13. Probability

(Previous Year Questions from solutions from 2015-2025)

2022 March :

1.

If A and B are two independent events with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then $P(B' \mid A)$ is equal to

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) $\frac{3}{4}$

(D) 1

2.

From the set { 1,2,3,4,5 }, two numbers a and b (a ≠ b) are chosen at random. The probability that $\frac{a}{b}$ is an integer is :

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{3}{5}$

A bag contains 3 white, 4 black and 2 red balls. If 2 balls are drawn at random (without replacement), then the probability that both the balls are white is

(a) $\frac{1}{18}$ (b) $\frac{1}{36}$ (c) $\frac{1}{12}$ (d) $\frac{1}{24}$

4.

A card is picked at random from a pack of 52 playing cards. Given that the picked card is a queen, the probability of this card to be a card of spade is

(a) $\frac{1}{3}$ (b) $\frac{4}{13}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

5.

Three dice are thrown simultaneously. The probability of obtaining a total score of 5 is

(a) $\frac{5}{216}$ (b) $\frac{1}{6}$ (c) $\frac{1}{36}$ (d) $\frac{1}{49}$

In Q. Nos. **11** to **15,** fill in the blanks with correct word/sentence :

6.

A die is thrown once. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then P(A∪B) is

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) 0 (d) 1

7.

A number is chosen randomly from numbers 1 to 60. The probability that the chosen number is a multiple of 2 or 5 is

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{7}{10}$ (d) $\frac{9}{10}$

8.

The probability of solving a specific question independently by A and B are $\frac{1}{3}$ and $\frac{1}{5}$ respectively. If both try to solve the question independently, the probability that the question is solved is

(A) $\frac{7}{15}$

(B) $\frac{8}{15}$

(C) $\frac{2}{15}$

(D) $\frac{14}{15}$

2023 March:

1.

If $P\left(\frac{A}{B}\right) = 0{\cdot}3$, $P(A) = 0{\cdot}4$ and $P(B) = 0{\cdot}8$, then $P\left(\frac{B}{A}\right)$ is equal to :

(a) 0·6 (b) 0·3

(c) 0·06 (d) 0·4

2.

If A and B are two events such that $P(A/B) = 2 \times P(B/A)$ and $P(A) + P(B) = \frac{2}{3}$, then P(B) is equal to

(A) $\frac{2}{9}$ (B) $\frac{7}{9}$

(C) $\frac{4}{9}$ (D) $\frac{5}{9}$

3.

If the sum of numbers obtained on throwing a pair of dice is 9, then the probability that number obtained on one of the dice is 4, is :

(A) $\frac{1}{9}$ (B) $\frac{4}{9}$

(C) $\frac{1}{18}$ (D) $\frac{1}{2}$

4.

If for two events A and B, $P(A - B) = \frac{1}{5}$ and $P(A) = \frac{3}{5}$, then $P\left(\frac{B}{A}\right)$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{3}{5}$

(C) $\frac{2}{5}$ (D) $\frac{2}{3}$

5.

If $P(A \cap B) = \frac{1}{8}$ and $P(\bar{A}) = \frac{3}{4}$, then $P\left(\frac{B}{A}\right)$ is equal to :

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{6}$ (d) $\frac{2}{3}$

6.

For any two events A and B, if $P(\overline{A}) = \frac{1}{2}$, $P(\overline{B}) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{4}$, then $P\left(\frac{\overline{A}}{\overline{B}}\right)$ equals :

(a) $\frac{3}{8}$ (b) $\frac{8}{9}$

(c) $\frac{1}{8}$ (d) $\frac{1}{4}$

7.

X and Y are independent events such that $P(X \cap \overline{Y}) = \frac{2}{5}$ and $P(X) = \frac{3}{5}$. Then P(Y) is equal to :

(a) $\frac{2}{3}$ (b) $\frac{2}{5}$

(c) $\frac{1}{3}$ (d) $\frac{1}{5}$

8.

The probability that A speaks the truth is $\frac{4}{5}$ and that of B speaking the truth is $\frac{3}{4}$. The probability that they contradict each other in stating the same fact is :

(a) $\frac{7}{20}$ (b) $\frac{1}{5}$

(c) $\frac{3}{20}$ (d) $\frac{4}{5}$

9.

For two events A and B, if $P(A) = 0{\cdot}4$, $P(B) = 0{\cdot}8$ and $P(B/A) = 0{\cdot}6$, then $P(A \cup B)$ is :

(a) 0·24 (b) 0·3

(c) 0·48 (d) 0·96

10.

The events E and F are independent. If P(E) = 0·3 and $P(E \cup F)$ = 0·5, then P(E/F) – P(F/E) equals :

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$

(c) $\frac{3}{35}$ (d) $\frac{1}{70}$

11.

If A and B are two events such that P(A)=0.2 , P(B)=0.4 and $P(A \cup B)$=0.5 , then value of P(A/B) is ?

(a)0.1 (b)0.25 (c)0.5 (d) 0.08

12.

If for any two events A and B, $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then P(B/A) is equal to

(a) $\frac{1}{10}$ (b) $\frac{1}{8}$

(c) $\frac{7}{8}$ (d) $\frac{17}{20}$

13.

Five fair coins are tossed simultaneously. The probability of the events that atleast one head comes up is

(a) $\frac{27}{32}$ (b) $\frac{5}{32}$

(c) $\frac{31}{32}$ (d) $\frac{1}{32}$

14.

Assertion (A) : Two coins are tossed simultaneously. The probability of getting two heads, if it is known that at least one head comes up, is $\frac{1}{3}$.

Reason (R) : Let E and F be two events with a random experiment, then $P(F/E) = \frac{P(E \cap F)}{P(E)}$.

15.

If A and B are two independent events such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then $P(B'/A)$ is

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$

(c) $\frac{3}{4}$ (d) 1

16.

A family has 2 children and the elder child is a girl. The probability that both children are girls is :

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

17.

A fair die is rolled. Events E and F are E = {1, 3, 5} and F = {2, 3} respectively. Value of $P(E|F)$ is :

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{6}$ (d) $\frac{1}{2}$

18.

Assertion (A) : Given two independent events A and B such that $P(A) = 0{\cdot}3$ and $P(B) = 0{\cdot}6$, then $P(A \text{ and not } B) = 0{\cdot}12$.

Reason (R) : For two independent events A and B, $P(A \text{ and } B) = P(A) \cdot P(B)$.

19.

Two events A and B will be independent, if :

(a) A and B are mutually exclusive

(b) $P(A) = P(B)$

(c) $P(A'B') = [1 - P(A)]\ [1 - P(B)]$

(d) $P(A) + P(B) = 1$

20.
An urn contains 6 balls of which two are red and four are black. Two balls are drawn at random. Probability that they are of the different colours is

(a) $\frac{2}{5}$ (b) $\frac{1}{15}$ (c) $\frac{8}{15}$ (d) $\frac{4}{15}$

2024 March :

1.
If $P(A|B) = P(A'|B)$, then which of the following statements is true ?

(A) $P(A) = P(A')$ (B) $P(A) = 2\,P(B)$

(C) $P(A \cap B) = \frac{1}{2}\,P(B)$ (D) $P(A \cap B) = 2\,P(B)$

2.
Let E be an event of a sample space S of an experiment, then $P(S|E) =$

(A) $P(S \cap E)$ (B) $P(E)$

(C) 1 (D) 0

3.

If A and B are events such that $P(A/B) = P(B/A) \neq 0$, then :

(A) $A \subset B$, but $A \neq B$ (B) $A = B$

(C) $A \cap B = \phi$ (D) $P(A) = P(B)$

4.
Let E and F be two events such that $P(E) = 0{\cdot}1$, $P(F) = 0{\cdot}3$, $P(E \cup F) = 0{\cdot}4$, then $P(F|E)$ is :

(A) 0·6 (B) 0·4 (C) 0·5 (D) 0

2025 March :

1.

If E and F are two independent events such that $P(E) = \frac{2}{3}$, $P(F) = \frac{3}{7}$, then $P(E/\overline{F})$ is equal to :

(A) $\frac{1}{6}$ (B) $\frac{1}{2}$

(C) $\frac{2}{3}$ (D) $\frac{7}{9}$

2.

Let M and N be two events such that $P(M) = 0.6$, $P(N) = 0.2$ and $P(M \cap N) = 0.5$, then $P(M'/N')$ is

(A) $\frac{7}{8}$ (B) $\frac{2}{5}$

(C) $\frac{1}{2}$ (D) $\frac{2}{3}$

3.

If E and F are two events such that $P(E) > 0$ and $P(F) \neq 1$, then $P(\overline{E}/\overline{F})$ is

(A) $\frac{P(\overline{E})}{P(\overline{F})}$ (B) $1 - P(\overline{E}/F)$

(C) $1 - P(E/F)$ (D) $\frac{1 - P(E \cup F)}{P(\overline{F})}$

4.

If $P(A) = \frac{1}{7}$, $P(B) = \frac{5}{7}$ and $P(A \cap B) = \frac{4}{7}$, then $P(\bar{A} \mid B)$ is :

(A) $\frac{6}{7}$ (B) $\frac{3}{4}$

(C) $\frac{4}{5}$ (D) $\frac{1}{5}$

5.

If $P(A) = \frac{1}{5}$, $P(B) = \frac{3}{5}$ and $P\left(\frac{A}{B}\right) = \frac{2}{5}$, then $P(A' \cap B')$ is :

(A) $\frac{11}{25}$ (B) $\frac{19}{25}$

(C) $\frac{8}{25}$ (D) $\frac{6}{25}$

6.

Assertion (A) : If A and B are two events such that $P(A \cap B) = 0$, then A and B are independent events.

Reason (R) : Two events are independent if the occurrence of one does not effect the occurrence of the other.

7.

If $P(A \cup B) = 0{\cdot}9$ and $P(A \cap B) = 0{\cdot}4$, then $P(\overline{A}) + P(\overline{B})$ is :

(A) 0·3 (B) 1

(C) 1·3 (D) 0·7

8.

A box has 4 green, 8 blue and 3 red pens. A student picks up a pen at random, checks its colour and replaces it in the box. He repeats this process 3 times. The probability that at least one pen picked was red is :

(A) $\frac{124}{125}$ (B) $\frac{1}{125}$

(C) $\frac{61}{125}$ (D) $\frac{64}{125}$

9.

Chances that three persons A, B, and C go to the market are 30%, 60% and 50% respectively. The probability that at least one will go to the market is :

(A) $\frac{14}{10}$ (B) $\frac{43}{50}$

(C) $\frac{9}{100}$ (D) $\frac{7}{50}$

10.

A meeting will be held only if all three members A, B and C are present. The probability that member A does not turn up is 0·10, member B does not turn up is 0·20 and member C does not turn up is 0·05. The probability of the meeting being cancelled is :

(A) 0·35 (B) 0·316

(C) 0·001 (D) 0·65

11.

A coin is tossed and a card is selected at random from a well shuffled pack of 52 playing cards. The probability of getting head on the coin and a face card from the pack is :

(A) $\frac{2}{13}$ (B) $\frac{3}{26}$

(C) $\frac{19}{26}$ (D) $\frac{3}{13}$

12.

If A and B are two events such that $P(B) = \frac{1}{5}$, $P(A \mid B) = \frac{2}{3}$ and $P(A \cup B) = \frac{3}{5}$, then P(A) is :

(A) $\frac{10}{15}$ (B) $\frac{2}{15}$

(C) $\frac{1}{5}$ (D) $\frac{8}{15}$

I.Conditional probability :

a. formula based :

$p(A\cup B)$ = P(A)+P(B)$-P(A\cap B)$

A but not B : $P(A\cap B') = P(A) - P(A\cap B)$

B but not A : $P(A'\cap B) = P(B) - P(A\cap B)$

Neither A nor B : $P(A'\cap B') = 1 - P(A\cup B)$

1. 2022

Let A and B be two events such that $P(A) = \frac{5}{8}$, $P(B) = \frac{1}{2}$ and $P(A/B) = \frac{3}{4}$.

Find the value of P(B/A).

Sol.

$$P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{3}{4} \Rightarrow P(A\cap B) = \frac{3}{8}$$

$$P(B/A) = \frac{P(B\cap A)}{P(A)} = \frac{\frac{3}{8}}{\frac{5}{8}} = \frac{3}{5}$$

2.

If P(A) = 0·6, P(B) = 0·5 and P(B | A) = 0·4, find P(A ∪ B) and P(A | B).

Sol.

$$P(B\mid A) = 0.4 \Rightarrow \frac{P(B\cap A)}{P(A)} = 0.4 \Rightarrow P(B\cap A) = 0.24$$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B) = 0.6 + 0.5 - 0.24 = 0.86$$

$$P(A\mid B) = \frac{P(A\cap B)}{P(B)} = \frac{0.24}{0.5} = 0.48$$

3.

Find [P(B/A) + P(A/B)], if P(A) = $\frac{3}{10}$, P(B) = $\frac{2}{5}$ and P(A∪B) = $\frac{3}{5}$.

Sol.

$$P(A\cap B)=\frac{3}{10}+\frac{2}{5}-\frac{3}{5}=\frac{1}{10}$$

Now, $P(B|A)+P(A|B)$

$$=\frac{P(A\cap B)}{P(A)}+\frac{P(A\cap B)}{P(B)}$$

$$=\frac{1}{3}+\frac{1}{4}=\frac{7}{12}$$

4.

If P(A) = 0·6, P(B) = 0·5 and P(B | A) = 0·4, find P(A ∪ B) and P(A | B).

4.b

Evaluate P(A ∪ B), if 2P(A) = P(B) = $\frac{5}{13}$ and P(A/B) = $\frac{2}{5}$.

Sol.

$$P(A/B)=\frac{P(A\cap B)}{P(B)} \text{ gives } P(A\cap B)=\frac{2}{13}$$

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$=\frac{5}{26}+\frac{5}{13}-\frac{2}{13}=\frac{11}{26}$$

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5.

If P(not A) = 0·7, P(B) = 0·7 and P(B/A) = 0·5, then find P(A/B).

Sol.

$$P(\bar{A}) = 0.7 \Rightarrow 1 - P(A) = 0.7 \Rightarrow P(A) = 0.3$$

$$P(A \cap B) = P(A) \cdot P(B|A) = 0.3 \times 0.5 = 0.15$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.7} = \frac{15}{70} \text{ or } \frac{3}{14}$$

6.

If P(A) = 0.6, P(B) = 0.5 and P(A|B) = 0.3, then find $P(A \cup B)$.

7.

Given that A and B are two independent events such that P(A) = 0·3 and P(B) = 0·5. Find P(A | B).

8.

Given that E and F are events such that P(E) = 0.8, P(F) = 0.7, P (E∩F) = 0.6
Find $P(\bar{E} \mid \bar{F})$

9. 2024

E and F are two independent events such that $P(\bar{E}) = 0·6$ and $P(E \cup F) = 0·6$. Find P(F) and $P(\bar{E} \cup \bar{F})$.

Sol.

$$P(\bar{E}) = 0.6 \Rightarrow P(E) = 0.4$$

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$\Rightarrow 0.6 = 0.4 + P(F) - 0.4\,P(F) \Rightarrow P(F) = \frac{1}{3}$$

$$P(\bar{E} \cup \bar{F}) = 1 - P(E \cap F)$$

$$= 1 - 0.4 \times \frac{1}{3} = \frac{13}{15}$$

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b.Conditional probability Word problems

1.a

A die is thrown three times. Events A and B are defined as below :

A : 5 on the first and 6 on the second throw.

B : 3 or 4 on the third throw.

Find the probability of B, given that A has already occurred.

Sol.

$$A = \{(5, 6, 1), (5, 6, 2), (5, 6, 3), (5, 6, 4), (5, 6, 5), (5, 6, 6),\}$$

$$P(A) = \frac{6}{6 \times 6 \times 6} = \frac{1}{36}, \ P(B) = P\text{(getting 3 or 4 on the third throw)}$$

$$A \cap B = \{(5, 6, 3), (5, 6, 4)\} \Rightarrow P(A \cap B) = \frac{2}{6 \times 6 \times 6} = \frac{1}{108}$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{1}{3}$$

1.b 2022

A pair of dice is thrown and the sum of the numbers appearing on the dice is observed to be 7. Find the probability that the number 5 has appeared on atleast one die.

Sol.

A: sum is 7

B: 5 has appeared at least on one die

$A \cap B = \{(2,5), (5,2)\}$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{2/36}{6/36} = \frac{1}{3}$$

1.c 2022

A pair of dice is thrown. It is given that the sum of numbers appearing on both dice is an even number. Find the probability that the number appearing on at least one die is 3.

Sol.

A: number appearing on at least one die is 3

B: sum of numbers appearing on both dice is even

Clearly, $A \cap B = \{(3,1),(3,5),(1,3),(5,3),(3,3)\}$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{5/36}{18/36}$$

$$= \frac{5}{18}$$

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1.d

A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Sol.

A: Getting a sum of 8, B: Red die resulted in no. < 4

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{2/36}{18/36} = \frac{1}{9}$$

1.e

A black die and a red die are rolled together. Find the conditional probability of obtaining a sum greater than 9 given that the black die resulted in a 5.

Ans: $P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{1}{3}$

2.a

Ten cards, numbered 1 to 10 are placed in a box, mixed up thoroughly and then one card is drawn randomly. If it is known that the number on the drawn card is 'more than 5', what is the probability that it is an even number ?

2.b

12 cards numbered 1 to 12 (one number on one card), are placed in a box and mixed up thoroughly. Then a card is drawn at random from the box. If it is known that the number on the drawn card is greater than 5, find the probability that the card bears an odd number.

Sol.

A: card bears odd number

B: Number on the card is greater than 5

$$A \cap B = \{7, 9, 11\}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{3/12}{7/12} = \frac{3}{7}$$

2.c

Eight cards numbered 1 to 8 (one number on one card) are placed in a box, mixed up thoroughly and then a card is drawn randomly. If it is known that the number on the drawn card is more than 2, then find the probability that it is an odd number.

Sol.

A: Number on the card is odd

B: Number on card > 2

$$B = \{3, 4, 5, 6, 7, 8\}$$

$$A \cap B: \{3, 5, 7\}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{3/8}{6/8} = \frac{1}{2}$$

3.a

Three friends A, B and C got their photograph clicked. Find the probability that B is standing at the central position, given that A is standing at the left corner.

