2 \Rightarrow f$ is one-one.

Let $f(x) = y \Rightarrow \log_a x = y \Rightarrow a^y = x$

$\therefore$ for every $y \in R$, there exists $x \in R^+$

$\therefore f$ is onto.

f is a bijection.

prepared by : BALAJI KANCHI

Case Study :

1. 2025

A class-room teacher is keen to assess the learning of her students the concept of "relations" taught to them. She writes the following five relations each defined on the set $A = \{1, 2, 3\}$:

$R_1 = \{(2, 3), (3, 2)\}$

$R_2 = \{(1, 2), (1, 3), (3, 2)\}$

$R_3 = \{(1, 2), (2, 1), (1, 1)\}$

$R_4 = \{(1, 1), (1, 2), (3, 3), (2, 2)\}$

$R_5 = \{(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)\}$

The students are asked to answer the following questions about the above relations :

(i) Identify the relation which is reflexive, transitive but not symmetric.

(ii) Identify the relation which is reflexive and symmetric but not transitive.

(iii) (a) Identify the relations which are symmetric but neither reflexive nor transitive.

OR

(iii) (b) What pairs should be added to the relation R_2 to make it an equivalence relation ?

2. 2025

A school is organizing a debate competition with participants as speakers $S = \{S_1, S_2, S_3, S_4\}$ and these are judged by judges $J = \{J_1, J_2, J_3\}$. Each speaker can be assigned one judge. Let R be a relation from set S to J defined as $R = \{(x, y) : \text{speaker } x \text{ is judged by judge } y, x \in S, y \in J\}$.

Based on the above, answer the following :

(i) How many relations can be there from S to J ?

(ii) A student identifies a function from S to J as $f = \{(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)\}$ Check if it is bijective.

(iii) (a) How many one-one functions can be there from set S to set J ?

OR

(iii) (b) Another student considers a relation $R_1 = \{(S_1, S_2), \{S_2, S_4)\}$ in set S. Write minimum ordered pairs to be included in R_1 so that R_1 is reflexive but not symmetric.

Sol.

Ans (i) The number of relations = $2^{4 \times 3} = 2^{12}$

Ans (ii) Since, S_2 and S_3 have been assigned the same judge J_2, the function is not one-one.

Hence, it is not bijective.

(iii) (a) There cannot exist any one-one function from S to J as n(S) > n(J). Hence, the number of one-one functions from S to J is 0.

(iii) (b) To make R_1 reflexive and not symmetric we need to add the following ordered pairs:

$(S_1, S_1), (S_2, S_2), (S_3, S_3), (S_4, S_4)$

3.2025

Let A be the set of 30 students of class XII in a school. Let $f : A \rightarrow N$, N is a set of natural numbers such that function f(x) = Roll Number of student x.

On the basis of the given information, answer the following :

(i) Is f a bijective function ?

(ii) Give reasons to support your answer to (i).

(iii) (a) Let R be a relation defined by the teacher to plan the seating arrangement of students in pairs, where

$R = \{(x, y) : x, y$ are Roll Numbers of students such that $y = 3x\}$.

List the elements of R. Is the relation R reflexive, symmetric and transitive ? Justify your answer.

OR

(iii) (b) Let R be a relation defined by

$R = \{(x, y) : x, y$ are Roll Numbers of students such that $y = x^3\}$.

List the elements of R. Is R a function ? Justify your answer.

Sol.

(i) **No, f is not bijective function**

(ii) **Range = $\{1, 2, 3, 4, \ldots\ldots\ldots\ldots, 30\}$ and codomain= N**

Since, Range ≠ codomain ⇒ f is not onto and hence f is not bijective.

(iii) **(a)**

$R = \{(1,3), (2,6), (3,9), (4,12), (5,15), (6,18), (7,21), (8,24), (9,27), (10,30)\}$

Since $(1,1) \notin R$ ⇒ R is not reflexive.

$(1,3) \in R$ but $(3,1) \notin R$ ⇒ R is not symmetric

$(1,3) \in R, (3,9) \in R$ but $(1,9) \notin R$ ⇒ R is not transitive.

OR

(iii) **(b) $R = \{(1,1), (2,8), (3,27)\}$**

∵ elements 4, 5, 6 … 30 do not have an image. Hence the above relation is not a function.

prepared by : BALAJI KANCHI

4.

During the festival season, there was a mela organized by the Resident Welfare Association at a park, near the society. The main attraction of the mela was a huge swing installed at one corner of the park. The swing is traced to follow the path of a parabola given by $x^2 = y$.

Based on the above information, answer the following questions :

(i) Let $f : N \longrightarrow R$ is defined by $f(x) = x^2$. What will be the range ?

(ii) Let $f : N \longrightarrow N$ is defined by $f(x) = x^2$. Check if the function is injective or not.

(iii) (a) Let $f : \{1, 2, 3, 4 .....\} \longrightarrow \{1, 4, 9, 16 .....\}$ be defined by $f(x) = x^2$. Prove that the function is bijective.

OR

(iii) (b) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ is defined by $f(x) = x^2$. Show that f is neither injective nor surjective.

Sol.

$(i)\, R_f = \{1, 4, 9, 16, ...\}$ i.e. set of perfect squares of natural numbers.

(ii) Let $x_1, x_2 \in N$ (domain)

Let $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = \pm x_2$

$\Rightarrow x_1 = x_2$ as $x_1, x_2 \in N$

$\therefore f$ is injective.

