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10. Vector Algebra

(Previou years Questions 2017 -2025 Solutions)

2022 March:

1.

If the projection of $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ on $\vec{b} = 2\hat{i} + \lambda\hat{k}$ is zero, then the value of λ is

(A) 0

(B) 1

(C) $\frac{-2}{3}$

(D) $\frac{-3}{2}$

2.

The vector equation of the line passing through the point (–1, 5, 4) and perpendicular to the plane z = 0 is

(A) $\vec{r} = -\hat{i} + 5\hat{j} + 4\hat{k} + \lambda(\hat{i} + \hat{j})$

(B) $\vec{r} = -\hat{i} + 5\hat{j} + (4 + \lambda)\hat{k}$

(C) $\vec{r} = \hat{i} - 5\hat{j} - 4\hat{k} + \lambda\hat{k}$

(D) $\vec{r} = \lambda\hat{k}$

3.

If $|\vec{a}| = 4$ and $-3 \le \lambda \le 2$, then $|\lambda\vec{a}|$ lies in

(A) [0, 12]

(B) [2, 3]

(C) [8, 12]

(D) [–12, 8]

4.

The vectors $3\hat{i} - \hat{j} + 2\hat{k}$, $2\hat{i} + \hat{j} + 3\hat{k}$ and $\hat{i} + \lambda\hat{j} - \hat{k}$ are coplanar if value of λ is

(A) -2

(B) 0

(C) 2

(D) Any real number

5.

The area of a triangle formed by vertices O, A and B, where $\vec{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{OB} = -3\hat{i} - 2\hat{j} + \hat{k}$ is

(A) $3\sqrt{5}$ sq. units

(B) $5\sqrt{5}$ sq. units

(C) $6\sqrt{5}$ sq. units

(D) 4 sq. units

q6.

The angle between the vectors $\hat{i} - \hat{j}$ and $\hat{j} - \hat{k}$ is

(A) $-\frac{\pi}{3}$

(B) 0

(C) $\frac{\pi}{3}$

(D) $\frac{2\pi}{3}$

7.

If $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{a} \times \vec{b}| = 6$, then the value of $\vec{a} \cdot \vec{b}$ is

(A) 12

(B) 6

(C) $3\sqrt{3}$

(D) $6\sqrt{3}$

8.

The value of p for which $p(\hat{i} + \hat{j} + \hat{k})$ is a unit vector is

(A) 0

(B) $\frac{1}{\sqrt{3}}$

(C) 1

(D) $\sqrt{3}$

9.

If $\vec{a} \cdot \vec{b} = \frac{1}{2} |\vec{a}| \, |\vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $0°$ (b) $30°$ (c) $60°$ (d) $90°$

10.

Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$. If $\vec{b}$ is a vector such that $\vec{a} \cdot \vec{b} = |\vec{b}|^2$ and $|\vec{a} - \vec{b}| = \sqrt{7}$, then $|\vec{b}|$ equals

(a) 7 (b) 14 (c) $\sqrt{7}$ (d) 21

11.

If $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along three mutually perpendicular directions, then

(a) $\hat{i} \cdot \hat{j} = 1$ (b) $\hat{i} \times \hat{j} = 1$ (c) $\hat{i} \cdot \hat{k} = 0$ (d) $\hat{i} \times \hat{k} = 0$

12

ABCD is a rhombus whose diagonals intersect at E. Then $\overrightarrow{EA} + \overrightarrow{EB} + \overrightarrow{EC} + \overrightarrow{ED}$ equals

(a) $\vec{0}$ (b) $\overrightarrow{AD}$ (c) $2\overrightarrow{BC}$ (d) $2\overrightarrow{AD}$

13.

If two vectors $\vec{a}$ *and* $\vec{b}$ are such that $|\vec{a}| = 2$, $|\vec{b}| = 3$ *and* $\vec{a}.\vec{b} = 4$, *then* $|\vec{a} - 2\vec{b}|$ is equal to

(a) $\sqrt{2}$ (b) $2\sqrt{6}$ (c) 24 (d) $2\sqrt{2}$

14.

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are the position vectors of the points A(2, 3, – 4), B(3, – 4, – 5) and C(3, 2, – 3) respectively, then $|\vec{a} + \vec{b} + \vec{c}|$ is equal to

(A) $\sqrt{113}$

(B) $\sqrt{185}$

(C) $\sqrt{203}$

(D) $\sqrt{209}$

15.

If $\vec{a} = \hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$,

then the value of λ is

(A) 1

(B) – 1

(C) 2

(D) – 2

2023 March:

1.

The value of p for which the vectors $2\hat{i} + p\hat{j} + \hat{k}$ and $-4\hat{i} - 6\hat{j} + 26\hat{k}$ are perpendicular to each other, is :

(a) 3 (b) -3

(c) $-\frac{17}{3}$ (d) $\frac{17}{3}$

2.

The value of $(\hat{i} \times \hat{j}) \cdot \hat{j} + (\hat{j} \times \hat{i}) \cdot \hat{k}$ is :

(a) 2 (b) 0

(c) 1 (d) -1

3.

If $\vec{a} + \vec{b} = \hat{i}$ and $\vec{a} = 2\hat{i} - 2\hat{j} + 2\hat{k}$, then $|\vec{b}|$ equals :

(a) $\sqrt{14}$ (b) 3

(c) $\sqrt{12}$ (d) $\sqrt{17}$

4.

If the vector $\hat{i} - b\hat{j} + \hat{k}$ is equally inclined to the coordinate axes, then the value of b is :

(a) -1 (b) 1

(c) $-\sqrt{3}$ (d) $-\frac{1}{\sqrt{3}}$

5.

For what value of λ, the projection of vector $\hat{i} + \lambda\hat{j}$ on vector $\hat{i} - \hat{j}$ is $\sqrt{2}$?

(a) -1 (b) 1

(c) 0 (d) 3

6.

Projection of vector $2\hat{i} + 3\hat{j}$ on the vector $3\hat{i} - 2\hat{j}$ is

(A) 0 (B) 12

(C) $\frac{12}{\sqrt{13}}$ (D) $\frac{-12}{\sqrt{13}}$

7.

Position vector of the mid-point of line segment AB is $3\hat{i} + 2\hat{j} - 3\hat{k}$. If position vector of the point A is $2\hat{i} + 3\hat{j} - 4\hat{k}$, then position vector of the point B is

(A) $\frac{5\hat{i}}{2} + \frac{5\hat{j}}{2} - \frac{7\hat{k}}{2}$ (B) $4\hat{i} + \hat{j} - 2\hat{k}$

(C) $5\hat{i} + 5\hat{j} - 7\hat{k}$ (D) $\frac{\hat{i}}{2} - \frac{\hat{j}}{2} + \frac{\hat{k}}{2}$

8.

Unit vector along $\overrightarrow{PQ}$, where coordinates of P and Q respectively are (2, 1, –1) and (4, 4, –7), is

(A) $2\hat{i} + 3\hat{j} - 6\hat{k}$ (B) $-2\hat{i} - 3\hat{j} + 6\hat{k}$

(C) $\frac{-2\hat{i}}{7} - \frac{3\hat{j}}{7} + \frac{6\hat{k}}{7}$ (D) $\frac{2\hat{i}}{7} + \frac{3\hat{j}}{7} - \frac{6\hat{k}}{7}$

9.

If in ΔABC, $\overrightarrow{BA} = 2\vec{a}$ and $\overrightarrow{BC} = 3\vec{b}$, then $\overrightarrow{AC}$ is

(A) $2\vec{a} + 3\vec{b}$ (B) $2\vec{a} - 3\vec{b}$

(C) $3\vec{b} - 2\vec{a}$ (D) $-2\vec{a} - 3\vec{b}$

10.

If $|\vec{a} \times \vec{b}| = \sqrt{3}$ and $\vec{a} \cdot \vec{b} = -3$, then angle between $\vec{a}$ and $\vec{b}$ is

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{6}$

(C) $\frac{\pi}{3}$ (D) $\frac{5\pi}{6}$

11.

If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$ and $|\vec{a} \times \vec{b}| = 3\sqrt{3}$, then the value of $\vec{a} \cdot \vec{b}$ is

(A) 9 (B) 3

(C) $\frac{1}{9}$ (D) $\frac{1}{3}$

12.

The position vectors of three consecutive vertices of a parallelogram ABCD are A $(4\hat{i} + 2\hat{j} - 6\hat{k})$, B $(5\hat{i} - 3\hat{j} + \hat{k})$ and C $(12\hat{i} + 4\hat{j} + 5\hat{k})$. The position vector of D is given by

(A) $-3\hat{i} - 5\hat{j} - 10\hat{k}$ (B) $21\hat{i} + 3\hat{j}$

(C) $11\hat{i} + 9\hat{j} - 2\hat{k}$ (D) $-11\hat{i} - 9\hat{j} + 2\hat{k}$

13.

If a vector makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and y-axis, then the angle which it makes with positive z-axis is :

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$

(c) $\frac{\pi}{2}$ (d) 0

14.

$\vec{a}$ and $\vec{b}$ are two non-zero vectors such that the projection of $\vec{a}$ on $\vec{b}$ is 0. The angle between $\vec{a}$ and $\vec{b}$ is :

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{\pi}{4}$ (d) 0

15.

In Δ ABC, $\overrightarrow{AB} = \hat{i} + \hat{j} + 2\hat{k}$ and $\overrightarrow{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$. If D is mid-point of BC, then vector $\overrightarrow{AD}$ is equal to :

(a) $4\hat{i} + 6\hat{k}$ (b) $2\hat{i} - 2\hat{j} + 2\hat{k}$

(c) $\hat{i} - \hat{j} + \hat{k}$ (d) $2\hat{i} + 3\hat{k}$

16.

If the angle between the vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$ and $|\vec{a} \times \vec{b}| = 1$, then $\vec{a} \cdot \vec{b}$ is equal to

(a) -1 (b) 1

(c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

17.

A unit vector along the vector $4\hat{i} - 3\hat{k}$ is :

(a) $\frac{1}{7}(4\hat{i} - 3\hat{k})$

(b) $\frac{1}{5}(4\hat{i} - 3\hat{k})$

(c) $\frac{1}{\sqrt{7}}(4\hat{i} - 3\hat{k})$

(d) $\frac{1}{\sqrt{5}}(4\hat{i} - 3\hat{k})$

18.

If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then $\vec{a} \cdot \vec{b} \geq 0$ only when :

(a) $0 < \theta < \frac{\pi}{2}$ (b) $0 \leq \theta \leq \frac{\pi}{2}$

(c) $0 < \theta < \pi$ (d) $0 \leq \theta \leq \pi$

19.

The sine of the angle between the vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$ is :

(a) $\sqrt{\frac{5}{21}}$ (b) $\frac{5}{\sqrt{21}}$

(c) $\sqrt{\frac{3}{21}}$ (d) $\frac{4}{\sqrt{21}}$

20.

The point (x, y, 0) on the xy-plane divides the line segment joining the points (1, 2, 3) and (3, 2, 1) in the ratio :

(a) 1 : 2 internally (b) 2 : 1 internally

(c) 3 : 1 internally (d) 3 : 1 externally

21.

Two vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are collinear if

(a) $a_1b_1 + a_2b_2 + a_3b_3 = 0$ (b) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

(c) $a_1 = b_1, a_2 = b_2, a_3 = b_3$ (d) $a_1 + a_2 + a_3 = b_1 + b_2 + b_3$

22.

The magnitude of the vector $6\hat{i} - 2\hat{j} + 3\hat{k}$ is

(a) 1 (b) 5

(c) 7 (d) 12

23.

A unit vector $\hat{a}$ makes equal but acute angles on the co-ordinate axes. The projection of the vector $\hat{a}$ on the vector $\vec{b} = 5\hat{i} + 7\hat{j} - \hat{k}$ is

(a) $\frac{11}{15}$ (b) $\frac{11}{5\sqrt{3}}$

(c) $\frac{4}{5}$ (d) $\frac{3}{5\sqrt{3}}$

24.

If $\vec{a} \cdot \hat{i} = \vec{a} \cdot (\hat{i} + \hat{j}) = \vec{a} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$, then $\vec{a}$ is

(a) $\hat{k}$ (b) $\hat{i}$

(c) $\hat{j}$ (d) $\hat{i} + \hat{j} + \hat{k}$

25.

If $\vec{a}$, $\vec{b}$ and $(\vec{a} + \vec{b})$ are all unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$, then the value of θ is :

(a) $\frac{2\pi}{3}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

26.

The projection of vector $\hat{i}$ on the vector $\hat{i} + \hat{j} + 2\hat{k}$ is :

(a) $\frac{1}{\sqrt{6}}$ (b) $\sqrt{6}$ (c) $\frac{2}{\sqrt{6}}$ (d) $\frac{3}{\sqrt{6}}$

27.

If ABCD is a parallelogram and AC and BD are its diagonals, then $\vec{AC} + \vec{BD}$ is :

(a) $2\vec{DA}$ (b) $2\vec{AB}$ (c) $2\vec{BC}$ (d) $2\vec{BD}$

28.

Let P and Q be two points with position vectors $\vec{a} - 2\vec{b}$ and $2\vec{a} + \vec{b}$ respectively. The position vector of a point which divides the join of P and Q externally in the ratio 3 : 2 is :

(a) $4\vec{a} + 7\vec{b}$ (b) $\frac{8\vec{a} + 7\vec{b}}{5}$

(c) $4\vec{a} - 7\vec{b}$ (d) $\vec{a} + 4\vec{b}$

29.

If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then the value of θ is :

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$

30.

The position vectors of two points A and B are $(2\vec{a} + \vec{b})$ and $(\vec{a} - 3\vec{b})$ respectively. The position vector of a point C which divides AB externally in the ratio 1 : 2 is :

(a) $-3\vec{a} - 5\vec{b}$ (b) $-7\vec{b}$

(c) $\frac{1}{3}(5\vec{a} - \vec{b})$ (d) $(3\vec{a} + 5\vec{b})$

31.

The value of λ for which the points A, B and C having position vectors $(3\hat{i} - 2\hat{j} + 4\hat{k})$, $(\hat{i} + \lambda\hat{j} + \hat{k})$ and $(-\hat{i} + 4\hat{j} - 2\hat{k})$ respectively are collinear, is :

(a) 4 (b) 1

(c) 3 (d) 2

32.

Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. $\vec{a} + \vec{b}$ is a unit vector, if :

(a) $\theta = \frac{\pi}{3}$ (b) $\theta = \frac{\pi}{4}$

(c) $\theta = \frac{\pi}{2}$ (d) $\theta = \frac{2\pi}{3}$

33.

If $(2\hat{i} + 6\hat{j} - 22\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0}$, then $\lambda - \mu$ is equal to :

(a) -8 (b) -14

(c) 14 (d) 8

34.

If $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + p\hat{j} + q\hat{k}) = \vec{0}$,then the values of p and q are ?

(a) p= 6 ,q=27(b)p=3,q=$\frac{27}{2}$ (c) p=6,q= $\frac{27}{2}$ (d) p=3 ,q=27

2024 March :

1.

Let θ be the angle between two unit vectors $\hat{a}$ and $\hat{b}$ such that $\sin\theta = \frac{3}{5}$.

Then, $\hat{a} \cdot \hat{b}$ is equal to :

(A) $\pm\frac{3}{5}$ (B) $\pm\frac{3}{4}$

(C) $\pm\frac{4}{5}$ (D) $\pm\frac{4}{3}$

2.

The vector with terminal point A (2, – 3, 5) and initial point B (3, – 4, 7) is :

(A) $\hat{i} - \hat{j} + 2\hat{k}$ (B) $\hat{i} + \hat{j} + 2\hat{k}$

(C) $-\hat{i} - \hat{j} - 2\hat{k}$ (D) $-\hat{i} + \hat{j} - 2\hat{k}$

For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true ?

(A) $\vec{a} \cdot \vec{b} \geq |\vec{a}||\vec{b}|$ (B) $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|$

(C) $\vec{a} \cdot \vec{b} \leq |\vec{a}||\vec{b}|$ (D) $\vec{a} \cdot \vec{b} < |\vec{a}||\vec{b}|$

4.

The unit vector perpendicular to both vectors $\hat{i} + \hat{k}$ and $\hat{i} - \hat{k}$ is :

(A) $2\hat{j}$ (B) $\hat{j}$

(C) $\frac{\hat{i} - \hat{k}}{\sqrt{2}}$ (D) $\frac{\hat{i} + \hat{k}}{\sqrt{2}}$

5.

Assertion (A) : For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$.

Reason (R) : For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$.

6.

Assertion (A) : $(\vec{b} \cdot \vec{c})\, \vec{a}$ is a scalar quantity.

Reason (R) : Dot product of two vectors is a scalar quantity.

7.

If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$, then $\vec{a}$ and $\vec{b}$ are :

(A) collinear vectors which are not parallel

(B) parallel vectors

(C) perpendicular vectors

(D) unit vectors

8.

