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2. Inverse Trigonometric Functions

(Previous Year Questions from 2008-2025)

McQ's :

1.

The principal value of $\tan^{-1}\left(\tan \frac{3\pi}{5}\right)$ is

(A) $\frac{2\pi}{5}$

(B) $\frac{-2\pi}{5}$

(C) $\frac{3\pi}{5}$

(D) $\frac{-3\pi}{5}$

2.

The principal value of $\cos^{-1}\left(\cos \frac{13\pi}{6}\right)$ is

(A) $\frac{13\pi}{6}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{6}$

3.

The domain of the function $f(x) = \sin^{-1}(2x)$ is

(A) $[0, 1]$

(B) $[-1, 1]$

(C) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(D) $[-2, 2]$

4.

The value of $\tan^{-1}(\tan\frac{7\pi}{6})$ is

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$

(D) $\frac{7\pi}{6}$

5.

$\tan^{-1}3 + \tan^{-1}\lambda = \tan^{-1}\left(\frac{3+\lambda}{1-3\lambda}\right)$ is valid for what values of λ ?

(A) $\lambda \in \left(-\frac{1}{3}, \frac{1}{3}\right)$

(B) $\lambda > \frac{1}{3}$

(C) $\lambda < \frac{1}{3}$

(D) All real values of λ

6.

If $\cos\left(\sin^{-1}\frac{2}{\sqrt{5}} + \cos^{-1}x\right) = 0$, then x is equal to

(A) $\frac{1}{\sqrt{5}}$

(B) $-\frac{2}{\sqrt{5}}$

(C) $\frac{2}{\sqrt{5}}$

(D) 1

7.

The principal value of $\cot^{-1}(-\sqrt{3})$ is

(A) $-\frac{\pi}{6}$

(B) $\frac{\pi}{6}$

(C) $\frac{2\pi}{3}$

(D) $\frac{5\pi}{6}$

8.

The value of $\sin^{-1}\left(\cos\frac{3\pi}{5}\right)$ is

(a) $\frac{\pi}{10}$ (b) $\frac{3\pi}{5}$ (c) $\frac{-\pi}{10}$ (d) $\frac{-3\pi}{5}$

9.

$\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$ is equal to:

a) $\frac{1}{2}$	b) $\frac{1}{3}$
c) -1	d) 1

5.

$\sin(\tan^{-1}x)$, where $|x| < 1$, is equal to:

a) $\frac{x}{\sqrt{1-x^2}}$	b) $\frac{1}{\sqrt{1-x^2}}$
c) $\frac{1}{\sqrt{1+x^2}}$	d) $\frac{x}{\sqrt{1+x^2}}$

10.

Simplest form of $\tan^{-1}\left(\frac{\sqrt{1+cosx}+\sqrt{1-cosx}}{\sqrt{1+cosx}-\sqrt{1-cosx}}\right), \pi < x < \frac{3\pi}{2}$ is:

a) $\frac{\pi}{4} - \frac{x}{2}$	b) $\frac{3\pi}{2} - \frac{x}{2}$
c) $-\frac{x}{2}$	d) $\pi - \frac{x}{2}$

11.

If $\tan^{-1} x = y$, then:

a) $-1 < y < 1$	b) $\frac{-\pi}{2} \leq y \leq \frac{\pi}{2}$
c) $\frac{-\pi}{2} < y < \frac{\pi}{2}$	d) $y \in \{\frac{-\pi}{2}, \frac{\pi}{2}\}$

2023 march :

1.

Assertion (A) : The range of the function $f(x) = 2\sin^{-1}x + \frac{3\pi}{2}$, where $x \in [-1, 1]$, is $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$.

Reason (R) : The range of the principal value branch of $\sin^{-1}(x)$ is $[0, \pi]$.

2.

Assertion (A) : The range of the function $f(x) = 2\sin^{-1}x + \frac{3\pi}{2}$, where $x \in [-1, 1]$, is $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$.

Reason (R) : The range of the principal value branch of $\sin^{-1}(x)$ is $[0, \pi]$.

3.

Assertion (A) : Maximum value of $(\cos^{-1}x)^2$ is π^2.

Reason (R) : Range of the principal value branch of $\cos^{-1}x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

4.

Assertion (A) : Range of $[\sin^{-1}x + 2\cos^{-1}x]$ is $[0, \pi]$.

Reason (R) : Principal value branch of $\sin^{-1}x$ has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

5.

Assertion (A) : All trigonometric functions have their inverses over their respective domains.

Reason (R) : The inverse of $\tan^{-1}x$ exists for some $x \in \mathbb{R}$.