Sol.

E_1 : B is standing at centre position

E_2 : A is standing at left corner

$$P(E_1 \mid E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)}$$

$$= \frac{\frac{1}{6}}{\frac{2}{6}} = \frac{1}{2}$$

3.b

Mother, father and son line up at random for a family photo. If A and B are two events given by A = Son on one end, B = Father in the middle, find P(B/A).

Sol.

A = {(S, F, M), (S, M, F), (M, F, S), (F, M, S)}

B = {(S, F, M), (M, F, S)}

Total number of possible arrangements = 6

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$= \frac{2/6}{4/6} = \frac{1}{2}$$

4.

A committee of 4 students is selected at random from a group consisting of 7 boys and 4 girls. Find the probability that there are exactly 2 boys in the committee, given that at least one girl must be there in the committee.

Sol.

Let A = exactly 2 boys in the committee

B = at least one girl must be there in the committee.

$$P(B) = \frac{{}^4C_1 \times {}^7C_3 + {}^4C_2 \times {}^7C_2 + {}^4C_3 \times {}^7C_1 + {}^4C_4}{{}^{11}C_4} = \frac{59}{66}$$

$$P(A \cap B) = \frac{{}^4C_2 \times {}^7C_2}{{}^{11}C_4} = \frac{21}{55}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{21/55}{59/66} = \frac{126}{295}$$

5. 2022

A bag contains 3 red and 4 white balls. Three balls are drawn at random, one-by-one without replacement from the bag. If the first ball drawn is red in colour, then find the probability that the remaining two balls drawn are also red in colour.

Sol.

A : First ball drawn is red

B : Remaining two balls are red

$$P(B / A) = \frac{P(B \cap A)}{P(A)}$$

$$= \frac{\frac{1}{35}}{\frac{3}{7}} = \frac{1}{15}$$

6. 2025

Consider the experiment of tossing a coin. If the coin shows head, toss it again; but if it shows a tail, then throw a die. Find the conditional probability of the event A : 'the die shows a number greater than 3' given that B : 'there is at least one tail'.

Sol.

Let A : The die shows a number greater than 3

and B : There is at least one tail

$$P(B\cap A)=P(T4,T5,T6)=\frac{3}{12}=\frac{1}{4}$$

$$P(B)=P(HT,T1,T2,T3,T4,T5,T6)=\frac{1}{4}+\frac{6}{12}=\frac{3}{4}$$

$$P(A\mid B)=\frac{P(B\cap A)}{P(B)}=\frac{1/4}{3/4}=\frac{1}{3}$$

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7. 2022

In a toss of three different coins, find the probability of coming up of three heads, if it is known that at least one head comes up.

Sol.

Let E_1 : getting three heads

E_2 : getting at least one head

$$P(E_2)=7/8\ ;\ P(E_1\cap E_2)=1/8$$

$$P(E_1\mid E_2)=\frac{P(E_1\cap E_2)}{P(E_2)}=\frac{\frac{1}{8}}{\frac{7}{8}}$$

$$=\frac{1}{7}$$

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7. 2025

The probability that a student buys a colouring book is 0·7 and that she buys a box of colours is 0·2. The probability that she buys a colouring book, given that she buys a box of colours, is 0·3. Find the probability that the student :

(i) Buys both the colouring book and the box of colours.

(ii) Buys a box of colours given that she buys the colouring book.

Sol.

Let A be the event of buying colouring book and

B be the event of buying coloured box.

$$P(A) = 0.7, \quad P(B) = 0.2, \quad P(A/B) = 0.3$$

(i) $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} \Rightarrow 0.3 = \frac{P(A \cap B)}{0.2}$

$\Rightarrow P(A \cap B) = 0.06 \text{ or } \frac{3}{50}$

(ii) $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

$= \frac{0.06}{0.7} = \frac{3}{35} \text{ or } 0.086$

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II.Independent events/ mutually exclusive events :

a.Formula based problems:

$P(A \cap B) = P(A)\,P(B)$

$P(A \cap B) = \emptyset$

$p(A \cup B) = P(A)+P(B) - P(A \cap B)$

Only A = A but not B : $P(A \cap B') = P(A) - P(A \cap B)$

Only B = B but not A : $P(A' \cap B) = P(B) - P(A \cap B)$

Neither A nor B : $P(A' \cap B') = 1 - P(A \cup B)$

1.a

If A and B are two independent events and $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{2}$, find $P(\bar{A} \mid \bar{B})$.

1.b

Given two independent events A and B such that P(A) = 0.3 and P(B) = 0.6, find $P(A' \cap B')$

Sol.

$$P(A' \cap B') = P(A')P(B')$$
$$= (0.7)(0.4) = 0.28$$

1.c

If A and B are independent events with $P(A) = \frac{3}{7}$ and $P(B) = \frac{2}{5}$,

then find $P(A' \cap B')$.

Sol.

$$P(A' \cap B') = P(A') \cdot P(B')$$
$$= \frac{4}{7} \times \frac{3}{5}$$
$$= \frac{12}{35}$$

1.d

Given that A and B are two independent events such that P(A) = 0·3 and P(B) = 0·5. Find P(A | B).

Sol.

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)\,\cancel{P(B)}}{\cancel{P(B)}}$$

$$= 0.3$$

1.c

If A and B are two events such that P(A) = 0·4, P(B) = 0·3 and P(A ∪ B) = 0·6, then find P(B′ ∩ A).

Sol.

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.1$$

$$P(B' \cap A) = P(A) - P(A \cap B) = 0.3$$

2.a

A and B are independent events such that $P(A \cap \bar{B}) = \frac{1}{4}$ and $P(\bar{A} \cap B) = \frac{1}{6}$. Find P(A) and P(B).

Sol.

$$P(A)\,P(\bar{B}) = \frac{1}{4} \qquad P(\bar{A})\,P(B) = \frac{1}{6}$$

Let P(A) = x P(B) = y

$$x(1-y) = \frac{1}{4}, \quad (1-x)y = \frac{1}{6} \Rightarrow x - y = \frac{1}{12}$$

eliminating y, we get $12x^2 - 13x + 3 = 0$

$$\text{gives } x = \frac{1}{3}, \frac{3}{4}$$

$$\left.\begin{aligned} P(A) = \frac{1}{3} \Rightarrow P(B) = \frac{1}{4} \\ P(A) = \frac{3}{4} \Rightarrow P(B) = \frac{2}{3} \end{aligned}\right\}$$

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2.b

If A and B are two independent events such that $P(\bar{A} \cap B) = \frac{2}{15}$ and $P(A \cap \bar{B}) = \frac{1}{6}$, then find P(A) and P(B).

Sol.

$$P(\bar{A} \cap B) = \frac{2}{15} \Rightarrow P(\bar{A}) \cdot P(B) = \frac{2}{15}$$

$$P(A \cap \bar{B}) = \frac{1}{6} \Rightarrow P(A) \cdot P(\bar{B}) = \frac{1}{6}$$

$$\therefore \ (1 - P(A))P(B) = \frac{2}{15} \text{ or } P(B) - P(A) \cdot P(B) = \frac{2}{15} \ \ldots\ldots\ldots\ldots\ldots\ldots (i)$$

$$P(A)(1 - P(B)) = \frac{1}{6} \text{ or } P(A) - P(A) \cdot P(B) = \frac{1}{6} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

From (i) and (ii) $P(A) - P(B) = \frac{1}{6} - \frac{2}{15} = \frac{1}{30}$

Let $P(A) = x$, $P(B) = y$ $\therefore$ $x = \left(\frac{1}{30} + y\right)$

$$(i) \Rightarrow \ y - \left(\frac{1}{30} + y\right) y = \frac{2}{15} \ \therefore \ 30y^2 - 29y + 4 = 0$$

Solving to get $y = 1/6$ or $y = 4/5$

$\therefore$ $x = 1/5$ or $x = 5/6$

Hence $P(A) = 1/5$, $P(B) = 1/6$ OR $P(A) = 5/6$, $P(B) = 4/5$

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3.a

Let A and B be the events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and P(not A or not B) = $\frac{1}{4}$. Find whether A and B are (i) mutually exclusive (ii) independent.

Sol.

$$P(A' \cup B') = \frac{1}{4}$$

$$\Rightarrow P(A \cap B)' = \frac{1}{4}$$

$$\Rightarrow 1 - P(A \cap B) = \frac{1}{4} \Rightarrow P(A \cap B) = \frac{3}{4} \neq 0,$$

$\therefore$ **A and B are not mutually exclusive.**

$$P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{7}{12} = \frac{7}{24} \neq P(A \cap B),$$

$\therefore$ **A and B are not independent.**

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3.b

Events A and B are such that

$P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(\overline{A} \cup \overline{B}) = \frac{1}{4}$

Find whether the events A and B are independent or not.

Sol.

$$P(A) = \frac{1}{2},\ P(B) = \frac{7}{12},\ P(\bar{A} \cup \bar{B}) = \frac{1}{4}$$

$$P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B)$$

$$P(A \cap B) = 1 - \frac{1}{4} = \frac{3}{4}$$

$$P(A) \times P(B) = \frac{1}{2} \times \frac{7}{12} = \frac{7}{24}$$

$$P(A) \times P(B) \neq P(A \cap B)$$

$\therefore$ A and B are not independent

prepared by : BALAJI KANCHI

4.

If P(A) = 0·4, P(B) = p, P(A ∪ B) = 0·6 and A and B are given to be independent events, find the value of ‘p’.

Sol.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A)\,P(B)$ as A and B are independent events

$\therefore\ 0.6 = 0.4 + p - (0.4)p$

$\Rightarrow p = \frac{1}{3}$

prepared by : **BALAJI KANCHI**

5.

If A and B are two independent events, then prove that the probability of occurrence of at least one of A and B is given by 1 – P(A') · P(B').

Sol.

Required probability $= P(A \cup B)$

$= P(A) + P(B) - P(A) \cdot P(B)$

$= P(A)\,[1 - P(B)] + 1 - P(B')$

$= P(A)\,P(B') - P(B') + 1$

$= (1 - P(B')\,(1 - P(A)) = 1 - P(A')\,P(B')$

prepared by : **BALAJI KANCHI**

6.a

Prove that if E and F are independent events, then the events E and F' are also independent.

Sol.

$P(E \cap F') = P(E) - P(E \cap F)$

$= P(E) - P(E) \cdot P(F)$

$= P(E)[1 - P(F)]$

$= P(E)P(F')$

$\Rightarrow$ E and F' are independent events.

b. Word problems :

1.

A and B are two independent events. The probability that both A and B occur is 1/6 and the probability that neither of them occur is 1/3. Find the probability of the occurrence of A.

2.

A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red". Find if A and B are independent events.

Sol.

$$A = \{2, 4, 6\}, \quad B = \{1, 2, 3\}, \quad A \cap B = \{2\}$$

$$\text{Now, } P(A) = \frac{1}{2}, \ P(B) = \frac{1}{2}, \ P(A \cap B) = \frac{1}{6}$$

$$\text{as } P(A) \times P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \neq P(A \cap B)$$

$\Rightarrow$ A and B are not independent.

3.

A fair coin and an unbiased die are tossed. Let A be the event, "Head appears on the coin" and B be the event, "3 comes on the die". Find whether A and B are independent events or not.

4. 2025

Two dice are thrown. Defined are the following two events A and B :

$A = \{(x, y) : x + y = 9\}$, $B = \{(x, y) : x \neq 3\}$, where (x, y) denote a point in the sample space.

Check if events A and B are independent or mutually exclusive.

Sol.

$A = \{(3,6), (4,5), (5,4), (6,3)$

$P(A) = \frac{4}{36} = \frac{1}{9}$, $P(B) = \frac{30}{36} = \frac{5}{6}$

$P(A \cap B) = \frac{3}{36} = \frac{1}{12}$

$P(A) \times P(B) = \frac{5}{54} \neq P(A \cap B)$

Therefore, A and B are not independent.

A and B are not mutually exclusive as $A \cap B \neq \emptyset$

5.

A card is randomly drawn from a well-shuffled pack of 52 playing cards. Events A and B are defined as under :

A : Getting a card of diamond

B : Getting a queen

Determine whether the events A and B are independent or not.

Sol.

$P(A) = P(\text{diamond}) = \frac{13}{52} = \frac{1}{4}$

$P(B) = P(\text{queen}) = \frac{4}{52} = \frac{1}{13}$

$P(A \cap B) = P(\text{queen of diamond}) = \frac{1}{52}$

Since $P(A).P(B) = \frac{1}{52} = P(A \cap B)$

$\Rightarrow$ A and B are independent events.

c. A or B or both A and B / atleast one :

$p(A \cup B) = P(A) + P(B) - P(A \cap B)$

A but not B : $P(A \cap B') = P(A) - P(A \cap B)$

A but not B : $P(A' \cap B) = P(B) - P(A \cap B)$

Neither A nor B : $P(A' \cap B') = 1 - P(A \cup B)$

1.

From a set of 100 cards numbered 1 to 100, one card is drawn at random. Find the probability that the number on the card is divisible by 6 or 8, but not by 24.

Sol.

No's divisible by 6 .... = 16

No's divisible by 8 .... = 12

No's not divisible by 24 .... = 20

Required probabilty $= \frac{20}{100} = \frac{1}{5}$

2.

Three distinct numbers are chosen randomly from the first 50 natural numbers. Find the probability that all the three numbers are divisible by both 2 and 3.

Sol.

Since there are only 8 numbers (in first 50 natural numbers) which are divisible by 6,

$\therefore$ favourable number of outcomes are ${}^{8}C_{3}$.

Total number of possible outcomes are ${}^{50}C_{3}$.

Required probability $= \frac{{}^{8}C_{3}}{{}^{50}C_{3}} = \frac{1}{350}$

3. 2025

In a city, a survey was conducted among residents about their preferred mode of commuting. It was found that 50% people preferred using public transport, 35% preferred using a bicycle and 20% use both public transport and a bicycle. If a person is selected at random, find the probability that :

(i) The person uses only public transport.

(ii) The person uses a bicycle, given that they also use the public transport.

(iii) The person uses neither public transport nor a bicycle.

Sol.

Let T :Person uses public transport

B :Person uses a bicycle

Given $P(T)=\frac{50}{100}, P(B)=\frac{35}{100}, P(T\cap B)=\frac{20}{100}$

$$(i)\ P(\text{only } T)=P(T)-P(T\cap B)$$

$$=\frac{50}{100}-\frac{20}{100}=\frac{30}{100}$$

$$(ii)\ P(B\,|\,T)=\frac{P(T\cap B)}{P(T)}$$

$$=\frac{\frac{20}{100}}{\frac{50}{100}}=\frac{2}{5}$$

$$(iii)\ P(T'\cap B')=1-P(T\cup B)$$

$$=1-\left[P(T)+P(B)-P(T\cap B)\right]$$

$$=1-\left[\frac{50}{100}+\frac{35}{100}-\frac{20}{100}\right]$$

$$=1-\frac{65}{100}=\frac{35}{100}$$

4.

In a village of 8000 people, 3000 go out of the village to work and 4000 are women. It is noted that 30% of women go out of the village to work. What is the probability that a randomly chosen individual is either a woman or a person working outside the village ?

Sol.

A = A randomly chosen person is a woman

B = A randomly chosen person works outside village.

$$P(A)=\frac{4000}{8000}=\frac{1}{2},\ P(B)=\frac{3000}{8000}=\frac{3}{8},\ P(A\cap B)=\frac{1200}{8000}=\frac{3}{20}$$

Required probability $= P(A\cup B)=P(A)+P(B)-P(A\cap B)=\frac{1}{2}+\frac{3}{8}-\frac{3}{20}=\frac{29}{40}$

prepared by : BALAJI KANCHI

d. if Two independent events A,B probabilities $P(A)$, $P(B)$ are given :

Finding probability of atleast one , Exactly one , both, neither A nor B :

atleast one , problem is solved = $p(A \cup B) = 1 - P(A')\, P(B')$
exactly one = $P(A)\, P(B') + P(A')P(B)$
both A and B= P(A) . P(B)
neither A nor B = P(A') . P(B')

1.

The probability of two students A and B coming to school on time are $\frac{2}{7}$ and $\frac{4}{7}$, respectively. Assuming that the events 'A coming on time' and 'B coming on time' are independent, find the probability of only one of them coming to school on time.

Sol.

Let E_1 : A coming on time.

E_2 : B coming on time.

$P(\bar{E}_1) = \frac{5}{7}, P(\bar{E}_2) = \frac{3}{7}$

P(only one on time)

$= P(E_1)\, P(\bar{E}_2) + P(\bar{E}_1)\, P(E_2)$

$= \frac{2}{7} \times \frac{3}{7} + \frac{5}{7} \times \frac{4}{7}$

$= \frac{26}{49}$

prepared by : BALAJI KANCHI

1.a

.Probabilities of solving problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that (i) the problem is solved (ii) exactly one of them solves the problem.

E_1: Problem is solved by A.

E_2: Problem is solved by B.

$$P(E_1)=\frac{1}{2}, P(E_2)=\frac{1}{3}, P(\bar{E}_1)=\frac{1}{2}, P(\bar{E}_2)=\frac{2}{3}$$

$$P(E_1 \cap E_2)=P(E_1)\cdot P(E_2)=\frac{1}{6}$$

$$P(\text{problem is solved}) = 1-P(\bar{E}_1)\cdot P(\bar{E}_2) = 1-\frac{1}{2}\cdot\frac{2}{3}=\frac{2}{3}$$

$$P(\text{one of them is solved}) = P(E_1)P(\bar{E}_2)+P(\bar{E}_1)P(E_2)$$

$$=\frac{1}{2}\cdot\frac{2}{3}+\frac{1}{2}\cdot\frac{1}{3}=\frac{1}{2}$$

1.b

Probabilities of A and B solving a specific problem are $\frac{2}{3}$ and $\frac{3}{5}$, respectively. If both of them try independently to solve the problem, then find the probability that the problem is solved.

sol.

P (Problem is solved)=1 – P (Problem not solved)

$$=1-P(\bar{A})P(\bar{B})$$

$$=1-\frac{1}{3}\cdot\frac{2}{5}$$

$$=\frac{13}{15}$$

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1.c

The probabilities of solving a specific problem independently by A and B are $\frac{1}{3}$ and $\frac{1}{5}$ respectively. If both try to solve the problem independently, find the probability that the problem is solved.

