11. 3D-Geometry

(Previou years Questions 2017 -2025 Solutions)

2022 March :

1.

The length of the perpendicular drawn from the point $(4, -7, 3)$ on the y-axis is

(A) 3 units
(B) 4 units
(C) 5 units
(D) 7 units

2.

The two lines $x = ay + b$, $z = cy + d$; and $x = a'y + b'$, $z = c'y + d'$ are perpendicular to each other, if

(a) $\frac{a}{a'} + \frac{c}{c'} = 1$ (b) $\frac{a}{a'} + \frac{c}{c'} = -1$ (c) $aa' + cc' = 1$ (d) $aa' + cc' = -1$

3.

The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{4-z}{k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{-2}$ are mutually perpendicular if the value of k is

(a) $-\frac{2}{3}$ (b) $\frac{2}{3}$ (c) -2 (d) 2

2023 March:

1.

Direction cosines of the line $\frac{x-1}{2} = \frac{1-y}{3} = \frac{2z-1}{12}$ are :

(a) $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$

(b) $\frac{2}{\sqrt{157}}, -\frac{3}{\sqrt{157}}, \frac{12}{\sqrt{157}}$

(c) $\frac{2}{7}, -\frac{3}{7}, -\frac{6}{7}$

(d) $\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}$

2.

Assertion (A) : Equation of a line passing through the points (1, 2, 3) and (3, – 1, 3) is $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-3}{0}$.

Reason (R) : Equation of a line passing through points (x_1, y_1, z_1), (x_2, y_2, z_2) is given by $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$.

3.

Direction cosines of a line perpendicular to both x-axis and z-axis are :

(a) 1, 0, 1

(b) 1, 1, 1

(c) 0, 0, 1

(d) 0, 1, 0

4.

Equation of a line passing through point (1, 1, 1) and parallel to z-axis is

(A) $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$

(B) $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-1}{1}$

(C) $\frac{x}{0} = \frac{y}{0} = \frac{z-1}{1}$

(D) $\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{1}$

5.

Assertion (A) : If a line makes angles α, β, γ with positive direction of the coordinate axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.

Reason (R) : The sum of squares of the direction cosines of a line is 1.

6.

Equation of line passing through origin and making 30°, 60° and 90° with x, y, z axes respectively is

(A) $\frac{2x}{\sqrt{3}}=\frac{y}{2}=\frac{z}{0}$ (B) $\frac{2x}{\sqrt{3}}=\frac{2y}{1}=\frac{z}{0}$

(C) $2x=\frac{2y}{\sqrt{3}}=\frac{z}{1}$ (D) $\frac{2x}{\sqrt{3}}=\frac{2y}{1}=\frac{z}{1}$

7.

Equation of a line passing through point (1, 2, 3) and equally inclined to the coordinate axis, is

(A) $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ (B) $\frac{x}{1}=\frac{y}{1}=\frac{z}{1}$

(C) $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-1}{3}$ (D) $\frac{x-1}{1}=\frac{y-2}{1}=\frac{z-3}{1}$

8.

The value of λ for which the angle between the lines

$\vec{r}=\hat{i}+\hat{j}+\hat{k}+p(2\hat{i}+\hat{j}+2\hat{k})$ and

$\vec{r}=(1+q)\hat{i}+(1+q\lambda)\hat{j}+(1+q)\hat{k}$ is $\frac{\pi}{2}$ is :

(a) -4 (b) 4

(c) 2 (d) -2

9.

Assertion (A) : A line through the points (4, 7, 8) and (2, 3, 4) is parallel to a line through the points $(-1, -2, 1)$ and (1, 2, 5).

Reason (R): Lines $\vec{r}=\vec{a_1}+\lambda\vec{b_1}$ and $\vec{r}=\vec{a_2}+\mu\vec{b_2}$ are parallel if $\vec{b_1}\cdot\vec{b_2}=0$.

10.

The direction cosines of vector $\overrightarrow{BA}$, where coordinates of A and B are (1, 2, – 1) and (3, 4, 0) respectively, are :

(a) $-2, -2, -1$ (b) $-\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}$

(c) $2, 2, 1$ (d) $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$

11.

If the point P(a, b, 0) lies on the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z+3}{4}$, then (a, b) is :

(a) $(1, 2)$ (b) $\left(\frac{1}{2}, \frac{2}{3}\right)$

(c) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (d) $(0, 0)$

12.

The equation of a line passing through point (2, – 1, 0) and parallel to the line $\frac{x}{1} = \frac{y-1}{2} = \frac{2-z}{2}$ is :

(a) $\frac{x+2}{1} = \frac{y-1}{2} = \frac{z}{2}$ (b) $\frac{x-2}{1} = \frac{y-1}{2} = \frac{z}{2}$

(c) $\frac{x+2}{1} = \frac{y-1}{2} = \frac{z}{-2}$ (d) $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z}{-2}$

13.

Distance of the point (p, q, r) from y-axis is :

(a) q (b) $|q|$

(c) $|q| + |r|$ (d) $\sqrt{p^2 + r^2}$

14.

If the direction cosines of a line are $\left(\frac{1}{a}, \frac{1}{a}, \frac{1}{a}\right)$, then :

(a) $0 < a < 1$ (b) $a > 2$

(c) $a > 0$ (d) $a = \pm\sqrt{3}$

15.

Assertion (A) : The lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ are perpendicular, when $\vec{b_1} \cdot \vec{b_2} = 0$.

Reason (R) : The angle θ between the lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is given by $\cos\theta = \frac{\vec{b_1} \cdot \vec{b_2}}{|\vec{b_1}||\vec{b_2}|}$

16.

If a line makes angles of 90°, 135° and 45° with the x, y and z axes respectively, then its direction cosines are

(a) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}$ (d) $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

17.

The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is

(a) 0° (b) 30°

(c) 45° (d) 90°

18.

Assertion (A) : Quadrilateral formed by vertices A(0, 0, 0), B(3, 4, 5), C(8, 8, 8) and D(5, 4, 3) is a rhombus.

Reason (R) : ABCD is a rhombus if AB = BC = CD = DA, AC ≠ BD.

18.

The vector equation of a line which passes through the point (2, – 4, 5) and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{2} = \frac{z+8}{6}$ is :

(a) $\vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$

(b) $\vec{r} = (2\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 6\hat{k})$

(c) $\vec{r} = (2\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$

(d) $\vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(3\hat{i} - 2\hat{j} - 6\hat{k})$

19.

The vector equation of a line which passes through the point (1, – 2, 3) and is parallel to the vector $3\hat{i} - 2\hat{j} + 4\hat{k}$ is :

(a) $\vec{r} = (-\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 4\hat{k})$

(b) $\vec{r} = (-3\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$

(c) $\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 4\hat{k})$

(d) $\vec{r} = (3\hat{i} - 2\hat{j} + 4\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$

20.

The angle between the lines $\frac{x+1}{1} = \frac{4-y}{-1} = \frac{z-5}{2}$ and $\frac{x+3}{-3} = \frac{y-2}{5} = \frac{z+5}{4}$ is :

(a) $\cos^{-1}\left(\frac{2}{3}\right)$ (b) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

21.

The direction ratios of a line whose Cartesian equations are $3x - 3 = 2y + 1 = 5 - 6z$, are :

(a) 2, 3, –1 (b) 3, –2, 1

(c) 2, 1, –3 (d) 3, 2, –1

22.

Assertion (A) : The vector equation of a line passing through the points (1, 2, 3) and (3, 0, 2) is $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(2\hat{i} - 2\hat{j} - \hat{k})$.

Reason (R) : Equation of a line passing through two points with position vectors $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$.

23.

If the position vectors of two points A and B are $\hat{i} + 2\hat{j} - 3\hat{k}$ and $-\hat{i} - 2\hat{j} + \hat{k}$ respectively, then the direction cosines of the vector $\vec{BA}$ are :

(a) $\frac{2}{6}, -\frac{4}{6}, -\frac{4}{6}$ (b) $\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}$

(c) $\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}, -\frac{2}{\sqrt{3}}$ (d) $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$

24.

The value of λ for which the lines $\frac{x-5}{7} = \frac{2-y}{5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{2y-1}{\lambda} = \frac{z}{3}$ are at right angles, is :

(a) 2 (b) 4

(c) -4 (d) -2

25.

Assertion (A) : The vector equation of a line passing through the points A(–1, 0, 2) and B(3, 4, 6) is $\vec{r} = -\hat{i} + 2\hat{k} + \lambda(\hat{i} + \hat{j} + \hat{k})$.

Reason (R) : The equation of a line passing through a point with position vector $\vec{a}$ and parallel to a vector $\vec{b}$, is $\vec{r} = \vec{a} + \lambda\vec{b}$.

26.

If the two lines

$$L_1 : x = 5, \ \frac{y}{3-\alpha} = \frac{z}{-2}$$

$$L_2 : x = 2, \ \frac{y}{-1} = \frac{z}{2-\alpha}$$

are perpendicular, then the value of α is

(A) $\frac{2}{3}$

(B) 3

(C) 4

(D) $\frac{7}{3}$

27.

The equation of the line in vector form passing through the point$(-1,3,5)$ and parallel to line $\frac{x-3}{2} = \frac{y-4}{3}, z = 2$. is

(a) $\vec{r} = (-\hat{i} + 3\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + \hat{k})$.

(b) $\vec{r} = (-\hat{i} + 3\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 3\hat{j})$

(c) $\vec{r} = (2\hat{i} + 3\hat{j} - 2\hat{k}) + \lambda(-\hat{i} + 3\hat{j} + 5\hat{k})$

(d) $\vec{r} = (2\hat{i} + 3\hat{j}) + \lambda(-\hat{i} + 3\hat{j} + 5\hat{k})$

March 2024 :

1.

If the direction cosines of a line are $\sqrt{3}\,k$, $\sqrt{3}\,k$, $\sqrt{3}\,k$, then the value of k is :

(A) ± 1 (B) $\pm \sqrt{3}$

(C) ± 3 (D) $\pm \frac{1}{3}$

2.

The distance of point P(a, b, c) from y-axis is :

(A) b (B) b^2

(C) $\sqrt{a^2 + c^2}$ (D) $a^2 + c^2$

3.

Assertion (A) : A line in space cannot be drawn perpendicular to x, y and z axes simultaneously.

Reason (R) : For any line making angles, α, β, γ with the positive directions of x, y and z axes respectively, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

4.

Direction ratios of a vector parallel to line $\frac{x-1}{2} = -y = \frac{2z+1}{6}$ are :

(A) 2, – 1, 6 (B) 2, 1, 6

(C) 2, 1, 3 (D) 2, – 1, 3

5.

If a line makes an angle of 30° with the positive direction of x-axis, 120° with the positive direction of y-axis, then the angle which it makes with the positive direction of z-axis is :

(A) 90° (B) 120°

(C) 60° (D) 0°

6.

The coordinates of the foot of the perpendicular drawn from the point (0, 1, 2) on the x-axis are given by :

(A) (1, 0, 0) (B) (2, 0, 0)

(C) $(\sqrt{5}, 0, 0)$ (D) (0, 0, 0)

7.

Direction ratios of a vector parallel to line $\frac{x-1}{2} = -y = \frac{2z+1}{6}$ are :

(A) 2, – 1, 6 (B) 2, 1, 6

(C) 2, 1, 3 (D) 2, – 1, 3

8.

If α, β and γ are the angles which a line makes with positive directions of x, y and z axes respectively, then which of the following is ***not*** true ?

(A) $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

(B) $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$

(C) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

(D) $\cos \alpha + \cos \beta + \cos \gamma = 1$

9.

The vector equation of a line passing through the point (1, –1, 0) and parallel to Y-axis is :

(A) $\vec{r} = \hat{i} - \hat{j} + \lambda(\hat{i} - \hat{j})$ (B) $\vec{r} = \hat{i} - \hat{j} + \lambda \hat{j}$

(C) $\vec{r} = \hat{i} - \hat{j} + \lambda \hat{k}$ (D) $\vec{r} = \lambda \hat{j}$

10.

The lines $\frac{1-x}{2} = \frac{y-1}{3} = \frac{z}{1}$ and $\frac{2x-3}{2p} = \frac{y}{-1} = \frac{z-4}{7}$ are perpendicular to each other for p equal to :

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$

(C) 2 (D) 3

11.

A vector perpendicular to the line $\vec{r} = \hat{i} + \hat{j} - \hat{k} + \lambda(3\hat{i} - \hat{j})$ is :

(A) $5\hat{i} + \hat{j} + 6k$ (B) $\hat{i} + 3\hat{j} + 5\hat{k}$

(C) $2\hat{i} - 2\hat{j}$ (D) $9\hat{i} - 3\hat{j}$

12.

The Cartesian equation of a line passing through the point with position vector $\vec{a} = \hat{i} - \hat{j}$ and parallel to the line $\vec{r} = \hat{i} + \hat{k} + \mu(2\hat{i} - \hat{j})$, is

(A) $\frac{x-2}{1} = \frac{y+1}{0} = \frac{z}{1}$

(B) $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{0}$

(C) $\frac{x+1}{2} = \frac{y+1}{-1} = \frac{z}{0}$

(D) $\frac{x-1}{2} = \frac{y}{-1} = \frac{z-1}{0}$

13.

The angle which the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{0}$ makes with the positive direction of Y-axis is :

(A) $\frac{5\pi}{6}$

(B) $\frac{3\pi}{4}$

(C) $\frac{5\pi}{4}$

(D) $\frac{7\pi}{4}$

14.

The Cartesian equation of the line passing through the point (1, –3, 2) and parallel to the line :

$\vec{r} = (2 + \lambda)\hat{i} + \lambda\hat{j} + (2\lambda - 1)\hat{k}$ is

(A) $\frac{x-1}{2} = \frac{y+3}{0} = \frac{z-2}{-1}$

(B) $\frac{x+1}{1} = \frac{y-3}{1} = \frac{z+2}{2}$

(C) $\frac{x+1}{2} = \frac{y-3}{0} = \frac{z+2}{-1}$

(D) $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$

2025 March :

1.

If a line makes angles of $\frac{3\pi}{4}, \frac{\pi}{3}$ and θ with the positive directions of x, y and z-axis respectively, then θ is

(A) $\frac{-\pi}{3}$ only (B) $\frac{\pi}{3}$ only

(C) $\frac{\pi}{6}$ (D) $\pm\frac{\pi}{3}$

2.

The equation of a line parallel to the vector $3\hat{i} + \hat{j} + 2\hat{k}$ and passing through the point (4, –3, 7) is :

(A) $x = 4t + 3, y = -3t + 1, z = 7t + 2$

(B) $x = 3t + 4, y = t + 3, z = 2t + 7$

(C) $x = 3t + 4, y = t - 3, z = 2t + 7$

(D) $x = 3t + 4, y = -t + 3, z = 2t + 7$

3.

The line $x = 1 + 5\mu, y = -5 + \mu, z = -6 - 3\mu$ passes through which of the following point ?

(A) (1, –5, 6) (B) (1, 5, 6)

(C) (1, –5, –6) (D) (–1, –5, 6)

4.

If P is a point on the line segment joining (3, 6, –1) and (6, 2, – 2) and y-coordinate of P is 4, then its z-coordinate is :

(A) $-\frac{3}{2}$ (B) 0

(C) 1 (D) $\frac{3}{2}$

5.

The coordinates of the foot of the perpendicular drawn from the point A(– 2, 3, 5) on the y-axis is :

(A) (0, 0, 5) (B) (0, 3, 0)

(C) (– 2, 0, 5) (D) (– 2, 0, 0)

6.

If the direction cosines of a line are λ, λ, λ, then λ is equal to :

(A) $-\dfrac{1}{\sqrt{3}}$ (B) 1

(C) $\dfrac{1}{\sqrt{3}}$ (D) $\pm\dfrac{1}{\sqrt{3}}$

I.Direction cosines/ratios of line :

1.a

If a line makes angles 90°, 135°, 45° with the x, y and z axes respectively, find its direction cosines.

Sol. Let $\alpha = 90°$

$\beta = 135°$

$\gamma = 45°$

$\because \quad l = \cos\alpha = \cos 90° = 0$

$m = \cos\beta = \cos 135° = -\dfrac{1}{\sqrt{2}}$

$n = \cos\gamma = \cos 45° = \dfrac{1}{\sqrt{2}}$

Direction cosines of the line are $0, -\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}$

2. 2022

If a line makes 60° and 45° angles with the positive directions of x-axis and z-axis respectively, then find the angle that it makes with the positive direction of y-axis. Hence, write the direction cosines of the line.