If $|\vec{a}| = 2$ and $-3 \le k \le 2$, then $|k\vec{a}| \in$:

(A) $[-6, 4]$ (B) $[0, 4]$

(C) $[4, 6]$ (D) $[0, 6]$

9.

If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and z-axis, then the angle which it makes with the positive direction of y-axis is :

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π

10.

The position vectors of points P and Q are $\vec{p}$ and $\vec{q}$ respectively. The point R divides line segment PQ in the ratio 3 : 1 and S is the mid-point of line segment PR. The position vector of S is :

(A) $\frac{\vec{p} + 3\vec{q}}{4}$ (B) $\frac{\vec{p} + 3\vec{q}}{8}$

(C) $\frac{5\vec{p} + 3\vec{q}}{4}$ (D) $\frac{5\vec{p} + 3\vec{q}}{8}$

11.

If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{3}$, then the angle between $2\vec{a}$ and $-\vec{b}$ is :

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\frac{5\pi}{6}$ (D) $\frac{11\pi}{6}$

12.

The vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 3\hat{j} - 5\hat{k}$ and $\vec{c} = -3\hat{i} + 4\hat{j} + 4\hat{k}$ represents the sides of

(A) an equilateral triangle (B) an obtuse-angled triangle

(C) an isosceles triangle (D) a right-angled triangle

13.

Let $\vec{a}$ be any vector such that $|\vec{a}| = a$. The value of

$$|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2 \text{ is :}$$

(A) a^2 (B) $2a^2$

(C) $3a^2$ (D) 0

14.

Assertion (A) : Projection of $\vec{a}$ on $\vec{b}$ is same as projection of $\vec{b}$ on $\vec{a}$.

Reason (R) : Angle between $\vec{a}$ and $\vec{b}$ is same as angle between $\vec{b}$ and $\vec{a}$ numerically.

15.

Assertion (A) : The vectors

$\vec{a} = 6\hat{i} + 2\hat{j} - 8\hat{k}$

$\vec{b} = 10\hat{i} - 2\hat{j} - 6\hat{k}$

$\vec{c} = 4\hat{i} - 4\hat{j} + 2\hat{k}$

represent the sides of a right angled triangle.

Reason (R) : Three non-zero vectors of which none of two are collinear forms a triangle if their resultant is zero vector or sum of any two vectors is equal to the third.

2025 March :

1.

If vector $\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}$ and vector $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, then which of the following is correct ?

(A) $\vec{a} \parallel \vec{b}$

(B) $\vec{a} \perp \vec{b}$

(C) $|\vec{b}| > |\vec{a}|$

(D) $|\vec{a}| = |\vec{b}|$

2.

If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $|\vec{a}| = \sqrt{37}$, $|\vec{b}| = 3$ and $|\vec{c}| = 4$, then angle between $\vec{b}$ and $\vec{c}$ is

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

3.

If $\vec{\alpha} = \hat{i} - 4\hat{j} + 9\hat{k}$ and $\vec{\beta} = 2\hat{i} - 8\hat{j} + \lambda\hat{k}$ are two mutually parallel vectors, then λ is equal to :

(A) -18

(B) 18

(C) $\frac{-34}{9}$

(D) $\frac{34}{9}$

4.

The unit vector perpendicular to the vectors $\hat{i} - \hat{j}$ and $\hat{i} + \hat{j}$ is

(A) $\hat{k}$

(B) $-\hat{k} + \hat{j}$

(C) $\frac{\hat{i} - \hat{j}}{\sqrt{2}}$

(D) $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$

5.

The projection vector of vector $\vec{a}$ on vector $\vec{b}$ is

(A) $\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2}\right)\vec{b}$

(B) $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

(C) $\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$

(D) $\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2}\right)\vec{b}$

4.

Let $\vec{p}$ and $\vec{q}$ be two unit vectors and α be the angle between them. Then $(\vec{p} + \vec{q})$ will be a unit vector for what value of α ?

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$ (D) $\frac{2\pi}{3}$

5.

The values of x for which the angle between the vectors $\vec{a} = 2x^2\hat{i} + 4x\hat{j} + \hat{k}$ and $\vec{b} = 7\hat{i} - 2\hat{j} + x\hat{k}$ is obtuse, is :

(A) 0 or $\frac{1}{2}$ (B) $x > \frac{1}{2}$

(C) $\left(0, \frac{1}{2}\right)$ (D) $\left[0, \frac{1}{2}\right]$

6.

If $\overrightarrow{PQ} \times \overrightarrow{PR} = 4\hat{i} + 8\hat{j} - 8\hat{k}$, then the area (ΔPQR) is

(A) 2 sq units (B) 4 sq units

(C) 6 sq units (D) 12 sq units

7.

If $\vec{p}$ and $\vec{q}$ are unit vectors, then which of the following values of $\vec{p} \cdot \vec{q}$ is not possible ?

(A) $\frac{-1}{2}$ (B) $\frac{1}{\sqrt{2}}$

(C) $\frac{\sqrt{3}}{2}$ (D) $\sqrt{3}$

8.

If projection of $\vec{a} = \alpha\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units, then α is

(A) –13 (B) –5

(C) 13 (D) 5

10.

If the sides AB and AC of Δ ABC are represented by vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ respectively, then the length of the median through A on BC is :

(A) $2\sqrt{2}$ units (B) $\sqrt{18}$ units

(C) $\frac{\sqrt{34}}{2}$ units (D) $\frac{\sqrt{48}}{2}$ units

11.

Let $\vec{a}$ be a position vector whose tip is the point (2, – 3). If $\overrightarrow{AB} = \vec{a}$, where coordinates of A are (– 4, 5), then the coordinates of B are :

(A) (– 2, – 2) (B) (2, – 2) (C) (– 2, 2) (D) (2, 2)

12.

The respective values of $|\vec{a}|$ and $|\vec{b}|$, if given $(\vec{a} - \vec{b}) \cdot (\vec{a} + \vec{b}) = 512$ and $|\vec{a}| = 3|\vec{b}|$, are :

(A) 48 and 16 (B) 3 and 1

(C) 24 and 8 (D) 6 and 2

13.

Let $|\vec{a}| = 5$ and $-2 \le \lambda \le 1$. Then, the range of $|\lambda \vec{a}|$ is :

(A) [5, 10] (B) [– 2, 5]

(C) [– 2, 1] (D) [–10, 5]

14.

Assertion (A) : If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 256$ and $|\vec{b}| = 8$, then $|\vec{a}| = 2$.

Reason (R) : $\sin^2\theta + \cos^2\theta = 1$ and

$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$ and $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$.

15.

A student tries to tie ropes, parallel to each other from one end of the wall to the other. If one rope is along the vector $3\hat{i} + 15\hat{j} + 6\hat{k}$ and the other is along the vector $2\hat{i} + 10\hat{j} + \lambda\hat{k}$, then the value of λ is :

(A) 6 (B) 1

(C) $\frac{1}{4}$ (D) 4

12.

If $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$ for any two vectors, then vectors $\vec{a}$ and $\vec{b}$ are :

(A) orthogonal vectors (B) parallel to each other

(C) unit vectors (D) collinear vectors

I. vector/Magnitudes based :

1.a

Find all the vectors of magnitude $3\sqrt{3}$ which are collinear to vector $\hat{i}+\hat{j}+\hat{k}$.

Unit vector along $\hat{i}+\hat{j}+\hat{k}$ is $\frac{\hat{i}}{\sqrt{3}}+\frac{\hat{j}}{\sqrt{3}}+\frac{\hat{k}}{\sqrt{3}}$

Required vectors are $3\hat{i}+3\hat{j}+3\hat{k}$ and $-3\hat{i}-3\hat{j}-3\hat{k}$

1.b 2023

Find a vector $\vec{r}$ equally inclined to the three axes and whose magnitude is $3\sqrt{3}$ units.

Sol.

Ans: Let the vector $\vec{r} = a\hat{i}+a\hat{j}+a\hat{k}$

$\therefore \quad \sqrt{3a^2} = 3\sqrt{3}$

required vector is $3\hat{i}+3\hat{j}+3\hat{k}$ or $-3\hat{i}-3\hat{j}-3\hat{k}$

1.c

Find all the possible vectors of magnitude $5\sqrt{3}$ which are equally inclined to the coordinate axes.

Sol.

Let the required vector be $x\hat{i}+x\hat{j}+x\hat{k}$

$\sqrt{3x^2} = 5\sqrt{3}$

$x^2 = 25 \Rightarrow x = \pm 5$

Required vectors are $5\hat{i}+5\hat{j}+5\hat{k}$ or $-5\hat{i}-5\hat{j}-5\hat{k}$.

2. 2024

Find a vector of magnitude 21 units in the direction opposite to that of $\overrightarrow{AB}$ where A and B are the points A(2, 1, 3) and B(8, –1, 0) respectively.

Sol.

$$\overrightarrow{BA} = -6\hat{i} + 2\hat{j} + 3\hat{k}$$

Required unit vector of magnitude 21

$$= 21 \times \left(\frac{-6\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{36+4+9}} \right)$$

$$= 3\left(-6\hat{i} + 2\hat{j} + 3\hat{k}\right) \text{ or } -18\hat{i} + 6\hat{j} + 9\hat{k}$$

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3.2025

A vector $\vec{a}$ makes equal angles with all the three axes. If the magnitude of the vector is $5\sqrt{3}$ units, then find $\vec{a}$.

Sol.

Let α be the angle which the vector $\vec{a}$ makes with all the three axes.

Then $3\cos^2\alpha = 1$

$$\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$$

The unit vector along the vector $\vec{a} = \frac{1}{\sqrt{3}}(\hat{\imath} + \hat{\jmath} + \hat{k})$

$$\vec{a} = 5(\hat{\imath} + \hat{\jmath} + \hat{k})$$

4. 2025

Vector $\vec{r}$ is inclined at equal angles to the three axes x, y and z. If magnitude of $\vec{r}$ is $5\sqrt{3}$ units, then find $\vec{r}$.

Sol.

Unit vector equally inclined along coordinate axes is $\frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{3}}$

$$\vec{r} = 5\sqrt{3}\left(\frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{3}}\right) = 5\hat{i} + 5\hat{j} + 5\hat{k} \qquad (\text{or } -5\hat{i} - 5\hat{j} - 5\hat{k})$$

5. 2017

If $\vec{a} = 2\hat{i} - \hat{j} - 2\hat{k}$ and $\vec{b} = 7\hat{i} + 2\hat{j} - 3\hat{k}$, then express $\vec{b}$ in the form of $\vec{b} = \vec{b}_1 + \vec{b}_2$, where $\vec{b}_1$ is parallel to $\vec{a}$ and $\vec{b}_2$ is perpendicular to $\vec{a}$.

Sol.

$\vec{b}_1 \,||\, \vec{a} \Rightarrow$ let $\vec{b}_1 = \lambda(2\hat{i} - \hat{j} - 2\hat{k})$

$\vec{b}_2 = \vec{b} - \vec{b}_1 = (7\hat{i} + 2\hat{j} - 3\hat{k}) - (2\lambda i - \lambda\hat{j} - 2\lambda\hat{k})$

$= (7 - 2\lambda)\hat{i} + (2 + \lambda)\hat{j} - (3 - 2\lambda)\hat{k}$

$\vec{b}_2 \perp \vec{a} \Rightarrow 2(7 - 2\lambda) - 1(2 + \lambda) + 2(3 - 2\lambda) = 0$

$\Rightarrow \lambda = 2$

$\therefore\ \vec{b}_1 = 4\hat{i} - 2\hat{j} - 4\hat{k}$ and $\vec{b}_2 = 3\hat{i} + 4\hat{j} + \hat{k}$

$\Rightarrow (7\hat{i} + 2\hat{j} - 3\hat{k}) = (4\hat{i} - 2\hat{j} - 4\hat{k}) + (3\hat{i} + 4\hat{j} + \hat{k})$

6. 2023

Express the vector $\vec{a} = 5\hat{i} - 2\hat{j} + 5\hat{k}$ as the sum of two vectors such that one is parallel to the vector $\vec{b} = (3\hat{i} + \hat{k})$ and the other is perpendicular to $\vec{b}$.

Ans.

Let $\vec{a} = \vec{c} + \vec{d}$, $\vec{c} \,||\, \vec{b} \Rightarrow \vec{c} = \lambda\vec{b}$ $\therefore \vec{c} = 3\lambda\hat{i} + \lambda\hat{k}$ and $\vec{d} = (5 - 3\lambda)\hat{i} - 2\hat{j} + (5 - \lambda)\hat{k}$

$\vec{b} \cdot \vec{d} = 0 \Rightarrow 15 - 9\lambda + 5 - \lambda = 0 \Rightarrow \lambda = 2$

$\vec{a} = (6\hat{i} + 2\hat{k}) + \lambda(-\hat{i} - 2\hat{j} + 3\hat{k})$

7. 2025

Let $\vec{p} = 2\hat{i} - 3\hat{j} - \hat{k}$, $\vec{q} = -3\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{r} = \hat{i} + \hat{j} + 2\hat{k}$. Express $\vec{r}$ in the form of $\vec{r} = \lambda\vec{p} + \mu\vec{q}$ and hence find the values of λ and μ.

Sol.

$\vec{r} = \lambda\vec{p} + \mu\vec{q}$

$\Rightarrow 1 = 2\lambda - 3\mu, 1 = -3\lambda + 4\mu, 2 = -\lambda + \mu$

$\Rightarrow \lambda = -7, \mu = -5$

II. Dividing vector section formula internally/externally :

1.a

X and Y are two points with position vectors $3\vec{a} + \vec{b}$ and $\vec{a} - 3\vec{b}$ respectively. Write the position vector of a point Z which divides the line segment XY in the ratio 2 : 1 externally.

Sol.

$$\text{Position vector of } z = \frac{2(\vec{a} - 3\vec{b}) - 1(3\vec{a} + \vec{b})}{2 - 1}$$

$$= -\vec{a} - 7\vec{b}$$

2. 2022

Using vectors, find the value of 'b' if the points A(–1, –1, 2), B(2, b, 5) and C(3, 11, 6) are collinear . Also, determine the ratio in which the point B divides the line-segment AC internally.

Ans.

Let $\vec{a}, \vec{b}, \vec{c}$ be the position vectors of points A, B, C respectively

$\vec{a} = -\hat{\imath} - \hat{\jmath} + 2\hat{k}$

$\vec{b} = 2\hat{\imath} + b\hat{\jmath} + 5\hat{k}$

$\vec{c} = 3\hat{\imath} + 11\hat{\jmath} + 6\hat{k}$

$\overrightarrow{AB} = \vec{b} - \vec{a} = 3\hat{\imath} + (b+1)\hat{\jmath} + 3\hat{k}$

$\overrightarrow{AC} = \vec{c} - \vec{a} = 4\hat{\imath} + 12\hat{\jmath} + 4\hat{k}$

As A, B, C are collinear

$$\frac{3}{4} = \frac{b+1}{12} = \frac{3}{4}$$

$$\Rightarrow b = 8$$

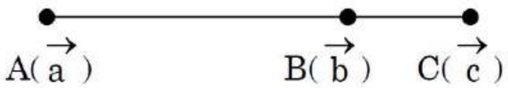
$$|\overrightarrow{AB}| = \sqrt{9+81+9} = \sqrt{99}$$
$$= 3\sqrt{11}$$

$$|\overrightarrow{AC}| = \sqrt{16+144+16}$$
$$= \sqrt{176} = 4\sqrt{11}$$

Here, B divides AC in the ratio 3 : 1

3.2024

Position vectors of the points A, B and C as shown in the figure below are $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively.



If $\vec{AC} = \frac{5}{4}\vec{AB}$, express $\vec{c}$ in terms of $\vec{a}$ and $\vec{b}$.

Sol.

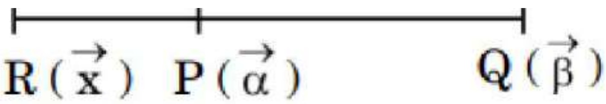
According to question, $\vec{c} - \vec{a} = \frac{5}{4}(\vec{b} - \vec{a})$

$\therefore \vec{c} = \frac{5\vec{b}}{4} - \frac{\vec{a}}{4}$

4.a 2025

If $\vec{\alpha}$ and $\vec{\beta}$ are position vectors of two points P and Q respectively, then find the position vector of a point R in QP produced such that $QR = \frac{3}{2}QP$.

Sol.



$\frac{QR}{QP} = \frac{3}{2}$

Hence, R divides PQ, externally, in the ratio 1:3.

The Position vector of R = $\vec{x} = \frac{\vec{\beta}-3\vec{\alpha}}{1-3} = \frac{3\vec{\alpha}-\vec{\beta}}{2}$

4.b 2025

If $\vec{a}$ and $\vec{b}$ are position vectors of point A and point B respectively, find the position vector of point C on BA produced such that BC = 3BA.

Sol.