Sol.

P(Problem is solved) = 1 – P(Problem is not solved)

$$= 1 - \frac{2}{3} \times \frac{4}{5}$$

$$= \frac{7}{15}$$

prepared by : BALAJI KANCHI

1.d 2022

The probability that A hits the target is $\frac{1}{3}$ and the probability that B hits it, is $\frac{2}{5}$. If both try to hit the target independently, find the probability that the target is hit.

Sol.

$E_1 = A$ hits the target

$E_2 = B$ hits the target

P(target is hit) = 1 – P(target is not hit)

$$= 1 - \left(\frac{2}{3} \cdot \frac{3}{5}\right) = \frac{3}{5}$$

prepared by : BALAJI KANCHI

1.e

A person is Head of two independent selection committees I and II. If the probability of making a wrong selection in committee I is 0·03 and that in committee II is 0·01, then find the probability that the person makes the correct decision of selection :

(i) in both committees

(ii) in only one committee

Sol.

(i) P(correct decision in both committees) =(1- 0.03) . (1- 0.01) = 0.9603

(ii) P(correct decision in one committee) = 0.03 . (1- 0.01) + (1- 0.03). 0.01
= 0.0394

1.f

A speaks truth in 80% cases and B speaks truth in 90%cases. In what percentage of cases are they likely to agree with each other in stating the same fact?

2.

An unbiased coin is tossed 4 times. Find the probability of getting at least one head.

e.Three events probabilities given , atleast one, atmost one :

atleast one , problem is solved = $p(A \cup B \cup C) = 1 - P(A')\, P(B')P(C')$
exactly one = $P(A)\, P(B')P(C') + P(A')P(B)P(C') + P(A')P(B')P(C)$
exactly two = $P(A)\, P(B)P(C') + P(A')P(B)P(C) + P(A)P(B')P(C)$
atmost one = $P(A)\, P(B')P(C') + P(A')P(B)P(C') + P(A')P(B')P(C) + P(A')P(B')P(C')$
Three of them = P(A) . P(B). P(C)
neither A nor B nor C = P(A') . P(B') . P(C')

1.a

A problem is given to three students whose probabilities of solving it are $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{6}$ respectively. If the events of solving the problem are independent, find the probability that at least one of them solves it.

1.b

The probabilities of A, B and C solving a problem independently are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. If all the three try to solve the problem independently, find the probability that the problem is solved.

Sol.

Required probability = 1 – P(problem is not solved)

$$= 1 - P(A' \cap B' \cap C')$$

$$= 1 - P(A') \cdot P(B') \cdot P(C')$$

$$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{3}{4}$$

1.c

A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. If the events of their solving the problem are independent then the probability that the problem will be solved, is

1.d

A coach is training 3 players. He observes that player A can hit a target 4 times in 5 shots, player B can hit 3 times in 4 shots and player C can hit 2 times in 3 shots.

Based on the above, answer the following questions :

(i) Find the probability that all three players miss the target.

(ii) Find the probability that all of them hit the target.

(iii) (a) Find the probability that only one of them hits the target.

OR

(iii) (b) Find the probability that exactly two of them hit the target.

Sol.

(i) P(all will miss the target) $= \left(\frac{1}{5}\right)\left(\frac{1}{4}\right)\left(\frac{1}{3}\right) = \frac{1}{60}$

(ii)P(all hit the target) $= \left(\frac{4}{5}\right)\left(\frac{3}{4}\right)\left(\frac{2}{3}\right) = \frac{2}{5}$

(iii)(a) P(only one hit the target)

$$= \left(\frac{4}{5}\right)\left(\frac{1}{4}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{5}\right)\left(\frac{3}{4}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{5}\right)\left(\frac{1}{4}\right)\left(\frac{2}{3}\right)$$

$$= \frac{4}{60} + \frac{3}{60} + \frac{2}{60} = \frac{9}{60} = \frac{3}{20}$$

OR

(iii)(b) P(Exactly two hits)

$$= \left(\frac{4}{5}\right)\left(\frac{3}{4}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{5}\right)\left(\frac{3}{4}\right)\left(\frac{2}{3}\right) + \left(\frac{4}{5}\right)\left(\frac{1}{4}\right)\left(\frac{2}{3}\right)$$

$$= \frac{12}{60} + \frac{6}{60} + \frac{8}{60} = \frac{26}{60} = \frac{13}{30}$$

2.

A problem in mathematics is given to 4 students A, B, C, D. Their chances of solving the problem, respectively, are 1/3, 1/4, 1/5 and 2/3. What is the probability that (i) the problem will be solved? (ii) at most one of them will solve the problem?

3.

Four students of class XII are given a problem to solve independently. Their chances of solving the problem respectively are $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}$ and $\frac{1}{5}$. Find the probability that at most one of them will solve the problem.

Sol.

$$P(\text{at most one of them will solve the problem})$$
$$= P(\text{none of them solves the problem}) + P(\text{only one of them solves the problem})$$
$$= \left(\frac{1}{2}\times\frac{2}{3}\times\frac{1}{3}\times\frac{4}{5}\right) + \left(\frac{1}{2}\times\frac{2}{3}\times\frac{1}{3}\times\frac{4}{5} + \frac{1}{2}\times\frac{1}{3}\times\frac{1}{3}\times\frac{4}{5} + \frac{1}{2}\times\frac{2}{3}\times\frac{2}{3}\times\frac{4}{5} + \frac{1}{2}\times\frac{2}{3}\times\frac{1}{3}\times\frac{1}{5}\right)$$
$$= \frac{19}{45}$$

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III.Multiplication Theorem:

selecting multiple objects from single section :

(Two balls drawn, three cards, two bulbs likewise)

1.a

Four cards are drawn successively with replacement from a well shuffled deck of 52 cards. What is the probability that

(i) all the four cards are spades ?

(ii) only 2 cards are spades ?

Sol.

(i) P (all four spades) = $^{4}C_{4}\left(\frac{13}{52}\right)^{4}\left(\frac{39}{52}\right)^{0}=\frac{1}{256}$

(ii) P (only 2 are spades) = $^{4}C_{2}\left(\frac{1}{4}\right)^{2}\left(\frac{3}{4}\right)^{2}=\frac{27}{128}$

1.b

Two cards are drawn at random and one-by-one without replacement from a well-shuffled pack of 52 playing cards. Find the probability that one card is red and the other is black.

Sol.

Ans: $\frac{^{26}C_{1}\times {^{26}C_{1}}}{^{52}C_{2}}=\frac{26}{51}$

1.c

Two cards are drawn at random without replacement from a well-shuffled deck of 52 playing cards. Find the probability of getting both cards of the same colour.

1.c

From a pack of 52 cards, 3 cards are drawn at random (without replacement). The probability that they are two red cards and one black card, is __________ .

1.d

Two cards are drawn at random from a pack of 52 cards one-by-one without replacement. What is the probability of getting first card red and second card Jack?

2.

A bag contains cards numbered 1 to 25. Two cards are drawn at random, one after the other, without replacement. Find the probability that the number on each card is a multiple of 7.

Sol.

Multiples of 7 from 1 to 25 are 7, 14, 21

P (number on each card is a multiple of 7)

$$= \frac{3}{25} \times \frac{2}{24} = \frac{1}{100}$$

3.

A bag contains 3 black, 4 red and 2 green balls. If three balls are drawn simultaneously at random, then the probability that the balls are of different colours is __________ .

3.b

A bag contains 3 white and 2 red balls, another bag contains 4 white and 3 red balls. One ball is drawn at random from each bag. Find the probability that the balls drawn are one white and one red.

Sol.

$$\text{Required probability} = \frac{3}{5} \times \frac{3}{7} + \frac{2}{5} \times \frac{4}{7} = \frac{17}{35}$$

4.

The probability of finding a green signal on a busy crossing X is 30%. What is the probability of finding a green signal on X on two consecutive days out of three ?

Sol.

Probability of green signal on crossing $X = \frac{30}{100} = \frac{3}{10}$

Probability of not a green signal on crossing $X = 1 - \frac{3}{10} = \frac{7}{10}$

Probability of a green signal on X on two concecutative days out of three

$$= \frac{3}{10} \times \frac{3}{10} \times \frac{7}{10} + \frac{7}{10} \times \frac{3}{10} \times \frac{3}{10} = \frac{63}{500}$$

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IV.Total probability theorem :

selecting object(s) from multiple sections :

1.a

There are two bags. Bag I contains 1 red and 3 white balls, and Bag II contains 3 red and 5 white balls. A bag is selected at random and a ball is drawn from it. Find the probability that the ball so drawn is red in colour.

Sol.

E_1 : Bag 1 is selected

E_2 : Bag 2 is selected

A : Ball drawn is red in colour

$$P(A) = P(E_1)P(A|E_1) + P(E_2)P(A|E_2)$$

$$= \frac{1}{2}\times\frac{1}{4}+\frac{1}{2}\times\frac{3}{8}$$

$$= \frac{5}{16}$$

1.b

A purse contains 3 silver and 6 copper coins and a second purse contains 4 silver and 3 copper coins. If a coin is drawn at random from one of the two purses, find the probability that it is a silver coin.

2.

A bag A contains 4 black and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown. If 1 or 2 appears on it, then bag A is chosen, otherwise bag B. If two balls are drawn at random (without replacement) from the selected bag, find the probability of one of them being red and another black.

3.a

A bag contains 3 white and 2 red balls, another bag contains 4 white and 3 red balls. One ball is drawn at random from each bag. Find the probability that the balls drawn are one white and one red.

3.b 2022

A box B_1 contains 1 white ball and 3 red balls. Another box B_2 contains 2 white balls and 3 red balls. If one ball is drawn at random from each of the boxes B_1 and B_2, then find the probability that the two balls drawn are of the same colour.

Sol.

P (both balls drawn are of same colour)

$= P$ (both white) $+ P$ (both red)

$$= \frac{1}{4} \times \frac{2}{5} + \frac{3}{4} \times \frac{3}{5} = \frac{11}{20}$$

prepared by : **BALAJI KANCHI**

4.

An urn contains 3 red and 5 black balls. A ball is drawn at random, its colour is noted and returned to the urn. Moreover, 2 additional balls of the colour noted down, are put in the urn and then two balls are drawn at random (without replacement) from the urn. Find the probability that both the balls drawn are of red colour.

5.a

One bag contains 3 red and 5 black balls. Another bag contains 6 red and 4 black balls. A ball is transferred from first bag to the second bag and then a ball is drawn from the second bag. Find the probability that the ball drawn is red.

5.b 2022

One bag contains 4 white and 5 black balls. Another bag contains 6 white and 7 black balls. A ball, drawn at random, is transferred from the first bag to the second bag and then a ball is drawn at random from the second bag. Find the probability that the ball drawn is white.

Sol.

Case I: White ball is transferred from bag I to bag II

$$P(\text{white ball from bag II}) = \frac{4}{9} \times \frac{7}{14}$$

Case II: Black ball is transferred from bag I to bag II

$$P(\text{white ball from bag II}) = \frac{5}{9} \times \frac{6}{14}$$

$$\text{Total Probability} = \frac{4}{9} \times \frac{7}{14} + \frac{5}{9} \times \frac{6}{14}$$

$$= \frac{29}{63}$$

6.a2024

Bag I contains 3 red and 4 black balls, Bag II contains 5 red and 2 black balls. Two balls are transferred at random from Bag I to Bag II and then a ball is drawn at random from Bag II. Find the probability that the drawn ball is red in colour.

Sol.

E_1: Two Balls transferred from Bag I are Red.
E_2: Two Balls transferred from Bag I are Black.
E_3: Two Balls transferred from Bag I are Red and Black.
A: Ball drawn from Bag II is Red

$$P(E_1)=\frac{1}{7},\ P(E_2)=\frac{2}{7},\ P(E_3)=\frac{4}{7},$$

$$P(A|E_1)=\frac{7}{9},\quad P(A|E_2)=\frac{5}{9},\ P(A|E_3)=\frac{6}{9}$$

$$P(A)=\frac{1}{7}\cdot\frac{7}{9}+\frac{2}{7}\cdot\frac{5}{9}+\frac{4}{7}\cdot\frac{6}{9}=\frac{41}{63}$$

prepared by : BALAJI KANCHI

6.b 2025

Bag I contains 4 white and 5 black balls. Bag II contains 6 white and 7 black balls. A ball drawn randomly by from bag I is transferred to bag II and then a ball is drawn randomly from bag II. Find the probability that the ball drawn is white.

Sol.

P(white ball transferred) $= \frac{4}{9}$, Probability(black ball transferred) $= \frac{5}{9}$

P(white ball drawn from bag II) $= \frac{4}{9}\cdot\frac{7}{14}+\frac{5}{9}\cdot\frac{6}{14}$
$= \frac{29}{63}$

V.Bayes theorem :

(Probability of a section when the object is selected) :

a. Direct Model :

(both sections and objects probabilities are given directly):

1.a

Three machines E_1, E_2 and E_3 in a certain factory producing electric bulbs, produce 50%, 25% and 25% respectively, of the total daily output of electric bulbs. It is known that 4% of the bulbs produced by each of machines E_1 and E_2 are defective and that 5% of those produced by machine E_3 are defective. If one bulb is picked up at random from a day's production, calculate the probability that it is defective.

Sol.

Let B_1, B_2, B_3 be the events that the bolts produced by machines E_1, E_2, E_3 and A be the event that the selected bulb is defective

$$\therefore\ P(B_1) = \frac{1}{2},\ P(B_2) = P(B_3) = \frac{1}{4}$$

$$P\left(A/B_1\right) = \frac{1}{25},\ P\left(A/B_2\right) = \frac{1}{25},\ P\left(A/B_3\right) = \frac{1}{20}$$

$$P(A) = \sum_{c=1}^{3} P(B_c) P\left(A/B_c\right) = \frac{1}{2}\times\frac{1}{25}+\frac{1}{4}\times\frac{1}{20}+\frac{1}{4}\times\frac{1}{25} = \frac{17}{400}$$

prepared by : BALAJI KANCHI

1.b

A manufacturer has three machine operators A, B and C. The first operator A produces 1% of defective items, whereas the other two operators B and C produces 5% and 7% defective items respectively. A is on the job for 50% of the time, B on the job 30% of the time and C on the job for 20% of the time. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by A ?

Sol.

$$\left.\begin{aligned} \text{Let } E_1 &: \text{item is produced by A} \\ E_2 &: \text{item is produced by B} \\ E_3 &: \text{item is produced by C} \\ A &: \text{defective item is found.} \end{aligned}\right\}$$

$$P(E_1)=\frac{50}{100},\ P(E_2)=\frac{30}{100},\ P(E_3)=\frac{20}{100}$$

$$P(A/E_1)=\frac{1}{100},\ P(A/E_2)=\frac{5}{100},\ P(A\mid E_3)=\frac{7}{100}$$

$$P(E_1\mid A)=\frac{\frac{50}{100}\times\frac{1}{100}}{\frac{50}{100}\times\frac{1}{100}+\frac{30}{100}\times\frac{5}{100}+\frac{20}{100}\times\frac{7}{100}}$$

$$=\frac{5}{34}$$

1.c 2022

In a factory which manufactures bolts, machines A, B and C manufacture respectively 30%, 50% and 20% of the bolts. Of their outputs 3, 4 and 1 percent respectively are defective bolts. A bolts is drawn at random from the product and is found to be defective. Find the probability that this is not manufactured by machine B.

Sol.

E_1 : Item was produced by A

E_2 : Item was produced by B

E_3 : Item was produce by C

F : Item was defective

$$P(E_1)=\frac{30}{100},\ P(E_2)=\frac{25}{100},\ P(E_3)=\frac{45}{100}$$

$$P(F\mid E_1)=\frac{1}{100},\ P(F\mid E_2)=\frac{1\cdot 2}{100},\ P(F\mid E_3)=\frac{2}{100}$$

$$P(E_2\mid F)=\frac{P(E_2)P(F\mid E_2)}{P(E_1)P(F\mid E_1)+P(E_2)P(F\mid E_2)+P(E_3)P(F\mid E_3)}$$

$$=\frac{\frac{25}{100}\times\frac{1\cdot 2}{100}}{\frac{30}{100}\times\frac{1}{100}+\frac{25}{100}\times\frac{1.2}{100}+\frac{45}{100}\times\frac{2}{100}}$$

$$=\frac{3}{3+3+9}=\frac{1}{5}$$

prepared by : **BALAJI KANCHI**

1.d

Two groups are competing for the position on the Board of Directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product was introduced by the second group.

Sol.

Let E_1 = First group wins, E_2 = Second group wins

H = Introduction of new product.

$P(E_1) = 0.6,\ P(E_2) = 0.4,$

$P(H/E_2) = 0.3,\ P(H/E_1) = 0.7$

$$\text{Now, } P(E_2/H) = \frac{P(E_2)\,P(H/E_2)}{P(E_2)\,P(H/E_2) + P(E_1)\,P(H/E_1)}$$

$$= \frac{0.4 \times 0.3}{0.4 \times 0.3 + 0.6 \times 0.7} = \frac{2}{9}$$

prepared by : BALAJI KANCHI

1.e

40% students of a college reside in hostel and the remaining reside outside. At the end of the year, 50% of the hostelers got A grade while from outside students, only 30% got A grade in the examination. At the end of the year, a student of the college was chosen at random and was found to have gotten A grade. What is the probability that the selected student was a hosteler ?

Sol.

Let E_1, E_2 and E be the events such that

E_1 : students residing in hostel

E_2 : students residing outside hostel

E_3 : students getting 'A' grade

$$\therefore \quad P(E_1) = \frac{40}{100}, \quad P(E/E_1) = \frac{50}{100}$$

$$P(E_2) = \frac{60}{100}, \quad P(E/E_2) = \frac{30}{100}$$

$$P(E_1/E) = \frac{P(E_1) \cdot P(E/E_1)}{P(E_1) \cdot P(E/E_1) + P(E_2)\ P(E/E_2)}$$

$$= \frac{\frac{40}{100} \times \frac{50}{100}}{\frac{40}{100} \times \frac{50}{100} + \frac{30}{100} \times \frac{60}{100}} = \frac{10}{19}$$

prepared by : BALAJI KANCHI

b. Indirect model:

(Section probabilities not given but object probabilities are given)

2.a

An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter, a car and a truck are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

2.b

An insurance company insured 3000 cyclists, 6000 scooter drivers and 9000 car drivers. The probability of an accident involving a cyclist, a scooter driver and a car driver are 0·3, 0·05 and 0·02 respectively. One of the insured persons meets with an accident. What is the probability that he is a cyclist ?