Sol.

$$l = \cos 60^\circ = \frac{1}{2},\ n = \cos 45^\circ = \frac{1}{\sqrt{2}}$$

Now, $l^2 + m^2 + n^2 = 1$

$$\Rightarrow \frac{1}{4} + m^2 + \frac{1}{2} = 1$$

$$\Rightarrow m^2 = \frac{1}{4} \Rightarrow m = \pm\frac{1}{2}$$

$\theta = 60^\circ$

Required direction cosines are $\left\langle \frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}} \right\rangle$

3.a 2023

If a line makes angles α, β and γ with x-axis, y-axis and z-axis respectively, then prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.

Sol.

d.c. are $\cos\alpha, \cos\beta, \cos\gamma$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\Rightarrow (1-\sin^2\alpha) + (1-\sin^2\beta) + (1-\sin^2\gamma) = 1$$

$$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$$

3.b 2022

If a line makes an angle α, β, γ with the coordinate axes, then find the value of $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$.

Sol.

$l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma \Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$\begin{aligned}\cos 2\alpha + \cos 2\beta + \cos 2\gamma &= 2\cos^2\alpha - 1 + 2\cos^2\beta - 1 + 2\cos^2\gamma - 1 \\ &= 2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) - 3 \\ &= 2 - 3 \\ &= -1\end{aligned}$$

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4.

If a line has the direction ratios -18, 12, -4, then what are its direction cosines ?

Sol.

$$\sqrt{(-18)^2 + (12)^2 + (-4)^2} = 22$$

$$\therefore \text{ DC's are } \frac{-18}{22}, \frac{12}{22}, \frac{-4}{22} \text{ or } \frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$$

5.a

Find the direction cosines of the line whose Cartesian equations are $5x - 3 = 15y + 7 = 3 - 10z$.

Sol.

Equations of given line can be written as

$$\frac{x - \frac{3}{5}}{\frac{1}{5}} = \frac{y + \frac{7}{15}}{\frac{1}{15}} = \frac{z - \frac{3}{10}}{-\frac{1}{10}}$$

$$\text{d.r.'s are } < \frac{1}{5}, \frac{1}{15}, -\frac{1}{10} > \text{ or } < 6, 2, -3 >$$

$$\text{d.c.'s are } < \frac{6}{7}, \frac{2}{7}, -\frac{3}{7} >$$

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5.b

The equations of a line are 5x − 3 = 15y + 7 = 3 − 10z. Write the direction cosines of the line and find the coordinates of a point through which it passes.

Sol.

The equation of the given line is

$$\frac{x - 3/5}{1/5} = \frac{y + 7/15}{1/15} = \frac{z - 3/10}{-1/10}$$

Its direction ratios are.

$$\langle \frac{1}{5}, \frac{1}{15}, -\frac{1}{10} \rangle \text{or } \langle 6, 2, -3 \rangle$$

Direction cosines are $\langle \pm\frac{6}{7}, \pm\frac{2}{7}, \mp\frac{3}{7} \rangle$

The point through which it passes is $\left(\frac{3}{5}, \frac{-7}{15}, \frac{3}{10}\right)$

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5. c

Find the direction cosines of a line whose cartesian equation is given as 3x + 1 = 6y − 2 = 1 − z.

Sol.

Given equation is $3x+1=6y-2=1-z$

$$\Rightarrow \frac{x+\frac{1}{3}}{\frac{1}{3}} = \frac{y-\frac{1}{3}}{\frac{1}{6}} = \frac{z-1}{-1}$$

$$\Rightarrow \text{DRs are } \left\langle \frac{1}{3}, \frac{1}{6}, -1 \right\rangle \text{ or } < 2, 1, -6 >$$

$$\Rightarrow \text{DCs are } \frac{2}{\sqrt{41}}, \frac{1}{\sqrt{41}}, -\frac{6}{\sqrt{41}}$$

$$\text{Or } -\frac{2}{\sqrt{41}}, -\frac{1}{\sqrt{41}}, \frac{6}{\sqrt{41}}$$

5.d

The Cartesian equation of a line AB is :

$$\frac{2x-1}{12}=\frac{y+2}{2}=\frac{z-3}{3}$$

Find the direction cosines of a line parallel to line AB.

Sol.

Equation of line

$$\frac{x-\frac{1}{2}}{\frac{12}{2}}=\frac{y+2}{2}=\frac{z-3}{3}$$

Direction ratios of line are 6, 2, 3

Direction cosines of line are $\left\langle\frac{6}{7},\frac{2}{7},\frac{3}{7}\right\rangle$ **prepared by : BALAJI KANCHI**

6.

If the equations of a line are $\frac{x-2}{2}=\frac{2y-5}{-3}$, z = – 1, then find the direction ratios of the line and also a point on the line.

Sol.

$$\frac{x-2}{2}=\frac{2y-5}{-3}, z=-1 \Rightarrow \frac{x-2}{2}=\frac{y-\frac{5}{2}}{-\frac{3}{2}}=\frac{z+1}{0}$$

Direction Ratios $\left(2,-\frac{3}{2},0\right)$ or $(4,-3,0)$

A Point on the line = $\left(2,\frac{5}{2},-1\right)$ or any other point on the given line

7. 2023

If the equation of a line is $x = ay + b$, $z = cy + d$, then find the direction ratios of the line and a point on the line.

Sol.

The equation of the line can be written as: $\frac{x-b}{a}=\frac{y}{1}=\frac{z-d}{c}$

$\therefore$ **the direction ratios are a, 1, c**

A point on the line is $(b,0,d)$

8.

Find the direction cosines of the line joining the points $P(4,\ 3,\ -5)$ and $Q(-2,1,-8)$.

Sol.

DRs are (6, 2, 3) $\therefore$ DC's are $\left(\frac{6}{7}, \frac{2}{7}, \frac{3}{7}\right)$

a.Perpendicular distance of point from the x,y,z axis :

1.a

What is the distance of the point (p, q, r) from the x-axis?

Ans .

Distance from X -axis $= \sqrt{q^2 + r^2}$

1.b

Find the distance of the point (a, b, c) from the x-axis.

Ans: $\sqrt{b^2 + c^2}$

2.a

Find the length of the perpendicular drawn from the point $P(3, -4, 5)$ on the z-axis.

Sol.

Required length $= \sqrt{3^2 + (-4)^2} = 5$

2.b

Write the distance of the point $(3, -5, 12)$ from x-axis.

Ans.

$\sqrt{(-5)^2 + (12)^2} = 13$

b.Write vector form :

1.

The Cartesian equation of a line is $\frac{x-5}{3}=\frac{2y+4}{7}=\frac{6-z}{2}$. Write its vector equation.

Ans: $\vec{r}=5\hat{i}-2\hat{j}+6\hat{k}+\lambda\left(3\hat{i}+\frac{7}{2}\hat{j}-2\hat{k}\right)$

2.

Find the vector equation of the line which passes through the point (3, 4, 5) and is parallel to the vector $2\hat{i}+2\hat{j}-3\hat{k}$.

Ans.

The equation of line passing through $A(\bar{a})$ and parallel to the vector $\bar{b}$ is

$\vec{r}=\vec{a}+\lambda\vec{b}$ where $\lambda \in R$

Here $\vec{a}=3\hat{i}+4\hat{j}+5\hat{k}$

and $\vec{b}=2\hat{i}+2\hat{j}-3\hat{k}$

So equation of line will be

$\vec{r}=(3\hat{i}+4\hat{j}+5\hat{k})+\lambda(2\hat{i}+2\hat{j}-3\hat{k})$

3. 2024

Find the vector equation of the line passing through the point (2, 3, –5) and making equal angles with the co-ordinate axes.

Sol.

$$\cos\alpha=\cos\beta=\cos\gamma=l \Rightarrow l^2+l^2+l^2=1 \Rightarrow 3l^2=1,\ \therefore l=1/\sqrt{3}$$

Direction cosines of the line are $\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$ $\Rightarrow$ **the direction ratios are 1,1,1**

$\therefore$ **Vector equation of the line is:** $\vec{r}=2\hat{i}+3\hat{j}-5\hat{k}+\lambda\left(\hat{i}+\hat{j}+\hat{k}\right)$

c.Write cartesian form :

1.

A line passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction of the vector $\hat{i} + \hat{j} - 2\hat{k}$. Find the equation of the line in cartesian form.

Sol.

Equation of the line is:

$$\frac{x-2}{1} = \frac{y+1}{1} = \frac{z-4}{-2}$$

2. 2024

Find the Cartesian equation of the line passing through the origin, perpendicular to y-axis and making equal acute angles with x and z axes.

Sol.

Let the line makes angle α with the x and z axes respectively.

$$\therefore \cos^2\alpha + \cos^2 \pi/2 + \cos^2\alpha = 1 \Rightarrow \cos\alpha = \pm\frac{1}{\sqrt{2}}$$

$$\frac{x}{1/\sqrt{2}} = \frac{y}{0} = \frac{z}{1/\sqrt{2}} \text{ or } \frac{x}{1} = \frac{y}{0} = \frac{z}{1}$$

prepared by : BALAJI KANCHI

d.Finding the unknown coordinates of a point if one of the coordinate is given :

1.a

The x-coordinate of a point on the line joining the points P(2, 2, 1) and Q(5, 1, – 2) is 4. Find its z-coordinate.

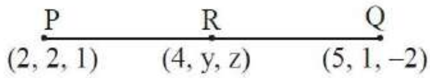
Sol.

Equation of line PQ is $\frac{x-2}{3} = \frac{y-2}{-1} = \frac{z-1}{-3}$

Any point on the line is $(3\lambda + 2, -\lambda + 2, -3\lambda + 1)$

$3\lambda + 2 = 4 \Rightarrow \lambda = \frac{2}{3}$ $\therefore$ z coord. $= -3\left(\frac{2}{3}\right)+1 = -1.$

OR



Let R(4, y, z) lying on PQ divides PQ in the ratio k : 1

$$\Rightarrow 4 = \frac{5k+2}{k+1} \Rightarrow k = 2.$$

$$\therefore\ z = \frac{2(-2)+1(1)}{2+1} = \frac{-3}{3} = -1.$$

1.b 2022

Write the cartesian equation of the line PQ passing through points P(2, 2, 1) and Q(5, 1, – 2). Hence, find the y-coordinate of the point on the line PQ whose z-coordinate is – 2.

Sol.

Required equation of line is given by

$$\frac{x-2}{3} = \frac{y-2}{-1} = \frac{z-1}{-3}$$

putting $z = -2$, we get $\frac{y-2}{-1} = \frac{-3}{-3} = 1$

$$y-2 = -1 \Rightarrow y = 1$$

1.c 2022

Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the zx-plane.

Sol.

Equations of line

$$\frac{x-5}{-2} = \frac{y-1}{3} = \frac{z-6}{-5}$$

Any point on the line

$(-2\lambda + 5, 3\lambda + 1, -5\lambda + 6)$

The point lies on ZX- plane i.e., $y = 0$

$\therefore\ 3\lambda + 1 = 0$

$\lambda = -\frac{1}{3}$

$\therefore$ Point is $\left(\frac{17}{3}, 0, \frac{23}{3}\right)$

1.d 2022

Find the coordinates of the point where the line through the points (– 1, 1, – 8) and (5, – 2, 10) crosses the ZX-plane.

Sol.

Equation of line through (–1, 1, –8) and (5, –2, 10)

is $\dfrac{x+1}{5-(-1)}=\dfrac{y-1}{-2-1}=\dfrac{z+8}{10-(-8)}$

i.e. $\dfrac{x+1}{6}=\dfrac{y-1}{-3}=\dfrac{z+8}{18}=\lambda$

Any point on this line is $(6\lambda - 1, -3\lambda + 1, 18\lambda - 8)$

Line crosses ZX-plane i.e. $y = 0$

$\Rightarrow -3\lambda + 1 = 0 \Rightarrow \lambda = \dfrac{1}{3}$

Required point is (1, 0, –2)

2.

Find the co-ordinates of the point, where the line $\dfrac{x+2}{1}=\dfrac{y-5}{3}=\dfrac{z+1}{5}$ cuts the yz-plane.

Sol.

Any point on $\dfrac{x+2}{1}=\dfrac{y-5}{3}=\dfrac{z+1}{5}=\lambda$ ...(1)

is $(\lambda-2, 3\lambda+5, 5\lambda-1)$

Line (1) cuts yz plane at $\lambda - 2 = 0$ i.e., $\lambda = 2$

hence required point is (0, 11, 9)

3.

Find the coordinates of the point where the line $\frac{x-1}{3}=\frac{y+4}{7}=\frac{z+4}{2}$ cuts the line xy-plane.

Ans: Putting z = 0 in given equation gives $\frac{x-1}{3}=\frac{y+4}{7}=2$

Coordinates of required point are (7, 10, 0)

II.Angle based :

a. Angle between the two lines :

1.

Find the angle between the lines

$2x = 3y = -z$ and $6x = -y = -4z$.

Ans.

The given lines are

$$\frac{x-0}{1/2}=\frac{y-0}{1/3}=\frac{z-0}{-1} \text{ and } \frac{x-0}{1/6}=\frac{y-0}{-1}=\frac{z-0}{-1/4}$$

$$\text{Or } \frac{x}{3}=\frac{y}{2}=\frac{z}{-6} \text{ and } \frac{x}{2}=\frac{y}{-12}=\frac{z}{-3}$$

Let θ be the angle between the two lines, then.

$$\cos\theta = \left|\frac{(3\times2)+(2\times-12)+(-6)(-3)}{\sqrt{9+4+36}\sqrt{4+144+9}}\right|$$

$$= \left|\frac{6-24+18}{7\times\sqrt{157}}\right|$$

$$= 0$$

$$\Rightarrow \theta = 90^\circ$$

prepared by : BALAJI KANCHI

2.a

Find the angle between the pair of lines given by

$$\frac{x-3}{1}=\frac{y+2}{-2}=\frac{z+4}{2};\ \frac{x-5}{-3}=\frac{y+2}{2}=\frac{z}{6}.$$

Sol.

D-ratios of the two lines are 1, -2, 2 and -3, 2, 6

$$\cos\theta=\frac{-3-4+12}{3\cdot 7}=\frac{5}{21}\Rightarrow\theta=\cos^{-1}\left(\frac{5}{21}\right)$$

2.b

Find the angle between the lines

$$\frac{5-x}{-7}=\frac{y+2}{-5}=\frac{z}{1}\text{ and }\frac{x}{1}=\frac{y}{2}=\frac{z}{3}.$$

Sol.

The direction ratios of the two lines are: $7,-5,1$ and $1,2,3$ respectively.
$\therefore$ The angle between the two lines is given by:

$$\theta=\cos^{-1}\left(\frac{7(1)-5(2)+1(3)}{\sqrt{49+25+1}\sqrt{1+4+9}}\right)$$

$$\Rightarrow\theta=\cos^{-1}0=\frac{\pi}{2}$$

2.c 2025

Find the angle at which the given lines are inclined to each other :

$$l_1:\ \frac{x-5}{2}=\frac{y+3}{1}=\frac{z-1}{-3}$$

$$l_2:\ \frac{x}{3}=\frac{y-1}{2}=\frac{z+5}{-1}$$

Sol.

Let θ be the angle between given lines

$$\therefore\cos\theta=\frac{(2\times 3)+(1\times 2)+(-3\times -1)}{\sqrt{14}\sqrt{14}}$$

$$\Rightarrow\cos\theta=\frac{11}{14}$$

$$\Rightarrow\ \theta=\cos^{-1}\left(\frac{11}{14}\right)$$

3.a 2023

Find the angle between the following two lines :

$$\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})\,;$$

$$\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$$

Ans.

Let θ be the angle between the given lines. Then

$$\cos\theta = \left|\frac{(3i + 2j + 6k).\,(i + 2j + 2k)}{\sqrt{9 + 4 + 36}\,\sqrt{1 + 4 + 4}}\right| = \frac{19}{21}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{19}{21}\right)$$

prepared by : BALAJI KANCHI

3.b 2023

Find the angle between the pair of lines given by

$\vec{r} = \hat{i} + 2\hat{j} - 2\hat{k} + \lambda(\hat{i} - 2\hat{j} - 2\hat{k})$ and

$\vec{r} = 3\hat{i} - 5\hat{j} + \hat{k} + \mu(3\hat{i} + 2\hat{j} - 6\hat{k})$.