C divides BA in the ratio 3 : 2 externally

Required vector = $\vec{c} = \frac{3\vec{a}-2\vec{b}}{3-2} = 3\vec{a} - 2\vec{b}$

5. 2024

Find the position vector of point C which divides the line segment joining points A and B having position vectors $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively in the ratio 4 : 1 externally. Further, find $|\overrightarrow{AB}| : |\overrightarrow{BC}|$.

Sol.

Position vector of $C = \vec{r} = \dfrac{4\vec{b} - \vec{a}}{3}$

i.e. $\vec{r} = \dfrac{1}{3}\left(-5\hat{i} + 2\hat{j} + 5\hat{k}\right)$

Now, $\overrightarrow{AB} = -2\hat{i} - \hat{j} + 2\hat{k} \Rightarrow \left|\overrightarrow{AB}\right| = 3$

$\overrightarrow{BC} = -\dfrac{1}{3}\left(2\hat{i} + \hat{j} - 2\hat{k}\right) \Rightarrow \left|\overrightarrow{BC}\right| = 1$

$\left|\overrightarrow{AB}\right| : \left|\overrightarrow{BC}\right| = 3 : 1$

6. 2025

A man needs to hang two lanterns on a straight wire whose end points have coordinates A (4, 1, – 2) and B (6, 2, – 3). Find the coordinates of the points where he hangs the lanterns such that these points trisect the wire AB.

Sol.

Let P and Q trisect the wire AB.

P divides AB in the ratio 1:2 then, coordinate of point P = $\left(\frac{14}{3}, \frac{4}{3}, -\frac{7}{3}\right)$

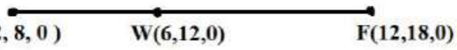
Q divides AB in the ratio 2:1 then, coordinate of point Q = $\left(\frac{16}{3}, \frac{5}{3}, -\frac{8}{3}\right)$

7. 2025

During a cricket match, the position of the bowler, the wicket keeper and the leg slip fielder are in a line given by $\vec{B} = 2\hat{i} + 8\hat{j}$, $\vec{W} = 6\hat{i} + 12\hat{j}$ and $\vec{F} = 12\hat{i} + 18\hat{j}$ respectively. Calculate the ratio in which the wicketkeeper divides the line segment joining the bowler and the leg slip fielder.

sol.

Let the wicket keeper divides the line segment in ratio $k:1$



$$\therefore \vec{W} = \frac{k\vec{F} + 1.\vec{B}}{k+1}$$

$$\Rightarrow 6\hat{i} + 12\hat{j} = \left(\frac{12k+2}{k+1}\right)\hat{i} + \left(\frac{18k+8}{k+1}\right)\hat{j}$$

$$\Rightarrow k = \frac{2}{3}$$

Hence, the required ratio is $2:3$

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III. Direction cosines of a vector :

1.

What is the cosine of the angle which the vector $\sqrt{2}\hat{i} + \hat{j} + k$ makes with y-axis?

Sol.

Let β be angle which $\sqrt{2}\hat{i} + \hat{j} + \hat{k}$ makes with y axix

$$\cos\beta = \frac{1}{\sqrt{(\sqrt{2})^2 + 1^2 + 1^2}} = \frac{1}{2}$$

IV. Two vectors parallel/collinear :

1.

If $\vec{a} = \alpha\hat{i} + 3\hat{j} - 6\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} - \beta\hat{k}$, find the value of α and β so that $\vec{a}$ and $\vec{b}$ may be collinear.

Ans. $\vec{a}$ and $\vec{b}$ are collinear.

$$\therefore \frac{\alpha}{2} = \frac{3}{-1} = \frac{-6}{-\beta} \Rightarrow \alpha = -6, \beta = -2$$

2. 2025

If $\vec{a}$ and $\vec{b}$ are two non-collinear vectors, then find x, such that $\vec{\alpha} = (x - 2)\vec{a} + \vec{b}$ and $\vec{\beta} = (3 + 2x)\vec{a} - 2\vec{b}$ are collinear.

Sol.

$\vec{\alpha}$ and $\vec{\beta}$ are collinear

$$\Rightarrow \frac{x-2}{3+2x} = \frac{1}{-2}$$

$$\Rightarrow x = \frac{1}{4}$$

V. Three points are collinear :

1.

Using vectors, prove that the points (2, – 1, 3), (3, – 5, 1) and (– 1, 11, 9) are collinear.

Sol.

Let $P(2,-1,3), Q(3,-5,1)$ and $R(-1,11,9)$ be three point.

$$\overrightarrow{PQ} = \hat{i} - 4\hat{j} - 2\hat{k}$$

$$\overrightarrow{PR} = -3\hat{i} + 12\hat{j} + 6\hat{k} = -3(\hat{i} - 4\hat{j} - 2\hat{k})$$

$\because \overrightarrow{PR} = -3\overrightarrow{PQ}$, since P is common.

Therefore the points P, Q and R are collinear.

2. 2025

Find the value of λ, if the points $(-1, -1, 2)$, $(2, 8, \lambda)$ and $(3, 11, 6)$ are collinear.

Sol.

$A(-1,-1,2), B(2,8,\lambda), C(3,11,6)$

$\overrightarrow{AB} = 3\hat{\imath} + 9\hat{\jmath} + (\lambda - 2)\hat{k}$ and $\overrightarrow{BC} = \hat{\imath} + 3\hat{\jmath} + (6 - \lambda)\hat{k}$

Since A, B and C are collinear, $\frac{3}{1} = \frac{9}{3} = \frac{\lambda - 2}{6 - \lambda}$

$\Rightarrow \lambda = 5$

3.

Using vectors, find the value of x such that the four points A(x, 5, – 1), B(3, 2, 1), C(4, 5, 5) and D(4, 2, – 2) are coplanar.

Sol.

A(x, 5, –1), B(3, 2, 1), C(4, 5, 5), D(4, 2, –2)

$$\left.\begin{aligned} \overrightarrow{BA} &= (x-3)\hat{i} + 3\hat{j} - 2\hat{k} \\ \overrightarrow{BC} &= 1\hat{i} + 3\hat{j} + 4\hat{k} \\ \overrightarrow{BD} &= 1\hat{i} + 0\hat{j} - 3\hat{k} \end{aligned}\right\}$$

$$\begin{vmatrix} x-3 & 3 & -2 \\ 1 & 3 & 4 \\ 1 & 0 & -3 \end{vmatrix} = 0$$

i.e., $(x - 3)(-9) - 3(-7) - 2(-3) = 0$

$\Rightarrow x = 6$

4.

Prove that three points A, B and C with position vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively are collinear if and only if $(\vec{b} \times \vec{c}) + (\vec{c} \times \vec{a}) + (\vec{a} \times \vec{b}) = \vec{0}$

Ans. Points $A(\vec{a}), B(\vec{b})$ and $C(\vec{c})$ are collinear

$$\Rightarrow \overrightarrow{AB} \times \overrightarrow{AC} = \vec{0}$$

$$\Rightarrow (\vec{b} - \vec{a}) \times (\vec{c} - \vec{a}) = \vec{0}$$

$$\Rightarrow \vec{b} \times \vec{c} - \vec{b} \times \vec{a} - \vec{a} \times \vec{c} + \vec{a} \times \vec{a} = \vec{0}$$

$$\Rightarrow \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \vec{0}$$

Similarly, converse can be proved

VI. Three vectors forms right angle triangle :

1.a

Show that the three vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$, and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form the vertices of a right-angled triangle.

Ans: Let $A(2\hat{i} - \hat{j} + \hat{k}), B(\hat{i} - 3\hat{j} - 5\hat{k}), C(3\hat{i} - 4\hat{j} - 4\hat{k})$

then $\overrightarrow{AB} = -\hat{i} - 2\hat{j} - 6\hat{k}, \overrightarrow{BC} = 2\hat{i} - \hat{j} + \hat{k}, \overrightarrow{CA} = -\hat{i} + 3\hat{j} + 5\hat{k}$

$$|\overrightarrow{AB}| = \sqrt{41}, |\overrightarrow{BC}| = \sqrt{6}, \ |\overrightarrow{CA}| = \sqrt{35}, \ |\overrightarrow{CA}|^2 + |\overrightarrow{BC}|^2 = |\overrightarrow{AB}|^2$$

$\therefore$ A, B, C are vertices of a righy angle. prepared by : BALAJI KANCHI

1.b

Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $3\hat{i} + 7\hat{j} + \hat{k}$ and $5\hat{i} + 6\hat{j} + 2\hat{k}$ form the sides of a right-angled triangle.

Ans: Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} + 7\hat{j} + \hat{k}$ and $\vec{c} = 5\hat{i} + 6\hat{j} + 2\hat{k}$

Since $\vec{c} = \vec{a} + \vec{b}$, three vectors form a triangle.

Also, $\vec{a} \cdot \vec{b} = 0$.

So, triangle is a right angled triangle.

1.c 2024

Show that the vectors $3\hat{i} + \hat{j} - 2\hat{k}$, $2\hat{i} - \hat{j} - 8\hat{k}$ and $4\hat{i} - 2\hat{j} - 7\hat{k}$ form the vertices of a right triangle.

Sol.

Let $A(3\hat{i}+\hat{j}-2\hat{k})$, $B(2\hat{i}-\hat{j}-8\hat{k})$, and $C(4\hat{i}-2\hat{j}-7\hat{k})$, then

$\overrightarrow{AB} = -\hat{i} - 2\hat{j} - 6\hat{k}$, $\overrightarrow{BC} = 2\hat{i} - \hat{j} + \hat{k}$, $\overrightarrow{CA} = -\hat{i} + 3\hat{j} + 5\hat{k}$ **and** $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{O}$

$\overrightarrow{BC} \cdot \overrightarrow{CA} = -2 - 3 + 5 = 0 \Rightarrow \overrightarrow{BC} \perp \overrightarrow{CA}$, $\therefore$ **ABC is a right triangle.**

2.

If points A, B and C have position vectors $2\hat{i}$, $\hat{j}$ and $2\hat{k}$ respectively, then show that ΔABC is an isosceles triangle.

Sol.

$\overrightarrow{BA} = 2\hat{i} - \hat{j}$; $\overrightarrow{BC} = 2\hat{k} - \hat{j}$

$|\overrightarrow{BA}| = |\overrightarrow{BC}| = \sqrt{5} \Rightarrow$ **ΔABC is an isosceles triangle**

prepared by : BALAJI KANCHI

VII. scalar(dot) product :

1.

Find the magnitude of each of the two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such that the angle between them is $60°$ and their scalar product is $\frac{9}{2}$.

Ans.

$|\vec{a}| = |\vec{b}| = 3$

2.a 2025

The scalar product of the vector $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ with a unit vector along sum of vectors $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = \lambda\hat{i} - 2\hat{j} - 3\hat{k}$ is equal to 1. Find the value of λ.

Sol.

Let $\vec{d} = \vec{b} + \vec{c} = (2+\lambda)\hat{i} - 6\hat{j} + 2\hat{k}$

$$\hat{d} = \frac{(2+\lambda)\hat{i} - 6\hat{j} + 2\hat{k}}{\sqrt{(2+\lambda)^2 + 40}}$$

$$\vec{a} \cdot \hat{d} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) \cdot \frac{(2+\lambda)\hat{i} - 6\hat{j} + 2\hat{k}}{\sqrt{(2+\lambda)^2 + 40}} = 1$$

$$\Rightarrow (2+\lambda) + 6 + 4 = \sqrt{(2+\lambda)^2 + 40} \Rightarrow \lambda = -5$$

2.b

If the scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to 1, find the value of λ.

Sol.

Sum of given vectors is $(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}$

Unit vector is $\dfrac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+\lambda)^2 + 40}}$

Acc. to Question $(\hat{i} + \hat{j} + \hat{k}) \cdot \dfrac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+\lambda)^2 + 40}} = 1$

$\Rightarrow 2 + \lambda + 6 - 2 = \sqrt{(2+\lambda)^2 + 40}$

$\Rightarrow 8\lambda = 8 \Rightarrow \lambda = 1$

2.c

The scalar product of the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of the vectors $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to 1. Find the value of λ and hence find the unit vector along $\vec{b} + \vec{c}$.

Sol.

$$\vec{a}\cdot\frac{(\vec{b}+\vec{c})}{|\vec{b}+\vec{c}|}=1$$

$$\Rightarrow \quad (\hat{i}+\hat{j}+\hat{k})\cdot\{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}\}=\sqrt{(2+\lambda)^2+36+4}$$

$$\Rightarrow \quad \lambda + 6 = \sqrt{(2+\lambda)^2+40}$$

Squaring to get

$$\lambda^2+12\lambda+36=\lambda^2+4\lambda+44 \;\Rightarrow\; \lambda=1$$

$\therefore$ Unit vector along $(\vec{b}+\vec{c})$ is $\frac{3}{7}\hat{i}+\frac{6}{7}\hat{j}-\frac{2}{7}\hat{k}$

a. Projection of vectors :

1.

Find the projection of the vector $7\hat{i} - \hat{j} + 8\hat{k}$ on the vector $\hat{i} + 2\hat{j} + 2\hat{k}$.

Sol.

$$p=\frac{(7\hat{i}-\hat{j}+8\hat{k}).(\hat{i}+2\hat{j}+2\hat{k})}{|\hat{i}+2\hat{j}+2\hat{k}|}$$

$$=\frac{(7)(1)+(-1)(2)+(8)(2)}{\sqrt{1^2+2^2+2^2}}=7$$

2.a

Write the projection of the vector $(\vec{b} + \vec{c})$ on the vector $\vec{a}$, where $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$.

Sol.

$\vec{b} + \vec{c} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$

Projection of $\vec{b} + \vec{c}$ on $\vec{a} = \frac{(\vec{b}+\vec{c}).\vec{a}}{|\vec{a}|}$

$$= \frac{(3\hat{i} + \hat{j} + 2\hat{k}) \cdot (2\hat{i} - 2\hat{j} + \hat{k})}{\sqrt{9}}$$

$$= \frac{6-2+2}{3} = \frac{6}{3} = 2$$

prepared by : BALAJI KANCHI

2.b 2024

Find the projection of vector $(\vec{b} + \vec{c})$ on vector $\vec{a}$, where $\vec{a} = 2\hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + \hat{k}$.

Sol.

$\vec{b} + \vec{c} = 2\hat{i} + 3\hat{j} + 2\hat{k}$

$$\text{Projection of } (\vec{b} + \vec{c}) \text{ on } \vec{a} = \frac{(\vec{b}+\vec{c}) \cdot \vec{a}}{|\vec{a}|} = \frac{4+6+2}{\sqrt{4+4+1}}$$

$$= 4$$

3.

If $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 5\hat{i} - 3\hat{j} - 4\hat{k}$, then find the ratio

$$\frac{\text{projection of vector } \vec{a} \text{ on vector } \vec{b}}{\text{projection of vector } \vec{b} \text{ on vector } \vec{a}}.$$

Ans: $$\frac{\text{projection of vector } \vec{a} \text{ on vector } \vec{b}}{\text{projection of vector } \vec{b} \text{ on vector } \vec{a}} = \frac{|\vec{a}|}{|\vec{b}|}$$

$$= \frac{3}{5\sqrt{2}}$$

4. 2023

If the projection of the vector $\hat{i} + \hat{j} + \hat{k}$ on the vector $p\hat{i} + \hat{j} - 2\hat{k}$ is $\frac{1}{3}$ then find the value(s) of p.

Sol.

Here, $\left[\frac{(\hat{i} + \hat{j} + \hat{k}) \cdot (p\hat{i} + \hat{j} - 2\hat{k})}{\sqrt{p^2 + 1 + 4}}\right] = \frac{1}{3}$

$\Rightarrow \frac{p - 1}{\sqrt{p^2 + 5}} = \frac{1}{3}$

$\Rightarrow 8p^2 - 18p + 4 = 0$

$4p^2 - 9p + 2 = 0$

$4p^2 - 8p - p + 2 = 0$

$(4p - 1)(p - 2) = 0$

$\Rightarrow p = 2 \text{ or } p = \frac{1}{4}$

prepared by : BALAJI KANCHI

5. 2022

If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{c} = \hat{i} + 3\hat{j} - \hat{k}$ and the projection of vector $\vec{c} + \lambda\vec{b}$ on vector $\vec{a}$ is $2\sqrt{6}$, then find the value of λ.

Sol.

According to question

$\frac{(\vec{c} + \lambda\vec{b}) \cdot \vec{a}}{|\vec{a}|} = 2\sqrt{6}$

$\Rightarrow \frac{\vec{c} \cdot \vec{a} + \lambda\vec{b} \cdot \vec{a}}{\sqrt{6}} = 2\sqrt{6}$

$\Rightarrow -2 + \lambda(-1) = 12$

$\Rightarrow \lambda = -14$

prepared by : BALAJI KANCHI

b. Angle between two vectors :

1.

Find the cosine of the angle which the vector $\sqrt{2}\,\hat{i} + \hat{j} + \hat{k}$ makes with y-axis.

Sol.