Sol.

E_1: Selected person is cyclist

E_2: Selected person is scooter driver

E_3: Selected person is car driver

A: insured person met with an accident

$$P(E_1)=\frac{3}{18},\ P(E_2)=\frac{6}{18},\ P(E_3)=\frac{9}{18}$$

$$P(A|E_1)=0.3,\ P(A|E_2)=0.05,\ P(A|E_3)=0.02$$

$$P(E_1/A)=\frac{P(E_1)\,P(A|E_1)}{P(E_1)\,P(A|E_1)+P(E_2)\,P(A|E_2)+P(E_3)\,P(A|E_3)}$$

$$=\frac{\frac{3}{18}\times\frac{30}{100}}{\frac{3}{18}\times\frac{30}{100}+\frac{6}{18}\times\frac{5}{100}+\frac{9}{18}\times\frac{2}{100}}$$

$$=\frac{90}{138}\text{ or }\frac{15}{23}$$

2.c 2025

In a group of 400 people, 160 are smokers and non-vegetarian, 100 are smokers and vegetarian and the remaining are non-smokers and vegetarian. The probabilities of getting a special chest disease are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the disease. What is the probability that the selected person is a smoker and non-vegetarian ? What value is reflected in this question ?

3.

There are three categories of students in a class of 60 students :
A : Very hard working students
B : Regular but not so hard working
C : Careless and irregular
10 students are in category A, 30 in category B and rest in category C. It is found that probability of students of category A, unable to get good marks in the final year examination is, 0.002, of category B it is 0.02 and of category C, this probability is 0.20. A student selected at random was found to be the one who could not get good marks in the examination. Find the probability that this student is of category C. What values need to be developed in students of category C ?

Sol.

E_1: Student selected from category A

E_2: Student selected from category B

E_3: Student selected from category C

S: Student could not get good marks

$P(E_1) = \frac{1}{6} \quad P(E_2) = \frac{3}{6} \quad P(E_3) = \frac{2}{6}$

$P(S/E_1) = 0.002 \; P(S/E_2) = 0.02, P(S/E_3) = 0.2$

$$P(E_3/S) = \frac{P(E_3)\,P(S/E_3)}{P(E_1)\,P(S/E_1) + P(E_2)\,P(S/E_2) + P(E_3)\,P(S/E_3)}$$

$$= \frac{\frac{2}{6} \times 0.2}{\frac{1}{6} \times .002 + \frac{3}{6} \times .02 + \frac{2}{6} \times 0.2} = \frac{200}{231}$$

Value: Hardwork and Regularity

prepared by : BALAJI KANCHI

4.a

Three persons A, B and C apply for a job of Manager in a Private Company. Chances of their selection (A, B and C) are in the ratio 1 : 2 : 4. The probabilities that A, B and C can introduce changes to improve profits of the company are 0.8, 0.5 and 0.3 respectively. If the change does not take place, find the probability that it is due to the appointment of C.

Sol.

Let events are:

E_1 : A is selected

E_2 : B is selected

E_3 : C is selected

A : Change is not introduced

$$P(E_1) = \frac{1}{7}, P(E_2) = \frac{2}{7}, P(E_3) = \frac{4}{7}$$

$$P(A/E_1) = 0.2,\ P(A/E_2) = 0.5,\ P(A/E_3) = 0.7$$

$$\therefore\quad P(E_3/A) = \frac{\frac{4}{7}\times\frac{7}{10}}{\frac{1}{7}\times\frac{2}{10}+\frac{2}{7}\times\frac{5}{10}+\frac{4}{7}\times\frac{7}{10}}$$

$$= \frac{28}{40} = \frac{7}{10}$$

4.b 2022

Three persons A, B and C apply for a job of manager in a private company. Chances of their selection are in the ratio 1 : 2 : 4. The probability that A, B and C can introduce changes to increase the profits of a company are 0.8, 0.5 and 0.3 respectively. If increase in the profit does not take place, find the probability that it is due to the appointment of A.

Sol.

E_1 : A is selected

E_2 : B is selected

E_3 : C is selected

F : increase in profit does not take place

$P(E_1)=1/7$, $P(E_2)=2/7$, $P(E_3)=4/7$

$P(F \mid E_1)=0\cdot2$, $P(F \mid E_2)=0\cdot5$, $P(F \mid E_3)=0\cdot7$

$$P(E_1 \mid F)=\frac{P(E_1)P(F \mid E_1)}{P(E_1)P(F \mid E_1)+P(E_2)P(F \mid E_2)+P(E_3)P(F \mid E_3)}$$

$$=\frac{\frac{1}{7}\times\frac{2}{10}}{\frac{1}{7}\times\frac{2}{10}+\frac{2}{7}\times\frac{5}{10}+\frac{4}{7}\times\frac{7}{10}}$$

$$=\frac{2}{40}=\frac{1}{20}$$

prepared by : BALAJI KANCHI

4.c 2024

The chances of P, Q and R getting selected as CEO of a company are in the ratio 4 : 1 : 2 respectively. The probabilities for the company to increase its profits from the previous year under the new CEO, P, Q or R are 0·3, 0·8 and 0·5 respectively. If the company increased the profits from the previous year, find the probability that it is due to the appointment of R as CEO.

Sol.

Let $E_1 : P$ is appointed as CEO,

$E_2 : Q$ is appointed as CEO,

$E_3 : R$ is appointed as CEO

A : company increase profits from previous year

here, $P(E_1)=\frac{4}{7}, P(E_3)=\frac{1}{7}, P(E_1)=\frac{2}{7}$

$P(A|E_1)=0.3, P(A|E_2)=0.8, P(A|E_3)=0.5$

$$P(E_3|A)=\frac{P(E_3)P(A|E_3)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+P(E_3)P(A|E_3)}$$

$$=\frac{\frac{2}{7}\times 0.5}{\frac{4}{7}\times 0.3+\frac{1}{7}\times 0.8+\frac{2}{7}\times 0.5}$$

$$=\frac{1}{3}$$

prepared by : BALAJI KANCHI

i. cards based:

(a card is lost from 52 cards then a card is drawn from the remaining)

5.a

A card form a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn at random and are found to be both clubs. Find the probability of the lost card being of clubs.

5.b

A card from a pack of 52 playing cards is lost. From the remaining cards of the pack, two cards are drawn at random (without replacement) and both are found to be spades. Find the probability of the lost card being a spade.

Sol.

Let E_1 : spade card is lost

E_2 : non spade card is lost.

A : Two cards drawn are spade

$$P(E_1) = \frac{1}{4}, P(E_2) = \frac{3}{4}$$

$$P(A \mid E_1) = \frac{{}^{12}C_2}{{}^{51}C_2} = \frac{22}{425}$$

$$P(A \mid E_2) = \frac{{}^{13}C_2}{{}^{51}C_2} = \frac{26}{425}$$

$$P(E_1 \mid A) = \frac{\frac{1}{4} \times \frac{22}{425}}{\frac{1}{4} \times \frac{22}{425} + \frac{3}{4} \times \frac{26}{425}} = \frac{11}{50}$$

5.b 2022

A card from a pack of 52 playing cards is lost. From the remaining cards, 2 cards are drawn at random without replacement, and are found to be both aces. Find the probability that lost card being an ace.

Sol.

E_1 : Lost card is an ace

E_2 : Lost card is not an ace

A: 2 ace cards are drawn

$$P(E_1)=\frac{1}{13} \qquad P(E_2)=\frac{12}{13}$$

$$P(A/E_1)=\frac{{}^3C_2}{{}^{51}C_2} \qquad P(A/E_2)=\frac{{}^4C_2}{{}^{51}C_2}$$

$$P(E_1/A)=\frac{P(E_1)\,P(A/E_1)}{P(E_1)\,P(A/E_1)+P(E_2)\,P(A/E_2)}$$

$$=\frac{\frac{1}{13}\frac{{}^3C_2}{{}^{51}C_2}}{\frac{1}{13}\cdot\frac{{}^3C_2}{{}^{51}C_2}+\frac{12}{13}\cdot\frac{{}^4C_2}{{}^{51}C_2}} \quad =\frac{3}{75} \text{ or } \frac{1}{25}$$

5.c 2024

A card from a well shuffled deck of 52 playing cards is lost. From the remaining cards of the pack, a card is drawn at random and is found to be a King. Find the probability of the lost card being a King.

Sol.

Let E_1 be the event of lost card is King,

E_2 be the event of lost card not a King and

A be the event of drawing a King from remaining 51 cards.

so, $P(E_1)=\frac{1}{13}, P(E_2)=\frac{12}{13}, P(A|E_1)=\frac{3}{51}, P(A|E_2)=\frac{4}{51}$

Now, Required probability is $P(E_1 | A)$,

$$P(E_1 | A)=\frac{P(A|E_1)\times P(E_1)}{P(A|E_1)\times P(E_1)+P(A|E_2)\times P(E_2)}=\frac{\frac{1}{13}\times\frac{3}{51}}{\frac{1}{13}\times\frac{3}{51}+\frac{12}{13}\times\frac{4}{51}}=\frac{1}{17}$$

prepared by : BALAJI KANCHI

ii. die based

(first throws a die and then tosses a coin(s)):

6.a

Suppose a boy throws a die. If he gets a 1 or 2, he tosses a coin three times and notes down the number of heads. If he gets 3, 4, 5 or 6 he tosses the coin once and notes down whether a head or a tail is obtained. If he obtains exactly one head, what is the probability that he obtained 3, 4, 5, or 6 with the die ?

6.b

Suppose a girl throws a die. If she gets a 1 or 2, she tosses a coin three times and notes the number of 'tails'. If she gets 3, 4, 5 or 6, she tosses a coin once and notes whether a 'head' or 'tail' is obtained. If she obtained exactly one 'tail', what is the probability that she threw 3, 4, 5 or 6 with the die ?

Sol.

E_1 : She gets 1 or 2 on die.

E_2 : She gets 3, 4, 5 or 6 on die.

A: She obtained exactly 1 tail

$$P(E_1) = \frac{1}{3},\ P(E_2) = \frac{2}{3}$$

$$P(A/E_1) = \frac{3}{8},\ P(A/E_2) = \frac{1}{2}$$

$$P(E_2/A) = \frac{P(E_2)\cdot P(A/E_2)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)}$$

$$= \frac{\frac{2}{3}\times\frac{1}{2}}{\frac{1}{3}\times\frac{3}{8}+\frac{2}{3}\times\frac{1}{2}} = \frac{8}{11}$$

prepared by : BALAJI KANCHI

iii. coins biased / unbiased :

(a coin is selected and tosses , notes the outcome)

7.a

A bag contains two coins, one biased and the other unbiased. When tossed, the biased coin has a 60% chance of showing heads. One of the coins is selected at random and on tossing it shows tails. What is the probability it was an unbiased coin ?

Sol.

Let E_1 be the event that unbiased coin is tossed.

E_2 be the event that biased coin is tossed.

A be the event that coin tossed shows tail

$$P(E_1)=\frac{1}{2}, P(E_2)=\frac{1}{2}, P(A|E_1)=\frac{1}{2}, P(A|E_2)=\frac{2}{5}$$

$$P(E_1|A)=\frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)}$$

$$=\frac{\frac{1}{2}\times\frac{1}{2}}{\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{2}{5}}=\frac{5}{9}$$

7.b

There are three coins. One is a coin having tails on both faces, another is a biased coin that comes up tails 70% of the time and the third is an unbiased coin. One of the coins is chosen at random and tossed, it shows tail. Find the probability that it was a coin with tail on both the faces.

Sol.

E_1: Selected coin has tail on both faces

E_2: Selected coin is biased

E_3: Selected coin is unbiased

A: Tail comes up

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

$$P(A|E_1) = 1,\ P(A|E_2) = \frac{7}{10},\ P(A|E_3) = \frac{1}{2}$$

$$P(E_1|A) = \frac{P(E_1)\,P(A|E_1)}{P(E_1)\,P(A|E_1) + P(E_2)\,P(A|E_2) + P(E_3)\,P(A|E_3)}$$

$$= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1 + \frac{1}{3}\times\frac{7}{10} + \frac{1}{3}\times\frac{1}{2}}$$

$$= \frac{10}{22} \text{ or } \frac{5}{11}$$

7.c

There are three coins. One is a two-headed coin, another is a biased coin that comes up heads 75% of the time and the third is an unbiased coin. One of the three coins is chosen at random and tossed. If it shows heads, what is the probability that it is the two-headed coin ?

Sol.

Let E_1 = Event that two-headed coin is chosen

E_2 = Event that biased coin is chosen

E_3 = Event that unbiased coin is chosen

A = Event that coin tossed shows head

Then, $P(E_1) = P(E_2) = P(E_3) = 1/3$

$$P(A|E_1) = 1,\ P(A\mid E_2) = \frac{75}{100} = \frac{3}{4},\ P(A\mid E_3) = \frac{1}{2}$$

$$P(E_1|A) = \frac{P(E_1)\cdot P(A\mid E_1)}{P(E_1)\cdot P(A\mid E_1) + P(E_2)\cdot P(A\mid E_2) + P(E_3)\cdot P(A\mid E_3)}$$

$$= \frac{\frac{1}{3}\cdot 1}{\frac{1}{3}\cdot 1 + \frac{1}{3}\cdot\frac{3}{4} + \frac{1}{3}\cdot\frac{1}{2}} = \frac{4}{9}$$

7.

In a set of 10 coins, 2 coins are with heads on both the sides. A coin is selected at random from this set and tossed five times. If all the five times, the result was heads, find the probability that the selected coin had heads on both the sides.

Sol.

Let E_1 : two headed coin is chosen

E_2 : unbiased coin is chosen

A : All 5 tosses are heads

$$P(E_1) = \frac{1}{5},\ P(E_2) = \frac{4}{5},\ P\left(A/E_1\right) = 1,\ P\left(A/E_2\right) = \frac{1}{32}$$

$$P\left(E_1/A\right) = \frac{P(E_1)P\left(A/E_1\right)}{P(E_1)P\left(A/E_1\right) + P(E_2)P\left(A/E_2\right)}$$

$$P\left(E_1/A\right) = \frac{\frac{1}{5}\times 1}{\frac{1}{5}\times 1+\frac{4}{5}\cdot\frac{1}{32}} = \frac{8}{9}$$

8.

There are two coins. One of them is a biased coin such that P (head) : P (tail) is 1 : 3 and the other coin is a fair coin. A coin is selected at random and tossed once. If the coin showed head, then find the probability that it is a biased coin.

Sol.

E_1 = Biased coin is selected $\Rightarrow P(E_1)=\frac{1}{2}$

E_2 = Fair coin is selected $\Rightarrow P(E_2)=\frac{1}{2}$

A = Head appeared on tossing a selected coin .

$$P\left(\frac{A}{E_1}\right)=\frac{1}{4},\ P\left(\frac{A}{E_2}\right)=\frac{1}{2}$$

By Bayes' Theorem $P\left(\frac{E_1}{A}\right)=\frac{P(E_1)\,P\left(\frac{A}{E_1}\right)}{P(E_1)\,P\left(\frac{A}{E_1}\right)+P(E_2)\,P\left(\frac{A}{E_2}\right)}$

$$=\frac{\frac{1}{2}\cdot\frac{1}{4}}{\frac{1}{2}\cdot\frac{1}{4}+\frac{1}{2}\cdot\frac{1}{2}}=\frac{1}{3}$$

prepared by : BALAJI KANCHI

iv. Bags and balls :

(selecting a bag and then a ball is drawn from it):

9.a 2023

Out of two bags, bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag B.

Sol.

Let E_1 :event of choosing bag A, E_2 :event of choosing bag B,
A :red ball is found

here, $P(E_1)=P(E_2)=\frac{1}{2}; P(A|E_1)=\frac{3}{5}, P(A|E_2)=\frac{5}{9}$

$$P(E_2|A)=\frac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$$

$$=\frac{\frac{5}{9}\times\frac{1}{2}}{\frac{3}{5}\times\frac{1}{2}+\frac{5}{9}\times\frac{1}{2}}=\frac{25}{52}$$

9.b

Two bags I and II are given. Bag I contains 3 red and 4 black balls while bag II contains 5 red and 6 black balls. A ball is drawn at random from one of the bags and is found to be black. Find the probability that it was drawn from bag II.

9.c

There are two boxes, namely box-I and box-II. Box-I contains 3 red and 6 black balls. Box-II contains 5 red and 5 black balls. One of the two boxes, is selected at random and a ball is drawn at random. The ball drawn is found to be red. Find the probability that this red ball comes out from box-II.

Sol.

E_1 : Box I is selected

E_2 : Box II is selected

A : A red ball is drawn from the selected bag

$$P(E_1) = P(E_2) = \frac{1}{2}$$

$$P(A|E_1) = \frac{3}{9} = \frac{1}{3}, P(A|E_2) = \frac{5}{10} = \frac{1}{2}$$

$$P(E_2|A) = \frac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{2}}{\frac{1}{2}\times\frac{1}{3}+\frac{1}{2}\times\frac{1}{2}}$$

$$= \frac{\frac{1}{2}}{\frac{1}{3}+\frac{1}{2}} = \frac{3}{5}$$

prepared by : BALAJI KANCHI

9.d

A purse contains 3 silver and 6 copper coins and a second purse contains 4 silver and 3 copper coins. If a coin is drawn at random from one of the two purses, find the probability that it is a silver coin.

Sol.

E_1 : coin is drawn from purse 1.

E_2 : coin is drawn from purse 2.

A : Silver coin is drawn

$$P(E_1)=\frac{1}{2}, P(E_2)=\frac{1}{2}, P\left(\frac{A}{E_1}\right)=\frac{3}{9}, P\left(\frac{A}{E_2}\right)=\frac{4}{7}$$

$$P(A)=\frac{1}{2}\times\frac{3}{9}+\frac{1}{2}\times\frac{4}{7}$$

$$=\frac{19}{42}$$

prepared by : BALAJI KANCHI

(selecting a bag and then two/three balls are drawn from it):

10.a

A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and two balls are drawn at random without replacement from the bag and are found to be both red. Find the probability that the balls are drawn from the first bag.