Sol.

$$\cos\theta = \frac{\left(\hat{i} - 2\hat{j} - 2\hat{k}\right).\left(3\hat{i} + 2\hat{j} - 6\hat{k}\right)}{\left|\hat{i} - 2\hat{j} - 2\hat{k}\right|.\left|3\hat{i} + 2\hat{j} - 6\hat{k}\right|}$$

$$= \frac{(1)(3) + (-2)(2) + (-2)(-6)}{\sqrt{(1)^2 + (-2)^2 + (-2)^2}\sqrt{(3)^2 + (2)^2 + (-6)^2}} = \frac{11}{21}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{11}{21}\right)$$

3.c 2025

Find the angle between the lines

$\vec{r} = (3 + 2\lambda)\hat{i} - (2 - 2\lambda)\hat{j} + (6 + 2\lambda)\hat{k}$ and

$\vec{r} = (2\hat{j} - 5\hat{k}) + \mu(6\hat{i} + 3\hat{j} + 2\hat{k})$.

Sol.

Given lines are: $\vec{r} = \left(3\hat{i} - 2\hat{j} + 6\hat{k}\right) + \lambda\left(2\hat{i} + 2\hat{j} + 2\hat{k}\right)$

and $\vec{r} = \left(2\hat{j} - 5\hat{k}\right) + \mu\left(6\hat{i} + 3\hat{j} + 2\hat{k}\right)$

Let θ be the angle between these two lines.

$$\cos\theta = \frac{2(6)+2(3)+2(2)}{\sqrt{4+4+4}\sqrt{36+9+4}} = \frac{22}{2\sqrt{3}\times 7}$$

$$\Rightarrow \cos\theta = \frac{11}{21}\sqrt{3} \Rightarrow \theta = \cos^{-1}\left(\frac{11}{21}\sqrt{3}\right)$$

4.

If the angle between the lines $\frac{x-5}{\alpha} = \frac{y+2}{-5} = \frac{z + \frac{24}{5}}{\beta}$ and $\frac{x}{1} = \frac{y}{0} = \frac{z}{1}$ is $\frac{\pi}{4}$,

find the relation between α and β.

Sol.

$$\cos\frac{\pi}{4} = \frac{|\alpha \cdot 1 + 0 + \beta|}{\sqrt{\alpha^2 + \beta^2 + 25}\,\sqrt{2}} \Rightarrow \frac{1}{\sqrt{2}} = \frac{|\alpha + \beta|}{\sqrt{\alpha^2 + \beta^2 + 25}\,\sqrt{2}}$$

Squaring both sides, we get

$$\alpha^2 + \beta^2 + 2\alpha\beta = \alpha^2 + \beta^2 + 25$$
$$\Rightarrow \alpha\beta = \frac{25}{2}$$

prepared by : BALAJI KANCHI

b.Two lines are perpendicular :

1.a 2023

Find the value of p, so that lines $\frac{x-1}{-2}=\frac{y-4}{3p}=\frac{z-3}{4}$ and $\frac{x-2}{4p}=\frac{y-5}{2}=\frac{1-z}{7}$ are perpendicular to each other.

Sol.

d.r.'s of lines are $< -2, 3p, 4 >$ and $< 4p, 2, -7 >$

As lines are perpendicular

$-8p + 6p - 28 = 0$

$\Rightarrow p = -14$

1.b

If the lines $\frac{x-1}{-3}=\frac{2y-2}{4k}=\frac{3-z}{-2}$ and $\frac{x-1}{3k}=\frac{3y-1}{6}=\frac{z-6}{-5}$ are perpendicular to each other, find the value of k.

Sol.

The equation of the lines in standard form are.

$$\frac{x-1}{-3}=\frac{y-1}{2k}=\frac{z-3}{2} \text{ and } \frac{x-1}{3k}=\frac{y-1/3}{2}=\frac{z-6}{-5}$$

Lines are perpendicular $\therefore -9k+4k-10=0 \Rightarrow k=-2$

2.a 2025

Find the value of λ if the following lines are perpendicular to each other :

$$l_1: \frac{1-x}{-3} = \frac{3y-2}{2\lambda} = \frac{z-3}{3}$$

$$l_2: \frac{x-1}{3\lambda} = \frac{1-y}{1} = \frac{2z-5}{3}$$

Sol.

$$l_1: \frac{x-1}{3} = \frac{y-\frac{2}{3}}{\frac{2}{3}\lambda} = \frac{z-3}{3}$$

$$l_2: \frac{x-1}{3\lambda} = \frac{y-1}{-1} = \frac{z-\frac{5}{2}}{\frac{3}{2}}$$

lines are perpendicular $\Rightarrow 3(3\lambda) + \frac{2}{3}\lambda(-1) + 3.\frac{3}{2} = 0$

$$\lambda = \frac{-27}{50}$$

2.b

Find the value of p for which the following lines are perpendicular :

$$\frac{1-x}{3} = \frac{2y-14}{2p} = \frac{z-3}{2}; \quad \frac{1-x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$$

Ans.

$$\frac{x-1}{-3} = \frac{y-7}{p} = \frac{z-3}{2}; \frac{x-1}{-3p} = \frac{y-5}{1} = \frac{z-6}{-5}$$

$$\Rightarrow 9p + p - 10 = 0 \Rightarrow p = 1$$

3.a

Find the value of k so that the lines $x = -y = kz$ and $x - 2 = 2y + 1 = -z + 1$ are perpendicular to each other.

Ans: The lines, $\dfrac{x}{1} = \dfrac{y}{-1} = \dfrac{z}{\frac{1}{k}}$ and $\dfrac{x-2}{1} = \dfrac{y+\frac{1}{2}}{\frac{1}{2}} = \dfrac{z-1}{-1}$

are perpendicular $\therefore\ 1 - \dfrac{1}{2} - \dfrac{1}{k} = 0 \Rightarrow k = 2$

3.b 2023

Check whether the lines given by equations $x = 2\lambda + 2,\ y = 7\lambda + 1,\ z = -3\lambda - 3$ and $x = -\mu - 2,\ y = 2\mu + 8,\ z = 4\mu + 5$ are perpendicular to each other or not.

Sol.

D.r.s. of lines are $< 2, 7, -3 >$ and $< -1, 2, 4 >$

Now $2. -1 + 7 \cdot 2 + -3 \cdot 4 = 0$

$\therefore$ given lines are perpendicular

prepared by : **BALAJI KANCHI**

4.

Find the value of k so that the lines joining the points (1, – 1, 2) and (3, 4, k) is perpendicular to the line joining the points (0, 3, 2) and (3, 5, 6).

Sol.

The direction ratios of the two lines are: $2, 5, k-2$ and $3, 2, 4$ respectively.

The lines are perpendicular $\therefore$, $2(3) + 5(2) + 4(k-2) = 0 \Rightarrow k = -2$

III.Shortest Distance:

a.Shortest distance between skew lines :

1.

Find the shortest distance between the lines

$\vec{r} = (\lambda + 1)\hat{i} + (\lambda + 4)\hat{j} - (\lambda - 3)\hat{k}$, and

$\vec{r} = (3 - \mu)\hat{i} + (2\mu + 2)\hat{j} + (\mu + 6)\hat{k}$.

Ans.

Lines are $\vec{r} = \hat{i} + 4\hat{j} + 3\hat{k} + \lambda(\hat{i} + \hat{j} - \hat{k})$ and $\vec{r} = 3\hat{i} + 2\hat{j} + 6\hat{k} + \mu(-\hat{i} + 2\hat{j} + \hat{k})$

$\vec{a}_2 - \vec{a}_1 = 2\hat{i} - 2\hat{j} + 3\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ -1 & 2 & 1 \end{vmatrix} = 3\hat{i} + 0\hat{j} + 3\hat{k}$$

$$\text{SD} = \frac{\left|(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)\right|}{|\vec{b}_1 \times \vec{b}_2|} = \frac{6+9}{\sqrt{18}} = \frac{15}{3\sqrt{2}} = \frac{5}{\sqrt{2}}$$

prepared by : BALAJI KANCHI

2.a 2022

Find the shortest distance between the following lines :

$\vec{r} = 3\hat{i} + 5\hat{j} + 7\hat{k} + \lambda(\hat{i} - 2\hat{j} + \hat{k})$ and $\vec{r} = (-\hat{i} - \hat{j} - \hat{k}) + \mu(7\hat{i} - 6\hat{j} + \hat{k})$.

Sol.

$\vec{a}_1 = 3\hat{\imath} + 5\hat{\jmath} + 7\hat{k} \qquad \vec{b}_1 = (\hat{\imath} - 2\hat{\jmath} + \hat{k})$

$\vec{a}_2 = -\hat{\imath} - \hat{\jmath} - \hat{k}\vec{b}_2 = 7\hat{\imath} - 6\hat{\jmath} + \hat{k}$

$$(\vec{a}_2 - \vec{a}_1) = -4\hat{\imath} - 6\hat{\jmath} - 8\hat{k}, \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} i & \hat{\jmath} & \hat{k} \\ 1 & -2 & 1 \\ 7 & -6 & 1 \end{vmatrix} = 4\hat{\imath} + 6\hat{\jmath} + 8\hat{k}$$

$(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = -16 - 36 - 64 = -116$

$|\vec{b}_1 \times \vec{b}_2| = \sqrt{16 + 36 + 64} = \sqrt{116}$

$\text{S.D.} = \frac{|-116|}{\sqrt{116}} = \sqrt{116}$

prepared by : BALAJI KANCHI

2.b

Find the shortest distance between the lines whose vector equations are :

$$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}) \text{ and}$$

$$\vec{r} = 3\hat{i} - 3\hat{j} - 5\hat{k} + \mu(-2\hat{i} + 3\hat{j} + 8\hat{k})$$

Sol.

$\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k},\ \vec{a}_2 = 3\hat{i} - 3\hat{j} - 5k$

$\vec{b}_1 = 2\hat{i} + 3\hat{j} + 6\hat{k},\ \vec{b}_2 = -2\hat{i} + 3\hat{j} + 8\hat{k}$

here, $\vec{a}_2 - \vec{a}_1 = 2\hat{i} - 5\hat{j} - \hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ -2 & 3 & 8 \end{vmatrix} = 6\hat{i} - 28\hat{j} + 12\hat{k}$$

$$\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{964}$$

$$\text{S.D.} = \left|\frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right|$$

$$= \left|\frac{(2)(6) + (-5)(-28) + (-1)(12)}{\sqrt{964}}\right| = \frac{140}{\sqrt{964}} \text{ or } \frac{70}{\sqrt{241}}$$

2.c

Find the shortest distance between the lines

$\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 4\hat{j} - 5\hat{k})$.

Sol.

Here $\vec{a}_1 = 4\hat{i} - \hat{j}, \vec{a}_2 = \hat{i} - \hat{j} + 2\hat{k}, \vec{a}_2 - \vec{a}_1 = -3\hat{i} + 2\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix} = 2\hat{i} - \hat{j}$$

$$\text{Shortest distance} = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$$

$$= \left|\frac{-6}{\sqrt{5}}\right| = \frac{6}{\sqrt{5}} \quad \text{or} \quad \frac{6\sqrt{5}}{5}$$

2.d 2022

Find the distance between the following parallel lines :

$$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k})$$

$$\vec{r} = (\hat{i} - 2\hat{j} + \hat{k}) + \mu(\hat{i} + \hat{j} - \hat{k})$$

Sol.

$\vec{a}_2 - \vec{a}_2 = -\hat{i} - 3\hat{j} + 2\hat{k}, \ \vec{b} = \hat{i} + \hat{j} - \hat{k}$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -3 & 2 \\ 1 & 1 & -1 \end{vmatrix} = \hat{i} + \hat{j} + 2\hat{k}$$

$$\text{Required distance} = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$$

$$= \frac{\sqrt{1+1+4}}{\sqrt{1+1+1}} = \sqrt{\frac{6}{3}} = \sqrt{2}$$

prepared by : BALAJI KANCHI

2.c 2025

Find the shortest distance between the lines :

$$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$$

$$\vec{r} = (\hat{i} + 4\hat{k}) + \mu(3\hat{i} - 6\hat{j} + 9\hat{k}).$$

Sol.

The two given lines are parallel with,

$$\vec{a}_1 = 2\hat{i} - \hat{j} + 3\hat{k},\ \vec{a}_2 = \hat{i} + 4\hat{k}$$

Then $\vec{a}_2 - \vec{a}_1 = -\hat{i} + \hat{j} + \hat{k}$ **and the parallel vector is** $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$

$$\vec{b} \times (\vec{a}_2 - \vec{a}_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ -1 & 1 & 1 \end{vmatrix} = -5\hat{i} - 4\hat{j} - \hat{k}$$

$$\text{Shortest Distance} = \frac{\left|\vec{b} \times (\vec{a}_2 - \vec{a}_1)\right|}{\left|\vec{b}\right|} = \frac{\sqrt{42}}{\sqrt{14}} = \sqrt{3}$$

2.d

Find the shortest distance between the lines whose vector equations are given below :

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$ and

$\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$.

Sol.

Here, $\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j} + 3\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix} = -9\hat{i} + 3\hat{j} + 9\hat{k}$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{81 + 9 + 81} = \sqrt{171}$$

$$\text{S.D.} = \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$= \frac{3(-9) + 3(3) + 3(9)}{\sqrt{171}} = \frac{3}{\sqrt{19}} \text{ units}$$

3.a 2022

Find the distance between the lines $x = \frac{y-1}{2} = \frac{z-2}{3}$ and $x+1 = \frac{y+2}{2} = \frac{z-1}{3}$.

Sol.

For lines $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and $\frac{x+1}{1} = \frac{y+2}{2} = \frac{z-1}{3}$

Let $\overrightarrow{a_1} = \hat{j} + 2\hat{k}, \vec{b} = \hat{\imath} + 2\hat{j} + 3\hat{k}$

$\overrightarrow{a_2} = -\hat{\imath} - 2\hat{j} + \hat{k}, \vec{b} = \hat{\imath} + 2\hat{j} + 3\hat{k}$

Clearly lines are parallel

Hence, Shortest distance or distance is given by

$$\frac{\left|(\overrightarrow{a_2} - \overrightarrow{a_1}) \times \vec{b}\right|}{|\vec{b}|}$$

$$\overrightarrow{a_2} - \overrightarrow{a_1} = -\hat{\imath} - 3\hat{j} - \hat{k}$$

$$(\overrightarrow{a_2} - \overrightarrow{a_1}) \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{j} & \hat{k} \\ -1 & -3 & -1 \\ 1 & 2 & 3 \end{vmatrix}$$

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$$= -7\hat{i} + 2\hat{j} + \hat{k}$$

Required distance $= \frac{\sqrt{49+4+1}}{\sqrt{1+4+9}} = \frac{\sqrt{27}}{\sqrt{7}}$ or $\frac{3\sqrt{21}}{7}$

3.b 2023

Find the shortest distance between the pair of lines $\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{y-2}{1}$; $z = 2$.

Sol.

Given lines are $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-0}{1}$ and $\frac{x+1}{5} = \frac{y-2}{1} = \frac{z-2}{0}$

In vector form, lines are

$$\vec{r} = \left(\hat{i} - \hat{j}\right) + \lambda\left(2\hat{i} + 3\hat{j} + \hat{k}\right) = \vec{a}_1 + \lambda\vec{b}_1 \text{ and}$$

$$\vec{r} = \left(-\hat{i} + 2\hat{j} + 2\hat{k}\right) + \mu\left(5\hat{i} + \hat{j}\right) = \vec{a}_2 + \lambda\vec{b}_2$$

now, $\vec{a}_2 - \vec{a}_1 = -2\hat{i} + 3\hat{j} + 2\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 5 & 1 & 0 \end{vmatrix} = -\hat{i} + 5\hat{j} - 13\hat{k}$$

$$\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{195}$$

$$\text{S.D.} = \left|\frac{\left(\vec{a}_2 - \vec{a}_1\right).\left(\vec{b}_1 \times \vec{b}_2\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right|$$

$$= \left|\frac{2 + 15 - 26}{\sqrt{195}}\right| = \frac{9}{\sqrt{195}}$$

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4.a 2025

Find the shortest distance between the lines :

$$\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3} \text{ and}$$

$$\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}.$$

Sol.