Let β be angle which $\sqrt{2}\hat{i} + \hat{j} + \hat{k}$ makes with y axix

$$\cos\beta = \frac{1}{\sqrt{(\sqrt{2})^2 + 1^2 + 1^2}} = \frac{1}{2}$$

2. 2024

The position vectors of vertices of Δ ABC are $A(2\hat{i} - \hat{j} + \hat{k})$, $B(\hat{i} - 3\hat{j} - 5\hat{k})$ and $C(3\hat{i} - 4\hat{j} - 4\hat{k})$. Find all the angles of Δ ABC.

Sol.

$$cosA = \frac{\overrightarrow{AB}.\overrightarrow{AC}}{|\overrightarrow{AB}||\overrightarrow{AC}|} = \frac{(-\hat{\imath} - 2\hat{\jmath} - 6\hat{k}).(\hat{\imath} - 3\hat{\jmath} - 5\hat{k})}{\sqrt{41}\sqrt{35}} = \frac{35}{\sqrt{41}\sqrt{35}} = \frac{\sqrt{35}}{\sqrt{41}}$$

$$A = cos^{-1}\left(\frac{\sqrt{35}}{\sqrt{41}}\right)$$

$$cosB = \frac{\overrightarrow{BA}.\overrightarrow{BC}}{|\overrightarrow{BA}||\overrightarrow{BC}|} = \frac{(\hat{\imath} + 2\hat{\jmath} + 6\hat{k}).(2\hat{\imath} - \hat{\jmath} + \hat{k})}{\sqrt{41}\sqrt{6}} = \frac{6}{\sqrt{41}\sqrt{6}} = \frac{\sqrt{6}}{\sqrt{41}}$$

$$B = cos^{-1}\left(\frac{\sqrt{6}}{\sqrt{41}}\right)$$

$$cosC = \frac{\overrightarrow{CB}.\overrightarrow{CA}}{|\overrightarrow{CB}||\overrightarrow{CA}|} = \frac{(-2\hat{\imath} + \hat{\jmath} - \hat{k}).(-\hat{\imath} + 3\hat{\jmath} + 5\hat{k})}{|\overrightarrow{CB}||\overrightarrow{CA}|} = 0$$

$$\cos C = 0 \quad \Rightarrow C = \frac{\pi}{2}$$

3.

If θ is the angle between two vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$, find $\sin\theta$.

Sol.

$$\sin\theta = \frac{|(\hat{i} - 2\hat{j} + 3\hat{k}) \times (3\hat{i} - 2\hat{j} + \hat{k})|}{|\hat{i} - 2\hat{j} + 3\hat{k}|\,|3\hat{i} - 2\hat{j} + \hat{k}|}$$

$$|(\hat{i} - 2\hat{j} + 3\hat{k}) \times (3\hat{i} - 2\hat{j} + \hat{k})| = |4\hat{i} + 8\hat{j} + 4\hat{k}| = 4\sqrt{6}$$

$$\sin\theta = \frac{4\sqrt{6}}{14} = \frac{2\sqrt{6}}{7}$$

4. 2025

Two friends while flying kites from different locations, find the strings of their kites crossing each other. The strings can be represented by vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$. Determine the angle formed between the kite strings. Assume there is no slack in the strings.

Sol.

Let the required angle between the kite strings be θ.

$$\text{Then, } \cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$

$$\Rightarrow \cos\theta == \frac{(3\hat{i} + \hat{j} + 2\hat{k})(2\hat{i} - 2\hat{j} + 4\hat{k})}{\sqrt{9+1+4}\sqrt{4+4+16}} = \frac{12}{\sqrt{336}} = \frac{3}{\sqrt{21}}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{12}{\sqrt{336}}\right) \text{ or } \cos^{-1}\left(\frac{3}{\sqrt{21}}\right)$$

prepared by : BALAJI KANCHI

5.

Find the angle between unit vectors $\vec{a}$ and $\vec{b}$ so that $\sqrt{3}\,\vec{a}-\vec{b}$ is also a unit vector.

Ans: Using $\left|\sqrt{3}\,\vec{a}-\vec{b}\right|=1$ i.e. $\left|\sqrt{3}\,\vec{a}-\vec{b}\right|^2=1$

& getting $\vec{a}\cdot\vec{b}=\dfrac{\sqrt{3}}{2}$

getting angle $\dfrac{\pi}{6}$ or $30°$

6.a

If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=5$, $|\vec{b}|=6$ and $|\vec{c}|=9$, then find the angle between $\vec{a}$ and $\vec{b}$.

Sol.

$$\vec{a}+\vec{b}+\vec{c}=0$$

$$\vec{a}+\vec{b}=-\vec{c}$$

$$\vec{a}^2+\vec{b}^2+2\vec{a}\cdot\vec{b}=\vec{c}^2$$

$$\Rightarrow \vec{a}\cdot\vec{b}=\frac{|\vec{c}|^2-|\vec{a}|^2-|\vec{b}|^2}{2}$$

$$\cos\theta=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}=\frac{|\vec{c}|^2-|\vec{a}|^2-|\vec{b}|^2}{2|a||b|}$$

$$=\frac{9^2-5^2-6^2}{2(5)(6)}$$

$$\cos\theta=\frac{81-25-36}{60}=\frac{1}{3}$$

$$\theta=\cos^{-1}\left(\frac{1}{3}\right)$$

6.b

If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ such that $|\vec{a}|=3$, $|\vec{b}|=5$, $|\vec{c}|=7$, then find the angle between $\vec{a}$ and $\vec{b}$.

Sol.

$$\text{Given } \vec{a}+\vec{b}+\vec{c}=\vec{0} \implies |\vec{a}+\vec{b}| = |-\vec{c}|$$

$$\implies |\vec{a}+\vec{b}|^2 = |\vec{c}|^2 \implies |\vec{a}|^2+|\vec{b}|^2+2\,\vec{a}.\vec{b} = |\vec{c}|^2$$

$$\implies 9+25+2\,\vec{a}.\vec{b} = 49$$

$$\implies 2|\vec{a}||\vec{b}|\cos\theta = 15$$

$$\implies \cos\theta = \frac{1}{2} \implies \theta = \frac{\pi}{3}$$

prepared by : BALAJI KANCHI

6.c 2024

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a}+\vec{b}-\vec{c}=\vec{0}$, find the angle between vectors $\vec{a}$ and $\vec{c}$.

Sol.

$$\text{Given } |\vec{a}| = |\vec{b}| = |\vec{c}| = 1$$

$$\text{Now } \vec{a}-\vec{c} = -\vec{b}$$

$$(\vec{a}-\vec{c}).(\vec{a}-\vec{c}) = (-\vec{b}).(-\vec{b})$$

$$\Rightarrow |\vec{a}|^2+|\vec{c}|^2-2\vec{a}.\vec{c} = |\vec{b}|^2$$

$$\Rightarrow 1+1-2|\vec{a}||\vec{c}|\cos\theta = 1$$

$$\Rightarrow 2-2(1)(1)\cos\theta = 1$$

$$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

7.a

If $|\vec{a}| = 2$, $|\vec{b}| = 7$ and $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, find the angle between $\vec{a}$ and $\vec{b}$.

Ans.

Let θ be the angle between $\vec{a}$ & $\vec{b}$, then

$$\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|} = \frac{7}{2.7} = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}$$

7.b

If the vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3$, $|\vec{b}| = \frac{2}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.

Ans.

Let θ be the angle between $\vec{a}$ and $\vec{b}$

Since $\vec{a} \times \vec{b}$ is a unit vector, we have $|\vec{a} \times \vec{b}| = 1$

$\Rightarrow |\vec{a}|\ |\vec{b}| \sin\theta = 1$

$\Rightarrow \sin\theta = \frac{1}{2}$, or $\theta = 30°$ (or $\frac{\pi}{6}$)

prepared by : BALAJI KANCHI

7.c

For the vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - 5\hat{k}$, verify that the angle between $\vec{a}$ and $\vec{a} \times \vec{b}$ is $\frac{\pi}{2}$.

Sol.

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 3 & -5 \end{vmatrix} = \hat{i} + 11\hat{j} + 7\hat{k}$$

$$\vec{a}.\left(\vec{a}\times\vec{b}\right) = 1 - 22 + 21 = 0$$

$$|\vec{a}|\left|\vec{a}\times\vec{b}\right|\cos\theta = 0$$

$$\Rightarrow \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$$

8.a 2023

For two non-zero vectors $\vec{a}$ and $\vec{b}$, if $|\vec{a} - \vec{b}| = |\vec{a} + \vec{b}|$, then find the angle between $\vec{a}$ and $\vec{b}$.

Sol.

$|\vec{a} - \vec{b}|^2 = |\vec{a} + \vec{b}|^2$

$\Rightarrow |\vec{a}|^2 - 2\vec{a}.\vec{b} + |\vec{b}|^2 = |\vec{a}|^2 + 2\vec{a}.\vec{b} + |\vec{b}|^2$

$\Rightarrow 4\vec{a}.\vec{b} = 0$

$\Rightarrow$ angle between $\vec{a}$ and $\vec{b}$ is 90°.

8.b

Show that for any two non-zero vectors $\vec{a}$ and $\vec{b}$, $\left|\vec{a}+\vec{b}\right| = \left|\vec{a}-\vec{b}\right|$ iff $\vec{a}$ and $\vec{b}$ are perpendicular vectors.

Ans: $\left|\vec{a}+\vec{b}\right| = \left|\vec{a}-\vec{b}\right|$

$\Rightarrow \quad \left|\vec{a}+\vec{b}\right|^2 = \left|\vec{a}-\vec{b}\right|^2$

$\Rightarrow \quad 4\vec{a}\cdot\vec{b} = 0$ or $\vec{a}\cdot\vec{b} = 0$ or $\vec{a}\perp\vec{b}$

Let $\vec{a}\perp\vec{b}$

Then $\vec{a}\cdot\vec{b} = 0$

Thus, $\left|\vec{a}+\vec{b}\right|^2 = \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2$ and $\left|\vec{a}-\vec{b}\right|^2 = \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2$

$\Rightarrow \quad \left|\vec{a}+\vec{b}\right| = \left|\vec{a}-\vec{b}\right|$

8.c

$\vec{a}$ and $\vec{b}$ are two unit vectors such that $|2\vec{a} + 3\vec{b}| = |3\vec{a} - 2\vec{b}|$. Find the angle between $\vec{a}$ and $\vec{b}$.

Sol.

$|2\vec{a} + 3\vec{b}| = |3\vec{a} - 2\vec{b}|$

$\Rightarrow |2\vec{a} + 3\vec{b}|^2 = |3\vec{a} - 2\vec{b}|^2$

$\Rightarrow 4|\vec{a}|^2 + 12\,\vec{a}.\vec{b} + 9|\vec{b}|^2 = 9|\vec{a}|^2 - 12\,\vec{a}.\vec{b} + 4|\vec{b}|^2$

As $|\vec{a}| = |\vec{b}| = 1$

$\therefore 24\,\vec{a}.\vec{b} = 5|\vec{a}|^2 - 5|\vec{b}|^2 = 0 \Rightarrow \vec{a}.\vec{b} = 0$

So, $\vec{a} \perp \vec{b}$ or Angle between them is $\frac{\pi}{2}$

prepared by : BALAJI KANCHI

8.d 2023

If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero unequal vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$, then find the angle between $\vec{a}$ and $\vec{b} - \vec{c}$.

Sol.

$$\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} \Rightarrow \vec{a} \cdot \left(\vec{b} - \vec{c}\right) = 0 \Rightarrow \vec{a} = \vec{0}\ ;\ \vec{b} = \vec{c} \text{ or } \vec{a} \perp \left(\vec{b} - \vec{c}\right)$$

As, $\vec{a} \neq \vec{0}\ ;\ \vec{b} \neq \vec{c}$ $\therefore$ the angle between $\vec{a}$ and $\vec{b} - \vec{c}$ is $\frac{\pi}{2}$

9.a

Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then find the angle between the vectors $\hat{a}$ and $\hat{b}$.

Ans: $\vec{c} \perp \vec{d} \Rightarrow \vec{c} \cdot \vec{d} = 0$

$$\Rightarrow \hat{a} \cdot \hat{b} = \frac{1}{2}$$

$$\Rightarrow \text{Angle between vectors } \hat{a} \ \&\ \hat{b} = \frac{\pi}{3} \text{ or } 60^\circ$$

9.b 2024

If vectors $\vec{a}$, $\vec{b}$ and $2\vec{a} + 3\vec{b}$ are unit vectors, then find the angle between $\vec{a}$ and $\vec{b}$.

Sol.

$$|\vec{a}| = |\vec{b}| = |2\vec{a} + 3\vec{b}| = 1$$

$$(2\vec{a} + 3\vec{b})^2 = |2\vec{a} + 3\vec{b}|^2$$

$$\Rightarrow 4|\vec{a}|^2 + 12\vec{a}.\vec{b} + 9|\vec{b}|^2 = 1$$

$\Rightarrow cos\theta = -1$, where θ is angle between $\vec{a}$ and $\vec{b}$

Hence, $\theta = \pi$

10.

If $\hat{a}$ and $\hat{b}$ are two unit vectors and $|\hat{a}-\hat{b}|=1$, then find the acute angle between $\hat{a}$ and $\hat{b}$.

Sol.

$$|\hat{a}-\hat{b}|=1 \Rightarrow |\hat{a}-\hat{b}|^2=1 \Rightarrow |\hat{a}|^2+|\hat{b}|^2-2|\hat{a}||\hat{b}|\cos\theta=1$$

$$\Rightarrow \quad \cos\theta=\frac{1}{2} \Rightarrow \theta=\frac{\pi}{3}$$

11. 2024

$\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular unit vectors. If θ is the angle between $\vec{a}$ and $(2\vec{a}+3\vec{b}+6\vec{c})$, find the value of $\cos\theta$.

Sol.

Given $|\vec{a}|=|\vec{b}|=|\vec{c}|=1$ and $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=0$

Now, $|2\vec{a}+3\vec{b}+6\vec{c}|^2=4|\vec{a}|^2+9|\vec{b}|^2+36|\vec{c}|^2=49$

$\Rightarrow |2\vec{a}+3\vec{b}+6\vec{c}|=7$

$$\cos\theta=\frac{\vec{a}.(2\vec{a}+3\vec{b}+6\vec{c})}{|\vec{a}||2\vec{a}+3\vec{b}+6\vec{c}|}=\frac{2|\vec{a}|^2}{|\vec{a}||2\vec{a}+3\vec{b}+6\vec{c}|}$$

$$\therefore \cos\theta=\frac{2}{7}$$

12. 2025

If $\vec{a}$ and $\vec{b}$ are unit vectors inclined with each other at an angle θ, then prove that $\frac{1}{2}|\vec{a}-\vec{b}| = \sin\frac{\theta}{2}$.

Sol.

$$|\vec{a}| = |\vec{b}| = 1$$

$$|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b}$$

$$= 1 + 1 - 2|\vec{a}||\vec{b}|\cos\theta$$

$$= 2 - 2\cos\theta$$

$$= 2\left(2\sin^2\frac{\theta}{2}\right) = 4\sin^2\frac{\theta}{2}$$

$$\Rightarrow \sin\frac{\theta}{2} = \frac{1}{2}|\vec{a}-\vec{b}|$$

c. Two vectors perpendicular :

1.2022

If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ.

Sol.

$$(\vec{a} + \lambda\vec{b}).\vec{c} = 0 \Rightarrow [(2-\lambda)\hat{\imath} + (2+2\lambda)\hat{\jmath} + (3+\lambda)\hat{k}].(3\hat{\imath} + \hat{\jmath}) = 0$$

$$\Rightarrow 3(2-\lambda) + (2+2\lambda)\cdot 1 = 0$$

$$\Rightarrow -3\lambda + 2\lambda + 6 + 2 = 0$$

$$\Rightarrow \lambda = 8$$

prepared by : BALAJI KANCHI

2.a

Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$ be two vectors. Show that the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are perpendicular to each other.

Sol.

$$\vec{a} + \vec{b} = 4\hat{i} + \hat{j} - \hat{k}$$

$$\vec{a} - \vec{b} = -2\hat{i} + 3\hat{j} - 5\hat{k}$$

$$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = -8 + 3 + 5 = 0$$

so $(\vec{a} + \vec{b}) \perp (\vec{a} - \vec{b})$

2.b 2022

If $\vec{a} = 2\hat{i} + y\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ are two vectors for which the vector $(\vec{a} + \vec{b})$ is perpendicular to the vector $(\vec{a} - \vec{b})$, then find all the possible values of y.

Sol.

$$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$$

$$\Rightarrow |\vec{a}|^2 - |\vec{b}|^2 = 0$$

$$\Rightarrow y^2 + 5 - 14 = 0$$

$$\Rightarrow y = +3 \quad \text{or} \quad -3$$

3.

Show that $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$, for any two non-zero vectors $\vec{a}$ and $\vec{b}$.