Sol.

Let: E_1 = Event selecting bag with 4 red & 4 black balls

E_2 = Event selecting bag with 2 red & 6 black balls

A = Event selecting 2 red balls without replacement

Then, $P(E_1) = P(E_2) = \frac{1}{2}$, $P(A/E_1)=\frac{{}^4c_2}{{}^8c_2}=\frac{3}{14}$, $P(A/E_2)=\frac{{}^2c_2}{{}^8c_2}=\frac{1}{28}$

$$P\left(\frac{E_1}{A}\right)=\frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1)+P(E_2)\cdot P(A/E_2)}=\frac{\frac{1}{2}\cdot\frac{3}{14}}{\frac{1}{2}\cdot\frac{3}{14}+\frac{1}{2}\cdot\frac{1}{28}}=\frac{6}{7}$$

10.b

A bag X contains 4 white balls and 2 black balls, while another bag Y contains 3 white balls and 3 black balls. Two balls are drawn (without replacement) at random from one of the bags and were found to be one white and one black. Find the probability that the balls were drawn from bag Y.

Sol.

Let A be the event of drawing one white and one black ball from any one of the bag without replacement. Then,

$$\Rightarrow \quad P(A/E_1) = \frac{4}{6}\times\frac{2}{5}+\frac{2}{6}\times\frac{4}{5}=\frac{16}{30}$$

$$P(A/E_2) = \frac{3}{6}\times\frac{3}{5}+\frac{3}{6}\times\frac{3}{5}=\frac{18}{30}$$

Using Bayes' Theorem, we have

$$P(E_2/A) = \frac{P(E_2)\,P(A/E_2)}{P(E_1)\,P(A/E_1)+P(E_2)\,P(A/E_2)}$$

$$= \frac{\frac{1}{2}\times\frac{18}{30}}{\frac{1}{2}\times\frac{16}{30}+\frac{1}{2}\times\frac{18}{30}} = \frac{9}{17}$$

prepared by : BALAJI KANCHI

10.c

A bag contains 5 red and 3 black balls and another bag contains 2 red and 6 black balls. Two balls are drawn at random (without replacement) from one of the bags and both are found to be red. Find the probability that balls are drawn from the first bag.

Sol.

E_1 : ball drawn from first bag

E_2 : ball drawn from second bag

A : both drawn balls are red

$$P(E_1) = P(E_2) = \frac{1}{2}$$

$$P(A \mid E_1) = \frac{5}{8} \times \frac{4}{7} = \frac{20}{56}$$

$$P(A \mid E_2) = \frac{2}{8} \times \frac{1}{7} = \frac{2}{56}$$

$$P(E_1 \mid A) = \frac{\frac{1}{2} \cdot \frac{20}{56}}{\frac{1}{2} \cdot \frac{20}{56} + \frac{1}{2} \cdot \frac{2}{56}} = \frac{\frac{20}{112}}{\frac{22}{112}} = \frac{10}{11}$$

prepared by : **BALAJI KANCHI**

10.d

A bag contains 5 red and 4 black balls, a second bag contains 3 red and 6 black balls. One of the two bags is selected at random and two balls are drawn at random (without replacement), both of which are found to be red. Find the probability that these two balls are drawn from the second bag.

Sol.

Let E_1 : Bag I is selected

E_2 : Bag II is selected

A : Two balls drawn at random both are red.

$$P(E_1)=P(E_2)=\frac{1}{2}, P\left(\frac{A}{E_1}\right)=\frac{^5C_2}{^9C_2}=\frac{5}{18}, P\left(\frac{A}{E_2}\right)=\frac{^3C_2}{^9C_2}=\frac{1}{12}$$

$$P\left(\frac{E_1}{A}\right)=\frac{\frac{1}{2}\cdot\frac{1}{12}}{\frac{1}{2}\cdot\frac{5}{18}+\frac{1}{2}\cdot\frac{1}{12}}=\frac{3}{13}$$

prepared by : BALAJI KANCHI

10.e

There are two bags A and B. Bag A contains 3 white and 4 red balls whereas bag B contains 4 white and 3 red balls. Three balls are drawn at random (without replacement) from one of the bags and are found to be two white and one red. Find the probability that these were drawn from bag B.

Sol.

Let E_1 : selecting bag A, E_2 : selecting bag B

A : getting 2 white and 1 red out of 3 drawn (without replacement)

$$\therefore\ P(E_1) = P(E_2) = \frac{1}{2}$$

$$P\left(\frac{A}{E_1}\right) = \frac{{}^3C_2 \cdot {}^4C_1}{{}^7C_3} = \frac{12}{35}$$

$$P\left(\frac{A}{E_2}\right) = \frac{{}^4C_2 \cdot {}^3C_1}{{}^7C_3} = \frac{18}{35}$$

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)\,P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2}\cdot\frac{18}{35}}{\frac{1}{2}\cdot\frac{12}{35} + \frac{1}{2}\cdot\frac{18}{35}} = \frac{3}{5}$$

10.f

A bag A contains 4 black and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown. If 1 or 2 appears on it, then bag A is chosen, otherwise bag B. If two balls are drawn at random (without replacement) from the selected bag, find the probability of one of them being red and another black.

Sol.

Let E_1 : selecting bag A, and E_2 : selecting bag B.

$\therefore\ P(E_1) = \frac{1}{3},\ P(E_2) = \frac{2}{3}$

Let A : Getting one Red and one balck ball

$$\therefore P(A|E_1) = \frac{{}^4C_1 \cdot {}^6C_1}{{}^{10}C_2} = \frac{8}{15},\ P(A|E_2) = \frac{{}^7C_1 \cdot {}^3C_1}{{}^{10}C_2} = \frac{7}{15}$$

$$P(A) = P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)$$

$$= \frac{1}{3}\cdot\frac{8}{15} + \frac{2}{3}\cdot\frac{7}{15} = \frac{22}{45}$$

prepared by : BALAJI KANCHI

v. Men &women :

(Select a man/woman and then person is having something):

11.a

Suppose 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that there are equal number of males and females.

11.b

Suppose that 5 men out of 100 and 25 women out of 1000 are good orators. Assuming that there are equal number of men and women, find the probability of choosing a good orator.

Sol.

Let M be an event of choosing a man and

N be an event of choosing a women.

A be an event of choosing a good orator.

$$P(M) = P(W) = \frac{1}{2};$$

$$P(A|M) = \frac{5}{100} = \frac{1}{20}, P(A|W) = \frac{25}{1000} = \frac{1}{40}$$

$$P(A) = P(A|M).P(M) + P(A|W).P(W)$$

$$= \frac{1}{20} \times \frac{1}{2} + \frac{1}{40} \times \frac{1}{2} = \frac{3}{80}$$

prepared by : BALAJI KANCHI

12. 2025

For the vacancy advertised in the newspaper, 3000 candidates submitted their applications. From the data it was revealed that two third of the total applicants were females and other were males. The selection for the job was done through a written test. The performance of the applicants indicates that the probability of a male getting a distinction in written test is 0.4 and that a female getting a distinction is 0.35. Find the probability that the candidate chosen at random will have a distinction in the written test.

Sol.

Let E_1 : The applicant is a male

E_2 : The applicant is a female

A : The candidate chosen will have distinction in the written test.

$$P(E_1)=\frac{1}{3}, P(E_2)=\frac{2}{3}, P(A|E_1)=0.4, P(A|E_2)=0.35$$

$$\therefore P(A)=P(E_1)P(A|E_1)+P(E_2)P(A|E_2)$$

$$=\frac{1}{3}\times 0.4+\frac{2}{3}\times 0.35$$

$$=\frac{11}{30}$$

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c. Logical problems :

13.

Of the students in a school, it is known that 30% have 100% attendance and 70% students are irregular. Previous year results report that 70% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one student is chosen at random from the school and he was found to have an A grade. What is the probability that the student has 100% attendance ? Is regularity required only in school ? Justify your answer.

Sol.

Let E_1: Selecting a student with 100% attendance

E_2: Selecting a student who is not regular

A: selected student attains A grade.

$$P(E_1) = \frac{30}{100} \text{ and } P(E_2) = \frac{70}{100}$$

$$P(A/E_1) = \frac{70}{100} \text{ and } P(A/E_2) = \frac{10}{100}$$

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1) + P(E_2)P(A/E_2)}$$

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\frac{30}{100}\times\frac{70}{100}}{\frac{30}{100}\times\frac{70}{100}+\frac{70}{100}\times\frac{10}{100}} = \frac{3}{4}$$

14.

It is known that 20% of the students in a school have above 90% attendance and 80% of the students are irregular. Past year results show that 80% of students who have above 90% attendance and 20% of irregular students get 'A' grade in their annual examination. At the end of a year, a student is chosen at random from the school and is found to have an 'A' grade. What is the probability that the student is irregular ?

Sol.

Let B : Student getting 'A' grade

E_1 : Student having above 90% attendance

E_2 : Student being irregular

$$P(E_1)=\frac{20}{100};P(E_2)=\frac{80}{100}\ P(B|E_1)=\frac{80}{100};P(B|E_2)=\frac{20}{100}$$

$$P(E_2|B)=\frac{P(E_2)P(B|E_2)}{P(E_1)P(B|E_1)+P(E_2)P(B|E_2)}$$

$$=\frac{0.8\times0.2}{0.2\times0.8+0.8\times0.2}=\frac{1}{2}$$

prepared by : BALAJI KANCHI

15.a

In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$, what is the probability that the student knows the answer given that he answered it correctly ?

Sol.

E_1: student knows the answer

E_2: student guesses the answer

A: answers correctly.

$$P(E_1) = \frac{3}{5}, \quad P(E_2) = \frac{2}{5}$$

$$P\left(\frac{A}{E_1}\right) = 1, \quad P\left(\frac{A}{E_2}\right) = \frac{1}{3}$$

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)}$$

$$= \frac{\frac{3}{5} \cdot 1}{\frac{3}{5} \cdot 1 + \frac{2}{5} \cdot \frac{1}{3}} = \frac{9}{11}$$

15.b

In answering a question on a multiple choice questions test with four choices in each question, out of which only one is correct, a student either guesses or copies or knows the answer. The probability that he makes a guess is $\frac{1}{4}$ and the probability the he copies is also $\frac{1}{4}$. The probability that the answer is correct, given that he copied it is $\frac{3}{4}$. Find the probability that he knows the answer to the question, given that he correctly answered it.

Sol.

Let E_1 = Student guesses the answer

E_2 = Student copies the answer

E_3 = Student knows the answer

A = Student answers the question correctly.

$$P(E_1)=\frac{1}{4},\ P(E_2)=\frac{1}{4},\ P(E_3)=1-\left(\frac{1}{4}+\frac{1}{4}\right)=\frac{1}{2}$$

$$P(A\,|\,E_1)=\frac{1}{4},\ P(A\,|\,E_2)=\frac{3}{4},\ P(A\,|\,E_3)=1$$

The required probability

$$=\ P(E_3\,|\,A)=\frac{P(E_3)\times P(A\,|\,E_3)}{\sum_{i=1}^{3} P(E_i)\times P(A\,|\,E_i)}$$

$$=\frac{\frac{1}{2}\times 1}{\frac{1}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{3}{4}+\frac{1}{2}\times 1}$$

$$=\frac{1}{\frac{1}{8}+\frac{3}{8}+1}=\frac{8}{12}=\frac{2}{3}$$

prepared by : BALAJI KANCHI

15.c

In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. What is the probability that the student knows the answer, given that he answered it correctly ?

Sol.

Let events A, B and E be defined as:

A : Student knows the answer

B : Student guesses the answer

E : student answered correctly

$P(A) = \frac{3}{5}$, $P(B) = \frac{2}{5}$

Here, $P\left(\frac{E}{A}\right) = 1$ and $P\left(\frac{E}{B}\right) = \frac{1}{3}$

By Bayes' Theorem

$$P\left(\frac{A}{E}\right) = \frac{P(A).P\left(\frac{E}{A}\right)}{P(A).P\left(\frac{E}{A}\right) + P(B).P\left(\frac{E}{B}\right)}$$

$$= \frac{\frac{3}{5}\times 1}{\left(\frac{3}{5}\times 1\right)+\left(\frac{2}{5}\times\frac{1}{3}\right)} = \frac{9}{11}$$

16.

In a shop X, 30 tins of ghee of type A and 40 tins of ghee of type B which look alike, are kept for sale. While in shop Y, similar 50 tins of ghee of type A and 60 tins of ghee of type B are there. One tin of ghee is purchased from one of the randomly selected shop and is found to be of type B. Find the probability that it is purchased from shop Y.

Sol.

E_1 : selecting shop X

E_2 : selecting shop Y

A : purchased tin is of type B

$$P(E_1)=P(E_2)=\frac{1}{2}$$

$$P(A|E_1)=\frac{4}{7}, P(A|E_2)=\frac{6}{11}$$

$$P(E_2|A)=\frac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$$

$$=\frac{\frac{1}{2}.\frac{6}{11}}{\frac{1}{2}.\frac{4}{7}+\frac{1}{2}.\frac{6}{11}}$$

$$=\frac{21}{43}$$

OR

E_1 = Ghee purchased from shop X

E_2 = Ghee purchased from shop Y

A = Getting adultered ghee

$$P(E_1) = P(E_2) = \frac{1}{2}, P(A/E_1) = \frac{4}{7}, P(A/E_2) = \frac{6}{11}$$

$$P(E_2/A) = \frac{\frac{1}{2}\times\frac{6}{11}}{\frac{1}{2}\times\frac{6}{11}+\frac{1}{2}\times\frac{4}{7}} = \frac{21}{43}$$

II part: Stringent punishment for the adultrators or any suitable measure

d. Critical level :

17.a

A bag contains 4 balls. Two balls are drawn at random (without replacement) and are found to be white. What is the probability that all balls in the bag are white ?

Sol.

E_1 = Event that all balls are white,

E_2 = Event that 3 balls are white and 1 ball is non white

E_3 = Event that 2 balls are white and 2 balls are non-white

A = Event that 2 balls drawn without replacement are white

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

$$P(A/E_1) = 1,\ P(A/E_2) = \frac{3}{4}\cdot\frac{2}{3} = \frac{1}{2},\ P(A/E_3) = \frac{2}{4}\cdot\frac{1}{3} = \frac{1}{6}$$

$$P(E_1/A) = \frac{1.\frac{1}{3}}{1\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{1}{2} + \frac{1}{3}\cdot\frac{1}{6}} = \frac{3}{5}$$

17.b

There are two boxes I and II. Box I contains 3 red and 6 black balls. Box II contains 5 red and 'n' black balls. One of the two boxes, box I and box II is selected at random and a ball is drawn at random. The ball drawn is found to be red. If the probability that this red ball comes out from box II is $\frac{3}{5}$, find the value of 'n'.

Sol.

Let the events be

E_1 : bag I is selected

E_2 : bag II is selected

A : getting a red ball

$$P(E_1) = P(E_2) = \frac{1}{2}$$

$$P(A/E_1) = \frac{3}{9} = \frac{1}{3}; \; P(A/E_2) = \frac{5}{5+n}$$

$$P(E_2/A) = \frac{3}{5} = \frac{\frac{1}{2}\cdot\frac{5}{5+n}}{\frac{1}{2}\cdot\frac{1}{3}+\frac{1}{2}\cdot\frac{5}{5+n}}$$

$$\Rightarrow \frac{3}{5} = \frac{15}{5+n+15} \Rightarrow n = 5.$$

prepared by : **BALAJI KANCHI**

18.a

Bag A contains 3 red and 5 black balls, while bag B contains 4 red and 4 black balls. Two balls are transferred at random from bag A to bag B and then a ball is drawn from bag B at random. If the ball drawn from bag B is found to be red, find the probability that two red balls were transferred from A to B.

Sol.

Let H_1 be the event 2 red balls are transferred

H_2 be the event 1 red and 1 balck ball, transferred

H_3 be the event 2 black and 1 balck ball transferred

E be the event that ball drawn from B is red.

$$P(H_1) = \frac{^3C_2}{^8C_2} = \frac{3}{28} \qquad P(E/H_1) = \frac{6}{10}$$

$$P(H_2) = \frac{^3C_1 \times {^5C_1}}{^8C_2} = \frac{15}{28} \qquad P(E/H_2) = \frac{5}{10}$$

$$P(H_3) = \frac{^5C_2}{^8C_2} = \frac{10}{28} \qquad P(E/H_3) = \frac{4}{10}$$

$$P(H_1/E) = \frac{\frac{3}{28} \times \frac{6}{10}}{\frac{3}{28} \times \frac{6}{10} + \frac{15}{28} \times \frac{5}{10} + \frac{10}{28} \times \frac{4}{10}} = \frac{18}{133}$$

18.b

Bag A contains 3 red and 2 black balls, while bag B contains 2 red and 3 black balls. A ball drawn at random from bag A is transferred to bag B and then one ball is drawn at random from bag B. If this ball was found to be a red ball, find the probability that the ball drawn from bag A was red.

Sol.

Let the events be

E_1: tansferring a red ball from A to B

E_2: transferring a black ball from A to B

A: Getting a red ball from bag B

$$P(E_1) = \frac{3}{5},\ P(E_2) = \frac{2}{5}$$

$$P(A/E_1) = \frac{1}{2},\ P(A/E_2) = \frac{1}{3}$$

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)}$$

$$= \frac{\frac{3}{5}\cdot\frac{1}{2}}{\frac{3}{5}\cdot\frac{1}{2} + \frac{2}{5}\cdot\frac{1}{3}} = \frac{9}{13}$$

prepared by : BALAJI KANCHI

18.c

There are two bags, I and II. Bag I contains 3 red and 5 black balls and Bag II contains 4 red and 3 black balls. One ball is transferred randomly from Bag I to Bag II and then a ball is drawn randomly from Bag II. If the ball so drawn is found to be black in colour, then find the probability that the transferred ball is also black.

Sol.