The vector equations of the lines are

$$\vec{r} = -\hat{\imath} + \hat{\jmath} + 9\hat{k} + \lambda(2\hat{\imath} + \hat{\jmath} - 3\hat{k})$$

$$\vec{r} = 3\hat{\imath} - 15\hat{\jmath} + 9\hat{k} + \mu(2\hat{\imath} - 7\hat{\jmath} + 5\hat{k})$$

$$\overrightarrow{a_1} = -\hat{\imath} + \hat{\jmath} + 9\hat{k},\ \overrightarrow{a_2} = 3\hat{\imath} - 15\hat{\jmath} + 9\hat{k}$$

$$\overrightarrow{b_1} = 2\hat{\imath} + \hat{\jmath} - 3\hat{k}\ ,\ \overrightarrow{b_2} = 2\hat{\imath} - 7\hat{\jmath} + 5\hat{k}$$

$$\overrightarrow{a_2} - \overrightarrow{a_1} = 4\hat{\imath} - 16\hat{\jmath}$$

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & -3 \\ 2 & -7 & 5 \end{vmatrix} = -16\hat{\imath} - 16\hat{\jmath} - 16\hat{k}$$

$$\text{S.D.} = \frac{|(\overrightarrow{a_2}-\overrightarrow{a_1}).(\overrightarrow{b_1}\times\overrightarrow{b_2})|}{|\overrightarrow{b_1}\times\overrightarrow{b_2}|} = \frac{12}{\sqrt{3}} = 4\sqrt{3}$$

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5.a 2024

Write the vector equations of the following lines and hence find the shortest distance between them :

$$\frac{x+1}{2}=\frac{y+1}{-6}=\frac{z+1}{1} \text{ and } \frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$$

Sol.

Vector equations are

$$\vec{r} = (-\hat{\imath} - \hat{\jmath} - \hat{k}) + \lambda(2\hat{\imath} - 6\hat{\jmath} + \hat{k}) \text{ and}$$

$$\vec{r} = (3\hat{\imath} + 5\hat{\jmath} + 7\hat{k}) + \mu(\hat{\imath} - 2\hat{\jmath} + \hat{k})$$

$$\overrightarrow{a_2} - \overrightarrow{a_1} = (4\hat{\imath} + 6\hat{\jmath} + 8\hat{k})$$

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix} = -4\hat{\imath} - \hat{\jmath} + 2\hat{k}$$

$$\text{SD} = \frac{|(\overrightarrow{a_2}-\overrightarrow{a_1}).(\overrightarrow{b_1}\times\overrightarrow{b_2})|}{|\overrightarrow{b_1}\times\overrightarrow{b_2}|}$$

$$= \frac{|-16-6+16|}{\sqrt{16+1+4}} = \frac{6}{\sqrt{21}}$$

5.b 2024

Find the shortest distance between the lines $\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$.

Sol.

$$\vec{r}=\left(8\hat{i}-9\hat{j}+10\hat{k}\right)+\lambda\left(3\hat{i}-16\hat{j}+7\hat{k}\right)$$

$$\vec{r}=\left(15\hat{i}+29\hat{j}+5\hat{k}\right)+\mu\left(3\hat{i}+8\hat{j}-5\hat{k}\right)$$

We have, $\vec{a}_1=8\hat{i}-9\hat{j}+10\hat{k}$, $\vec{b}_1=3\hat{i}-16\hat{j}+7\hat{k}$, $\vec{a}_2=15\hat{i}+29\hat{j}+5\hat{k}$, $\vec{b}_2=3\hat{i}+8\hat{j}-5\hat{k}$

$$\vec{a}_2-\vec{a}_1=7\hat{i}+38\hat{j}-5\hat{k},\ \vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -16 & 7\\ 3 & 8 & -5\end{vmatrix}=24\hat{i}+36\hat{j}+72\hat{k}$$

$$\text{Shortest Distance}=\left|\frac{\left(\vec{b}_1\times\vec{b}_2\right)\cdot\left(\vec{a}_2-\vec{a}_1\right)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right|=\frac{98}{7}=14$$

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b.Shortest distance between the Parallel lines :

1. 2023

Find the distance between the lines :

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k});$$

$$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$$

Sol.

Here

$$\vec{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}, \quad \vec{b_1} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

$$\vec{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}, \quad \vec{b_2} = 4\hat{i} + 6\hat{j} + 12\hat{k}$$

Here, $\vec{b_1}$ and $\vec{b_2}$ are parallel vectors.

$$\vec{a_2} - \vec{a_1} = 2\hat{\imath} + \hat{\jmath} - \hat{k}, \qquad \vec{b} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$$

Thus, $(\vec{a_2} - \vec{a_1}) \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix} = 9\hat{i} - 14\hat{j} + 4\hat{k}$

Distance between the lines $= \left| \frac{(\vec{a_2} - \vec{a_1}) \times \vec{b}}{|\vec{b}|} \right|$

$$= \frac{\sqrt{81 + 196 + 16}}{\sqrt{4 + 9 + 36}}$$

$$= \frac{\sqrt{293}}{7} \text{ units.}$$

prepared by : BALAJI KANCHI

2.

Find the shortest distance between the lines l_1 and l_2 given by :

$$l_1 : \vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(4\hat{i} + 6\hat{j} + 12\hat{k})$$

and $l_2 : \vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(6\hat{i} + 9\hat{j} + 18\hat{k})$

Sol.

Given lines are : $\vec{r} = \left(\hat{i}+2\hat{j}-4\hat{k}\right)+2\lambda\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$

and $\vec{r} = \left(\hat{i}+2\hat{j}-4\hat{k}\right)+3\mu\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$

Clearly, the given lines are parallel.

Here, $\vec{a}_1 = \hat{i}+2\hat{j}-4\hat{k}$, $\vec{a}_2 = \hat{i}+2\hat{j}-4\hat{k}$ and $\vec{b} = 2\hat{i}+3\hat{j}+6\hat{k}$

$\vec{a}_2 - \vec{a}_1 = 2\hat{i}+\hat{j}-\hat{k}$

$$(\vec{a}_2 - \vec{a}_1)\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix} = 9\hat{i} - 14\hat{j} + 4\hat{k}$$

$\therefore \left|(\vec{a}_2 - \vec{a}_1)\times\vec{b}\right| = \sqrt{81+196+16} = \sqrt{293}$

Also, $\left|\vec{b}\right| = \sqrt{4+9+36} = 7$

$$\text{S.D.} = \frac{\left|(\vec{a}_2 - \vec{a}_1)\times\vec{b}\right|}{\left|\vec{b}\right|}$$

$$= \frac{\sqrt{293}}{7}$$

3.2024

Find the distance between the line $\frac{x}{2}=\frac{2y-6}{4}=\frac{1-z}{-1}$ and another line parallel to it passing through the point (4, 0, –5).

Sol.

Equation of the given line in standard form is

$$L_1 : \frac{x}{2}=\frac{y-3}{2}=\frac{z-1}{1}$$

Equation of the line parallel to L_1 & passing through $(4, 0, -5)$ is

$$L_2 : \frac{x-4}{2}=\frac{y}{2}=\frac{z+5}{1}$$

Vector Equation of Lines are $L_1 : \vec{r}=(0\hat{i}+3\hat{j}+\hat{k})+\lambda(2\hat{i}+2\hat{j}+\hat{k})$

$L_2 : \vec{r}=(4\hat{i}+0\hat{j}-5\hat{k})+\mu(2\hat{i}+2\hat{j}+\hat{k})$

Now, $\overrightarrow{a_2}-\overrightarrow{a_1}=(4\hat{i}+0\hat{j}-5\hat{k})-(0\hat{i}+3\hat{j}+\hat{k})=(4\hat{i}-3\hat{j}-6\hat{k})$

$\vec{b}=2\hat{i}+2\hat{j}+\hat{k}$

$$(\overrightarrow{a_2}-\overrightarrow{a_1})\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & -3 & -6\\ 2 & 2 & 1\end{vmatrix}=9\hat{i}-16\hat{j}+14\hat{k}$$

$|\vec{b}|=\sqrt{4+4+1}=3$

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Thus, distance between the lines is

$$S.D.=\frac{\left|(\overrightarrow{a_2}-\overrightarrow{a_1})\times\vec{b}\right|}{|\vec{b}|}=\frac{\sqrt{81+256+196}}{3}=\frac{\sqrt{533}}{3}\text{units}$$

4. 2024

Find the shortest distance between the lines L_1 & L_2 given below :

L_1 : The line passing through (2, –1, 1) and parallel to $\frac{x}{1} = \frac{y}{1} = \frac{z}{3}$

L_2 : $\vec{r} = \hat{i} + (2\mu + 1)\hat{j} - (\mu + 2)\hat{k}$.

Sol.

$$\textbf{Equation of } L_1 : \vec{r} = 2\hat{i} - \hat{j} + \hat{k} + \lambda\left(\hat{i} + \hat{j} + 3\hat{k}\right)$$

$$\textbf{Equation of } L_2 : \vec{r} = \left(\hat{i} + \hat{j} - 2\hat{k}\right) + \mu\left(2\hat{j} - \hat{k}\right)$$

Taking

$$\vec{a}_1 = 2\hat{i} - \hat{j} + \hat{k},\ \vec{b}_1 = \hat{i} + \hat{j} + 3\hat{k}$$

$$\vec{a}_2 = \hat{i} + \hat{j} - 2\hat{k},\ \vec{b}_2 = 2\hat{j} - \hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = -\hat{i} + 2\hat{j} - 3\hat{k},\ \vec{b}_1 \times \vec{b}_2 = -7\hat{i} + \hat{j} + 2\hat{k}$$

$$\textbf{Shortest Distance} = \frac{\left|\left(\vec{a}_2 - \vec{a}_1\right) \cdot \left(\vec{b}_1 \times \vec{b}_2\right)\right|}{\left|\vec{b}_1 \times \vec{b}_2\right|} = \frac{1}{\sqrt{6}}$$

5.

Find the vector equation of the line passing through (2, 1, – 1) and parallel to the line $\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$. Also, find the distance between these two lines.

Sol.

Required equation of the line is

$$\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(2\hat{i} - \hat{j} + \hat{k})$$

Let $\vec{a}_1 = 2\hat{i} + \hat{j} - \hat{k}, \vec{a}_2 = \hat{i} + \hat{j}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k}$

The required distance $= \left| \dfrac{(\vec{a}_2 - \vec{a}_1) \times \vec{b}}{|\vec{b}|} \right|$

$$= \frac{|(\hat{i} - \hat{k}) \times (2\hat{i} - \hat{j} + \hat{k})|}{|2\hat{i} - \hat{j} + \hat{k}|}$$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 2 & -1 & 1 \end{vmatrix} = -\hat{i} - 3\hat{j} - \hat{k}$$

$$\text{Required distance} = \frac{\sqrt{1+9+1}}{\sqrt{4+1+1}} = \frac{\sqrt{11}}{\sqrt{6}} \text{ or } \frac{\sqrt{66}}{6}$$

IV. Intersecting/skew lines

a.Skew lines or not :

1.

Check whether the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ are skew or not.

Sol.

Let, $\vec{a_1}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{a_2}=4\hat{i}+\hat{j}$

$\vec{b_1}=2\hat{i}+3\hat{j}+4\hat{k}$, $\vec{b_2}=5\hat{i}+2\hat{j}+\hat{k}$

$\vec{a_2}-\vec{a_1}=3\hat{i}-\hat{j}-3\hat{k}$

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$\vec{b_1}\times\vec{b_2}=-5\hat{i}+18\hat{j}-11\hat{k}$

Here, $(\vec{a_2}-\vec{a_1})\cdot(\vec{b_1}\times\vec{b_2})=-15-18+33=0$

Hence given lines are **not** skew lines.

2. 2025

Verify that lines given by $\vec{r} = (1 - \lambda)\hat{i} + (\lambda - 2)\hat{j} + (3 - 2\lambda)\hat{k}$ and $\vec{r} = (\mu + 1)\hat{i} + (2\mu - 1)\hat{j} - (2\mu + 1)\hat{k}$ are skew lines. Hence, find shortest distance between the lines.

Sol.

Rewriting the lines, we get

$\vec{r} = \left(\hat{i} - 2\hat{j} + 3\hat{k}\right) + \lambda\left(-\hat{i} + \hat{j} - 2\hat{k}\right)$ **and** $\vec{r} = \left(\hat{i} - \hat{j} - \hat{k}\right) + \mu\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$

Let $\vec{a}_1 = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{a}_2 = \hat{i} - \hat{j} - \hat{k}, \vec{b}_1 = -\hat{i} + \hat{j} - 2\hat{k}, \vec{b}_2 = \hat{i} + 2\hat{j} - 2\hat{k}$

Note that the dr's of given lines are not proportional so, they are not parallel lines.

The lines will be skew if they do not intersect each other also.

Here $\vec{a}_2 - \vec{a}_1 = \hat{j} - 4\hat{k}, \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix} = 2\hat{i} - 4\hat{j} - 3\hat{k}$

Consider $\left(\vec{a}_2 - \vec{a}_1\right).\left(\vec{b}_1 \times \vec{b}_2\right)$

$= \left(\hat{j} - 4\hat{k}\right).\left(2\hat{i} - 4\hat{j} - 3\hat{k}\right) = 8 \neq 0$

Hence lines will not intersect. So the lines are skew.

$$\text{Shortest Distance} = \frac{\left|\left(\vec{a}_2 - \vec{a}_1\right).\left(\vec{b}_1 \times \vec{b}_2\right)\right|}{\left|\vec{b}_1 \times \vec{b}_2\right|}$$

$$= \frac{8}{\sqrt{4 + 16 + 9}} = \frac{8}{\sqrt{29}}$$

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b.Intersecting or not/ intersecting point :

1. 2025

Prove that the line through A(0, –1, –1) and B(4, 5, 1) intersects the line through C(3, 9, 4) and D(–4, 4, 4).

Sol.

Drs of line passing through points A and B are < 4,6,2>

Drs of line passing through C and D are <-7, -5, 0>

$$\text{Consider}\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 3-0 & 9-(-1) & 4-(-1) \\ 4 & 6 & 2 \\ -7 & -5 & 0 \end{vmatrix}$$

$= 3(+10) -10(+14) +5(22)$
$= 0$

Hence lines intersect each other

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2.a

Find the shortest distance between the following lines and hence write whether the lines are intersecting or not.

$$\frac{x-1}{2} = \frac{y+1}{3} = z, \quad \frac{x+1}{5} = \frac{y-2}{1}, \quad z = 2$$

Ans. Let $\vec{a}_1 = \hat{i} - \hat{j}$; $\vec{a}_2 = -\hat{i} + 2\hat{j} + 2\hat{k}$
$\vec{b}_1 = 2\hat{i} + 3\hat{j} + \hat{k}$; $\vec{b}_2 = 5\hat{i} + \hat{j}$

then, $\vec{a}_2 - \vec{a}_1 = -2\hat{i} + 3\hat{j} + 2\hat{k}$, $\vec{b}_1 \times \vec{b}_2 = -\hat{i} + 5\hat{j} - 13\hat{k}$

$$\therefore \text{ Shortest distance} = \left| \frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right| = \frac{9}{\sqrt{195}} \neq 0$$

$\therefore$ lines are not intersecting

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2.b

Show that the following lines do not intersect each other :

$$\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-1}{5};\ \frac{x+2}{4}=\frac{y-1}{3}=\frac{z+1}{-2}$$

Sol.

$$\text{Shortest Distance} = \frac{\begin{vmatrix} -3 & 2 & -2 \\ 3 & 2 & 5 \\ 4 & 3 & -2 \end{vmatrix}}{\sqrt{(2\times-2-5\times3)^2+(3\times-2-4\times5)^2+(3\times3-2\times4)^2}}$$

$$=\frac{-3(-19)-2(-26)-2(1)}{\sqrt{361+676+1}}$$

$$=\frac{57+52-2}{\sqrt{1038}}$$

$$=\frac{107}{\sqrt{1038}}\neq 0$$

So, the line will not intersect each other. prepared by : BALAJI KANCHI

2.c

Show that the lines $\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = \hat{i} + \hat{j} + \hat{k} + \mu(\hat{i} - \hat{j} + 2\hat{k})$ do not intersect.

Sol.

$$(\vec{a}_2-\vec{a}_1) = -\hat{j} \text{ and } \vec{b}_1\times\vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & -1 & 2 \end{vmatrix} = -\hat{i}-\hat{j}$$

$$(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2) = 1\neq 0 \Rightarrow \textbf{Lines are not intersecting}$$

2.d 2025

Determine if the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ intersect with each other.

Sol.