Ans: $\left(|\vec{a}|\vec{b} + |\vec{b}|\vec{a}\right) \cdot \left(|\vec{a}|\vec{b} - |\vec{b}|\vec{a}\right)$

$$= \left(|\vec{a}|\vec{b}\right)^2 - \left(|\vec{b}|\vec{a}\right)^2$$

$$= |\vec{a}|^2 |\vec{b}|^2 - |\vec{b}|^2 |\vec{a}|^2 = 0$$

$$\therefore \left(|\vec{a}|\vec{b} + |\vec{b}|\vec{a}\right) \perp \left(|\vec{a}|\vec{b} - |\vec{b}|\vec{a}\right)$$

VIII. Cross product :

a.Find a Unit vector Perpendicular to two given vectors :

1.

If $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$, then find a unit vector along the vector $\vec{a} \times \vec{b}$.

Sol.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 1 \\ 2 & -2 & 1 \end{vmatrix} = \hat{i} - 2\hat{j} - 6\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{41}$$

unit vector along $\vec{a} \times \vec{b}$ is $\frac{1}{\sqrt{41}}(\hat{i} - 2\hat{j} - 6\hat{k})$

2.a

Find the unit vector perpendicular to each of the vectors $\vec{a} = 4\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$

Ans.

$\vec{a} \times \vec{b} = 7\hat{i} - 6\hat{j} - 10\hat{k}$ and $|\vec{a} \times \vec{b}| = \sqrt{185}$

Required unit vector $= \frac{1}{\sqrt{185}}\left(7\hat{i} - 6\hat{j} - 10\hat{k}\right)$

2.b

Find a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$, where $\vec{a} = 4\hat{i} - \hat{j} + 8\hat{k}$, $\vec{b} = -\hat{j} + \hat{k}$.

Sol.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 8 \\ 0 & -1 & 1 \end{vmatrix} = 7\hat{i} - 4\hat{j} - 4\hat{k}$$

$$\text{Required unit vector} = \frac{(\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|}$$

$$= \frac{1}{9}(7\hat{i} - 4\hat{j} - 4\hat{k})$$

2.c

Find a unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$, where $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

Sol.

A vector perpendicular to both $\vec{a}$ and $\vec{b}$ = $\vec{a}\times\vec{b} = 19\hat{j} + 19\hat{k}$ or $\hat{j}+\hat{k}$

$\therefore$ Unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ = $\frac{1}{\sqrt{2}}(\hat{j}+\hat{k})$

2.d

Find a unit vector perpendicular to each of the vectors $\vec{a}$ and $\vec{b}$ where $\vec{a} = 5\hat{i} + 6\hat{j} - 2\hat{k}$ and $\vec{b} = 7\hat{i} + 6\hat{j} + 2\hat{k}$.

Ans: $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 6 & -2 \\ 7 & 6 & 2 \end{vmatrix} = 24\hat{i} - 24\hat{j} - 12\hat{k}$

Unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ is $\frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}$

b. Find a vector with some magnitude which is perpendicular to two given vectors :

1.a 2025

Find a vector of magnitude 5 which is perpendicular to both the vectors $3\hat{i} - 2\hat{j} + \hat{k}$ and $4\hat{i} + 3\hat{j} - 2\hat{k}$.

Sol.

Let $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} + 3\hat{j} - 2\hat{k}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 1 \\ 4 & 3 & -2 \end{vmatrix} = \hat{i} + 10\hat{j} + 17\hat{k}$$

$$|\vec{a}\times\vec{b}| = \sqrt{1^2 + 10^2 + 17^2} = \sqrt{390}$$

Unit vector $\hat{n} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} = \frac{1}{\sqrt{390}}(\hat{i} + 10\hat{j} + 17\hat{k})$

$\therefore$ **Required vector** $= \frac{5}{\sqrt{390}}(\hat{i} + 10\hat{j} + 17\hat{k})$

1.b 2024

Find a vector of magnitude 4 units perpendicular to each of the vectors $2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}-\hat{k}$ and hence verify your answer.

Sol.

Let $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+\hat{j}-\hat{k}$ and $\vec{c}$ be the vector perpendicular to both $\vec{a}\,\&\,\vec{b}$

then, $\vec{c}=\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 1 & 1 & -1\end{vmatrix}=3\hat{j}+3\hat{k}$

Let $\vec{d}$ the vector perpendicular to both the vectors $\vec{a}\,\&\,\vec{b}$ and having magnitude 4,

$$\vec{d}=4\hat{c}=2\sqrt{2}\hat{j}+2\sqrt{2}\hat{k}\ \ (\text{or } -2\sqrt{2}\hat{j}-2\sqrt{2}\hat{k})$$

Verification: $|\vec{d}|=\sqrt{8+8}=4$, $\vec{d}\cdot\vec{a}=\vec{d}\cdot\vec{b}=0\Rightarrow\vec{d}\perp\vec{a}$ and $\vec{d}\perp\vec{b}$

1.c 2024

Find all vectors of magnitude $8\sqrt{14}$ units that are perpendicular to the vectors $2\hat{i}-\hat{k}$ and $2\hat{j}+3\hat{k}$.

Sol.

Vector parallel to the required vector is $\begin{vmatrix}i & j & k\\ 2 & 0 & -1\\ 0 & 2 & 3\end{vmatrix}=2\hat{i}-6\hat{j}+4\hat{k}$, $|2\hat{i}-6\hat{j}+4\hat{k}|=2\sqrt{14}$

The required vectors are $=\pm 8\sqrt{14}\left(\dfrac{-2\hat{i}-6\hat{j}+4\hat{k}}{2\sqrt{14}}\right)=\pm\left(8\hat{i}+24\hat{j}-16\hat{k}\right)$

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c. Find a unit/with some magnitude vector and Perpendicular to sum and difference of two given vectors :

1.a

If $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{j} - \hat{k}$ are three vectors, then find a vector perpendicular to both the vectors $(\vec{a} + \vec{b})$ and $(\vec{b} - \vec{c})$.

Sol.

$\vec{a} + \vec{b} = 3\hat{j}$, $\vec{b} - \vec{c} = 3\hat{k}$

Vector perpendicular to $\vec{a} + \vec{b}$ and $\vec{b} - \vec{c}$

$= (3\hat{j}) \times (3\hat{k}) = 9\hat{i}$

1.b

If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ then find a unit vector perpendicular to both $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$.

Ans.

Let $\vec{c} = \vec{a} + \vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, and $\vec{d} = \vec{a} - \vec{b} = -\hat{j} - 2\hat{k}$

$$\vec{c} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix} = -2\hat{i} + 4\hat{j} - 2\hat{k}$$

$$\left|\vec{c} \times \vec{d}\right| = \sqrt{24}$$

$$\text{Required vector} = 6\left(\frac{\vec{c} \times \vec{d}}{\left|\vec{c} \times \vec{d}\right|}\right)$$

$$= \frac{6}{\sqrt{24}}\left(-2\hat{i} + 4\hat{j} - 2\hat{k}\right) \text{ or } \sqrt{6}\left(-\hat{i} + 2\hat{j} - \hat{k}\right)$$

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c. Area of Triangle ABC/median length :

1.a

Using vectors, find the area of the triangle with vertices A(–1, 0, – 2), B(0, 2, 1) and C(–1, 4, 1).

Sol.

Taking any two sides (say AB and AC)

$\overrightarrow{AB} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\overrightarrow{AC} = 0\hat{i} + 4\hat{j} + 3\hat{k}$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix} = -6\hat{i} - 3\hat{j} + 4\hat{k}$$

Area of $\Delta = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\sqrt{36+9+16} = \frac{1}{2}\sqrt{61}$ prepared by : BALAJI KANCHI

1.b

Show that the points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ respectively, are the vertices of a right-angled triangle. Hence find the area of the triangle.

Sol.

$\overrightarrow{AB} = -\hat{i} - 2\hat{j} - 6\hat{k}, \overrightarrow{BC} = 2\hat{i} - \hat{j} + \hat{k}, \overrightarrow{CA} = -\hat{i} + 3\hat{j} + 5\hat{k}$

Since $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CA}$, are not parallel vectors,

and $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$ $\therefore$ A, B, C form a triangle

Also $\overrightarrow{BC} \cdot \overrightarrow{CA} = 0$ $\therefore$ A, B, C form a right triangle

Area of $\Delta = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{BC}| = \frac{1}{2}\sqrt{210}$ prepared by : BALAJI KANCHI

1.c

Using vectors, find the area of triangle ABC, with vertices A (1, 2, 3), B (2, –1, 4) and C (4, 5, –1).

Sol.

$\overrightarrow{AB} = \hat{i}-3\hat{j}+\hat{k},\ \overrightarrow{AC} = 3\hat{i}+3\hat{j}-4\hat{k}$

Area of $\Delta ABC = \frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$

$= \frac{1}{2}$ magnitude of $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 3 & 3 & -4 \end{vmatrix}$

$= \frac{1}{2}|9\hat{i}+7\hat{j}+12\hat{k}|$

$= \frac{1}{2}\sqrt{81+49+144} = \frac{1}{2}\sqrt{274}$ sq.units

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1.d

Using vectors, find the area of triangle ABC with vertices A(4, 3, 3), B(5, 5, 6) and C(4,7, 6).

Sol.

$\overrightarrow{AB} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k},\ \overrightarrow{AC} = 4\hat{\jmath} + 3\hat{k},$

$\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix} = -6\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

So, the area of the triangle $= \frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$

$= \frac{1}{2}\left|-6\hat{\imath} - 3\hat{\jmath} + 4\hat{k}\right| = \frac{1}{2}\sqrt{36+9+16} = \frac{1}{2}\sqrt{61}$

d. Area of parallelogram /finding sides and diagonals of parallelogram :

1.a

Find the area of a parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

Ans.

Here

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix} = 20\hat{i} + 5\hat{j} - 5\hat{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{400 + 25 + 25} = \sqrt{450}$$

Area of parallelogram $= |\vec{a} \times \vec{b}| = \sqrt{450} = 15\sqrt{2}$

2.a 2025

The diagonals of a parallelogram are given by $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$. Find the area of the parallelogram.

Sol.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{vmatrix} = -2\hat{i} + 3\hat{j} + 7\hat{k}$$

$$\text{Area of parallelogram} = \frac{1}{2}|\vec{a} \times \vec{b}|$$

$$= \frac{1}{2}\sqrt{(-2)^2 + 3^2 + 7^2} = \frac{\sqrt{62}}{2}$$

2.b

The adjacent sides of a parallelogram are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$. Find the vectors determining its diagonals and hence find the area of the parallelogram.

Sol.

Diagonals of parallelogram are $\vec{a}+\vec{b}$ and $\vec{b}-\vec{a}$ [or $\vec{a}-\vec{b}$]

$\vec{a}+\vec{b} = 3\hat{i}-8\hat{j}+4\hat{k}$ and $\vec{b}-\vec{a} = \hat{i}-6\hat{j}-2\hat{k}$

$$\text{Req. area} = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -8 & 4 \\ 1 & -6 & -2 \end{vmatrix} = \frac{1}{2}|40\hat{i}+10\hat{j}-10\hat{k}|$$

$$= \frac{1}{2}\sqrt{1800} = 15\sqrt{2} \text{ sq. units}$$

3.a

Find the area of a parallelogram ABCD whose side AB and the diagonal AC are given by the vectors $3\hat{i} + \hat{j} + 4\hat{k}$ and $4\hat{i} + 5\hat{k}$ respectively.

Sol.

$\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB} = \hat{i} - \hat{j} + \hat{k}$

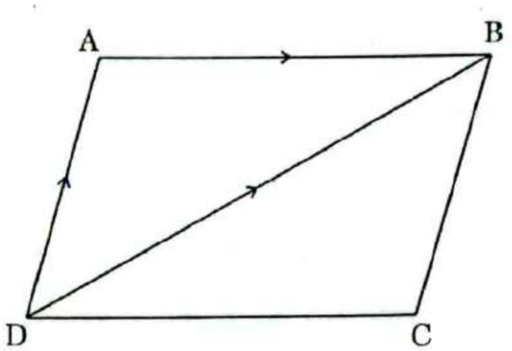
$$\text{Area} = \left|\overrightarrow{AB} \times \overrightarrow{BC}\right| = \text{magnitude of } \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= |5\hat{i} + \hat{j} - 4\hat{k}|$$

$$= \sqrt{42} \text{ sq. units}$$

3.b

In the given figure, ABCD is a parallelogram. If $\overrightarrow{AB} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\overrightarrow{DB} = 3\hat{i} - 6\hat{j} + 2\hat{k}$, then find $\overrightarrow{AD}$ and hence find the area of parallelogram ABCD.



Sol.

$\overrightarrow{AD} + \overrightarrow{DB} = \overrightarrow{AB}$

$\overrightarrow{AD} = (2\hat{\imath} - 4\hat{\jmath} + 5\hat{k}) - (3\hat{\imath} - 6\hat{\jmath} + 2\hat{k})$

$= -\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$$\overrightarrow{AD} \times \overrightarrow{AB} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 2 & 3 \\ 2 & -4 & 5 \end{vmatrix} = 22\hat{\imath} + 11\hat{\jmath}$$

$\text{Area} = |\overrightarrow{AD} \times \overrightarrow{AB}| = |22\hat{\imath} + 11\hat{\jmath}|$

$= \sqrt{605} \ or \ 11\sqrt{5}$

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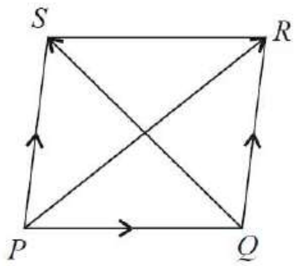
3.c

In a parallelogram PQRS, $\overrightarrow{PQ} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\overrightarrow{PS} = -\hat{i} - 2\hat{k}$. Find $|\overrightarrow{PR}|$ and $|\overrightarrow{QS}|$.

Sol.



$\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{PS} \quad \because \quad \overrightarrow{PS} = \overrightarrow{QR}$

$= 2\hat{i} - 2\hat{j}$

$|\overrightarrow{PR}| = 2\sqrt{2}$

$\overrightarrow{QS} = \overrightarrow{QP} + \overrightarrow{PS} = \overrightarrow{PS} - \overrightarrow{PQ}$

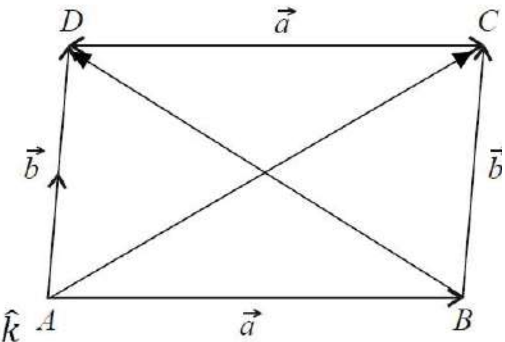
$= -4\hat{i} + 2\hat{j} - 4\hat{k}$

$|\overrightarrow{QS}| = 6$

4. 2022

ABCD is a parallelogram such that $\overrightarrow{AC} = \hat{i} + \hat{j}$ and $\overrightarrow{BD} = 2\hat{i} + \hat{j} + \hat{k}$. Find $\overrightarrow{AB}$ and $\overrightarrow{AD}$. Also, find the area of the parallelogram ABCD.

Sol.



Let $\overrightarrow{AB} = \vec{a}$ and $\overrightarrow{AD} = \vec{b}$

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \vec{a} + \vec{b} = \hat{\imath} + \hat{\jmath}$

$\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD} = \vec{b} - \vec{a} = 2\hat{\imath} + \hat{\jmath} + \hat{k}$

Adding we get, $2\overrightarrow{AD} = \overrightarrow{AC} + \overrightarrow{BD} = 3\hat{\imath} + 2\hat{\jmath} + \hat{k}$

$\Rightarrow \overrightarrow{AD} = \frac{3}{2}\hat{\imath} + \hat{\jmath} + \frac{1}{2}\hat{k}$

Subtracting, we get

$2\overrightarrow{AB} = \overrightarrow{AC} - \overrightarrow{BD} = -\hat{\imath} - \hat{k} \Rightarrow \overrightarrow{AB} = -\frac{1}{2}\hat{\imath} - \frac{1}{2}\hat{k}$

$$\left|\overrightarrow{AC} \times \overrightarrow{BD}\right| = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{vmatrix} = \hat{\imath} - \hat{\jmath} - \hat{k}$$

$$\text{Area} = \frac{1}{2}\left|\overrightarrow{AC} \times \overrightarrow{BD}\right|$$

$$= \frac{\sqrt{3}}{2}$$

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5.a

The two adjacent sides of a parallelogram are represented by vectors $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$. Find the unit vector parallel to one of its diagonals. Also, find the area of the parallelogram.

Sol.