6.

$\sin\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

7.

Assertion (A) : The principal value of $\cot^{-1}(\sqrt{3})$ is $\frac{\pi}{6}$.

Reason (R) : Domain of $\cot^{-1} x$ is $\mathbb{R} - \{-1, 1\}$.

8.

The domain of the function $\sin^{-1}(2x)$ is :

(a) $[-1, 1]$ (b) $[0, 1]$

(c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (d) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

8.

The domain of the function $\cos^{-1} x$ is :

(a) $[0, \pi]$ (b) $(-1, 1)$

(c) $[-1, 1]$ (d) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

2024 March:

9.

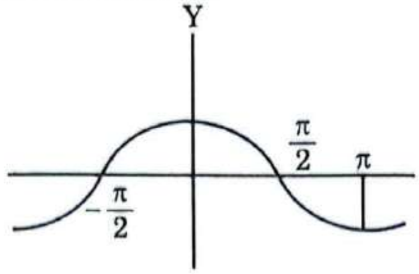
Assertion (A) : Domain of $y = \cos^{-1}(x)$ is $[-1, 1]$.

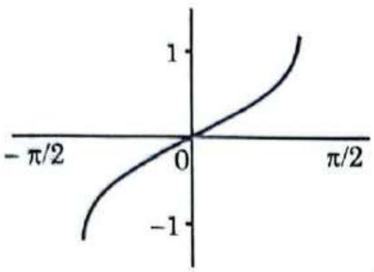
Reason (R) : The range of the principal value branch of $y = \cos^{-1}(x)$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.

2025 March :

1.

The graph of a trigonometric function is as shown. Which of the following will represent graph of its inverse ?



(A)


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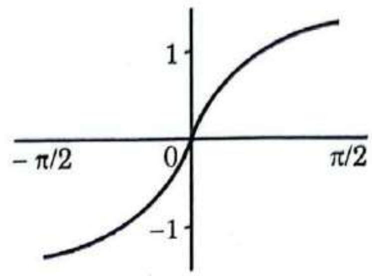
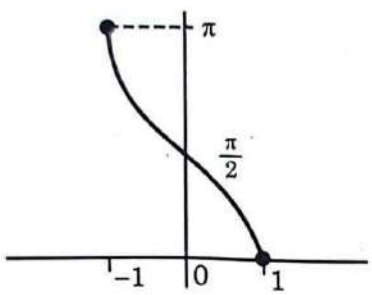
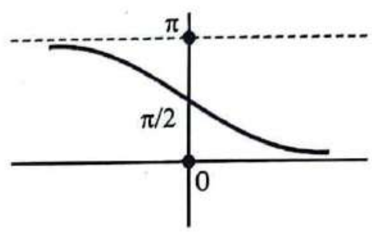


(C)


(D)


2.

The principal value of $\sin^{-1}\left(\cos\frac{43\pi}{5}\right)$ is

(A) $\frac{-7\pi}{5}$

(B) $\frac{-\pi}{10}$

(C) $\frac{\pi}{10}$

(D) $\frac{3\pi}{5}$

3.

If $y = \sin^{-1}x$, $-1 \le x \le 0$, then the range of y is

(A) $\left(\frac{-\pi}{2}, 0\right)$ (B) $\left[\frac{-\pi}{2}, 0\right]$

(C) $\left[\frac{-\pi}{2}, 0\right)$ (D) $\left(\frac{-\pi}{2}, 0\right]$

4.

The value of $\cos\left(\frac{\pi}{6} + \cot^{-1}(-\sqrt{3})\right)$ is

(A) -1 (B) $\frac{-\sqrt{3}}{2}$

(C) 0 (D) 1

5.

The principal value of $\sin^{-1}\left(\sin\left(-\frac{10\pi}{3}\right)\right)$ is :

(A) $-\frac{2\pi}{3}$ (B) $-\frac{\pi}{3}$

(C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

6.

Domain of $\sin^{-1}(2x^2 - 3)$ is :

(A) $(-1, 0) \cup (1, \sqrt{2})$ (B) $(-\sqrt{2}, -1) \cup (0, 1)$

(C) $[-\sqrt{2}, -1] \cup [1, \sqrt{2}]$ (D) $(-\sqrt{2}, -1) \cup (1, \sqrt{2})$

7.

The principal branch of $\cos^{-1} x$ is :

(A) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ (B) $[\pi, 2\pi]$

(C) $[0, \pi]$ (D) $[2\pi, 3\pi]$

8.