$\vec{b}_1 = 3\hat{i} - \hat{j}$, $\vec{b}_2 = 2\hat{i} + 3\hat{k}$, $\vec{a}_2 = 4\hat{i} - \hat{k}$, $\vec{a}_1 = \hat{i} + \hat{j} - \hat{k}$

$(\vec{a}_2 - \vec{a}_1) = 3\hat{i} - \hat{j}$

$\vec{b}_1 \times \vec{b}_2 = -3\hat{i} - 9\hat{j} + 2\hat{k}$

$(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = (3\hat{i} - \hat{j}).(-3\hat{i} - 9\hat{j} + 2\hat{k}) =$ 0, hence lines intersect

3.a

Find the value of λ, so that the lines $\frac{1-x}{3}=\frac{7y-14}{\lambda}=\frac{z-3}{2}$ and $\frac{7-7x}{3\lambda}=\frac{y-5}{1}=\frac{6-z}{5}$

are at right angles. Also, find whether the lines are intersecting or not.

Ans.

Given lines are: $\frac{x-1}{-3}=\frac{y-2}{\left(\frac{\lambda}{7}\right)}=\frac{z-3}{2}$ and $\frac{x-1}{\left(\frac{-3\lambda}{7}\right)}=\frac{y-5}{1}=\frac{z-6}{-5}$

As lines are perpendicular,

$$(-3)\left(\frac{-3\lambda}{7}\right)+\left(\frac{\lambda}{7}\right)(1)+2(-5)=0 \Rightarrow \lambda=7$$

So, lines are

$$\frac{x-1}{-3}=\frac{y-2}{1}=\frac{x-3}{2} \text{ and } \frac{x-1}{-3}=\frac{y-5}{1}=\frac{z-6}{-5}$$

$$\text{Consider } \Delta=\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=\begin{vmatrix} 0 & 3 & 3 \\ -3 & 1 & 2 \\ -3 & 1 & -5 \end{vmatrix}=-63$$

as $\Delta \neq 0 \Rightarrow$ lines are not intersecting.

3.b

If the lines $\frac{x-1}{-3}=\frac{y-2}{2\lambda}=\frac{z-3}{2}$ and $\frac{x-1}{3\lambda}=\frac{y-1}{2}=\frac{z-6}{-5}$ are perpendicular, find the value of λ. Hence find whether the lines are intersecting or not.

Ans.

Lines are perpendicular

$\therefore\ -3(3\lambda)+2\lambda(2)+2(-5)=0 \Rightarrow \lambda=-2$

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_3 \end{vmatrix} = \begin{vmatrix} 1-1 & 1-2 & 6-3 \\ -3 & 2(-2) & 2 \\ 3(-2) & 2 & -5 \end{vmatrix} = -63 \neq 0$$

$\therefore$ Lines are not intersecting

3.c

Find the value of λ for which the following lines are perpendicular to each other :

$$\frac{x-5}{5\lambda+2}=\frac{2-y}{5}=\frac{1-z}{-1};\quad \frac{x}{1}=\frac{y+\frac{1}{2}}{2\lambda}=\frac{z-1}{3}$$

Hence, find whether the lines intersect or not.

Sol.

Writing the equations of given lines in standard form, as

$$\frac{x-5}{5\lambda+2}=\frac{y-2}{-5}=\frac{z-1}{1};\frac{x}{1}=\frac{y+\frac{1}{2}}{2\lambda}=\frac{z-1}{3}$$

lines are perpendicular to each other,

$\Rightarrow\ (5\lambda+2)\cdot 1+(-5)(2\lambda)+1(3)=0$

$-5\lambda+5=0 \Rightarrow \lambda=1$

$\therefore$ lines are $\frac{x-5}{7}=\frac{y-2}{-5}=\frac{z-1}{1}$; $\frac{x}{1}=\frac{y+\frac{1}{2}}{2}=\frac{z-1}{3}$

Shortest distance between these lines $=\frac{|(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)|}{|\vec{b}_1\times\vec{b}_2|}=\frac{\left|\left(5\hat{i}+\frac{5}{2}\hat{j}\right)\cdot(-17\hat{i}-20\hat{j}+19\hat{k})\right|}{|\vec{b}_1\times\vec{b}_2|}$

$=\frac{135}{|\vec{b}_1\times\vec{b}_2|}\neq 0$

$\therefore$ lines are not intersecting.

4.a

Show that the lines

$\frac{x-2}{1}=\frac{y-2}{3}=\frac{z-3}{1}$ and $\frac{x-2}{1}=\frac{y-3}{4}=\frac{z-4}{2}$ intersect.

Also, find the coordinates of the point of intersection.

Ans: $\frac{x-2}{1}=\frac{y-2}{3}=\frac{z-3}{1}=\lambda$ (say)

and $\frac{x-2}{1}=\frac{y-3}{3}=\frac{z-4}{2}=\mu$ (say)

Arbitrary points on the lines are

$(\lambda+2, 3\lambda+2, \lambda+3)$ and $(\mu+2, 4\mu+3, 2\mu+4)$

$\Rightarrow$ $\lambda+2=\mu+2$, and $\lambda+3=2\mu+4$

$\Rightarrow$ $\lambda=\mu$, solving we get $\lambda=-1$, $\mu=-1$

$\lambda=-1$, $\mu=-1$ satisfying y-coordinates $3\lambda+2=4\mu+3$

$\therefore$ Point of intersection is $(1, -1, 2)$

4.b 2023

Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Also find their point of intersection.

Sol.

$$\text{line 1}: \frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} = \lambda \quad ...(1)$$

$$\text{line 2}: \frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = \mu \quad ...(2)$$

General points on (1) and (2) are

$(3\lambda-1, 5\lambda-3, 7\lambda-5)$ and $(\mu+2, 3\mu+4, 5\mu+6)$

for the lines to intersect,

$$3\lambda - 1 = \mu + 2 \quad ...(3)$$

$$5\lambda - 3 = 3\mu + 4 \quad ...(4)$$

$$7\lambda - 5 = 5\mu + 6 \quad ...(5)$$

solving (3) and (4) gives $\lambda = \frac{1}{2}$ and $\mu = -\frac{3}{2}$

clearly these values of λ and μ satisfies (5)

$\Rightarrow$ given lines intersect.

Point of intersection is $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$

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4.c 2025

Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$ intersect. Also, find their point of intersection.

Sol.

Let the given lines be

$l_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$ and $l_2 : \frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1} = \mu$

Any point on the line l_1 is $(2\lambda+1, 3\lambda+2, 4\lambda+3)$

Any point on the line l_2 is $(5\mu+4, 2\mu+1, \mu)$

For the given lines to intersect, there must be a common point.

$\therefore 2\lambda+1 = 5\mu+4 \Rightarrow 2\lambda-5\mu = 3 \quad ...(i)$

$3\lambda+2 = 2\mu+1 \Rightarrow 3\lambda-2\mu = -1 \quad ...(ii)$

$4\lambda+3 = \mu \Rightarrow 4\lambda-\mu = -3 \quad ...(iii)$

Solving (i) and (ii) gives, $\lambda = \mu = -1$

We notice that $\lambda = \mu = -1$ also satisfies equation (iii)

$\therefore$ The given lines intersect.

Point of intersection is $(2(-1)+1, 3(-1)+2, 4(-1)+3)$ i.e. $(-1,-1,-1)$

5. 2023

Find the equations of the diagonals of the parallelogram PQRS whose vertices are P(4, 2, –6), Q(5, –3, 1), R(12, 4, 5) and S(11, 9, –2). Use these equations to find the point of intersection of diagonals.

Sol.

Equation of diagonal PR: $\frac{x-4}{8}=\frac{y-2}{2}=\frac{z+6}{11}$

Equation of diagonal QS: $\frac{x-5}{6}=\frac{y+3}{12}=\frac{z-1}{-3}$

General points on PR & QS are $(8k+4,2k+2,11k-6)$ **and** $(6t+5,12t-3,-3t+1)$ **for real numbers 'k' and 't' respectively.**

For point of intersection of PR and QS: $8k+4=6t+5,\ 2k+2=12t-3$

Solving, we get $k=\frac{1}{2}, t=\frac{1}{2}$ $\therefore$ **The point of intersection is** $\left(8,3,-\frac{1}{2}\right)$

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6. 2024

Find the value of p for which the lines

$\vec{r}=\lambda\hat{i}+(2\lambda+1)\hat{j}+(3\lambda+2)\hat{k}$ and

$\vec{r}=\hat{i}-3\mu\hat{j}+(p\mu+7)\hat{k}$

are perpendicular to each other and also intersect. Also, find the point of intersection of the given lines.

Sol.

Given lines are

$\vec{r}=\hat{j}+2\hat{k}+\lambda(\hat{i}+2\hat{j}+3\hat{k})$ and

$\vec{r}=\hat{i}+7\hat{k}+\mu(-3\hat{j}+p\hat{k})$

The lines are perpendicular

$(\hat{i}+2\hat{j}+3\hat{k}).(-3\hat{j}+p\hat{k})=0 \Rightarrow -6+3p=0 \Rightarrow p=2$

The coordinates of any point on the two lines are

$(\lambda, 2\lambda+1, 3\lambda+2)$ and $(1, -3\mu, 2\mu+7)$

For the point of intersection, we must have

$\lambda = 1,\ 2\lambda+1=-3\mu,\ 3\lambda+2=2\mu+7$ for some λ and μ.

Solving first two equations, we get, $\lambda = 1, \mu = -1$, which satisfies the third equation as 5 = 5 is true.

Hence, the lines intersect each other.

The Point of intersection is (1, 3, 5)

7. 2025

Find the point of intersection of the lines

$\vec{r} = \hat{i} - \hat{j} + 6\hat{k} + \lambda(3\hat{i} - \hat{k})$, and

$\vec{r} = -3\hat{j} + 3\hat{k} + \mu(\hat{i} + 2\hat{j} - \hat{k})$.

Also, find the vector equation of the line passing through the point of intersection of the given lines and perpendicular to both the lines.

Sol.

If the given lines intersect then,

$$\hat{i} - \hat{j} + 6\hat{k} + \lambda\left(3\hat{i} - \hat{k}\right) = -3\hat{j} + 3\hat{k} + \mu\left(\hat{i} + 2\hat{j} - \hat{k}\right)$$

Solving the equations, $1 + 3\lambda = \mu, -1 = 2\mu - 3$ **, we get** $\lambda = 0, \mu = 1$

which do not satisfy the equation $, 6 - \lambda = 3 - \mu$.

$\therefore$ **The lines do not intersect, hence no point of intersection**

8. 2025

Find the distance of the point (–1, –5, –10) from the point of intersection of the lines $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$ and $\dfrac{x-4}{5} = \dfrac{y-1}{2} = z$.

Sol.

$l_1 : \dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4} = \lambda$

Any point on l_1 is $(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$

$l_2 : \dfrac{x-4}{5} = \dfrac{y-1}{2} = \dfrac{z-0}{1} = \mu$

Any point on l_2 is $(5\mu + 4, 2\mu + 1, \mu)$

For point of intersection,

$2\lambda + 1 = 5\mu + 4, 3\lambda + 2 = 2\mu + 1$

Solving, $\lambda = \mu = -1$

Since, $\lambda = \mu = -1$ satisfy $4\lambda + 3 = \mu$

$\therefore$ Point of intersection is $(-1, -1, -1)$

Now distance of $(-1, -5, -10)$ from $(-1, -1, -1)$ is:

$\sqrt{(-1+1)^2 + (-1+5)^2 + (-1+10)^2} = \sqrt{97}$ units

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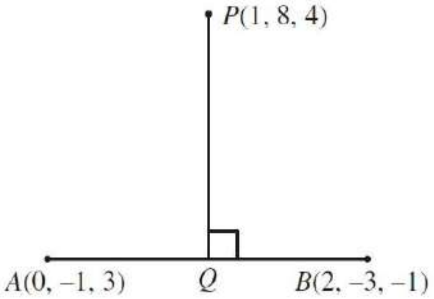
V. Foot /Image :

a.Foot of the perpendicular :

1.

Find the co-ordinates of the foot of perpendicular drawn from a point A (1, 8, 4) to the line joining the points B(0, –1, 3) and C(2, –3, –1).

Sol.



Equation of line AB $\frac{x}{2} = \frac{y+1}{-2} = \frac{z-3}{-4} = \lambda$

Coordinates of $Q(2\lambda, -2\lambda - 1, -4\lambda + 3)$ for some λ

Dr's of $PQ(2\lambda - 1, -2\lambda - 9, -4\lambda - 1)$

$PQ \perp AB \Rightarrow 2(2\lambda - 1) - 2(-2\lambda - 9) - 4(-4\lambda - 1) = 0$

gives $\lambda = \frac{-5}{6}$

So coordinates of Q are $\left(\frac{-5}{6}, \frac{+2}{3}, \frac{19}{3}\right)$

Length of perpendicular $PQ = \sqrt{\left(\frac{-8}{3}\right)^2 + \left(\frac{22}{3}\right)^2 + \left(\frac{7}{3}\right)^2} = \frac{\sqrt{597}}{3}$

2.

Find the equations of all the sides of the parallelogram ABCD whose vertices are A(4, 7, 8), B(2, 3, 4), C(– 1, – 2, 1) and D(1, 2, 5). Also, find the coordinates of the foot of the perpendicular from A to CD.

Sol.

Equation of the line AB : $\frac{x-4}{2} = \frac{y-7}{4} = \frac{z-8}{4}$

Equation of the line BC : $\frac{x-2}{3} = \frac{y-3}{5} = \frac{z-4}{3}$

Equation of the line CD : $\frac{x+1}{1} = \frac{y+2}{2} = \frac{z-1}{2}$

Equation of the line DA : $\frac{x-4}{3} = \frac{y-7}{5} = \frac{z-8}{3}$

Let P be foot of perpendicular from A to CD.

$\therefore$ Coordinates of P are $(\lambda - 1, 2\lambda - 2, 2\lambda + 1)$ for some λ

d.r.'s of AP are $(\lambda - 5, 2\lambda - 9, 2\lambda - 7)$

since $AP \perp CD$

$\Rightarrow 1(\lambda - 5) + 2(2\lambda - 9) + 2(2\lambda - 7) = 0$

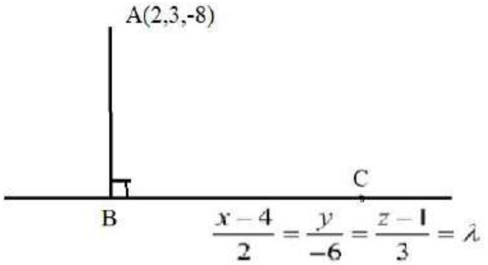
$\Rightarrow 9\lambda = 37 \quad \Rightarrow \lambda = \frac{37}{9}$

$\therefore$ Coordinates of P are $\left(\frac{28}{9}, \frac{56}{9}, \frac{83}{9}\right)$ prepared by : BALAJI KANCHI

3.a

Find the co-ordinates of the foot of perpendicular drawn from the point (2, 3, –8) to the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$.

Sol.



Any point on line is $(2\lambda+4,-6\lambda,3\lambda+1)$ for some λ. Let $B(2\lambda+4,-6\lambda,3\lambda+1)$

d.r. of $AB=<2\lambda+2,-6\lambda-3,3\lambda+9>$

d.r. of $BC=<2,-6,3>$

$AB\perp BC\Rightarrow 2(2\lambda+2)-6(-6\lambda-3)+3(3\lambda+9)=0$

$\Rightarrow \lambda=-1$

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$\therefore B(2,6,-2)$

3.b 2023

Find the coordinates of the foot of the perpendicular drawn from the point P(0, 2, 3) to the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$.

Sol.

General point on the given line is M $(5\lambda-3, 2\lambda+1, 3\lambda-4)$

Direction ratios of PM are $5\lambda-3, 2\lambda-1, 3\lambda-7$

If this point is the foot of the perpendicular from the point P (0, 2, 3), then PM is perpendicular to the line. Thus,

$$(5\lambda-3).5+(2\lambda-1).2+(3\lambda-7).3=0$$

$$\Rightarrow \lambda=1$$

Hence co-ordinates of M are (2, 3, – 1)

3.c 2023

Find the coordinates of the foot of the perpendicular drawn from point (5, 7, 3) to the line $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$.

Sol.

A general point on the given line is M$(3\lambda + 15, 8\lambda + 29, -5\lambda + 5)$.