$(iii)(a)\, f(x) = x^2$

Let $x_1, x_2 \in \{1, 2, 3, 4, ...\}$

Let $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = x_2$

$\therefore f$ is one-one.

As Co-domain = Range = $\{1, 4, 9, 16, ...\}$

$\therefore f$ is onto.

Since, f is one-one and onto, so f is bijective.

OR

$(iii)(b)\ f : R \to R,\ f(x) = x^2$

$-1, 1 \in R$ (domain)

As $f(-1) = f(1) = 1$ but $-1 \neq 1$

$\therefore f$ is not injective.

Co-domain $= R$, but Range $= [0, \infty)$

Since Co-domain $\neq$ Range, f is not surjective.

4. 2024

Students of a school are taken to a railway museum to learn about railways heritage and its history.

An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by

$R = \{(l_1, l_2) : l_1 \text{ is parallel to } l_2\}$

On the basis of the above information, answer the following questions :

(i) Find whether the relation R is symmetric or not.

(ii) Find whether the relation R is transitive or not.

(iii) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$, then find the set of rail lines in R related to it.

OR

Let S be the relation defined by S = $\{(l_1, l_2) : l_1$ is perpendicular to $l_2\}$ check whether the relation S is symmetric and transitive.

Sol.

(a) (i) Let $(l_1, l_2) \in R \Rightarrow l_1 \parallel l_2 \Rightarrow l_2 \parallel l_1 \Rightarrow (l_2, l_1) \in R$, $\therefore$ R is a symmetric relation

(ii) Let $(l_1, l_2), (l_2, l_3) \in R \Rightarrow l_1 \parallel l_2$, $l_2 \parallel l_3 \Rightarrow l_1 \parallel l_3 \Rightarrow (l_1, l_3) \in R$, $\therefore$ R is a transitive relation

(iii) The set is $\{l : l \text{ is a line of type } y = 3x + c, c \in R\}$

Or

(b) Let $(l_1, l_2) \in R \Rightarrow l_1 \perp l_2 \Rightarrow l_2 \perp l_1 \Rightarrow (l_2, l_1) \in R$, $\therefore$ R is a symmetric relation

Let $(l_1, l_2), (l_2, l_3) \in R \Rightarrow l_1 \perp l_2, l_2 \perp l_3 \Rightarrow l_1 \parallel l_3 \Rightarrow (l_1, l_3) \notin R$, $\therefore$ R is not a transitive relation

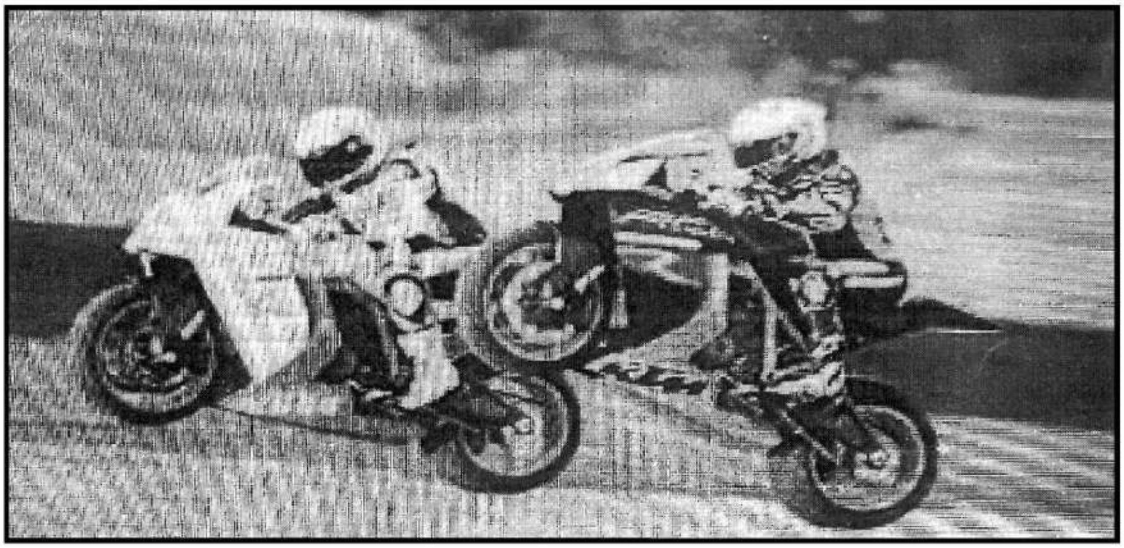
**** Due to printing error Part (a) or Part(b), both parts be taken as independent questions of 4 marks each**

5. 2023

Case Study-I

An organization conducted bike race under two different categories – Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.



Based on the above information, answer the following questions :

(I) How many relations are possible from B to G ?

(II) Among all the possible relations from B to G, how many functions can be formed from B to G ?

(III) Let $R : B \to B$ be defined by $R = \{(x, y) : x \text{ and } y \text{ are students of the same sex}\}$. Check if R is an equivalence relation.

OR

(III) A function $f : B \to G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$.

Check if f is bijective. Justify your answer.

Sol.

(I) Number of relations = $2^6 = 64$

(II) Number of possible functions = $2^3 = 8$

(III) R is an equivalence relation as it is reflexive, symmetric and transitive

OR

Since f is not one-one function

$\therefore f$ is not bijective

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