One diagonal of the parallelogram

$= (2\hat{\imath} - 4\hat{\jmath} + 5\hat{k}) + (\hat{\imath} - 2\hat{\jmath} - 3\hat{k})$

$= 3\hat{i} - 6\hat{j} + 2\hat{k}$

Unit vector parallel to the diagonal

$$= \frac{3\hat{i} - 6\hat{j} + 2\hat{k}}{\sqrt{9+36+4}}$$

$$= \frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k}$$

$$\text{Vector area of parallelogram} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 5 \\ 1 & -2 & -3 \end{vmatrix}$$

$$= \hat{i}(22) - \hat{j}(-11) + \hat{k}(0)$$

$$= 22\hat{i} + 11\hat{j}$$

$$\text{Area} = \sqrt{484+121} = \sqrt{605} = 11\sqrt{5}$$

5.b

The two adjacent sides of a parallelogram are represented by vectors $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $2\hat{i} + 2\hat{j} + 3\hat{k}$. Find the unit vector parallel to one of its diagonals. Also, find the area of the parallelogram.

Ans.

Let ABCD be a parallelogram with

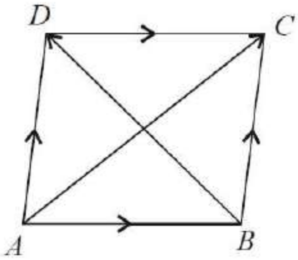
$$\overrightarrow{AB} = \overrightarrow{DC} = 2\hat{i} - 4\hat{j} - 5\hat{k}$$

and

$$\overrightarrow{BC} = \overrightarrow{AD} = 2\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = 4\hat{i} - 2\hat{j} - 2\hat{k}$$

and $\overrightarrow{BD} = 6\hat{j} + 8\hat{k}$



$\therefore |\overrightarrow{AC}| = 2\sqrt{6}$ and $|\overrightarrow{BD}| = 10$

$\therefore$ Required unit vectors d_1 and d_2 are

$$\hat{d}_1 = \frac{2}{\sqrt{6}}\hat{i} - \frac{1}{\sqrt{6}}\hat{j} - \frac{\hat{k}}{\sqrt{6}} \text{ and } \hat{d}_2 = \frac{3}{5}\hat{j} + \frac{4}{5}\hat{k}$$

Now,

$$\text{Area of } \| ABCD = \frac{1}{2}|\vec{d}_1 \times \vec{d}_2|$$

$$= \frac{1}{2}\left\| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & -2 \\ 0 & 6 & 8 \end{vmatrix} \right\|$$

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$$= \frac{1}{2}|-4\hat{i} - 32\hat{j} + 24\hat{k}|$$

$$= \frac{1}{2}\sqrt{1616} = 2\sqrt{101}$$

5.c

If the sides AB and BC of a parallelogram ABCD are represented as vectors $\vec{AB} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{BC} = \hat{i} + 2\hat{j} + 3\hat{k}$, then find the unit vector along diagonal AC.

Ans. $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = (2\hat{i} + 4\hat{j} - 5\hat{k}) + (\hat{i} + 2\hat{j} + 3\hat{k})$

$\therefore \ \overrightarrow{AC} = 3\hat{i} + 46 - 2\hat{k}$

$$\text{Unit vector along AC} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{|3\hat{i} + 6\hat{j} - 2\hat{k}|} = \frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}\hat{k}$$

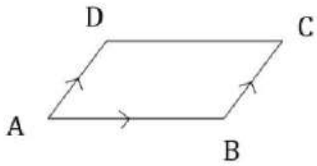
5.d

In a parallelogram ABCD, the sides AB and AD are represented by the vectors $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$ respectively. Find the unit vector parallel to its diagonal $\vec{AC}$.

Sol.

$\overline{AC} = \overline{AB} + \overline{BC} = \overline{AB} + \overline{AD} = 3\hat{i} - 6\hat{j} + 2\hat{k}$

$\text{Required unit vector} = \frac{1}{7}\left(3\hat{i} - 6\hat{j} + 2\hat{k}\right)$



5.e

If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ represent two adjacent sides of a parallelogram, find unit vectors parallel to the diagonals of the parallelogram.

Answer:

Diagonal vectors are: $\vec{a} + \vec{b} = 3\hat{i} + 6\hat{j} - 2\hat{k}$ and $\vec{a} - \vec{b} = -\hat{i} - 2\hat{j} + 8\hat{k}$

(or, $\vec{b} - \vec{a} = \hat{i} + 2\hat{j} - 8\hat{k}$)

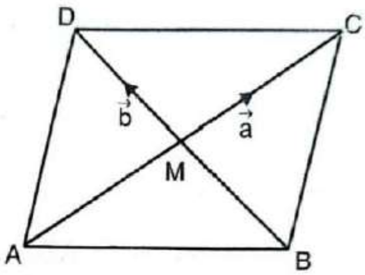
$$\therefore \text{unit vectors are} \frac{(\vec{a} + \vec{b})}{|\vec{a} + \vec{b}|} = \frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}\hat{k} \text{ and } \frac{(\vec{a} - \vec{b})}{|\vec{a} - \vec{b}|} = -\frac{1}{\sqrt{69}}\hat{i} - \frac{2}{\sqrt{69}}\hat{j} + \frac{8}{\sqrt{69}}\hat{k}$$

6. 2025

Show that the area of a parallelogram whose diagonals are represented by $\vec{a}$ and $\vec{b}$ is given by $\frac{1}{2}|\vec{a}\times\vec{b}|$. Also find the area of a parallelogram whose diagonals are $2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+3\hat{j}-\hat{k}$.

Sol.

Let $ABCD$ be the parallelogram with diagonals $\overrightarrow{AB}=\vec{a}$ and $\overrightarrow{BD}=\vec{b}$.



$\therefore \overrightarrow{AB}=\frac{1}{2}\left(\vec{a}-\vec{b}\right)$ and $\overrightarrow{AD}=\frac{1}{2}\left(\vec{a}+\vec{b}\right)$

Area of $ABCD$

$=\left|\overrightarrow{AB}\times\overrightarrow{AD}\right|$

$=\left|\frac{1}{2}\left(\vec{a}-\vec{b}\right)\times\frac{1}{2}\left(\vec{a}+\vec{b}\right)\right|$

$=\frac{1}{4}\left|\vec{a}\times\vec{a}+\vec{a}\times\vec{b}-\vec{b}\times\vec{a}-\vec{b}\times\vec{b}\right|$

$=\frac{1}{4}\left|\vec{a}\times\vec{b}+\vec{a}\times\vec{b}\right| \qquad \left(\because \vec{a}\times\vec{a}=\vec{0}\right)$

$=\frac{1}{4}\left|2\left(\vec{a}\times\vec{b}\right)\right|$

$=\frac{1}{2}\left|\vec{a}\times\vec{b}\right|$

Given $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+3\hat{j}-\hat{k}$

Area of parallelogram $=\frac{1}{2}\left|\vec{a}\times\vec{b}\right|$

Now $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 1 & 3 & -1\end{vmatrix}=-2\hat{i}+3\hat{j}+7\hat{k}$

$\left|\vec{a}\times\vec{b}\right|=\sqrt{62}$

prepared by : BALAJI KANCHI

Area of parallelogram $=\frac{1}{2}\sqrt{62}$

7. 2025

$\vec{a}$ and $\vec{b}$ are two co-initial vectors forming the adjacent sides of a parallelogram such that $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a}.\vec{b} = 12$. Find the area of the parallelogram.

Sol.

Let θ is the angle between $\vec{a}$ and $\vec{b}$.

$\vec{a}.\vec{b} = 12 \Rightarrow |\vec{a}||\vec{b}| \cos\theta = 12$

$\Rightarrow (10)(2)\cos\theta = 12 \Rightarrow \cos\theta = \dfrac{3}{5}$

$\therefore \sin\theta = \sqrt{1 - \left(\dfrac{3}{5}\right)^2} = \dfrac{4}{5}$

Now, area of parallelogram $= |\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$

$= (10)(2)\left(\dfrac{4}{5}\right) = 16$

$\therefore$ area of parallelogram $= 16$

IX. Sum of the two/three vectors magnitude based:

1.a 2022

If $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 4$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then find the value of $(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$.

Sol.

$\vec{a} + \vec{b} + \vec{c} = 0$ gives $|\vec{a} + \vec{b} + \vec{c}|^2 = 0$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}| + 2\vec{a} \cdot \vec{b} + 2\vec{b} \cdot \vec{c} + 2\vec{c} \cdot \vec{a} = 0$

$\Rightarrow 9 + 25 + 16 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$

$\Rightarrow \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -25$

prepared by : BALAJI KANCHI

1.b 2022

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

Sol.

$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$\vec{a} + \vec{b} + \vec{c} = 0 \Rightarrow (\vec{a} + \vec{b} + \vec{c})(\vec{a} + \vec{b} + \vec{c}) = 0$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$

$\Rightarrow 3 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$

$\Rightarrow \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = \frac{-3}{2}$

1.c

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $|\vec{a}| = 7$, $|\vec{b}| = 24$, $|\vec{c}| = 25$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

Sol.

$$\vec{a}+\vec{b}+\vec{c}=\vec{0} \Rightarrow \left(\vec{a}+\vec{b}+\vec{c}\right)^2=\left(\vec{0}\right)^2$$

$$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)=0$$

$$\Rightarrow 49+576+625+2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)=0$$

$$\Rightarrow \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=-625$$

1.d

Three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a} + \vec{b} + \vec{c} = \vec{0}$.

Evaluate the quantity $\mu = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$, if $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{c}| = 2$.

Ans.

$$(\vec{a} + \vec{b} + \vec{c})^2 = 0$$

$$\Rightarrow \vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2(\mu) = 0$$

$$\Rightarrow \mu = -\frac{29}{2}$$

1.e 2025

If $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 4$, then evaluate $|\vec{a} + 2\vec{b}|$.

Sol.

$$(\vec{a} + 2\vec{b})^2 = |\vec{a}|^2 + |2\vec{b}|^2 + 4\vec{a}.\vec{b}$$

$$= 56$$

$$\Rightarrow |\vec{a} + 2\vec{b}| = \sqrt{56}$$

2.a

If $\vec{a}, \vec{b}$ and $\vec{c}$ are three mutually perpendicular unit vectors, find the value of $|\vec{a}+2\vec{b}+3\vec{c}|$.

Ans. $|\vec{a}|=|\vec{b}|=|\vec{c}|=1$ and $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$

$$|\vec{a}+2\vec{b}+3\vec{c}|^2=\vec{a}^2+4\vec{b}^2+9\vec{c}^2=1+4+9=14$$

$\therefore$ $|\vec{a}+2\vec{b}+3\vec{c}|=\sqrt{14}$

2.b

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}|=1$, $|\vec{b}|=2$ and $|\vec{c}|=3$. If the projection of $\vec{b}$ along $\vec{a}$ is equal to the projection of $\vec{c}$ along $\vec{a}$; and $\vec{b}$, $\vec{c}$ are perpendicular to each other, then find $|3\vec{a}-2\vec{b}+2\vec{c}|$.

Sol.

Given $\dfrac{\vec{b}\cdot\vec{a}}{|\vec{a}|}=\dfrac{\vec{c}\cdot\vec{a}}{|\vec{a}|}$ $\therefore$ $\vec{b}\cdot\vec{a}=\vec{c}\cdot\vec{a}$ ...(i)

$\vec{b}\perp\vec{c} \Rightarrow \vec{b}\cdot\vec{c}=0$ ...(ii)

$$(|3\vec{a}-2\vec{b}+2\vec{c}|)^2=9|\vec{a}|^2+4|\vec{b}|^2+4|\vec{c}|^2-12\,\vec{a}\cdot\vec{b}-8\,\vec{b}\cdot\vec{c}+12\,\vec{a}\cdot\vec{c}$$

$$=9(1)^2+4(2)^2+4(3)^2 \quad \text{[using (i) and (ii)]}$$

$$=9+16+36=61$$

$\Rightarrow |3\vec{a}-2\vec{b}+2\vec{c}|=\sqrt{61}$

3.a

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular vectors of equal magnitude, then prove that the vector $(2\vec{a} + \vec{b} + 2\vec{c})$ is equally inclined to both $\vec{a}$ and $\vec{c}$. Also, find the angle between $\vec{a}$ and $(2\vec{a} + \vec{b} + 2\vec{c})$.

Sol.

Let, $|\vec{a}| = |\vec{b}| = |\vec{c}| = m$

$$|2\vec{a}+\vec{b}+2\vec{c}|^2 = 4|\vec{a}|^2 + |\vec{b}|^2 + 4|\vec{c}|^2 + 4\vec{a}\cdot\vec{b} + 4\vec{b}\cdot\vec{c} + 8\vec{a}\cdot\vec{c}$$

$$= 9m^2$$

$$|2\vec{a}+\vec{b}+2\vec{c}| = 3m$$

Let θ be angle between $2\vec{a}+\vec{b}+2\vec{c}$ and $\vec{a}$.

$$\cos\theta = \frac{(2\vec{a}+\vec{b}+2\vec{c})\cdot\vec{a}}{|2\vec{a}+\vec{b}+2\vec{c}||\vec{a}|} = \frac{2m^2}{3m\cdot m} = \frac{2}{3}$$

Let, α be angle between $2\vec{a}+\vec{b}+2\vec{c}$ and $\vec{c}$

Similarly, $\cos\alpha = \dfrac{2m^2}{3m\cdot m} = \dfrac{2}{3}$

$\therefore \cos\theta = \cos\alpha \Rightarrow \theta = \alpha$

prepared by : BALAJI KANCHI

$\therefore$ Required angle $= \cos^{-1}\left(\dfrac{2}{3}\right)$

3.b 2024

$\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular unit vectors. If θ is the angle between $\vec{a}$ and $(2\vec{a} + 3\vec{b} + 6\vec{c})$, find the value of cos θ.

Sol.

Given $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$ and $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

Now, $|2\vec{a} + 3\vec{b} + 6\vec{c}|^2 = 4|\vec{a}|^2 + 9|\vec{b}|^2 + 36|\vec{c}|^2 = 49$

$\Rightarrow |2\vec{a} + 3\vec{b} + 6\vec{c}| = 7$

$$\cos\theta = \frac{\vec{a}.\left(2\vec{a} + 3\vec{b} + 6\vec{c}\right)}{|\vec{a}||2\vec{a} + 3\vec{b} + 6\vec{c}|} = \frac{2|\vec{a}|^2}{|\vec{a}||2\vec{a} + 3\vec{b} + 6\vec{c}|}$$

$$\therefore \cos\theta = \frac{2}{7}$$

XI. Problems based on dot product and cross product :

1.a 2022

If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} - 3\hat{k}$, then find the vector $\vec{c}$, given that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 4$.

Ans.

Let $\vec{c} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\vec{a} \cdot \vec{c} = 4 \Rightarrow x - y + z = 4$

$\vec{a} \times \vec{c} = (-z - y)\hat{\imath} + (x - z)\hat{\jmath} + (x + y)\hat{k}$

$$\vec{a} \times \vec{c} = \vec{b} \Rightarrow \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 1 \\ x & y & z \end{vmatrix} = (2\vec{\imath} - \hat{\jmath} - 3\hat{k})$$

$\Rightarrow -(y + z)\hat{\imath} - (z - x)\hat{\jmath} + (y + x)\hat{k} = 2\vec{\imath} - \hat{\jmath} - 3\hat{k}$

$\Rightarrow y + z = -2, z - x = 1, y + x = -3$

Solving we get, $x = 0$, $y = -3$, $z = 1$

$$\therefore \vec{c} = -3\hat{\jmath} + \hat{k}$$

1.b

If $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{b} = 4\hat{i} - 7\hat{j} + \hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 6$.

Sol.

Let $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$; $\vec{a}\cdot\vec{c} = 6 \;\Rightarrow\; 2x + y - z = 6$

Now, $\vec{a} \times \vec{c} = \vec{b} \;\Rightarrow\; \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ x & y & z \end{vmatrix} = 4\hat{i} - 7\hat{j} + \hat{k}$

$\Rightarrow \quad \hat{i}(z+y) - \hat{j}(2z+x) + \hat{k}(2y-x) = 4\hat{i} - 7\hat{j} + \hat{k}$

$\Rightarrow \quad z + y = 4,\ 2z + x = 7,\ 2y - x = 1$

Solving and getting $x = 3$, $y = 2$, $z = 2$

$$\vec{c} = 3\hat{i} + 2\hat{i} + 2\hat{k}$$

2.a

For any two vectors $\vec{a}$ and $\vec{b}$, prove that

$$(\vec{a} \times \vec{b})^2 = \vec{a}^2\,\vec{b}^2 - (\vec{a} \cdot \vec{b})^2$$

Sol.

$(\vec{a}\times\vec{b})^2 = \vec{a}^2\vec{b}^2 - (\vec{a}\cdot\vec{b})^2$

LHS = $(\vec{a}\times\vec{b})^2$

$= (|\vec{a}||\vec{b}|\sin\theta\,\hat{n})^2 = |\vec{a}|^2|\vec{b}|^2\sin^2\theta$

$= |\vec{a}|^2|\vec{b}|^2(1-\cos^2\theta)$

$= |\vec{a}|^2|\vec{b}|^2 - (|\vec{a}||\vec{b}|\cos\theta)^2$

$= \vec{a}^2\,\vec{b}^2 - (\vec{a}\cdot\vec{b})^2$

2.b 2022

If $\left| \vec{a} \times \vec{b} \right|^2 + \left| \vec{a} \cdot \vec{b} \right|^2 = 400$ and $\left| \vec{a} \right| = 5$, then write the value of $\left| \vec{b} \right|$.