DRs of $\overrightarrow{MP}$ are $(3\lambda + 10, 8\lambda + 22, -5\lambda + 2)$

This general point for some specific value of λ will be the foot of the perpendicular drawn from (5, 7, 3) on the given line if PM $\perp$ line.

i.e. if $(3\lambda + 10)(3) + (8\lambda + 22)(8) + (-5\lambda + 2)(-5) = 0$

$\Rightarrow \quad \lambda = -2$

Hence, M is (9, 13, 15) is the required foot of the perpendicular.

4.

Find the coordinates of the foot of perpendicular drawn from the point (3, −1, 11) to the line $\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}$

Hence, write the equation of this perpendicular line.

Sol.

General point on the line, say, $P(2\lambda,\ 3\lambda+2,\ 4\lambda+3)$

Direction ratios of the perpendicular from the point $(3,\ -1,\ 11)$ to the line are

$$2\lambda-3,\ 3\lambda+3,\ 4\lambda-8$$

And direction ratios of the line are 2, 3, 4

$$\therefore 2(2\lambda-3)+3(3\lambda+3)+4(4\lambda-8)=0 \Rightarrow \lambda=1,$$

$$\therefore (2,\ 5,\ 7) \text{ is the foot of the perpendicular}$$

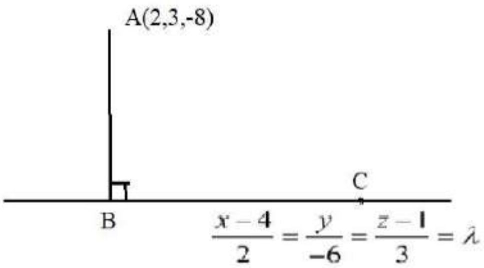
Equation of the perpendicular

$$\text{Cartesian form: } \frac{x-3}{1}=\frac{y+1}{-6}=\frac{z-11}{4}$$

5.a

Find the foot of the perpendicular drawn from the point $(2, 3, -8)$ to the line $\frac{x-4}{2}=\frac{y}{-6}=\frac{z-1}{3}$. Also, find the perpendicular distance of the given line from the given point.

Sol.



Any point on line is $(2\lambda+4,-6\lambda,3\lambda+1)$ for some λ. Let $B(2\lambda+4,-6\lambda,3\lambda+1)$

d.r. of $AB=<2\lambda+2,-6\lambda-3,3\lambda+9>$

d.r. of $BC=<2,-6,3>$

$AB\perp BC\Rightarrow 2(2\lambda+2)-6(-6\lambda-3)+3(3\lambda+9)=0$

$\Rightarrow \lambda=-1$

$\therefore B(2,6,-2)$

Now, $AB=\sqrt{(2-2)^2+(6-3)^2+(-2+8)^2}=\sqrt{45}$ or $3\sqrt{5}$ units

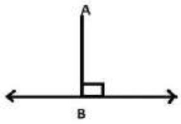
5.b 2024

Find the co-ordinates of the foot of the perpendicular drawn from the point (2, 3, –8) to the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$.

Also, find the perpendicular distance of the given point from the line.

Sol.

The standard form of the equation of the line is $\frac{x-4}{-2}=\frac{y}{6}=\frac{z-1}{-3}$



Let foot of the perpendicular from the point $A(2,3,-8)$ to the given line be $B(-2\lambda+4,6\lambda,-3\lambda+1)$

D-ratios of AB is: $-2\lambda+2,6\lambda-3,-3\lambda+9$

As AB is perpendicular to the given line: $-2(-2\lambda+2)+6(6\lambda-3)-3(-3\lambda+9)=0$

$$\Rightarrow\lambda=1$$

$\therefore$ Foot of the perpendicular is: $B(2,6,-2)$

Perpendicular distance $=AB=3\sqrt{5}$

5.c 2025

Find the foot of the perpendicular drawn from point (2, –1, 5) to the line $\frac{x-11}{10}=\frac{y+2}{-4}=\frac{z+8}{-11}$. Also, find the length of the perpendicular.

Sol.

Let l: $\frac{x-11}{10}=\frac{y+2}{-4}=\frac{z+8}{-11}=\lambda$

Coordinates of any point on l are $x = 10\lambda + 11$, $y = -4\lambda - 2$, $z = -11\lambda - 8$

Drs of perpendicular line are $(10\lambda + 9, -4\lambda - 1, -11\lambda - 13)$

Drs of given line are 10, – 4, – 11

As lines are perpendicular, so

$(10\lambda + 9)10 + (-4\lambda - 1)(-4) + (-11\lambda - 13)\times(-11) = 0$

$\Rightarrow\lambda = -1$

Hence coordinates of point are (1, 2, 3) which is the foot of the $\perp$ from P to l.

length of $\perp = \sqrt{(1-2)^2+(2+1)^2+(3-5)^2} = \sqrt{1+9+4} = \sqrt{14}$

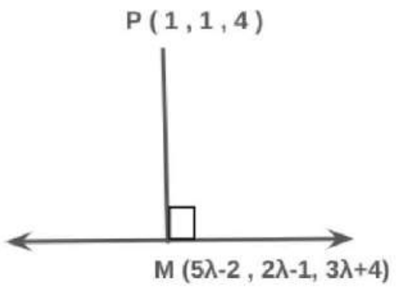
5.d

Find the foot of the perpendicular drawn from the point (1, 1, 4) on the line $\frac{x+2}{5} = \frac{y+1}{2} = \frac{-z+4}{-3}$.

Sol.

Let $\frac{x+2}{5} = \frac{y+1}{2} = \frac{z-4}{3} = \lambda$

Coordinate of general point on the given line are $M\,(5\lambda - 2, 2\lambda - 1, 3\lambda + 4)$



Direction ratios of PM vector are $< 5\lambda - 3, 2\lambda - 2, 3\lambda >$

Since, $PM \perp l$

$\Rightarrow 5(5\lambda - 3) + 2(2\lambda - 2) + 3(3\lambda) = 0$

$\Rightarrow \lambda = \frac{1}{2}$

Hence, coordinates of M are $\left(\frac{1}{2}, 0, \frac{11}{2}\right)$

b.Image of the of the point :

1.a 2024

The image of point P(x, y, z) with respect to line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ is P′ (1, 0, 7). Find the coordinates of point P.

So.

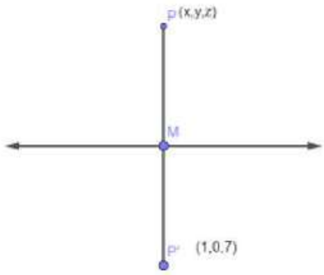
Let foot of the perpendicular on the given line from point P be M $(\lambda, 2\lambda+1, 3\lambda+2)$

D. ratios of PP' are $\lambda-1,\ 2\lambda+1,\ 3\lambda-5$

$1(\lambda-1)+2(2\lambda+1)+3(3\lambda-5)=0$

$\Rightarrow\ \lambda = 1$

Coordinates of M(1,3,5)



$\frac{x+1}{2} = 1,\ \frac{y+0}{2} = 3,\ \frac{z+7}{2} = 5$

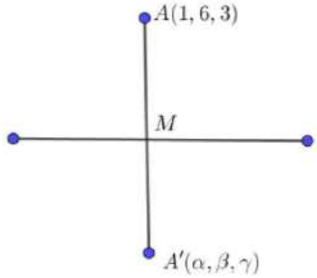
$\Rightarrow x = 1,\ y = 6,\ z = 3\ \Rightarrow P(1,6,3)$

1.b 2025

Find the image A' of the point A(1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also, find the equation of the line joining A and A'.

Sol.

The equation of given line is $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$



Any arbitrary point on the line is $M(\lambda, 2\lambda+1, 3\lambda+2)$

dr's of AM are $<\lambda-1, 2\lambda-5, 3\lambda-1>$

Here $1(\lambda-1)+2(2\lambda-5)+3(3\lambda-1)=0$

$\Rightarrow \lambda = 1$

$\therefore M(1,3,5)$ is the foot perpendicular of the point A to the given line.

Let image of point A in the line be $A'(\alpha, \beta, \gamma)$

Since M is the mid-point of AA', so $M\left(\frac{1+\alpha}{2}, \frac{6+\beta}{2}, \frac{3+\gamma}{2}\right) = M(1,3,5)$

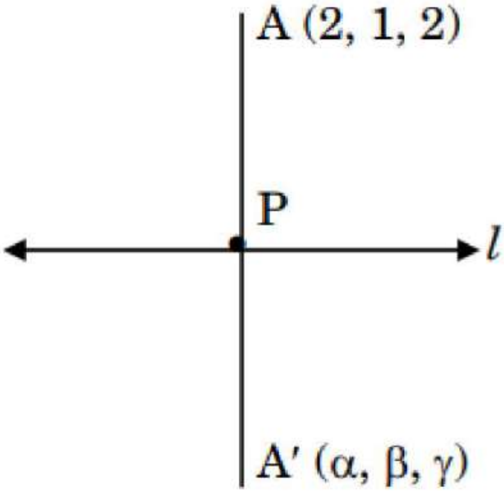
$\Rightarrow A'(1,0,7)$ is the image of A.

Also, Equation of AA' is $\frac{x-1}{0} = \frac{y-6}{-3} = \frac{z-3}{2}$

2. 2025

Find the image A′ of the point A(2, 1, 2) in the line $l : \vec{r} = 4\hat{i} + 2\hat{j} + 2\hat{k} + \lambda\ (\hat{i} - \hat{j} - \hat{k})$. Also, find the equation of line joining AA′. Find the foot of perpendicular from point A on the line l.

Sol.



Let the image of A in the line be $A'(\alpha, \beta, \gamma)$

The point P, which is the point of intersection of the lines l and AA', will have coordinates $(\lambda + 4, -\lambda + 2, -\lambda + 2)$ for some λ.

Drs of AP are $< \lambda + 2, -\lambda + 1, -\lambda >$

$AP \perp l$

$(\lambda + 2) - (-\lambda + 1) - (-\lambda) = 0$

$\Rightarrow \lambda = -\dfrac{1}{3}$

Therefore, the coordinates of P are $(\frac{11}{3}, \frac{7}{3}, \frac{7}{3})$

P is the mid-point of AA'

$\Rightarrow \dfrac{2+\alpha}{2} = \dfrac{11}{3}, \dfrac{1+\beta}{2} = \dfrac{7}{3}, \dfrac{2+\gamma}{2} = \dfrac{7}{3}$

$\Rightarrow \alpha = \dfrac{16}{3}, \beta = \dfrac{11}{3}, \gamma = \dfrac{8}{3}$

The coordinates of the image are $(\frac{16}{3}, \frac{11}{3}, \frac{8}{3})$

The equation of AA' is

$$\frac{x-2}{\frac{10}{3}}=\frac{y-1}{\frac{8}{3}}=\frac{z-2}{\frac{2}{3}}$$

or,

$$\frac{3(x-2)}{5}=\frac{3(y-1)}{4}=\frac{3(z-2)}{1}$$

3.2024

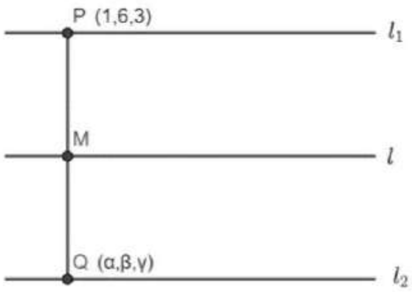
Find the equation of a line l_2 which is the mirror image of the line l_1 with respect to line $l : \frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$, given that line l_1 passes through the point P(1, 6, 3) and parallel to line l.

Sol.

D ratios of the line l i.e. $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ are 1, 2, 3

Let coordinates of foot of perpendicular M on line l be $(\lambda, 2\lambda+1, 3\lambda+2)$

D.ratios of PM are $\lambda-1, 2\lambda-5, 3\lambda-1$



$1(\lambda-1)+2(2\lambda-5)+3(3\lambda-1)=0$ ($\because$ PM $\perp l$)

$\Rightarrow \lambda=1$

Coordinates of M are (1, 3, 5)

Since M is midpoint of PQ $\therefore$ Coordinates of Q are (1, 0, 7)

Equation of line l_2 is $\frac{x-1}{1}=\frac{y}{2}=\frac{z-7}{3}$

4. 2025

Let the polished side of the mirror be along the line $\frac{x}{1} = \frac{1-y}{-2} = \frac{2z-4}{6}$.

A point P(1, 6, 3), some distance away from the mirror, has its image formed behind the mirror. Find the coordinates of the image point and the distance between the point P and its image.

Sol.

Equation of given line is $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

Let coordinates of point on the line be $(\lambda, 2\lambda + 1, 3\lambda + 2)$ for some λ

Drs of line perpendicular to line along mirror are $\langle \lambda-1, 2\lambda - 5, 3\lambda - 1 \rangle$

$(\lambda-1).1 + (2\lambda - 5).2 + (3\lambda - 1).3 = 0$ gives $\lambda = 1$

Coordinates of foot of perpendicular are (1,3,5)

For image

$\frac{x+1}{2} = 1, \frac{y+6}{2} = 3, \frac{z+3}{2} = 5$ gives image as (1, 0, 7)

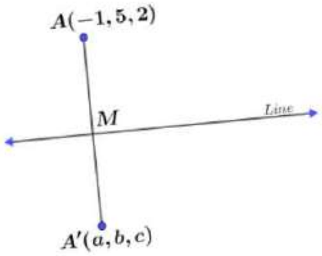
Required distance = $\sqrt{0+36+16} = 2\sqrt{13}$

5. 2025

Find the image of the point (–1, 5, 2) in the line $\frac{2x-4}{2} = \frac{y}{2} = \frac{2-z}{3}$. Find the length of the line segment joining the points (given point and the image point).

Sol.

Let $A'(a,b,c)$ be the image of the point $A(-1,5,2)$ in the given line, also assume 'M' as the point of intersection of AA' with the given line, then 'M' is the mid-point of the line segment AA'



The Line in the standard form is: $\frac{x-2}{1} = \frac{y}{2} = \frac{z-2}{-3}$, then

M is the point $(\lambda+2, 2\lambda, -3\lambda+2)$, for some $\lambda \in R$

Direction Ratios of AM are $\lambda+3, 2\lambda-5, -3\lambda$

$AM \perp$ Line, $\therefore 1(\lambda+3)+2(2\lambda-5)-3(-3\lambda)=0 \Rightarrow \lambda = \frac{1}{2}$

$$M\left(\frac{5}{2},1,\frac{1}{2}\right) = M\left(\frac{a-1}{2},\frac{b+5}{2},\frac{c+2}{2}\right) \Rightarrow a=6, b=-3, c=-1$$

$\therefore$ The Image of A in the line is $A'(6,-3,-1)$

And, $AA' = \sqrt{49+64+9} = \sqrt{122}$

6. 2024

Find the length and the coordinates of the foot of the perpendicular drawn from the point P(5, 9, 3) to the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$. Also, find the coordinates of the image of the point P in the given line.

Sol.

Any point on the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}=\lambda$ is

$Q(2\lambda+1, 3\lambda+2, 4\lambda+3)$

DRs of $\overrightarrow{PQ}$ are $(2\lambda-4, 3\lambda-7, 4\lambda)$

$\overrightarrow{PQ} \perp$ given line $\Rightarrow 2(2\lambda-4)+3(3\lambda-7)+4(4\lambda)=0$

$\Rightarrow \lambda = 1$

$\therefore$ Point Q is (3,5,7)

$PQ=\sqrt{4+16+16}=6$

coordinates of image P' are (1,1,11)

VI.Distance of a point from the line/lenth of the perpendicular:

1.

Find the distance of the point P(2, 4, –1) from the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$.

Sol.

Let $\vec{a}_2 = 2\hat{i} + 4\hat{j} - \hat{k}$, $\vec{a}_1 = -5\hat{i} - 3\hat{j} + 6\hat{k}$ and $\vec{b} = \hat{i} + 4\hat{j} - 9\hat{k}$

Distance between point and line is given by $d = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$

Here $(\vec{a}_2 - \vec{a}_1) = 7\hat{i} + 7\hat{j} - 7\hat{k}$

$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = -35\hat{i} + 56\hat{j} + 21\hat{k}$

$d = \frac{49\sqrt{2}}{7\sqrt{2}} = 7$

prepared by : BALAJI KANCHI

VII. Finding coordinates of a points if the "distance" of the point from the line is "Given" :

1.a

Find the point(s) on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance 5 *units* from the point (1,3,3).

Sol.