E_1 = Event that the ball transfered from Bag I is Black

E_2 = Event that the ball transfered from Bag I is Red

A = Event that the ball drawn from Bag II is Black

$$P(E_1)=\frac{5}{8};\ P(E_2)=\frac{3}{8};\ P\left(\frac{A}{E_1}\right)=\frac{4}{8}=\frac{1}{2};\ P\left(\frac{A}{E_2}\right)=\frac{3}{8}$$

Required Probability:

$$P\left(\frac{E_1}{A}\right)=\frac{P(E_1)\cdot P\left(\frac{A}{E_1}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right)+P(E_2)\cdot P\left(\frac{A}{E_2}\right)}=\frac{\frac{5}{8}\cdot\frac{1}{2}}{\frac{5}{8}\cdot\frac{1}{2}+\frac{3}{8}\cdot\frac{3}{8}}=\frac{20}{29}$$

prepared by : BALAJI KANCHI

18.d

Bag I contains 3 white and 4 black balls, while Bag II contains 5 white and 3 black balls. One ball is transferred at random from Bag I to Bag II and then a ball is drawn at random from Bag II. The ball so drawn is found to be white. Find the probability that the transferred ball is also white.

Sol.

Let E_1: Transferred ball is white

E_2: Transferred ball is black

A: white ball is found

Here, $P(E_1) = \frac{3}{7}, P(E_2) = \frac{4}{7}$

$P(A/E_1) = \frac{6}{9}, P(A/E_2) = \frac{5}{9}$

Using Baye's theorem

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)}$$

$$= \frac{\frac{3}{7} \times \frac{6}{9}}{\frac{3}{7} \times \frac{6}{9} + \frac{4}{7} \times \frac{5}{9}} = \frac{18}{18+20} = \frac{9}{19}$$

prepared by : **BALAJI KANCHI**

18.e

Bag I contains 4 red and 5 black balls and bag II contains 3 red and 4 black balls. One ball is transferred from bag I to bag II and then two balls are drawn at random (without replacement) from bag II. The balls so drawn are both found to be black. Find the probability that the transferred ball is black.

Sol.

Let E_1 : Event that transferred ball is black

E_2 : Event that transferred ball is Red

E_3 : Event that balls drawn are black

$$P(E_1) = \frac{5}{9}, \quad P(E_2) = \frac{4}{9}$$

$$P(A/E_1) = \frac{^5C_2}{^8C_2} = \frac{5}{14}, \quad P(A/E_2) = \frac{^4C_2}{^8C_2} = \frac{3}{14}$$

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1)\ P(A/E_1) + P(E_2)\ P(A/E_2)}$$

$$= \frac{\frac{5}{9} \times \frac{5}{14}}{\frac{5}{9} \times \frac{5}{14} + \frac{4}{9} \times \frac{3}{14}} = \frac{25}{37}$$

prepared by : BALAJI KANCHI

18.f

Bag I contains 4 red and 2 green balls and Bag II contains 3 red and 5 green balls. One ball is transferred at random from Bag I to Bag II and then a ball is drawn at random from Bag II. The ball so drawn is found to be green in colour. Find the probability that the transferred ball is also green.

Sol.

Let E_1: Transferred ball is green

E_2: Transferred ball is red

A: Green ball is found

Here, $P(E_1) = \frac{2}{6}, P(E_2) = \frac{4}{6}$

$$P(A/E_1) = \frac{6}{9}, P(A/E_2) = \frac{5}{9}$$

Using Baye's theorem.

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)}$$

$$= \frac{\frac{2}{6}\times\frac{6}{9}}{\frac{2}{6}\times\frac{6}{9}+\frac{4}{6}\times\frac{5}{9}} = \frac{12}{12+20} = \frac{3}{8}$$

prepared by : BALAJI KANCHI

19.

Assume that the chances of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduces the risk of heart attack by 30% and the prescription of a certain drug reduces its chance by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga.

Sol.

Let E_1 be the event of following course of meditation and yoga and E_2 be the event of following course of drugs

$$P(E_1) = \frac{1}{2},\ P(E_2) = \frac{1}{2}$$

$$P(A|E_1) = \frac{70 \times 40}{100 \times 100} \quad P(A|E_2) = \frac{75}{100} \times \frac{40}{100}$$

$$P(E_1|A) = \frac{\frac{40}{100}\left(\frac{1}{2} \times \frac{70}{100}\right)}{\frac{40}{100}\left(\frac{1}{2} \times \frac{70}{100} + \frac{1}{2} \times \frac{75}{100}\right)}$$

$$= \frac{70}{145} = \frac{14}{29}$$

prepared by : BALAJI KANCHI

A person Speaks Truth / lie :

20.a

A man is known to speak truth 3 out of 5 times. He throws a die and reports that it is 4. Find the probability that it is actually a 4.

20.b

A man is known to speak truth 7 out of 10 times. He threw a pair of dice and reports that doublet appeared. Find the probability that it was actually a doublet.

Sol.

E_1 : doublet appeared

E_2 : doublet did not appear

A : He reports doublet

$$P(E_1) = \frac{1}{6} \qquad P(E_2) = \frac{5}{6}$$

$$P(A/E_1) = \frac{7}{10} \qquad P(A/E_2) = \frac{3}{10}$$

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)}$$

$$= \frac{\frac{1}{6}\cdot\frac{7}{10}}{\frac{1}{6}\cdot\frac{7}{10} + \frac{5}{6}\cdot\frac{3}{10}}$$

$$= \frac{7}{22}$$

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20.c

Often it is taken that a truthful person commands, more respect in the society. A man is known to speak the truth 4 out of 5 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

Do you also agree that the value of truthfulness leads to more respect in the society ?

A man is known to speak the truth 4 out of 5 times.

He throws a die and reports that it is a six.

Find the probability that it is actually a six.

Sol.

Let H_1 be the event that 6 appears on throwing a die

H_2 be the event that 6 does not appear on throwing a die

E be the event that he reports it is six

$$P(H_1) = \frac{1}{6}, P(H_2) = 1 - \frac{1}{6} = \frac{5}{6}$$

$$P(E/H_1) = \frac{4}{5}, P(E/H_2) = \frac{1}{5}$$

$$P(H_1/E) = \frac{P(H_1) \cdot P(E/H_1)}{P(H_1) \cdot P(E/H_1) + P(H_2) P(E/H_2)}$$

$$= \frac{4}{9}$$

VI. Misc problems :

1.a

A and B throw a die alternatively till one of them gets a number greater than four and wins the game. If A starts the game, what is the probability of B winning ?

1.b

A and B throw a pair of dice alternately, till one of them gets a total of 10 and wins the game. Find their respective probabilities of winning, if A starts first.

Sol.

Let A_i and B_i be the events of throwing 10 by A and B in the respective ith turn. Then,

$$P(A_i) = P(B_i) = \frac{1}{12} \text{ and } P(\overline{A_i}) = P(\overline{B_i}) = \frac{11}{12}$$

Probability of winning A, when A starts first

$$= \frac{1}{12} + \left(\frac{11}{12}\right)^2 \frac{1}{12} + \left(\frac{11}{12}\right)^4 \frac{1}{12} + \ldots$$

$$= \frac{1/12}{1-(11/12)^2} = \frac{12}{23}$$

Probability of winning of B $= 1 - P(A) = 1 - \frac{12}{23} = \frac{11}{23}$

1.b

A and B throw a die alternately till one of them gets a '6' and wins the game. Find their respective probabilities of wining, if A starts the game first.

Sol.

$$P(\text{getting a six}) = \frac{1}{6}; P(\text{not getting a six}) = \frac{5}{6}$$

$$P(\text{A wins}) = \frac{1}{6} + \left(\frac{5}{6}\right)^2 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^4 \cdot \frac{1}{6} + \ldots = \frac{\frac{1}{6}}{1 - \frac{25}{36}} = \frac{6}{11}$$

$$P(\text{B wins}) = 1 - P(\text{A wins}) = 1 - \frac{6}{11} = \frac{5}{11}$$

prepared by : BALAJI KANCHI

1.c

A and B throw a pair of dice alternately till one of them gets the sum of the numbers as multiples of 6 and wins the game. If A starts first, find the probability of B winning the game.

Sol.

Let S denote Sum of numbers as multiples of six

$$P(S) = \frac{6}{36} \text{ or } \frac{1}{6}, \quad P(\bar{S}) = \frac{5}{6}$$

$$\begin{aligned} P(\text{B winning}) &= P(\bar{A})\,P(B) + P(\bar{A})P(\bar{B})\,P(\bar{A}).P(B) + \ldots \\ &= \frac{5}{6} \cdot \frac{1}{6} + \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} + \ldots \\ &= \frac{5}{11} \end{aligned}$$

1.d

A and B throw a pair of dice alternately till one of them gets a sum of 5, of the numbers on the two dice and wins the game. Find their respective probabilities of winning, if A starts the game.

Sol.

where W: Getting sum 5 $\quad L$: Not getting sum 5

$$P(W) = \frac{4}{36} = \frac{1}{9} \qquad P(L) = \frac{8}{9}$$

$$P(A \text{ winning}) = P(W) + P(LLW) + P(LLLLW) + \ldots.$$

$$= \frac{1}{9} + \frac{8}{9} \cdot \frac{8}{9} \cdot \frac{1}{9} + \frac{8}{9} \cdot \frac{8}{9} \cdot \frac{8}{9} \cdot \frac{8}{9} \cdot \frac{1}{9} + \ldots$$

$$= \frac{9}{17}$$

$$P(B \text{ winning}) = 1 - \frac{9}{17} = \frac{8}{17}$$

2.

A and B throw a pair of dice alternately. A wins the game if he gets a total of 7 and B wins the game if he gets a total of 10. If A starts the game, then find the probability that B wins.

Sol.

Prob. of success for A $= \frac{1}{6}$

Prob. of failure for A $= \frac{5}{6}$

Prob. of success for B $= \frac{1}{12}$

Prob. of failure for B $= \frac{11}{12}$

B can win in 2nd or 4th or 6th or....throw

$$\therefore \quad P(B) = \left(\frac{5}{6}.\frac{1}{12}\right) + \left(\frac{5}{6}.\frac{11}{12}.\frac{5}{6}.\frac{1}{12}\right) + \left(\frac{5}{6}.\frac{11}{12}.\frac{5}{6}.\frac{11}{12}.\frac{5}{6}.\frac{1}{12}\right) + ...$$

$$= \frac{5}{72}\left(1 + \frac{55}{72} + \left(\frac{55}{72}\right)^2 + ...\right)$$

$$= \frac{5}{72} \times \frac{1}{1 - \frac{55}{72}} = \frac{5}{72} \times \frac{72}{17} = \frac{5}{17}$$

3.

Out of 8 outstanding students of a school, in which there are 3 boys and 5 girls, a team of 4 students is to be selected for a quiz competition. Find the probability that 2 boys and 2 girls are selected.

4.

A coin is tossed once. If head comes up, a die is thrown, but if tail comes up, the coin is tossed again. Find the probability of obtaining head and number 6.

5.

An unbiased coin is tossed 4 times. Find the probability of getting at least one head.

Ans: $1 - \left(\frac{1}{2}\right)^4 = \frac{15}{16}$

6.

Out of 8 outstanding students of a school, in which there are 3 boys and 5 girls, a team of 4 students is to be selected for a quiz competition. Find the probability that 2 boys and 2 girls are selected.

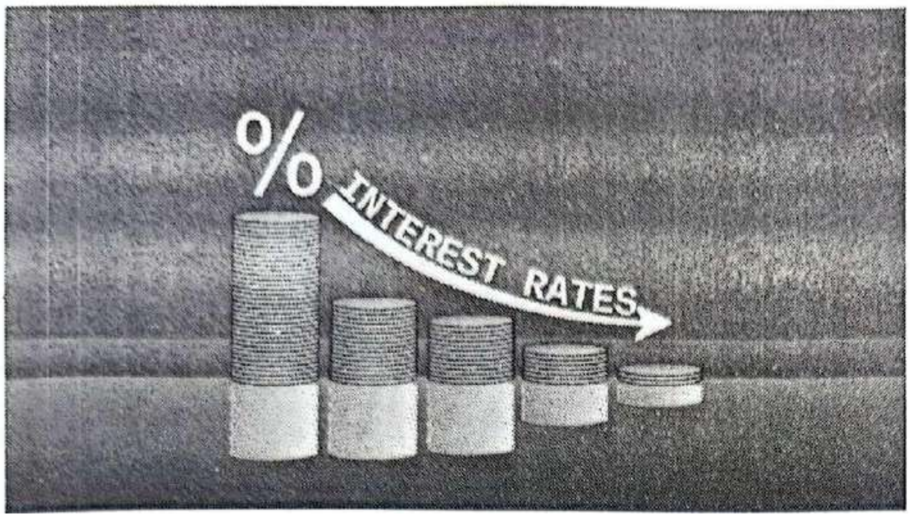
Sol.

Required probability $= \frac{{}^3C_2 \times {}^5C_2}{{}^8C_4}$

$$= \frac{3}{7}$$

Case study :

1. 2025



A bank offers loan to its customers on different types of interest namely, fixed rate, floating rate and variable rate. From the past data with the bank, it is known that a customer avails loan on fixed rate, floating rate or variable rate with probabilities 10%, 20% and 70% respectively. A customer after availing loan can pay the loan or default on loan repayment. The bank data suggests that the probability that a person defaults on loan after availing it at fixed rate, floating rate and variable rate is 5%, 3% and 1% respectively.

Based on the above information, answer the following :

(i) What is the probability that a customer after availing the loan will default on the loan repayment ?

(ii) A customer after availing the loan, defaults on loan repayment. What is the probability that he availed the loan at a variable rate of interest ?

Sol.

E_1 :customer avails loan on fixed rate

E_2 :customer avails loan on floating rate

E_3 :customer avails loan on variable rate

A:the person defaults on the loan

$$P(E_1)=\frac{1}{10},P(E_2)=\frac{2}{10},P(E_3)=\frac{7}{10}$$

$$P(A|E_1)=\frac{5}{100},P(A|E_2)=\frac{3}{100},P(A|E_3)=\frac{1}{100}$$

$$(i)P(A)=P(E_1).P(A|E_1)+P(E_2).P(A|E_2)+P(E_3).P(A|E_3)$$

$$=\frac{1}{10}\times\frac{5}{100}+\frac{2}{10}\times\frac{3}{100}+\frac{7}{10}\times\frac{1}{100}$$

$$=\frac{18}{1000}\text{ or }\frac{9}{500}$$

$$(ii)P(E_3|A)=\frac{P(E_3).P(A|E_3)}{P(E_1).P(A|E_1)+P(E_2).P(A|E_2)+P(E_3).P(A|E_3)}$$

$$=\frac{\frac{7}{10}\times\frac{1}{100}}{\frac{18}{1000}}$$

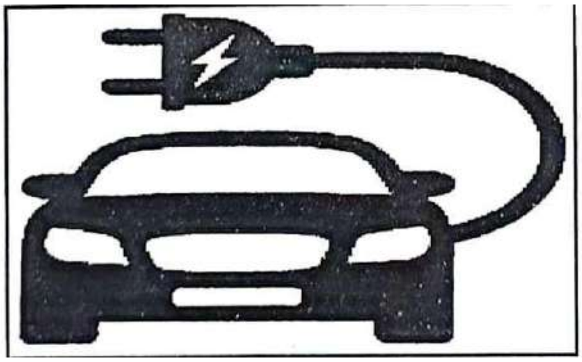
$$=\frac{7}{18}$$

prepared by : BALAJI KANCHI

2. 2025

Three persons viz. Amber, Bonzi and Comet are manufacturing cars which run on petrol and on battery as well. Their production share in the market is 60%, 30% and 10% respectively. Of their respective production capacities, 20%, 10% and 5% cars respectively are electric (or battery operated).

Based on the above, answer the following :



(i) (a) What is the probability that a randomly selected car is an electric car ?

OR

(i) (b) What is the probability that a randomly selected car is a petrol car ?

(ii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Comet ?

(iii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Amber or Bonzi ?

Sol.

(i).(a)

Let A = Amber manufactures the car

B = Bonzi manufactures the car

C = Comet manufactures the car

E = The selected car is electric

$$P(A) = \frac{60}{100}, P(B) = \frac{30}{100}, P(C) = \frac{10}{100}$$

$$P(E) = P(A) \times P\left(\frac{E}{A}\right) + P(B) \times P\left(\frac{E}{B}\right) + P(C) \times P(\frac{E}{C})$$

$$= \frac{60}{100} \times \frac{20}{100} + \frac{30}{100} \times \frac{10}{100} + \frac{10}{100} \times \frac{5}{100}$$

$$= \frac{155}{1000} \; or \; \frac{31}{200}$$

(i)(b)

Let A = Amber manufactures the car

B = Bonzi manufactures the car

C = Comet manufactures the car

E = The selected car is a petrol car

$$P(A) = \frac{60}{100}, P(B) = \frac{30}{100}, P(C) = \frac{10}{100}$$

$$P(E) = P(A) \times P\left(\frac{E}{A}\right) + P(B) \times P\left(\frac{E}{B}\right) + P(C) \times P(\frac{E}{C})$$

$$= \frac{60}{100} \times \frac{80}{100} + \frac{30}{100} \times \frac{90}{100} + \frac{10}{100} \times \frac{95}{100}$$

$$= \frac{845}{1000} \; or \; \frac{169}{200}$$

(ii)

$$P\left(\frac{C}{E}\right) = \frac{P(C) \times P(\frac{E}{C})}{P(E)}$$

$$= \frac{\frac{10}{100} \times \frac{5}{100}}{\frac{60}{100} \times \frac{20}{100} + \frac{30}{100} \times \frac{10}{100} + \frac{10}{100} \times \frac{5}{100}}$$

$$= \frac{\frac{50}{10000}}{\frac{1550}{10000}} = \frac{1}{31}$$

(iii) $\quad P\left(\frac{\text{A or B}}{\text{E}}\right) = 1 - P\left(\frac{C}{E}\right) = 1 - \frac{1}{31} = \frac{30}{31}$

3.2025

Some students are having a misconception while comparing decimals. For example, a student may mention that 78·56 > 78·9 as 7856 > 789. In order to assess this concept, a decimal comparison test was administered to the students of class VI through the following question : In the recently held Sports Day in the school, 5 students participated in a javelin throw competition. The distances to which they have thrown the javelin are shown below in the table :

Name of student	Distance of javelin (in meters)
Ajay	47·7
Bijoy	47·07
Kartik	43·09
Dinesh	43·9
Devesh	45·2

The students were asked to identify who has thrown the javelin the farthest.
Based on the test attempted by the students, the teacher concludes that 40% of the students have the misconception in the concept of decimal comparison and the rest do not have the misconception. 80% of the students having misconception answered Bijoy as the correct answer in the paper. 90% of the students who are identified with not having misconception, did not answer Bijoy as their answer.