Sol.

$$|\vec{a}|^2 |\vec{b}|^2 \sin^2\theta + |\vec{a}|^2 |\vec{b}|^2 \cos^2\theta = 400$$

$$|\vec{a}|^2 \cdot 25(1) = 400$$

$$|\vec{a}|^2 = 16$$

$$|\vec{a}| = 4$$

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2.c 2022

If $|\vec{a}| = 3$, $|\vec{b}| = 2\sqrt{3}$ and $\vec{a} \cdot \vec{b} = 6$, then find the value of $|\vec{a} \times \vec{b}|$.

Sol.

$$6 = |\vec{a}||\vec{b}|\cos\theta \Rightarrow 6 = 3 \cdot 2\sqrt{3} \cdot \cos\theta$$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{3}}$$

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta = (3)(2\sqrt{3})\sqrt{1 - \frac{1}{3}}$$

$$= 6\sqrt{3} \cdot \frac{\sqrt{2}}{\sqrt{3}} = 6\sqrt{2}$$

3. 2022

If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{a}.\vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$, then find $\left|\vec{b}\right|$

Sol.

Let $\vec{b} = x\hat{i} + y\hat{i} + z\hat{k}$

$$\vec{a} \cdot \vec{b} = 1 \Rightarrow x + y + z = 1 \qquad \dots (1)$$

$\vec{a} \times \vec{b} = \hat{j} - \hat{k}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = \hat{i}(z - y) - \hat{j}(z - x) + \hat{k}(y - x) = \hat{j} - \hat{k}$$

$$z - y = 0 \Rightarrow y = z$$

$$\Rightarrow x - z = 1 \qquad \dots (2)$$

$$x - y = 1 \qquad \dots (3)$$

Solving (1), (2), (3)

$$x = 1, \quad y = 0, \quad z = 0$$

$$\vec{b} = \hat{i}, \text{ so } |\vec{b}| = 1$$

prepared by : BALAJI KANCHI

4.a

If $\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k}$, find the value of $(\vec{r} \times \hat{j}).(\vec{r} \times \hat{k}) - 12$.

Sol.

If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, find $(\vec{r} \times \hat{i}) \cdot (\vec{r} \times j) + xy$

$$(\vec{r} \times \hat{j}).(\vec{r} \times \hat{k}) - 12 = (3\hat{k} - 6\hat{i}).(-3\hat{j} - 2\hat{i}) - 12$$

$$= 12 - 12 = 0$$

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4.b

If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$.

Ans.

Given $|\hat{a}+\hat{b}|=1$

As $|\hat{a}+\hat{b}|^2+|\hat{a}-\hat{b}|^2=2(|\hat{a}|^2+|\hat{b}|^2)$

$\Rightarrow 1+|\vec{a}-\vec{b}|^2=2(1+1)$

$\Rightarrow |\hat{a}-\hat{b}|^2=3 \Rightarrow |\hat{a}-\hat{b}|=\sqrt{3}$

XII. Prove that/ find/show that based on properties :

1.a 2025

If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three vectors such that $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c}$ and $\vec{a}\times\vec{b} = \vec{a}\times\vec{c}$, $\vec{a} \neq 0$, then show that $\vec{b} = \vec{c}$.

Sol.

$$\vec{a}.\vec{b} = \vec{a}.\vec{c} \Rightarrow \vec{a}.(\vec{b}-\vec{c}) = 0$$

$\Rightarrow$ either $\vec{b} = \vec{c}$ or $\vec{a} \perp (\vec{b}-\vec{c})$, since $\vec{a} \neq 0$

Also, $\vec{a}\times\vec{b} = \vec{a}\times\vec{c} \Rightarrow \vec{a}\times(\vec{b}-\vec{c}) = 0$

$\Rightarrow$ either $\vec{b} = \vec{c}$ or $\vec{a} \parallel (\vec{b}-\vec{c})$, since $\vec{a} \neq 0$

Since vectors $\vec{a}$ and $(\vec{b}-\vec{c})$ cannot be $\parallel$ and $\perp$ simultaneously

Hence $\vec{b} = \vec{c}$

2. 2022

If $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ are four non-zero vectors such that $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c} = 4\vec{b}\times\vec{d}$, then show that $(\vec{a} - 2\vec{d})$ is parallel to $(2\vec{b} - \vec{c})$ where $\vec{a} \neq 2\vec{d}$, $\vec{c} \neq 2\vec{b}$.

Sol.

Consider $(\vec{a} - 2\vec{d})\times(2\vec{b} - \vec{c})$

$$= \vec{a}\times 2\vec{b} - \vec{a}\times\vec{c} - 4\vec{d}\times\vec{b} + 2\vec{d}\times\vec{c}$$

$$= 0$$

$$\therefore (\vec{a} - 2\vec{d}) \parallel (2\vec{b} - \vec{c})$$

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3.

Show that $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$, for any two non-zero vectors $\vec{a}$ and $\vec{b}$.

Sol.

Ans: $\left(|\vec{a}|\vec{b}+|\vec{b}|\vec{a}\right)\cdot\left(|\vec{a}|\vec{b}-|\vec{b}|\vec{a}\right)$

$$= \left(|\vec{a}|\vec{b}\right)^2 - \left(|\vec{b}|\vec{a}\right)^2$$

$$= |\vec{a}|^2|\vec{b}|^2 - |\vec{b}|^2|\vec{a}|^2 = 0$$

$$\therefore \left(|\vec{a}|\vec{b}+|\vec{b}|\vec{a}\right) \perp \left(|\vec{a}|\vec{b}-|\vec{b}|\vec{a}\right)$$

4. 2022

If $\vec{a}$ and $\vec{b}$ are two vectors of equal magnitude and α is the angle between them, then prove that $\dfrac{|\vec{a}+\vec{b}|}{|\vec{a}-\vec{b}|} = \cot\left(\dfrac{\alpha}{2}\right)$.

Sol.

$$\text{Consider } \frac{|\vec{a}+\vec{b}|^2}{|\vec{a}-\vec{b}|^2} = \frac{|\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\cos\alpha}{|\vec{a}|^2+|\vec{b}|^2-2|\vec{a}||\vec{b}|\cos\alpha}$$

$$= \frac{2m^2(1+\cos\alpha)}{2m^2(1-\cos\alpha)} \quad \text{where } |\vec{a}| = |\vec{b}| = m$$

$$= \frac{2\cos^2\dfrac{\alpha}{2}}{2\sin^2\dfrac{\alpha}{2}}$$

$$= \cot^2\left(\frac{\alpha}{2}\right)$$

$$\therefore \frac{|\vec{a}+\vec{b}|}{|\vec{a}-\vec{b}|} = \cot\left(\frac{\alpha}{2}\right)$$

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5. 2022,2025

If $\hat{a}$ and $\hat{b}$ are unit vectors inclined at an angle θ, then prove tha

$$\sin\frac{\theta}{2} = \frac{1}{2}\,|\hat{a} - \hat{b}|.$$

Sol.

$$|\vec{a}| = |\vec{b}| = 1$$

$$|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b}$$

$$= 1 + 1 - 2|\vec{a}||\vec{b}|\cos\theta$$

$$= 2 - 2\cos\theta$$

$$= 2\left(2\sin^2\frac{\theta}{2}\right) = 4\sin^2\frac{\theta}{2}$$

$$\Rightarrow \sin\frac{\theta}{2} = \frac{1}{2}|\vec{a} - \vec{b}|$$

6.

Find $|\vec{a}|$ and $|\vec{b}|$, if $|\vec{a}| = 2|\vec{b}|$ and $(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 12$.

Answer:

$$(\vec{a} + \vec{b})(\vec{a} - \vec{b}) = 12 \Rightarrow |\vec{a}|^2 - |\vec{b}|^2 = 12$$

$$\Rightarrow 3|\vec{b}|^2 = 12 \Rightarrow |\vec{b}| = 2$$

$$\text{Now, } |\vec{a}|^2 = 12 + |\vec{b}|^2 = 16 \Rightarrow |\vec{a}| = 4$$

7. 2022

If $\vec{a}$ and $\vec{b}$ are two vectors such that $\left|\vec{a}+\vec{b}\right| = \left|\vec{b}\right|$, then prove that $\left(\vec{a}+2\vec{b}\right)$ is perpendicular to $\vec{a}$.

Ans.

$|\vec{a}+\vec{b}|=|\vec{b}|$

$\left(\vec{a}+\vec{b}\right)^2=\left(\vec{b}\right)^2$

$\vec{a}^2+\vec{b}^2+2\vec{a}.\vec{b}=\vec{b}^2$

$\vec{a}^2+2\vec{a}.\vec{b}=0$

$(\vec{a}+2\vec{b}).\vec{a}=0$

$\therefore(\vec{a}+2\vec{b})\perp\vec{a}$

prepared by : BALAJI KANCHI

8. 2022

Let $\vec{a}=\hat{i}+\hat{j}$, $\vec{b}=\hat{i}-\hat{j}$ and $\vec{c}=\hat{i}+\hat{j}+\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{a}.\hat{n}=0$ and $\vec{b}.\hat{n}=0$, then find $\left|\vec{c}.\hat{n}\right|$.

Sol.

$\vec{a}.\hat{n}=0, \vec{b}.\hat{n}=0 \Rightarrow \hat{n}$ is $\perp$ to both $\vec{a}$ and $\vec{b}$

Now, $\hat{n}=\dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

Here, $\vec{n}=\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 0\\ 1 & -1 & 0\end{vmatrix}=-2\hat{k}$

$\Rightarrow \hat{n}=-\hat{k}$ (or $\hat{n}=\hat{k}$)

$\therefore |\vec{c}.\hat{n}|=|\vec{c}.(-\hat{k})|=1$

prepared by : BALAJI KANCHI

9. 2022

If $\vec{a}$ and $\vec{b}$ are unit vectors inclined at an angle 30° to each other, then find the area of the parallelogram with $\left(\vec{a}+3\vec{b}\right)$ and $\left(3\vec{a}+\vec{b}\right)$ as adjacent sides.

Sol.

$$\text{Area of parallelogram} = \left|(\vec{a}+3\vec{b})\times(3\vec{a}+\vec{b})\right|$$

$$=\left|(\vec{a}\times 3\vec{a})+(3\vec{b}\times 3\vec{a})+(\vec{a}\times\vec{b})+(3\vec{b}\times\vec{b})\right|$$

$$=\left|9(\vec{b}\times\vec{a})+(\vec{a}\times\vec{b})\right|$$

$$=8\left|\vec{b}\times\vec{a}\right|$$

$$=8[|\vec{b}|.|\vec{a}|\sin\theta]$$

$$=8\times1\times1\times\frac{1}{2}$$

$$=4$$

10.

Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors.

Prove that $|\vec{a}\times\vec{b}| \le |\vec{a}||\vec{b}|$.

State the condition under which equality holds, i.e., $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|$.

Sol.

$$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}||\sin\theta|$$

$$\text{As, } 0 \le |\sin\theta| \le 1$$

$$\Rightarrow |\vec{a}||\vec{b}|\,|\sin\theta| \le |\vec{a}||\vec{b}|$$

$$\Rightarrow |\vec{a}\times\vec{b}| \le |\vec{a}||\vec{b}|$$

$$\text{For equality, } \sin\theta=1 \Rightarrow \theta=\frac{\pi}{2} \Rightarrow \vec{a} \text{ is perpendicular to } \vec{b}.$$

11. 2024

If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $(\vec{a} + \vec{b}) \perp \vec{a}$ and $(2\vec{a} + \vec{b}) \perp \vec{b}$, then prove that $|\vec{b}| = \sqrt{2}\,|\vec{a}|$.

Sol.

$(\vec{a} + \vec{b}).\vec{a} = 0 \Rightarrow |\vec{a}|^2 + \vec{b}.\vec{a} = 0$ -----------------(1)

$(2\vec{a} + \vec{b}).\vec{b} = 0 \Rightarrow 2\vec{a}.\vec{b} + |\vec{b}|^2 = 0$ -------------(2)

$2(-|\vec{a}|^2) + |\vec{b}|^2 = 0$ {Using (1) and (2)}

$|\vec{b}|^2 = 2|\vec{a}|^2 \Rightarrow |\vec{b}| = \sqrt{2}|\vec{a}|$

12. 2025

If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are unit vectors such that $\hat{a}.\hat{b} = \hat{a}.\hat{c} = 0$ and the angle between $\hat{b}$ and $\hat{c}$ is $\frac{\pi}{6}$, then prove that $\hat{a} = \pm 2(\hat{b} \times \hat{c})$.

Sol.

$\hat{a}.\hat{b} = 0 \Rightarrow \hat{a} \perp \hat{b}, \hat{a}.\hat{c} = 0 \Rightarrow \hat{a} \perp \hat{c}$

$\Rightarrow \hat{a}$ is perpendicular to both $\hat{b}$ and $\hat{c} \Rightarrow \hat{a} \,||\, (\hat{b} \times \hat{c})$

Let $\hat{a} = \lambda(\hat{b} \times \hat{c})$

$\Rightarrow |\hat{a}| = |\lambda|\left|(\hat{b} \times \hat{c})\right| \Rightarrow |\hat{a}| = |\lambda||\hat{b}||\hat{c}|\sin\frac{\pi}{6}$

$\Rightarrow |\lambda| = 2 \Rightarrow \lambda = \pm 2$

$\therefore \hat{a} = \pm 2(\hat{b} \times \hat{c})$

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13.a

If the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ represent the three sides of a triangle, then show that $\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a}$.

Sol.

$$\vec{a}+\vec{b}+\vec{c}=\vec{O}$$

$$\Rightarrow \vec{a}\times\left(\vec{a}+\vec{b}+\vec{c}\right)=\vec{O}$$

$$\Rightarrow \vec{a}\times\vec{a}+\vec{a}\times\vec{b}+\vec{a}\times\vec{c}=\vec{O}$$

$$\Rightarrow \vec{a}\times\vec{b}=-\vec{a}\times\vec{c},\ \therefore \vec{a}\times\vec{b}=\vec{c}\times\vec{a}$$

Similarly, $\vec{b}\times\vec{c}=\vec{c}\times\vec{a},\ \therefore \vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

13.b

Prove that three points A, B and C with position vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively are collinear if and only if $\left(\vec{b}\times\vec{c}\right)+\left(\vec{c}\times\vec{a}\right)+\left(\vec{a}\times\vec{b}\right)=\vec{0}$

Ans. Points $A(\vec{a}), B(\vec{b})$ and $C(\vec{c})$ are collinear

$$\Rightarrow \overrightarrow{AB}\times\overrightarrow{AC}=\vec{0}$$

$$\Rightarrow \left(\vec{b}-\vec{a}\right)\times\left(\vec{c}-\vec{a}\right)=\vec{0}$$

$$\Rightarrow \vec{b}\times\vec{c}-\vec{b}\times\vec{a}-\vec{a}\times\vec{c}+\vec{a}\times\vec{a}=\vec{0}$$

$$\Rightarrow \vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=\vec{0}$$

Similarly, converse can be proved

5 Mark problems :

1.a

Let $\vec{a} = 4\hat{i} + 5\hat{j} - \hat{k}$, $\vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - \hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{c}$ and $\vec{b}$ and $\vec{d} \cdot \vec{a} = 21$.

Sol.

$$\vec{d} = \lambda(\vec{c} \times \vec{b}) = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -1 \\ 1 & -4 & 5 \end{vmatrix}$$

$$\therefore \ \vec{d} = \lambda\hat{i} - 16\lambda\hat{j} - 13\lambda\hat{k}$$

$$\vec{d} \cdot \vec{a} = 21 \Rightarrow 4\lambda - 80\lambda + 13\lambda = 21 \Rightarrow \lambda = -\frac{1}{3}$$

$$\therefore \ \vec{d} = -\frac{1}{3}\hat{i} + \frac{16}{3}\hat{j} + \frac{13}{3}\hat{k}$$

1.b 2024

Given $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} - \hat{k}$ and $\vec{c} = 2\hat{i} + \hat{j} - 2\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot \vec{d} = 3$.

Sol.

Since $\vec{d} \perp \vec{a}$ and $\vec{d} \perp \vec{b} \Rightarrow \vec{d} = \lambda\left(\vec{a} \times \vec{b}\right)$

$$\vec{d} = \lambda\left(\hat{i} + 5\hat{j} + 3\hat{k}\right)$$

$$\vec{c} \cdot \vec{d} = 3 \Rightarrow 2\lambda + 5\lambda - 6\lambda = 3 \Rightarrow \lambda = 3$$

$$\Rightarrow \vec{d} = 3\hat{i} + 15\hat{j} + 9\hat{k}$$

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1.c 2025

Let $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{b} = 4\hat{i} - 3\hat{j} + 9\hat{k}$ and $\vec{c} = 3\hat{i} - 2\hat{j} + 6\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot \vec{d} = 39$.