Let the required point on given line be $(3\lambda-2,\ 2\lambda-1,\ 2\lambda+3)$ for some λ.
According to question

$$\sqrt{(3\lambda-3)^2+(2\lambda-4)^2+(2\lambda)^2}=5$$

$$17\lambda^2-34\lambda+25=25$$

$$17\lambda(\lambda-2)=0 \text{ gives } \lambda=0,\ \lambda=2$$

$\therefore$ Coordinates of required points are (–2, –1, 3), (4, 3, 7)

1.b 2023

Find the coordinates of points on line $\frac{x}{1}=\frac{y-1}{2}=\frac{z+1}{2}$ which are at a distance of $\sqrt{11}$ units from origin.

Sol.

General point on the curve is $P(k, 2k+1, 2k-1)$, $k \in R$

$$OP=\sqrt{11} \Rightarrow OP^2=11$$

$$\therefore k^2+(2k+1)^2+(2k-1)^2=11 \Rightarrow k=\pm 1$$

$$\therefore \text{ Coordinates of points are } (1,3,1)\ \&\ (-1,-1,-3)$$

1.c 2025

Find the point on the line $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-4}{3}$ at a distance of $2\sqrt{2}$ units from the point (–1, –1, 2).

Sol.

Equation of given line be $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-4}{3}=\lambda$ **(say)**

Coordinate of any general point on the line are $P(3\lambda+1, 2\lambda-1, 3\lambda+4)$.

Let distance of point P from $(-1,-1,2)$ **is** $2\sqrt{2}$.

$$\Rightarrow \sqrt{(3\lambda+2)^2+(2\lambda)^2+(3\lambda+2)^2}=2\sqrt{2}$$

$$\Rightarrow 22\lambda^2+24\lambda=0$$

$$\Rightarrow \lambda=0 \text{ or } \lambda=-\frac{12}{11}$$

Hence, coordinates of point P are $(1,-1,4)$ **or** $\left(-\frac{25}{11},-\frac{35}{11},\frac{8}{11}\right)$

1.d 2025

Find the point Q on the line $\frac{2x+4}{6}=\frac{y+1}{2}=\frac{-2z+6}{-4}$ at a distance of $3\sqrt{2}$ from the point P(1, 2, 3).

Sol.

The general point on the line $(3\lambda-2, 2\lambda-1, 2\lambda+3)$ **is Q, from some** $\lambda\in R$

$$PQ=3\sqrt{2}\Rightarrow (PQ)^2=18\Rightarrow (3\lambda-3)^2+(2\lambda-3)^2+(2\lambda)^2=18$$

$$17\lambda^2-30\lambda=0\Rightarrow \lambda=0 \text{ or } \lambda=\frac{30}{17}$$

Thus, the point is $Q(-2,-1,3)$ **or** $Q\left(\frac{56}{17},\frac{43}{17},\frac{111}{17}\right)$

1.e

Find a point P on the line $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}$ such that its distance from point Q(2, 4, –1) is 7 units. Also, find the equation of line joining P and Q.

Sol.

The given line is $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}=\lambda$ **and** $Q(2,4,-1)$

Any random point on the line will be given by $P(\lambda-5, 4\lambda-3, -9\lambda+6)$

Since $PQ=7 \Rightarrow \sqrt{(\lambda-7)^2+(4\lambda-7)^2+(-9\lambda+7)^2}=7$

$\Rightarrow 98(\lambda^2-2\lambda+1)=0 \Rightarrow \lambda=1$

Hence, the required point is $P(-4,1,-3)$

The equation of line PQ is $\frac{x+4}{6}=\frac{y-1}{3}=\frac{z+3}{2}$ **or** $\frac{x-2}{6}=\frac{y-4}{3}=\frac{z+1}{2}$

2. 2023

Find the equations of the line passing through the points A(1, 2, 3) and B(3, 5, 9). Hence, find the coordinates of the points on this line which are at a distance of 14 units from point B.

Sol.

Equation of line AB is $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{6}$

Let coordinates of required point on AB be $(2\lambda + 1, 3\lambda + 2, 6\lambda + 3)$ for some λ

According to Question

$(2\lambda - 2)^2 + (3\lambda - 3)^2 + (6\lambda - 6)^2 = 14^2$ gives $\lambda^2 - 2\lambda - 3 = 0$

Solving we get $\lambda = 3$ and -1

$\therefore$ required points are (7, 11, 21) and (– 1, – 1, – 3)

prepared by : BALAJI KANCHI

VIII. Finding Equation of line :

a. Equation of line parallel to another line :

1. 2023

Find the vector and the cartesian equations of a line that passes through the point A(1, 2, –1) and parallel to the line $5x - 25 = 14 - 7y = 35z$.

Sol.

The given line is

$$\frac{x-5}{\frac{1}{5}} = \frac{y-2}{-\frac{1}{7}} = \frac{z}{\frac{1}{35}}, \text{ or } \frac{x-5}{7} = \frac{y-2}{-5} = \frac{z}{1}$$

So, the required vector equation of the line passing through (1,2,-1) is

$$\vec{r} = (\hat{\imath} + 2\hat{\jmath} - \hat{k}) + \lambda(7\hat{\imath} - 5\hat{\jmath} + \hat{k})$$

prepared by : BALAJI KANCHI

Cartesian equation of the line is

$$\frac{x-1}{7} = \frac{y-2}{-5} = \frac{z+1}{1}$$

2.a

Find the vector equation of a line passing through a point with position vector $2\hat{i} - \hat{j} + \hat{k}$ and parallel to the line joining the points $-\hat{i} + 4\hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 2\hat{k}$.

sol.

Vector parallel to the required line is $2\hat{i} - 2\hat{j} + \hat{k}$

Required vector equation of the line

$$\vec{r} = (2\hat{i} - \hat{j} + \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$$

2.b 2023

Find the vector and the Cartesian equations of a line passing through the point (1, 2, – 4) and parallel to the line joining the points A(3, 3, – 5) and B(1, 0, – 11). Hence, find the distance between the two lines.

Sol.

Vector equation of required line through (1, 2, – 4) is

$$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

and cartesian equation: $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$

Equation of line through A(3, 3, – 5) and B(1, 0, – 11) is

$\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$

Distance between parallel lines is given by $d = \frac{|(\vec{a}_2 - \vec{a}_1)\times\vec{b}|}{|\vec{b}|}$

Here $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$, $\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}$, $\vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}$

$$(\vec{a}_2 - \vec{a}_1) = 2\hat{i} + \hat{j} - \hat{k}$$

$(\vec{a}_2 - \vec{a}_1)\times\vec{b} = 9\hat{i} - 14\hat{j} + 4\hat{k}$

$\therefore d = \frac{\sqrt{293}}{7}$

2.c 2025

Let the position vectors of the points A, B and C be $3\hat{i} - \hat{j} - 2\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$ and $\hat{i} + 5\hat{j} + 3\hat{k}$ respectively. Find the vector and cartesian equations of the line passing through A and parallel to line BC.

Sol.

Direction vector of line = $3\hat{j} + 4\hat{k}$

Vector equation is $\vec{r} = 3\hat{i} - \hat{j} - 2\hat{k} + \mu(3\hat{j} + 4\hat{k})$

Cartesian equation is $\frac{x-3}{0} = \frac{y+1}{3} = \frac{z+2}{4}$

3.a

Find the vector equation of a line passing through the point (2, 3, 2) and parallel to the line $\vec{r} = (-2\hat{i} + 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$. Also, find the distance between these two lines.

Sol.

Required equation of line is

$$\vec{r} = 2\hat{i} + 3\hat{j} + 2\hat{k} + \mu(2\hat{i} - 3\hat{j} + 6\hat{k})$$

$$\vec{a}_1 = -2\hat{i} + 3\hat{j}, \vec{a}_2 = 2\hat{i} + 3\hat{j} + 2\hat{k}, \quad \vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = 4\hat{i} + 2\hat{k}$$

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 0 & 2 \\ 2 & -3 & 6 \end{vmatrix}$$

$$= 6\hat{i} - 20\hat{j} - 12\hat{k}$$

$$d = \frac{\sqrt{580}}{7}$$

3.b

Find the cartesian equation of the line which passes through the point (– 2, 4, – 5) and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$.

Sol.

D·R's of required line are 3, –5, 6

Equation of line is $\frac{x+2}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$

b.Equation of a line which is perpendicular to two other lines :

1.

Find the vector and Cartesian equations of the line which is perpendicular to the lines with equations $\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4}$ and $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and passes through the point (1, 1, 1). Also find the angle between the given lines.

Answer:

Let equation of required line is $\frac{x-1}{a}=\frac{y-1}{b}=\frac{z-1}{c}$ ....(i)

Since this line is perpendicular to $\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4}$ and $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$,

$a+2b+4c=0$ ....(ii)

$2a+3b+4c=0$ ....(iii)

Solving (ii) and (iii) , $\frac{a}{-4}=\frac{b}{4}=\frac{c}{-1}$

$\therefore$ DR's of line in cartesian form is : $-4,4,-1$

Equation of line in Cartesian form is: $\frac{x-1}{-4}=\frac{y-1}{4}=\frac{z-1}{-1}$

Vector form of line is $\vec{r}=(\hat{i}+j+k)+\lambda(-4\hat{i}+4j-k)$

Let θ be the angle between given lines.

$$\cos\theta=\frac{1(2)+2(3)+4(4)}{\sqrt{1+4+16}\sqrt{4+9+16}}=\frac{24}{\sqrt{21}\sqrt{29}} \qquad \therefore \theta=\cos^{-1}\left(\frac{24}{\sqrt{21}\sqrt{29}}\right)$$

2. 2025

Find the equation of a line in vector and cartesian form which passes through the point (1, 2, – 4) and is perpendicular to the lines $\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}$, and

$\vec{r}=15\hat{i}+29\hat{j}+5\hat{k}+\mu(3\hat{i}+8\hat{j}-5\hat{k})$.

Sol.

Given lines are $\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}$

and $\vec{r}=\left(15\hat{i}+29\hat{j}+5\hat{k}\right)+\mu\left(3\hat{i}+8\hat{j}-5\hat{k}\right)$

The first line in vector form is $\vec{r}=\left(8\hat{i}-19\hat{j}+10\hat{k}\right)+\lambda\left(3\hat{i}-16\hat{j}+7\hat{k}\right)$

$\vec{a}_1=8\hat{i}-19\hat{j}+10\hat{k}, \vec{a}_2=15\hat{i}+29\hat{j}+5\hat{k}$

$\vec{b}_1=3\hat{i}-16\hat{j}+7\hat{k}, \vec{b}_2=3\hat{i}+8\hat{j}-5\hat{k}$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -16 & 7\\ 3 & 8 & -5\end{vmatrix}=24\hat{i}+36\hat{j}+72\hat{k}$$

$\therefore$ Equation of line passing through $(1,2,-4)$ and parallel to $\vec{b}$ is

$\vec{r}=\left(\hat{i}+2\hat{j}-4\hat{k}\right)+t\left(24\hat{i}+36\hat{j}+72\hat{k}\right)$ or $\vec{r}=\left(\hat{i}+2\hat{j}-4\hat{k}\right)+t'\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$

Cartesian form of line is $\frac{x-1}{24}=\frac{y-2}{36}=\frac{z+4}{72}$ or $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6}$

3. 2024

If the lines $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-1}{1}=\frac{z-6}{-7}$ are perpendicular to each other, find the value of k and hence write the vector equation of a line perpendicular to these two lines and passing through the point (3, –4, 7).

Sol.

$$L_1:\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}\Rightarrow \text{direction ratio's of } L_1 = <-3, 2k, 2>$$

$$L_2:\frac{x-1}{3k}=\frac{y-1}{1}=\frac{z-6}{-7}\Rightarrow \text{direction ratio's of } L_2 = <3k, 1, -7>$$

Since $L_1 \perp L_2$,

$$-9k+2k-14=0 \Rightarrow k=-2$$

Thus, d.r.'s of $L_1 = <-3,-4,2>$, d.r.'s of $L_2 = <-6,1,-7>$

Now the vector perpendicular to both L_1 & L_2 is given by

$$\vec{b}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -4 & 2 \\ -6 & 1 & -7 \end{vmatrix} = 26\hat{i}-33\hat{j}-27\hat{k}$$

Thus, Equation of the required line is

$$\vec{r}=(3\hat{i}-4\hat{j}+7\hat{k})+\lambda(26\hat{i}-33\hat{j}-27\hat{k})$$

prepared by : BALAJI KANCHI

4. 2024

Find the equation of the line which bisects the line segment joining points A(2, 3, 4) and B(4, 5, 8) and is perpendicular to the lines $\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$.

Sol.

Let direction ratios of the required line be a, b, c.

∵ the required line is perpendicular to both the given lines

$\therefore 3a-16b+7c=0$

and $3a+8b-5c=0$

$\Longrightarrow \frac{a}{24}=\frac{b}{36}=\frac{c}{72}$

$\Longrightarrow \frac{a}{2}=\frac{b}{3}=\frac{c}{6}$

The mid-point of the line-segment AB is (3, 4, 6)

Hence, the required equation of the line is

$$\frac{x-3}{2}=\frac{y-4}{3}=\frac{z-6}{6}$$

5.

Find the vector equation of the line passing through the point (2, 1, 3) and perpendicular to both the lines

$$\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3};\quad \frac{x}{-3}=\frac{y}{2}=\frac{z}{5}.$$

Sol.

Vector equation of the line passing through (2, 1, 3) is

$$\vec{r}=(2\hat{i}+\hat{j}+3\hat{k})+\lambda(a\hat{i}+b\hat{j}+c\hat{k})$$

Line $\vec{r}$ is perpendicular to the given lines then

$a+2b+3c=0;\ -3a+2b+5c=0$

$$\Rightarrow \frac{a}{4}=\frac{b}{-14}=\frac{c}{8}=k'(\text{say})$$

$\Rightarrow a=2k,\ b=-7k$ and $c=4k.$

Thus, the required vector equation is

$$\vec{r}=(2\hat{i}+\hat{j}+3\hat{k})+\lambda(2\hat{i}-7\hat{j}+4\hat{k})$$

6.

A line l passes through point (–1, 3, –2) and is perpendicular to both the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$. Find the vector equation of the line l. Hence, obtain its distance from origin.

Sol.

Let direction ratios of the required line be a, b, c

Since it is perpendicular to the two given lines, $a+2b+3c=0;\ -3a+2b+5c=0$

Solving together, $a=4k,\ b=-14k,\ c=8k$

$\therefore$ **Equation of line is:** $\frac{x+1}{4k}=\frac{y-3}{-14k}=\frac{z+2}{8k}\Rightarrow\frac{x+1}{2}=\frac{y-3}{-7}=\frac{z+2}{4}$

Vector equation: $\vec{r}=-\hat{i}+3\hat{j}-2\hat{k}+\lambda\left(2\hat{i}-7\hat{j}+4\hat{k}\right)$

$$\textbf{Distance from origin}=\frac{\left|\left(-\hat{i}+3\hat{j}-2\hat{k}\right)\times\left(2\hat{i}-7\hat{j}+4\hat{k}\right)\right|}{\left|2\hat{i}-7\hat{j}+4\hat{k}\right|}=\frac{\left|-2\hat{i}+\hat{k}\right|}{\left|2\hat{i}-7\hat{j}+4\hat{k}\right|}=\frac{\sqrt{5}}{\sqrt{69}}\text{ or }\sqrt{\frac{5}{69}}$$

7. 2025

Determine the vector equation of a line passing through the point (1, 2, – 3) and perpendicular to both the given lines

$$\frac{x-8}{3}=\frac{y+16}{-16}=\frac{x-10}{7} \text{ and } \frac{x-15}{3}=\frac{y-29}{-8}=\frac{z-5}{-5}.$$

Sol.

Let direction ratios of required line be a, b, c

Therefore, $3a-16b+7c=0$

$3a-8b-5c=0$

Solving we get $\frac{a}{136}=\frac{b}{36}=\frac{c}{24}$

DRs are 136, 36, 24 or 34, 9, 6

Equation is $\frac{x-1}{136}=\frac{y-2}{36}=\frac{z+3}{24}$ or $\frac{x-1}{34}=\frac{y-2}{9}=\frac{z+3}{6}$

$\vec{r}=\hat{i}+2\hat{j}-3\hat{k}+\lambda(34\hat{i}+9\hat{j}+6\hat{k})$

8.2024

Find the equation of the line passing through the point of intersection of the lines $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ and $\frac{x-1}{0}=\frac{y}{-3}=\frac{z-7}{2}$ and perpendicular to these given lines.

Sol.