On the basis of the above information, answer the following questions :

(i) What is the probability of a student not having misconception but still answers Bijoy in the test ?

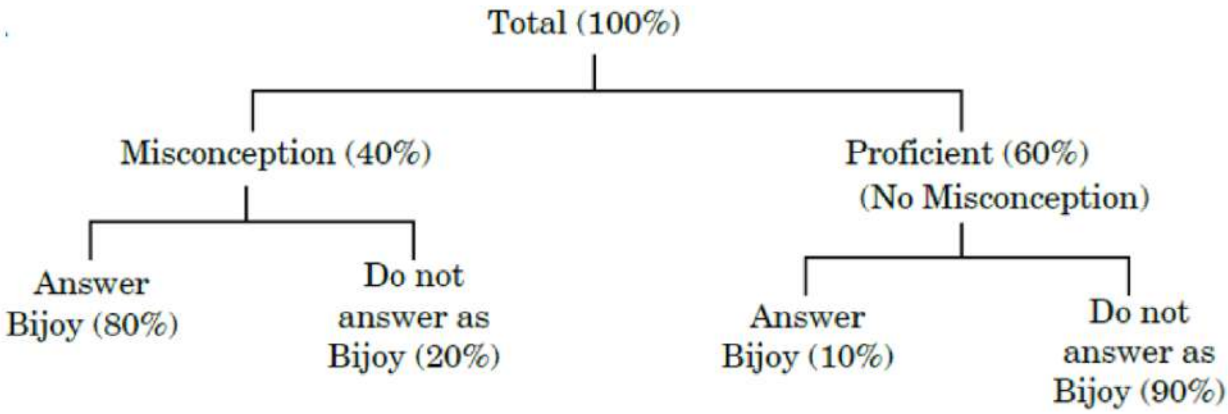
(ii) What is the probability that a randomly selected student answers Bijoy as his answer in the test ?

(iii) (a) What is the probability that a student who answered as Bijoy is having misconception ?

OR

(iii) (b) What is the probability that a student who answered as Bijoy is amongst students who do not have the misconception ?

Sol.



Let E_1 : Student has a misconception

E_2 :Student does not have misconception

A: Student answered Bijoy as correct

$$\therefore P(E_1)=\frac{40}{100}, P(E_2)=\frac{60}{100}$$

$$P(A|E_1)=\frac{80}{100}, P(A|E_2)=\frac{10}{100}$$

$$P(\bar{A}|E_1)=\frac{20}{100}, P(\bar{A}|E_2)=\frac{90}{100}$$

$$(i) P(A|E_2)=\frac{10}{100} \text{ or } \frac{1}{10}$$

$$(i) P(A|E_2)=\frac{10}{100} \text{ or } \frac{1}{10}$$

$$(ii) P(A)=P(E_1)P(A|E_1)+P(E_2)P(A|E_2)$$

$$=\frac{40}{100}\times\frac{80}{100}+\frac{60}{100}\times\frac{10}{100}$$

$$=\frac{38}{100} \text{ or } \frac{19}{50}$$

$$(iii)(a) P(E_1|A)=\frac{P(E_1)P(A|E_1)}{P(A)}$$

$$=\frac{\frac{40}{100}\times\frac{80}{100}}{\frac{38}{100}}=\frac{16}{19}$$

$$(iii)(b)\ P(E_2 \mid A) = \frac{P(E_2)P(A \mid E_2)}{P(A)}$$

$$= \frac{\frac{60}{100} \times \frac{10}{100}}{\frac{38}{100}} = \frac{3}{19}$$

4. 2025

A gardener wanted to plant vegetables in his garden. Hence he bought 10 seeds of brinjal plant, 12 seeds of cabbage plant and 8 seeds of radish plant. The shopkeeper assured him of germination probabilities of brinjal, cabbage and radish to be 25%, 35% and 40% respectively. But before he could plant the seeds, they got mixed up in the bag and he had to sow them randomly.


Radish


Cabbage


Brinjal

Based upon the above information, answer the following questions :

(i) Calculate the probability of a randomly chosen seed to germinate.

(ii) What is the probability that it is a cabbage seed, given that the chosen seed germinates ?

Sol.

Let A: Event that chosen seed germinates.

B: Event that Brinjal seed is chosen.

C: Event that Cabbage seed is chosen.

R: Event that Radish seed is chosen.

$$P(B) = \frac{10}{30};\ P(C) = \frac{12}{30};\ P(R) = \frac{8}{30};$$

$$P\left(\frac{A}{B}\right) = \frac{25}{100};\ P\left(\frac{A}{C}\right) = \frac{35}{100};\ P\left(\frac{A}{R}\right) = \frac{40}{100}$$

(i) $$P(A) = P(B).P\left(\frac{A}{B}\right) + P(C).P\left(\frac{A}{C}\right) + P(R).P\left(\frac{A}{R}\right)$$

$$= \frac{10}{30} \times \frac{25}{100} + \frac{12}{30} \times \frac{35}{100} + \frac{8}{30} \times \frac{40}{100}$$

$$= \frac{990}{3000} \text{ or } \frac{33}{100}$$

(ii) (a) $$P\left(\frac{C}{A}\right) = \frac{P(C).P\left(\frac{A}{C}\right)}{P(B).P\left(\frac{A}{B}\right)+P(C).P\left(\frac{A}{C}\right)+P(R).P\left(\frac{A}{R}\right)}$$

$$= \frac{\frac{12}{30}\times\frac{35}{100}}{\frac{990}{3000}}$$

$$= \frac{42}{99} \text{ or } \frac{14}{33}$$

prepared by : BALAJI KANCHI

5. 2025

A shop selling electronic items sells smartphones of only three reputed companies A, B and C because chances of their manufacturing a defective smartphone are only 5%, 4% and 2% respectively. In his inventory he has 25% smartphones from company A, 35% smartphones from company B and 40% smartphones from company C.

A person buys a smartphone from this shop.

(i) Find the probability that it was defective.

(ii) What is the probability that this defective smartphone was manufactured by company B ?

Sol.

(i) $P(\text{defective smartphone}) = 0.25\times0.05 + 0.35\times 0.04 + 0.40\times0.02$

$$= 0.0345$$

(ii) $P(B/\text{ Defective}) = \dfrac{0.35\times 0.04}{0.25\times 0.05 + 0.35\times0.04 + 0.40\times0.02}$

$$= \frac{140}{345} \text{ or } \frac{28}{69}$$

6. 2025

Based upon the results of regular medical check-ups in a hospital, it was found that out of 1000 people, 700 were very healthy, 200 maintained average health and 100 had a poor health record.

Let A_1 : People with good health,

A_2 : People with average health,

and A_3 : People with poor health.

During a pandemic, the data expressed that the chances of people contracting the disease from category A_1, A_2 and A_3 are 25%, 35% and 50%, respectively.

Based upon the above information, answer the following questions :

(i) A person was tested randomly. What is the probability that he/she has contracted the disease ?

(ii) Given that the person has not contracted the disease, what is the probability that the person is from category A_2 ?

Sol.

(i) Let A: Person contracted the disease

$$P(A)=P(A_1)\cdot P(A|A_1)+P(A_2)\cdot P(A|A_2)+P(A_3)\cdot P(A|A_3)$$

$$=\frac{7}{10}\left(\frac{25}{100}\right)+\frac{2}{10}\left(\frac{35}{100}\right)+\frac{1}{10}\left(\frac{50}{100}\right)$$

$$=\frac{295}{1000}=0.295 \text{ or } \left(\frac{59}{200}\right)$$

(ii) $$P(A_2|\bar{A})=\frac{P(A_2)\cdot P(\bar{A}/A_2)}{P(A_1)\cdot P(\bar{A}/A_1)+P(A_2)\cdot P(\bar{A}/A_2)}$$

$$=\frac{\frac{2}{10}\times\frac{65}{100}}{\frac{7}{10}\times\frac{75}{100}+\frac{2}{10}\times\frac{65}{100}+\frac{1}{10}\times\frac{50}{100}}$$

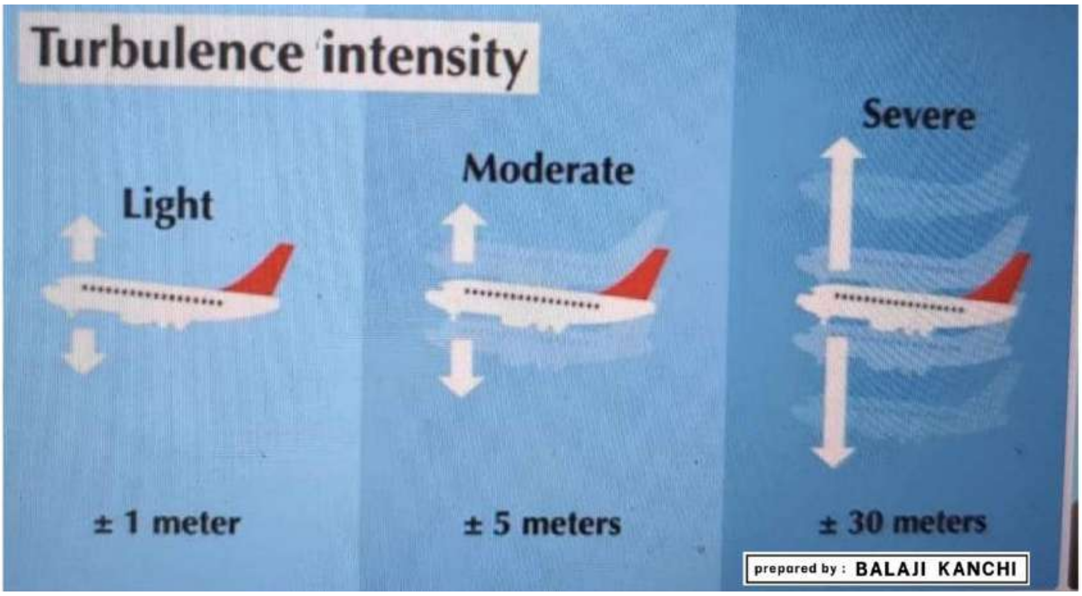
$$=\frac{2\times 13}{7\times 15+2\times 13+1\times 10}=\frac{26}{141}$$

prepared by : BALAJI KANCHI

7. 2024

According to recent research, air turbulence has increased in various regions around the world due to climate change. Turbulence makes flights bumpy and often delays the flights.

Assume that, an airplane observes severe turbulence, moderate turbulence or light turbulence with equal probabilities. Further, the chance of an airplane reaching late to the destination are 55%, 37% and 17% due to severe, moderate and light turbulence respectively.



On the basis of the above information, answer the following questions :

(i) Find the probability that an airplane reached its destination late.

(ii) If the airplane reached its destination late, find the probability that it was due to moderate turbulence.

Sol.

(i) Let A denote the event of airplane reaching its destination late

E_1 = severe turbulence

E_2 = moderate turbulence

E_3 = light turbulence

$$P(A) = P(E_1)\,P(A|E_1) + P(E_2)P(A|E_2) + P(E_3)P(A|E_3)$$

$$= \frac{1}{3} \times \frac{55}{100} + \frac{1}{3} \times \frac{37}{100} + \frac{1}{3} \times \frac{17}{100}$$

$$= \frac{1}{3}\left(\frac{109}{100}\right) = \frac{109}{300}$$

(ii) $$P(E_2|A) = \frac{P(E_2)P(A|E_2)}{P(A)}$$

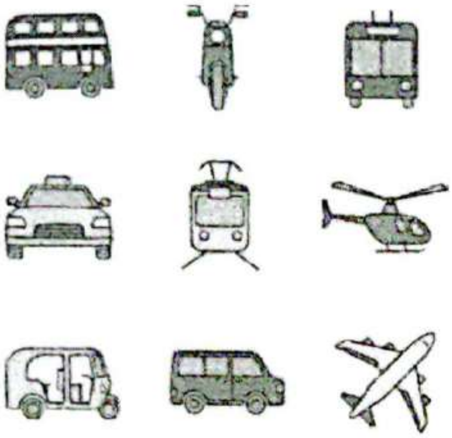
$$= \frac{\frac{1}{3} \times \frac{37}{100}}{\frac{109}{300}}$$

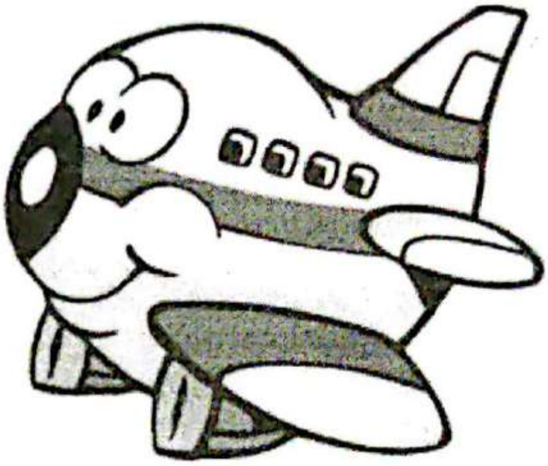
$$= \frac{37}{109}$$

prepared by : BALAJI KANCHI

8. 2024

Airplanes are by far the safest mode of transportation when the number of transported passengers are measured against personal injuries and fatality totals.





Previous records state that the probability of an airplane crash is 0·00001%. Further, there are 95% chances that there will be survivors after a plane crash. Assume that in case of no crash, all travellers survive.

Let E_1 be the event that there is a plane crash and E_2 be the event that there is no crash. Let A be the event that passengers survive after the journey.

On the basis of the above information, answer the following questions :

(i) Find the probability that the airplane will not crash.

(ii) Find $P(A \mid E_1) + P(A \mid E_2)$.

(iii) (a) Find P(A).

OR

(iii) (b) Find $P(E_2 \mid A)$.

Sol.

(i) $P(E_2) = 1 - 0.0000001$

$= 0.9999999$

(ii) $P(A/E_1) + P(A/E_2) = \frac{95}{100} + 1 = \frac{195}{100}$

(iii)(a) $P(A) = P(E_1) \times P(A/E_1) + P(E_2) \times P(A/E_2)$

$= \frac{1}{10000000} \times \frac{95}{100} + \frac{9999999}{10000000} \times 1$

$= \frac{95+999999900}{1000000000} = \frac{999999995}{1000000000}$

(iii)(b)

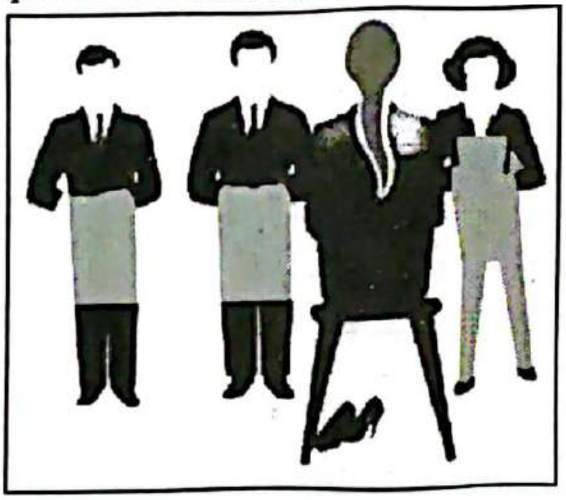
$P(E_2/A) = \frac{P(E_2) \times P(A/E_2)}{P(E_1) \times P(A/E_1) + P(E_2) \times P(A/E_2)}$

$= \frac{\frac{9999999}{10000000}}{\frac{999999995}{1000000000}} = \frac{999999900}{999999995}$

prepared by : BALAJI KANCHI

9. 2024

Rohit, Jaspreet and Alia appeared for an interview for three vacancies in the same post. The probability of Rohit's selection is $\frac{1}{5}$, Jaspreet's selection is $\frac{1}{3}$ and Alia's selection is $\frac{1}{4}$. The event of selection is independent of each other.



Based on the above information, answer the following questions :

(i) What is the probability that at least one of them is selected ?

(ii) Find $P(G \mid \overline{H})$ where G is the event of Jaspreet's selection and $\overline{H}$ denotes the event that Rohit is not selected.

(iii) Find the probability that exactly one of them is selected.

OR

(iii) Find the probability that exactly two of them are selected.

Sol.

Given $P(\text{Rohit}) = \frac{1}{5}, P(\text{Jaspreet}) = \frac{1}{3}, P(\text{Alia}) = \frac{1}{4}$

(i) P(atleast one of them is selected) = 1 – P(no one is selected)

$$= 1 - \left(\frac{4}{5} \times \frac{2}{3} \times \frac{3}{4}\right) = \frac{3}{5}$$

(ii) $P(G|\overline{H}) = \dfrac{P(G \cap \overline{H})}{P(\overline{H})} = \dfrac{1}{3}$

(iii) P(exactly one of them selected)

$$= P(R) \times P(\overline{J}) \times P(\overline{A}) + P(\overline{R}) \times P(J) \times P(\overline{A}) + P(\overline{R}) \times P(\overline{J}) \times P(A)$$

$$= \frac{6+12+8}{60} = \frac{13}{30}$$

OR

(iii) P(exactly two of them selected)

$$= P(R) \times P(J) \times P(\overline{A}) + P(R) \times P(\overline{J}) \times P(A) + P(\overline{R}) \times P(J) \times P(A)$$

$$= \frac{3+2+4}{60} = \frac{3}{20}$$

prepared by : BALAJI KANCHI

10. 2024

A departmental store sends bills to charge its customers once a month. Past experience shows that 70% of its customers pay their first month bill in time. The store also found that the customer who pays the bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month.

Based on the above information, answer the following questions :

(i) Let E_1 and E_2 respectively denote the event of customer paying or not paying the first month bill in time.
Find $P(E_1)$, $P(E_2)$.

(ii) Let A denotes the event of customer paying second month's bill in time, then find $P(A|E_1)$ and $P(A|E_2)$.

(iii) Find the probability of customer paying second month's bill in time.

OR

(iii) Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time.

Sol.