Sol.

Vector $\vec{d}$ is perpendicular to both $\vec{a}$ and $\vec{b}$

So $\vec{d} = \lambda(\vec{a} \times \vec{b})$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 4 & -3 & 9 \end{vmatrix} = 39\hat{i} - 2\hat{j} - 18\hat{k}$$

$\Rightarrow \vec{d} = \lambda(39\hat{i} - 2\hat{j} - 18\hat{k})$

As $\vec{c}.\vec{d} = 39$

$\Rightarrow (3 \times 39\lambda - 2 \times (-2\lambda) + 6 \times (-18\lambda)) = 39$

$\Rightarrow 13\,\lambda = 39 \Rightarrow \lambda = 3$

$\Rightarrow \vec{d} = 3(39\hat{i} - 2\hat{j} - 18\hat{k}) = 117\hat{i} - 6\hat{j} - 54\hat{k}$

2. 2022

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular vectors of equal magnitude, then prove that the vector $(2\vec{a} + \vec{b} + 2\vec{c})$ is equally inclined to both $\vec{a}$ and $\vec{c}$. Also, find the angle between $\vec{a}$ and $(2\vec{a} + \vec{b} + 2\vec{c})$.

Sol.

Let, $|\vec{a}| = |\vec{b}| = |\vec{c}| = m$

$$|2\vec{a} + \vec{b} + 2\vec{c}|^2 = 4|\vec{a}|^2 + |\vec{b}|^2 + 4|\vec{c}|^2 + 4\vec{a}\cdot\vec{b} + 4\vec{b}\cdot\vec{c} + 8\vec{a}\cdot\vec{c}$$

$$= 9m^2$$

$$|2\vec{a} + \vec{b} + 2\vec{c}| = 3m$$

Let θ be angle between $2\vec{a} + \vec{b} + 2\vec{c}$ and $\vec{a}$.

$$\cos\theta = \frac{(2\vec{a} + \vec{b} + 2\vec{c})\cdot\vec{a}}{|2\vec{a} + \vec{b} + 2\vec{c}||\vec{a}|} = \frac{2m^2}{3m\cdot m} = \frac{2}{3}$$

Let, α be angle between $2\vec{a} + \vec{b} + 2\vec{c}$ and $\vec{c}$

Similarly, $\cos\alpha = \frac{2m^2}{3m\cdot m} = \frac{2}{3}$

$\therefore \cos\theta = \cos\alpha \Rightarrow \theta = \alpha$

$\therefore$ Required angle $= \cos^{-1}\left(\frac{2}{3}\right)$

3.

If $\vec{a}$, $\vec{b}$, $\vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$. Also, find the angle which $\vec{a} + \vec{b} + \vec{c}$ makes with $\vec{a}$ or $\vec{b}$ or $\vec{c}$.

Sol.

$$|\vec{a}| = |\vec{b}| = |\vec{c}| \text{ and } \vec{a}\cdot\vec{b} = 0 = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} \qquad \text{...(i)}$$

Let α, β and γ be the angles made by $(\vec{a}+\vec{b}+\vec{c})$ with $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively

$$(\vec{a}+\vec{b}+\vec{c})\cdot\vec{a} = |\vec{a}+\vec{b}+\vec{c}|\,|\vec{a}|\cos\alpha$$

$$\Rightarrow \alpha = \cos^{-1}\left(\frac{|\vec{a}|}{|\vec{a}+\vec{b}+\vec{c}|}\right)$$

Similarly, $\beta = \cos^{-1}\left(\frac{|\vec{b}|}{|\vec{a}+\vec{b}+\vec{c}|}\right)$ and $\gamma = \cos^{-1}\left(\frac{|\vec{c}|}{|\vec{a}+\vec{b}+\vec{c}|}\right)$

using (i), we get $\alpha = \beta = \gamma$

Now $|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$

$$\Rightarrow |\vec{a}+\vec{b}+\vec{c}|^2 = 3|\vec{a}|^2 \text{ (using (i))}$$

$$\Rightarrow |\vec{a}+\vec{b}+\vec{c}| = \sqrt{3}\,|\vec{a}|$$

$$\therefore \alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = \beta = \gamma$$

4.

If $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+5\hat{j}$, $3\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-6\hat{j}-\hat{k}$ respectively are the position vectors of points A, B, C and D, then find the angle between the straight lines AB and CD. Find whether $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear or not.

Sol.

$\overrightarrow{AB}=\hat{i}+4\hat{j}-\hat{k}$

$\overrightarrow{CD}=-2\hat{i}-8\hat{j}+2\hat{k}$

Let required angle be θ.

Then $\cos\theta=\dfrac{\overrightarrow{AB}\cdot\overrightarrow{CD}}{|\overrightarrow{AB}||\overrightarrow{CD}|}=\dfrac{-2-32-2}{\sqrt{18}\sqrt{72}}=-1$

$\Rightarrow \theta = 180^\circ$ or π

Since $\theta=\pi$ so $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear.

Case Study :

1.

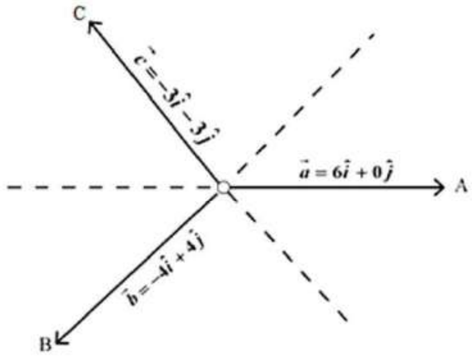
Read the following passage and answer the questions given below:

Teams A, B, C went for playing a tug of war game. Teams A, B, C have attached a rope to a metal ring and is trying to pull the ring into their own area.

Team A pulls with force $F_1 = 6\hat{i} + 0\hat{j}\ kN$,

Team B pulls with force $F_2 = -4\hat{i} + 4\hat{j}\ kN$,

Team C pulls with force $F_3 = -3\hat{i} - 3\hat{j}\ kN$,



(i) What is the magnitude of the force of Team A ?

(ii) Which team will win the game?

(iii) Find the magnitude of the resultant force exerted by the teams.

OR

(iii) In what direction is the ring getting pulled?

Sol.

We have ,

$$\left|\overrightarrow{F_1}\right| = \sqrt{6^2+0^2} = 6kN, \left|\overrightarrow{F_2}\right| = \sqrt{(-4)^2+4^2} = \sqrt{32} = 4\sqrt{2}\,kN, \left|\overrightarrow{F_3}\right| = \sqrt{(-3)^2+(-3)^2} = \sqrt{18} = 3\sqrt{2}\,kN.$$

(i) Magnitude of force of Team $A = 6\,kN$.

(ii) Since $\vec{a}+\vec{c} = 3(\hat{i}-\hat{j})$ *and* $\vec{b} = -4\ (\hat{i}-\hat{j})$

So, $\vec{b}$ **and** $\vec{a}+\vec{c}$ **are unlike vectors having same intial point**

and $\left|\vec{b}\right| = 4\sqrt{2}$ **&** $\left|\vec{a}+\vec{c}\right| = 3\sqrt{2}$

Thus $\left|\overrightarrow{F_2}\right| > \left|\overrightarrow{F_1}+\overrightarrow{F_3}\right|$ **also** $\overrightarrow{F_2}$ **and** $\overrightarrow{F_1}+\overrightarrow{F_3}$ **are unlike**

Hence B will win the game

(iii) $\overrightarrow{F} = \overrightarrow{F_1}+\overrightarrow{F_2}+\overrightarrow{F_3} = 6\hat{i}+0\hat{j}-4\hat{i}+4\hat{j}-3\hat{i}-3\hat{j} = -\hat{i}+\hat{j}$

$$\therefore \left|\overrightarrow{F}\right| = \sqrt{(-1)^2+(1)^2} = \sqrt{2}\,kN.$$

OR

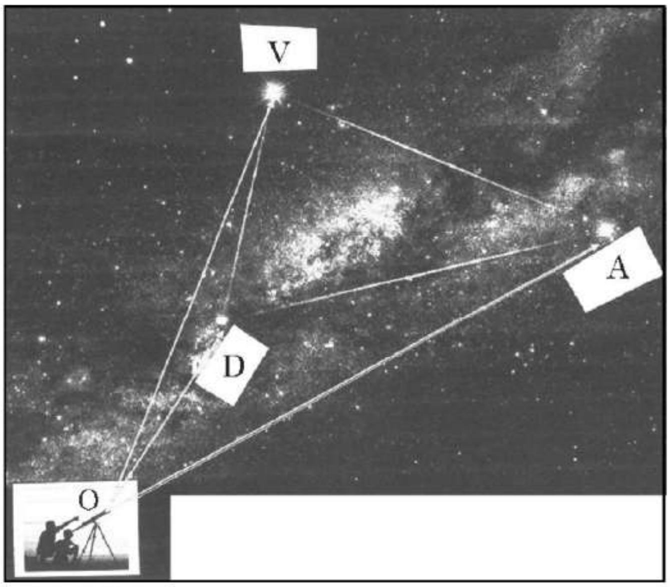
$\overrightarrow{F} = -\hat{i}+\hat{j}$

$\therefore \theta = \pi - \tan^{-1}\left(\frac{1}{1}\right) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$; **where** '$\theta$' is the angle made by the resultant force with the $+ve$ direction of the $x-$axis.

2.2024

In this section, there are **3** case study based questions of **4** marks each.

An instructor at the astronomical centre shows three among the brightest stars in a particular constellation. Assume that the telescope is located at O(0, 0, 0) and the three stars have their locations at the points D, A and V having position vectors $2\hat{i} + 3\hat{j} + 4\hat{k}$, $7\hat{i} + 5\hat{j} + 8\hat{k}$ and $-3\hat{i} + 7\hat{j} + 11\hat{k}$ respectively.



Based on the above information, answer the following questions :

(i) How far is the star V from star A ?

(ii) Find a unit vector in the direction of $\overrightarrow{DA}$.

(iii) Find the measure of $\angle VDA$.

OR

(iii) What is the projection of vector $\overrightarrow{DV}$ on vector $\overrightarrow{DA}$?

Sol.

(i) $\overrightarrow{AV}$ = Position Vector of V − Position Vector of A

$$= -3\hat{i}+7\hat{j}+11\hat{k}-7\hat{i}-5\hat{j}-8\hat{k} = -10\hat{i}+2\hat{j}+3\hat{k}$$

Thus, $|\overrightarrow{AV}| = \sqrt{100+4+9} = \sqrt{113}$ units

(ii) $\overrightarrow{DA}$ = Position Vector of A − Position Vector of D

$$= 7\hat{i}+5\hat{j}+8\hat{k}-2\hat{i}-3\hat{j}-4\hat{k} = 5\hat{i}+2\hat{j}+4\hat{k}$$

Unit vector in the direction of $\overrightarrow{DA} = \dfrac{5\hat{i}+2\hat{j}+4\hat{k}}{3\sqrt{5}}$

(iii) $\overrightarrow{DV} = -5\hat{i}+4\hat{j}+7\hat{k}$

$$\angle VDA = \cos^{-1}\left(\frac{\overrightarrow{DV}.\overrightarrow{DA}}{|\overrightarrow{DV}||\overrightarrow{DA}|}\right) = \cos^{-1}\left(\frac{11\sqrt{2}}{90}\right)$$

OR

(iii) $\overrightarrow{DV} = -5\hat{i}+4\hat{j}+7\hat{k}$

Projection of $\overrightarrow{DV}$ on $\overrightarrow{DA} = \left(\dfrac{\overrightarrow{DV}.\overrightarrow{DA}}{|\overrightarrow{DA}|}\right) = \dfrac{11\sqrt{5}}{15}$

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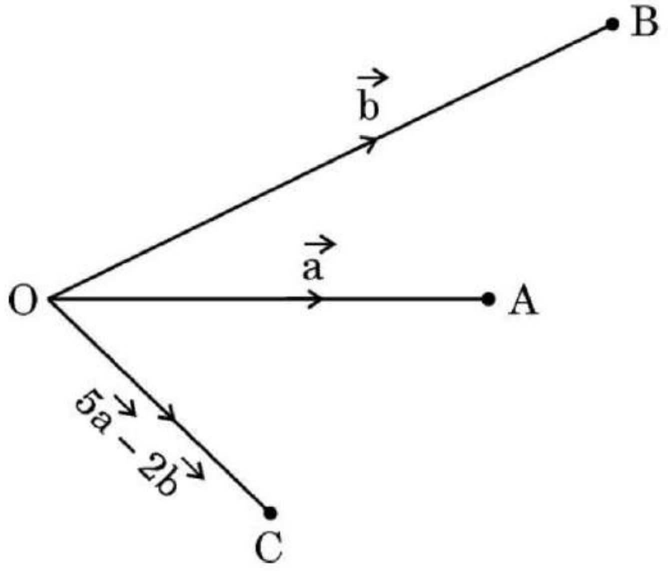
3. 2025

Three friends A, B and C move out from the same location O at the same time in three different directions to reach their destinations. They move out on straight paths and decide that A and B after reaching their destinations will meet up with C at his predecided destination, following straight paths from A to C and B to C in such a way that $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and $\overrightarrow{OC} = 5\vec{a} - 2\vec{b}$ respectively.



Based upon the above information, answer the following questions :

(i) Complete the given figure to explain their entire movement plan along the respective vectors.

(ii) Find vectors $\overrightarrow{AC}$ and $\overrightarrow{BC}$.

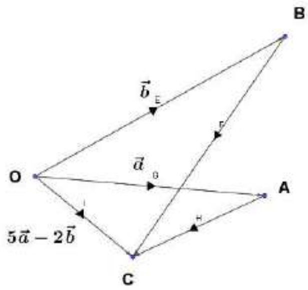
(iii) (a) If $\vec{a} \cdot \vec{b} = 1$, distance of O to A is 1 km and that from O to B is 2 km, then find the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Also, find $|\vec{a} \times \vec{b}|$.

OR

(iii) (b) If $\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, then find a unit vector perpendicular to $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$.

Sol.

(i) The Complete figure of their entire movement plan is:



(ii) $\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=4\vec{a}-2\vec{b}$, $\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}=5\vec{a}-3\vec{b}$

(iii) (a) we are given: $|\vec{a}|=1, |\vec{b}|=2$, assuming 'θ' as the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

$$\theta=\cos^{-1}\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}\right)=\cos^{-1}\frac{1}{1\times 2}=\cos^{-1}\frac{1}{2}=\frac{\pi}{3}$$

$$|\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|\sin\theta=1(2)\frac{\sqrt{3}}{2}=\sqrt{3}$$

OR

(iii) (b) $\vec{a}+\vec{b}=2\hat{i}+3\hat{k}$, $\vec{a}-\vec{b}=2\hat{i}-2\hat{j}+5\hat{k}$, let $\vec{c}$ be $\perp$ to both ($\vec{a}+\vec{b}$) and ($\vec{a}-\vec{b}$)

Then, $\vec{c}=\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 0 & 3\\ 2 & -2 & 5\end{vmatrix}=6\hat{i}-4\hat{j}-4k$ and $|\vec{c}|=\sqrt{68}$

The required unit vector is, $\hat{c}=\frac{1}{2\sqrt{17}}\left(6\hat{i}-4\hat{j}-4\hat{k}\right)=\frac{1}{\sqrt{17}}\left(3\hat{i}-2\hat{j}-2\hat{k}\right)$

4.

A cricket match is organised between two clubs P and Q for which a team from each club is chosen. Remaining players of club P and club Q are respectively sitting along the lines AB and CD, where the points are A(3, 4, 0), B(5, 3, 3), C(6, – 4, 1) and D(13, – 5, – 4).

Based on the above, answer the following questions :

(i) Write the direction ratios of vector $\overrightarrow{AB}$.

(ii) Write a unit vector in the direction of $\overrightarrow{CD}$.

(iii) (a) Find the angle between vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$.

OR

(iii) (b) Write a vector perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{CD}$.

Sol.

(i) $\overrightarrow{AB} = \vec{b} - \vec{a} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$

$\therefore$ DRs of vector $\overrightarrow{AB}$ are $2, -1, 3$

(ii) $\overrightarrow{CD} = 7\hat{\imath} - \hat{\jmath} - 5\hat{k}$,

$\widehat{CD} = \frac{7}{5\sqrt{3}}\hat{\imath} - \frac{1}{5\sqrt{3}}\hat{\jmath} - \frac{5}{5\sqrt{3}}\hat{k}$

(iii) (a) $\overrightarrow{AB}.\overrightarrow{CD} = 14 + 1 - 15 = 0$

$\overrightarrow{AB} \perp \overrightarrow{CD} \Rightarrow \theta = 90°$

OR

(iii) (b) $\overrightarrow{AB} \times \overrightarrow{CD} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -1 & 3 \\ 7 & -1 & -5 \end{vmatrix}$

$= 8\hat{\imath} + 31\hat{\jmath} + 5\hat{k}$