$l_1:\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}=\lambda$; $l_2:\frac{x-1}{0}=\frac{y}{-3}=\frac{z-7}{2}=\mu$

any point on l_1 is $(\lambda, 2\lambda+1, 3\lambda+2)$ & any point on l_2 is $(1, -3\mu, 2\mu+7)$

If l_1 and l_2 intersect,

$\lambda=1, 2\lambda+1=-3\mu$ and $3\lambda+2=2\mu+7 \Rightarrow \lambda=1$ and $\mu=-1$

Point of intersection of l_1 and l_2 is $(1,3,5)$.

Let d.r.'s of required line be $<a,b,c>$. Then,

$a+2b+3c=0$ and $-3b+2c=0 \Rightarrow \frac{a}{13}=\frac{b}{-2}=\frac{c}{-3}$

Required equation of line is $\frac{x-1}{13}=\frac{y-3}{-2}=\frac{z-5}{-3}$

9.

Find the point of intersection of the lines

$\vec{r} = \hat{i} - \hat{j} + 6\hat{k} + \lambda(3\hat{i} - \hat{k})$, and

$\vec{r} = -3\hat{j} + 3\hat{k} + \mu(\hat{i} + 2\hat{j} - \hat{k})$.

Also, find the vector equation of the line passing through the point of intersection of the given lines and perpendicular to both the lines.

Sol.

If the given lines intersect then,

$$\hat{i} - \hat{j} + 6\hat{k} + \lambda\left(3\hat{i} - \hat{k}\right) = -3\hat{j} + 3\hat{k} + \mu\left(\hat{i} + 2\hat{j} - \hat{k}\right)$$

Solving the equations, $1 + 3\lambda = \mu, -1 = 2\mu - 3$ **, we get** $\lambda = 0, \mu = 1$

which do not satisfy the equation $, 6 - \lambda = 3 - \mu$ **.**

$\therefore$ **The lines do not intersect, hence no point of intersection**

c. Equation of line which is intersecting two other lines:

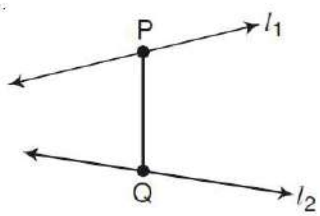
1.

A line with direction ratios < 2, 2, 1 > intersects the lines

$$\frac{x-7}{3}=\frac{y-5}{2}=\frac{z-3}{1} \text{ and } \frac{x-1}{2}=\frac{y+1}{4}=\frac{z+1}{3}$$

at the points P and Q respectively. Find the length and the equation of the intercept PQ.

Sol.



Let $P(3\lambda + 7, 2\lambda + 5, \lambda + 3)$ and

$Q(2\mu + 1, 4\mu - 1, 3\mu - 1)$

Now, d.r′.s. of PQ = $3\lambda - 2\mu + 6, 2\lambda - 4\mu + 6, \lambda - 3\mu + 4$

According to question,

$$\frac{3\lambda-2\mu+6}{2}=\frac{2\lambda-4\mu+6}{2}=\frac{\lambda-3\mu+4}{1}$$

$\Rightarrow \lambda + 2\mu = 0$ and $2\mu = 2 \Rightarrow \mu = 1$

$\Rightarrow \lambda = -2\mu$

$\therefore \mu = 1, \lambda = -2$

$\therefore P(1, 1, 1)$ and $Q(3, 3, 2)$

$$PQ = \sqrt{(3-1)^2+(3-1)^2+(2-1)^2}=\sqrt{4+4+1}=3$$

Equation of PQ is $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-1}{1}$

IX. Application based problems :

a. Equation of sides of a triangle, if three vertices are triangle are given/Finding area , length of the altitude/parallelogram sides equation , diagonal equation :

1 .2024

The vertices of Δ ABC are A(1, 1, 0), B(1, 2, 1) and C(−2, 2, −1). Find the equations of the medians through A and B. Use the equations so obtained to find the coordinates of the centroid.

Sol.

The mid-point of the BC is $(\frac{-1}{2}, 2, 0)$

The equation of the median through A is

$$\frac{x-1}{\frac{-1}{2}-1} = \frac{y-1}{2-1} = \frac{z}{0}$$

$$\Rightarrow \frac{x-1}{-3} = \frac{y-1}{2} = \frac{z}{0} \dots\dots\dots\dots\dots\dots. (1)$$

The mid-point of the AC is $(\frac{-1}{2}, \frac{3}{2}, \frac{-1}{2})$

The equation of the median through B is

$$\frac{x-1}{\frac{-1}{2}-1} = \frac{y-2}{\frac{3}{2}-2} = \frac{z-1}{\frac{-1}{2}-1}$$

$$\Rightarrow \frac{x-1}{-3} = \frac{y-2}{-1} = \frac{z-1}{-3} \dots\dots\dots. (2)$$

Any point on the line (1) is $(-3\lambda + 1, 2\lambda + 1, 0)$

Any point on the line (2) is $(-3\mu + 1, -\mu + 2, -3\mu + 1)$

For the point of intersection,

$-3\lambda + 1 = -3\mu + 1, 2\lambda + 1 = -\mu + 2, 0 = -3\mu + 1$

$$\Rightarrow \lambda = \mu = \frac{1}{3}$$

The coordinates of the centroid are $(0, \frac{5}{3}, 0)$

2. 2024

Equations of sides of a parallelogram ABCD are as follows :

AB : $\dfrac{x+1}{1} = \dfrac{y-2}{-2} = \dfrac{z-1}{2}$

BC : $\dfrac{x-1}{3} = \dfrac{y+2}{-5} = \dfrac{z-5}{3}$

CD : $\dfrac{x-4}{1} = \dfrac{y+7}{-2} = \dfrac{z-8}{2}$

DA : $\dfrac{x-2}{3} = \dfrac{y+3}{-5} = \dfrac{z-4}{3}$

Find the equation of diagonal BD.

Sol.

Let coordinates of B (from AB) = $(\lambda - 1, 2 - 2\lambda, 2\lambda + 1)$

Also coordinates of B (from BC) = $(3\mu + 1, -5\mu - 2, 3\mu + 5)$

Solving we get $\lambda = 2, \mu = 0$

$\therefore$ Point of intersection is B(1,−2,5)

Similarly, point of intersection from CD and DA is D (2, -3,4)

Direction ratios of BD are 2−1, −3+2, 4−5 i.e. 1, −1, −1

Equation of BD is

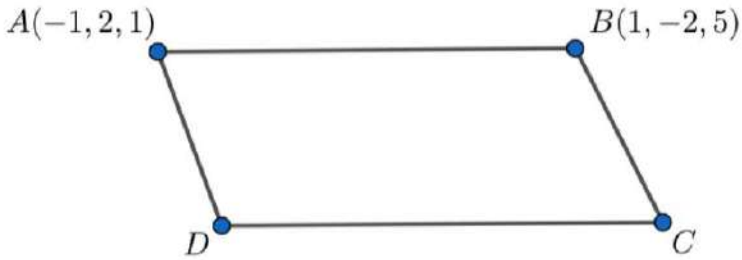
$\frac{x-1}{1} = \frac{y+2}{-1} = \frac{z-5}{-1}$

prepared by : BALAJI KANCHI

3.2024

Two vertices of the parallelogram ABCD are given as A(−1, 2, 1) and B(1, −2, 5). If the equation of the line passing through C and D is $\frac{x-4}{1} = \frac{y+7}{-2} = \frac{z-8}{2}$, then find the distance between sides AB and CD. Hence, find the area of parallelogram ABCD.

Sol.



d.r's of CD are < 1, – 2, 2 >

$\therefore$ d.r's of AB are < 1, – 2, 2 >

$\therefore$ Equation of AB is $\frac{x+1}{1} = \frac{y-2}{-2} = \frac{z-1}{2}$

$\therefore$ Equation of CD is $\frac{x-4}{1} = \frac{y+7}{-2} = \frac{z-8}{2}$

Let $\vec{a}_1 = -\hat{i} + 2\hat{j} + \hat{k}$, $\vec{a}_2 = 4\hat{i} - 7\hat{j} + 8\hat{k}$ & $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$

Now, $\vec{a}_2 - \vec{a}_1 = 5\hat{i} - 9\hat{j} + 7\hat{k}$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -9 & 7 \\ 1 & -2 & 2 \end{vmatrix} = -4\hat{i} - 3\hat{j} - \hat{k}$$

Distance between AB and CD is given by $d = \frac{\left|(\vec{a}_2 - \vec{a}_1) \times \vec{b}\right|}{\left|\vec{b}\right|}$

$$d = \frac{\sqrt{16+9+1}}{\sqrt{1+4+4}} = \frac{\sqrt{26}}{3}$$

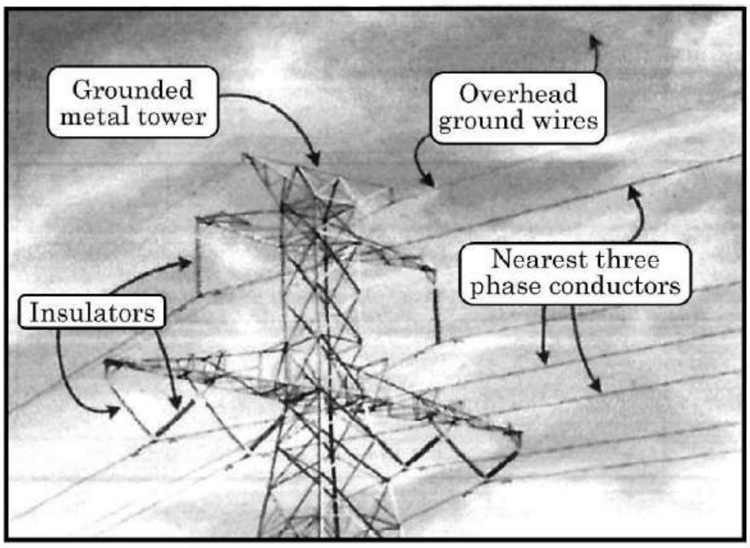
$$CD = \sqrt{2^2 + (-4)^2 + (4)^2} = 6$$

Area of parallelogram ABCD $= b \times h = 6 \times \frac{\sqrt{26}}{3} = 2\sqrt{26}$

Case study :

1. 2022

Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.



Two such wires lie along the following lines :

$$l_1 : \frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1}$$

$$l_2 : \frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2}$$

Based on the given information, answer the following questions :

(i) Are the lines l_1 and l_2 coplanar ? Justify your answer.

(ii) Find the point of intersection of the lines l_1 and l_2.

Sol.

(i) Consider $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix}$

$$= \begin{vmatrix} +1 & 4 & -5 \\ 3 & -2 & -1 \\ -1 & 3 & -2 \end{vmatrix}$$

$$= 1(+7) - 4(-7) - 5(7)$$

$$= 0$$

$\therefore$ lines l_1 and l_2 are coplanar

(ii) Any point on $l_1 : (3\lambda - 1, -2\lambda + 3, -\lambda - 2)$

Substituting in equation of l_2,

$$\frac{3\lambda - 1}{-1} = \frac{-2\lambda + 3 - 7}{3}$$

$$\Rightarrow 9\lambda - 3 = 2\lambda + 4$$

$$\Rightarrow 7\lambda = 7 \Rightarrow \lambda = 1$$

Point is $(2, 1, -3)$

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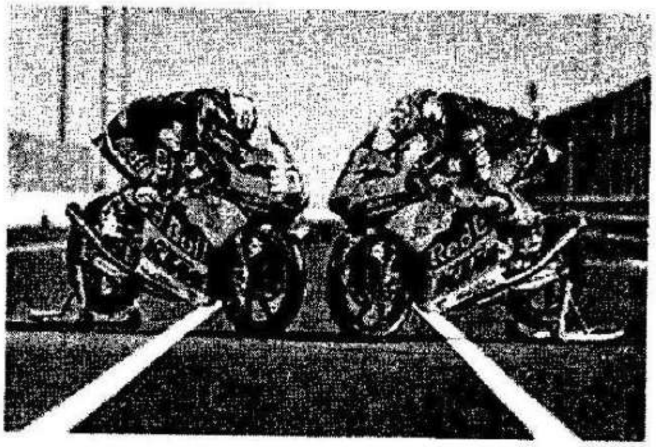
2. 2022

Two motorcycles A and B are running at the speed more than the allowed speed on the roads represented by the lines $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k})$ respectively.



Based on the above information, answer the following questions :

(a) Find the shortest distance between the given lines.

(b) Find the point at which the motorcycles may collide.

Sol.

(a).

$\vec{a}_1 = 0\hat{i} + 0\hat{j} + 0\hat{k}$, $\vec{a}_2 = 3\hat{i} + 3\hat{j}$

$$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{vmatrix}$$

$$= 3\hat{i} - 3\hat{j} - 3\hat{k}$$

$$\text{SD} = \frac{\left|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)\right|}{|\vec{b}_1 \times \vec{b}_2|}$$

Now, $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (3\hat{i} + 3\hat{j})(3\hat{i} - 3\hat{j} - 3\hat{k})$

$$= 9 - 9 = 0$$

Shortest distance between two lines = 0

(b).

Any point on the line $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ is $\lambda\hat{i} + 2\lambda\hat{j} - \lambda\hat{k}$

Any point on the line $\vec{r} = 3\hat{i} + 3\hat{j} + \mu(2\hat{i} + \hat{j} + \hat{k})$ is

$(2\mu + 3)\hat{i} + (\mu + 3)\hat{j} + \mu\hat{k}$

As the lines are intersecting,

$$\lambda = 2\mu + 3, 2\lambda = \mu + 3$$

On solving $\mu = -1$, $\lambda = 1$

Point of intersection is $\hat{i} + 2\hat{j} - \hat{k}$ or $(1, 2, -1)$

3. 2025

An engineer is designing a new metro rail network in a city.

Initially, two metro lines, Line A and Line B, each consisting of multiple stations are designed. The track for Line A is represented by $l_1 : \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-3}{4}$, while the track for Line B is represented by $l_2 : \frac{x-1}{2} = \frac{y-3}{1} = \frac{z+2}{-3}$.

Based on the above information, answer the following questions :

(i) Find whether the two metro tracks are parallel.

(ii) Solar panels are to be installed on the rooftop of the metro stations. Determine the equation of the line representing the placement of solar panels on the rooftop of Line A's stations, given that panels are to be positioned parallel to Line A's track (l_1) and pass through the point $(1, -2, -3)$.

(iii) (a) To connect the stations, a pedestrian pathway perpendicular to the two metro lines is to be constructed which passes through point (3, 2, 1). Determine the equation of the pedestrian walkway.

OR

(iii) (b) Find the shortest distance between Line A and Line B.

Sol.

$$l_1: \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-3}{4} \quad ; \quad l_2: \frac{x-1}{2} = \frac{y-3}{1} = \frac{z+2}{-3}$$

(i) Drs of l_1 are $<3, -2, 4>$, drs of l_2 are $< 2, 1, -3 >$

as drs are not proportional, hence l_1 is not parallel to l_2.

(ii) Equations of line parallel to l_1 and passing through $(1, -2, -3)$ is

$$\frac{x-1}{3} = \frac{y+2}{-2} = \frac{z+3}{4} \text{ or } \vec{r} = \left(\hat{i} - 2\hat{j} - 3\hat{k}\right) + \lambda\left(3\hat{i} - 2\hat{j} + 4\hat{k}\right)$$

$(iii)(a)$ The pathway is perpendicular to l_1 and l_2. $\therefore$ It is parallel to $\vec{b}_1 \times \vec{b}_2$

$$\vec{b} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 4 \\ 2 & 1 & -3 \end{vmatrix} = 2\hat{i} + 17\hat{j} + 7\hat{k}$$

$\therefore$ Equation of pathway is $\vec{r} = \left(3\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} + 17\hat{j} + 7\hat{k}\right)$

OR

$(iii)(b)$ $\vec{a}_1 = 2\hat{i} - \hat{j} + 3\hat{k}$, $\vec{a}_2 = \hat{i} + 3\hat{j} - 2\hat{k}$

$\vec{b}_1 = 3\hat{i} - 2\hat{j} + 4\hat{k}$, $\vec{b}_2 = 2\hat{i} + \hat{j} - 3\hat{k}$

$$d = \left|\frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$$

$$= \left|\frac{\left(-\hat{i} + 4\hat{j} - 5\hat{k}\right).\left(2\hat{i} + 17\hat{j} + 7\hat{k}\right)}{\sqrt{4 + 289 + 49}}\right|$$

$$= \frac{31}{\sqrt{342}}$$

prepared by : BALAJI KANCHI

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