(i) $P(E_1) = \frac{7}{10} = 0.7,\ P(E_2) = \frac{3}{10} = 0.3$

(ii) $P(A|E_1) = 0.8, P(A|E_2) = 0.4$

(iii) $P(A) = P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)$

$$= 0.7\times 0.8 + 0.3\times 0.4 = 0.68 \text{ or } \frac{17}{25}$$

Or

(iii) $P(A) = \dfrac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)} = \dfrac{14}{17}$

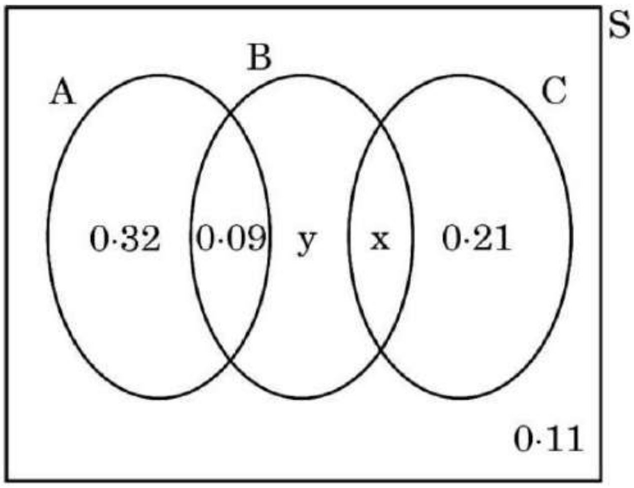
prepared by : BALAJI KANCHI

11.2023

There are different types of Yoga which involve the usage of different poses of Yoga Asanas, Meditation and Pranayam as shown in the figure below :

Types of Yoga

The Venn diagram below represents the probabilities of three different types of Yoga, A, B and C performed by the people of a society. Further, it is given that probability of a member performing type C Yoga is 0·44.



On the basis of the above information, answer the following questions :

(i) Find the value of x.

(ii) Find the value of y.

(iii) (a) Find $P\left(\frac{C}{B}\right)$.

OR

(iii) (b) Find the probability that a randomly selected person of the society does Yoga of type A or B but not C.

Sol.

(i) $x + 0{\cdot}21 = 0{\cdot}44 \Rightarrow x = 0{\cdot}23$

(ii) $0{\cdot}41 + y + 0{\cdot}44 + 0.11 = 1 \Rightarrow y = 0{\cdot}04$

(iii) (a) $P\left(\frac{C}{B}\right) = \frac{P(C \cap B)}{P(B)}$

$P(B) = 0{\cdot}09 + 0{\cdot}04 + 0{\cdot}23 = 0{\cdot}36$

$P\left(\frac{C}{B}\right) = \frac{0{\cdot}23}{0{\cdot}36} = \frac{23}{36}$

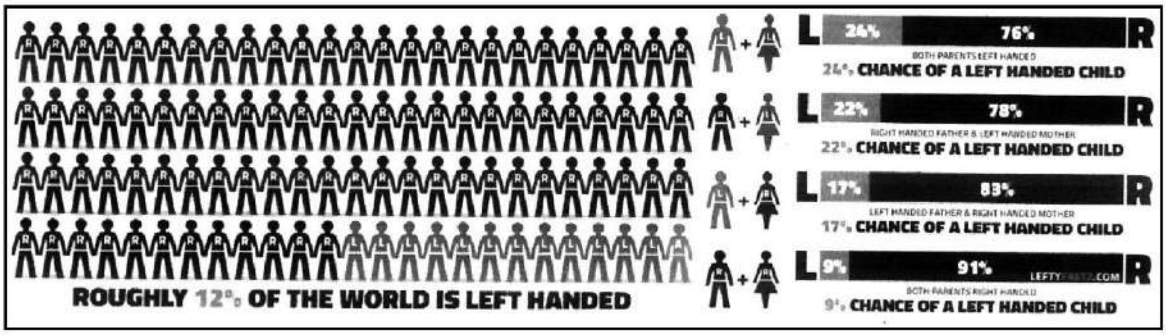
(iii) (b) P(A or B but not C)

$= 0{\cdot}32 + 0.09 + 0.04$

$= 0{\cdot}45$

12.2023

Recent studies suggest that roughly 12% of the world population is left handed.



Depending upon the parents, the chances of having a left handed child are as follows :

A : When both father and mother are left handed :
Chances of left handed child is 24%.

B : When father is right handed and mother is left handed :
Chances of left handed child is 22%.

C : When father is left handed and mother is right handed :
Chances of left handed child is 17%.

D : When both father and mother are right handed :
Chances of left handed child is 9%.

Assuming that $P(A) = P(B) = P(C) = P(D) = \frac{1}{4}$ and L denotes the event that child is left handed.

Based on the above information, answer the following questions :

(i) Find $P(L/C)$ **1**

(ii) Find $P(\overline{L}/A)$ **1**

(iii) (a) Find $P(A/L)$ **2**

OR

(b) Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed. **2**

Sol.

(i) $P(L \mid C) = \dfrac{17}{100}$

(ii) $P(\bar{L} \mid A) = 1 - P(L \mid A) = 1 - \dfrac{24}{100} = \dfrac{76}{100}$ or $\dfrac{19}{25}$

(iii) $P(A \mid L) = \dfrac{\frac{1}{4} \times \frac{24}{100}}{\frac{1}{4} \times \frac{24}{100} + \frac{1}{4} \times \frac{22}{100} + \frac{1}{4} \times \frac{17}{100} + \frac{1}{4} \times \frac{9}{100}} = \dfrac{24}{72} = \dfrac{1}{3}$

Or

Probability that a randomly selected child is left-handed given that exactly one of the parents is left-handed.

$= P(L \mid B \cup C) = \dfrac{22}{100} + \dfrac{17}{100} = \dfrac{39}{100}$

prepared by : BALAJI KANCHI

13.2023

A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0·65. The probability that many are not present and still the work gets completed on time is 0·35. The probability that work will be completed on time when all workers are present is 0·80.

Let : E_1 : represent the event when many workers were not present for the job;

E_2 : represent the event when all workers were present; and

E : represent completing the construction work on time.

Based on the above information, answer the following questions :

(i) What is the probability that all the workers are present for the job ?

(ii) What is the probability that construction will be completed on time ?

(iii) (a) What is the probability that many workers are not present given that the construction work is completed on time ?

OR

(iii) (b) What is the probability that all workers were present given that the construction job was completed on time ?

Sol.

(i) $P(E_2) = 1 - P(E_1) = 1 - 0{\cdot}65 = 0{\cdot}35$

(ii) $P(E) = P(E_1) \, . \, P\left(\frac{E}{E_1}\right) + P(E_2) \, . \, P\left(\frac{E}{E_2}\right)$

$= 0{\cdot}65 \times 0{\cdot}35 + 0{\cdot}35 \times 0{\cdot}8$

$= 0{\cdot}35 \times 1{\cdot}45$

$= 0{\cdot}51$

(iii) (a) $P\left(\frac{E_1}{E}\right) = \frac{P(E_1) \, . \, P(E/E_1)}{P(E_1).P(E/E_1)+P(E_2).P(E/E_2)} = \frac{0{\cdot}65 \times 0{\cdot}35}{0{\cdot}51} = 0{\cdot}45$

OR

(iii) (b) $P\left(\frac{E_2}{E}\right) = \frac{P(E_2) \, . \, P(E/E_2)}{P(E_1).P(E/E_1)+P(E_2).P(E/E_2)} = \frac{0{\cdot}35 \times 0{\cdot}8}{0{\cdot}51} = 0{\cdot}55$

prepared by : **BALAJI KANCHI**

14.2022

A shopkeeper sells three types of flower seeds A1, A2, A3. They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2, respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.

Based on the above information :

(a) Calculate the probability that a randomly chosen seed will germinate;

(b) Calculate the probability that the seed is of type A2, given that a randomly chosen seed germinates.

Sol.

E_1 : Seed is of type A_1

E_2 : Seed is of type A_2

E_3 : Seed is of type A_3

A : Seed germinates

$P(E_1) = \frac{4}{10}, P(E_2) = \frac{4}{10}, P(E_3) = \frac{2}{10}$

$P(A|E_1) = \frac{45}{100}, P(A|E_2) = \frac{60}{100}, P(A|E_3) = \frac{35}{100}$

(a) $P(A) = P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + P(E_3)P(A|E_3)$

$$= \frac{4}{10} \times \frac{45}{100} + \frac{4}{10} \times \frac{60}{100} + \frac{2}{10} \times \frac{35}{100}$$

$$= \frac{180 + 240 + 70}{1000}$$

$$= \frac{490}{1000} = \frac{49}{100}$$

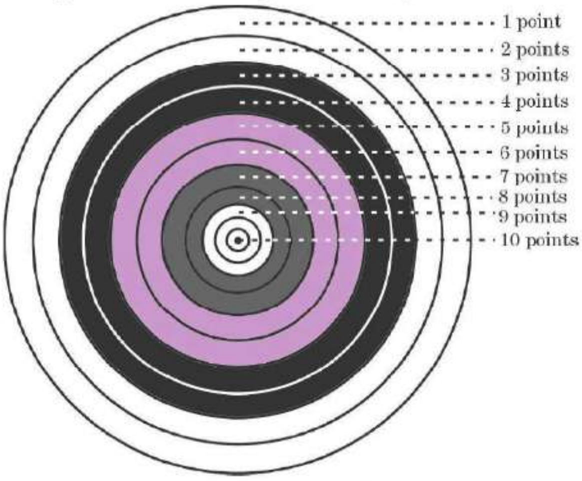
(b) $P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P(A \mid E_2)}{P(A)}$

$$= \frac{\frac{4}{10} \times \frac{60}{100}}{\frac{490}{1000}} = \frac{24}{49}$$

15. 2022

In a game of Archery, each ring of the Archery target is valued. The centremost ring is worth 10 points and rest of the rings are allotted points 9 to 1 in sequential order moving outwards.

Archer A is likely to earn 10 points with a probability of 0·8 and Archer B is likely the earn 10 points with a probability of 0·9.



Based on the above information, answer the following questions :

If both of them hit the Archery target, then find the probability that

(a) exactly one of them earns 10 points.

(b) both of them earn 10 points.

Sol.

$P(A)=0\cdot 8\ ,P(B)=0\cdot 9$

(a) P (exactly one of them earns 10 points) $= P(A)P(\bar{B})+P(\bar{A})P(B)$

$= 0\cdot 8\times 0\cdot 1+0\cdot 2\times 0\cdot 9$

$= 0\cdot 26$

(b) $P(A\cap B)=P(A)P(B)$

$= 0\cdot 8\times 0\cdot 9$

$= 0\cdot 72$

prepared by : **BALAJI KANCHI**

16.

In answering a multiple choice test for class XII, a student either knows or guesses or copies the answer to a multiple choice question with four choices. The probability that he makes a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct given that he copied is $\frac{1}{8}$.
Let E_1, E_2, E_3 be the events that the student guesses, copies or knows the answer respectively and A is the event that the student answers correctly.

Based on the above information, answer ***any four*** of the following ***five*** questions :

(i) What is the probability that the student knows the answer ?

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) $\frac{1}{4}$

(ii) What is the probability that he answers correctly given that he knew the answer ?

(A) 1

(B) 0

(C) $\frac{1}{4}$

(D) $\frac{1}{8}$

(iii) What is the probability that he answers correctly given that he had made a guess ?

(A) $\frac{1}{4}$

(B) 0

(C) 1

(D) $\frac{1}{8}$

(iv) What is the probability that he knew the answer to the question, given that he answered it correctly ?

(A) $\frac{24}{29}$

(B) $\frac{4}{29}$

(C) $\frac{1}{29}$

(D) $\frac{3}{29}$

17.

. **Case-Study** 3: Read the following passage and answer the questions given below.

There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

(i) What is the probability that the shell fired from exactly one of them hit the plane?

(ii) If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B?

Sol.

(i)P(Shell fired from exactly one of them hits the plane)

= P[(Shell from A hits the plane and Shell from B does not hit the plane) or (Shell from A does not hit the plane and Shell from B hits the plane)]

$= 0.3 \times 0.8 + 0.7 \times 0.2 = 0.38$

(ii)P(Shell fired from B hit the plane/Exactly one of them hit the plane)

$$= \frac{\text{P(Shell fired from B hit the plane} \cap \text{Exactly one of them hit the plane)}}{\text{P(Exactly one of them hit the plane)}}$$

$$= \frac{P(Shell\ from\ only\ B\ hit\ the\ plane)}{P(Exactly\ one\ of\ them\ hit\ the\ plane)}$$

$$= \frac{0.14}{0.38} = \frac{7}{19}$$

prepared by : BALAJI KANCHI

18.

A laboratory blood test is 98% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0·4% of the healthy person tested. From a large population, it is given that 0·2% of the population actually has the disease.

Based on the above, answer the following questions :

(a) One person, from the population, is taken at random and given the test. Find the probability of his getting a positive test result.

(b) What is the probability that the person actually has the disease, given that his test result is positive ?

Sol.

Let

E_1 : *person has the disease*

E_2 : *person does not have the disease*

A : *getting a +ve test report*

$$P(E_1)=\frac{2}{1000},\quad P(E_2)=\frac{998}{1000}$$

$$P(A|E_1)=\frac{98}{100},\quad P(A|E_2)=\frac{4}{1000}$$

(a) $$P(A)=P(E_1)P(A|E_1)+P(E_2)P(A|E_2)$$

$$=\frac{2}{1000}\times\frac{98}{100}+\frac{998}{1000}\times\frac{4}{1000}=\frac{5952}{1000000}$$

(b) $$P(E_1|A)=\frac{P(E_1)P(A|E_1)}{\Sigma P(E_i)P(A|E_i)}$$

$$=\frac{\frac{2}{1000}\times\frac{980}{1000}}{\frac{5952}{1000000}}=\frac{1960}{5952}=\frac{245}{744}$$

19.

In a class, 5% of the boys and 10% of the girls have an IQ more than 150. In this class, 60% of the students are boys. One student is selected at random from the class.

Based on the above,

(a) Find the probability that the selected student has an IQ more than 150.

(b) If it is given that the selected student has IQ more than 150, find the probability that the student is a girl.

20.

There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

Based on the above information answer the following questions:

(i) What is the probability that the shell fired from exactly one of them hit the plane?

(ii) If it is known that the shell fired exactly one of them hit the plane, then what is the probability that it was fired from B?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21.

Read the following passage and answer the questions given below :

There are ten cards numbered 1 to 10 and they are placed in a box and then mixed up thoroughly. Then one card is drawn at random from the box.

Based on the above, answer the following questions :

(i) What is the probability that the number on the drawn card is greater than 4 ?

(ii) If it is known that the number on the drawn card is greater than 4, then what is the probability that it is an even number ?

22.

A shopkeeper sells three types of flower seeds A_1, A_2 and A_3. They are sold as a mixture where the proportions are 4 : 4 : 2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35%.

Based on the above information, answer the following questions :

(i) What is the probability of a randomly chosen seed to germinate ?

(ii) What is the probability that the randomly selected seed is of type A_1, given that it germinates ?

Sol.

(i) **Probability of randomly chosen seed to germinate**

$$= \frac{4}{10} \times \frac{45}{100} + \frac{4}{10} \times \frac{60}{100} + \frac{2}{10} \times \frac{35}{100} == 0.49$$

(ii) **Probability that the randomly selected seed is of type A_1, given that it germinates**

$$= \frac{\frac{4}{10} \times \frac{45}{100}}{\frac{490}{1000}} = \frac{18}{49}$$

23.

In an office three employees Vinay, Sonia and Iqbal process incoming copies c a certain form. Vinay process 50% of the forms. Sonia processes 20% and Iqba the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has a error rate of 0.04 and Iqbal has an error rate of 0.03

Based on the above information answer the following:
(i) The conditional probability that an error is committed in processing given that Sonia processed the form is :

a) 0.0210
b) 0.04
c) 0.47
d) 0.06

(ii)The probability that Sonia processed the form and committed an error is :

a) 0.005
b) 0.006
c) 0.008
d) 0.68

(iii)The total probability of committing an error in processing the form is

a) 0
b) 0.047
c) 0.234

(iv)The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is **NOT** processed by Vinay is :

a) 1
b) 30/47
c) 20/47
d) 17/47

(v)Let A be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that Vinay, Sonia and Iqbal processed the form. The value of $\sum_{i=1}^{3} P\,(E_i|\,\mathrm{A})$ is

a) 0
b) 0.03
c) 0.06
d) 1

24.

An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. The company's statistics show that an accident-prone person will have an accident at sometime within a fixed one-year period with probability 0.6, whereas this probability is 0.2 for a person who is not accident prone. The company knows that 20 percent of the population is accident prone.

Based on the given information, answer the following questions.

(i)what is the probability that a new policyholder will have an accident within a year of purchasing a policy?

(ii) Suppose that a new policyholder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone?

25.

Read the following passage and answer the questions given below:

In an Office three employees Jayant, Sonia and Oliver process incoming copies of a certain form. Jayant processes **50%** of the forms, Sonia processes **20%** and Oliver the remaining **30%** of the forms. Jayant has an error rate of **0.06**, Sonia has an error rate of **0.04** and Oliver has an error rate of **0.03**.

Based on the above information, answer the following questions.

(i) Find the probability that Sonia processed the form and committed an error.

(ii) Find the total probability of committing an error in processing the form.

(iii) The manager of the Company wants to do a quality check. During inspection, he selects a form at random from the days output of processed form. If the form selected at random has an error, find the probability that the form is **not** processed by Jayant.

26.

Senior students tend to stay up all night and therefore are not able to wake up on time in morning. Not only this but their dependence on tuitions further leads to absenteeism in school. Of the students in class XII, it is known that 30% of the students have 100% attendance. Previous year results report that 80% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one student is chosen at random from the class XII.

Using above information answer the following:

(i). Find the conditional probability that a student attains A grade given that he is not 100 % regular student.

(ii) Find the probability of attaining A grade by the students of class XII

(iii) Find the probability that student is 100% regular given that he attains A grade.

OR

Find the probability that student is irregular given that he attains A grade.

Resource Materials by BALAJI KANCHI :

Class XII Maths Sample Papers (2025-26) KB Challenge series: click here

Class XII Maths Sample Papers (2025-26) KB Challenge series key : click here

Class XII Math Drive 2025-26 (Prepared by BALAJI KANCHI): Click here

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