The principal value of $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ is :

(A) $-\frac{\pi}{3}$

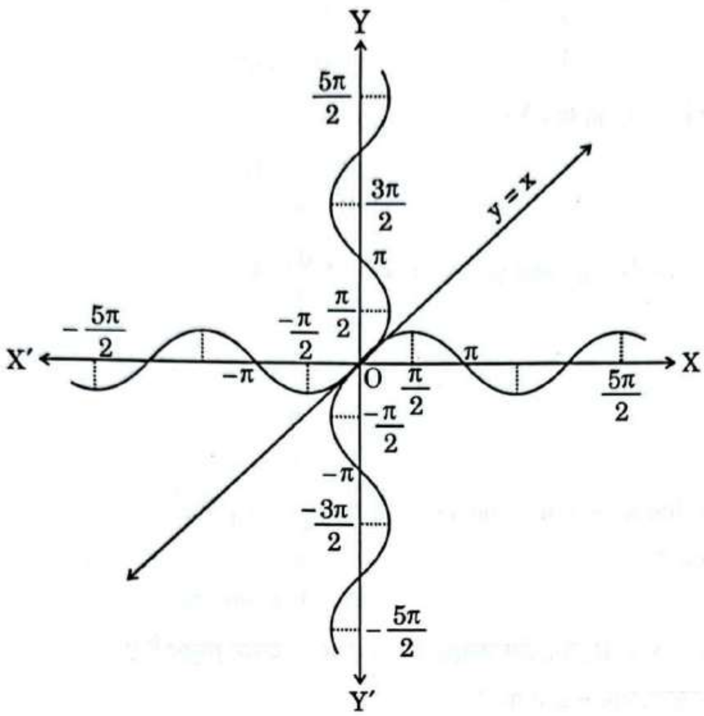
(B) $-\frac{2\pi}{3}$

(C) $\frac{\pi}{3}$

(D) $\frac{2\pi}{3}$

9.

The following graph is a combination of :



(A) $y = \sin^{-1} x$ and $y = \cos^{-1} x$

(B) $y = \cos^{-1} x$ and $y = \cos x$

(C) $y = \sin^{-1} x$ and $y = \sin x$

(D) $y = \cos^{-1} x$ and $y = \sin x$

10.

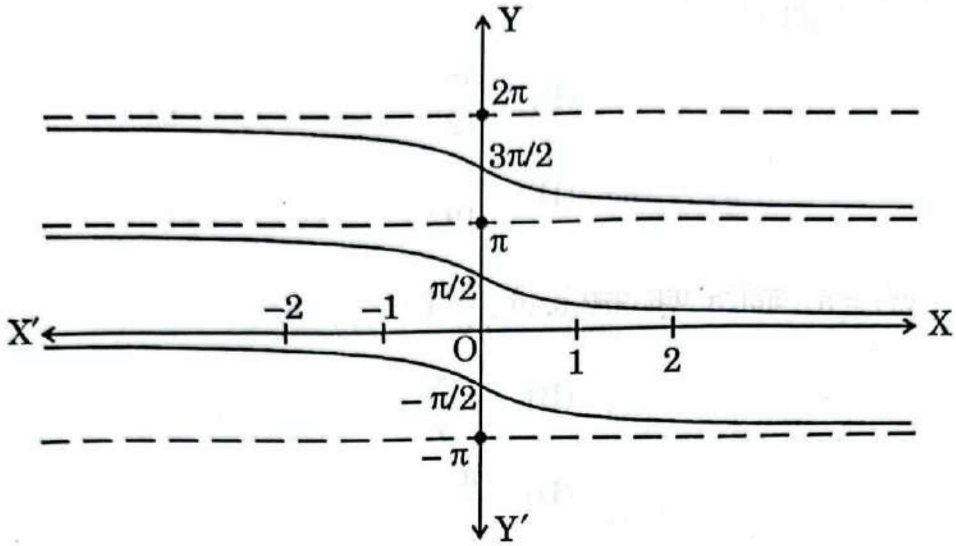
$\left[\sec^{-1}(-\sqrt{2}) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$ is equal to :

(A) $\frac{11\pi}{12}$ (B) $\frac{5\pi}{12}$

(C) $-\frac{5\pi}{12}$ (D) $\frac{7\pi}{12}$

11.

The graph shown below depicts :



(A) $y = \cot x$ (B) $y = \cot^{-1} x$

(C) $y = \tan x$ (D) $y = \tan^{-1} x$

12.

Domain of $f(x) = \cos^{-1} x + \sin x$ is :

(A) R (B) $(-1, 1)$

(C) $[-1, 1]$ (D) ϕ

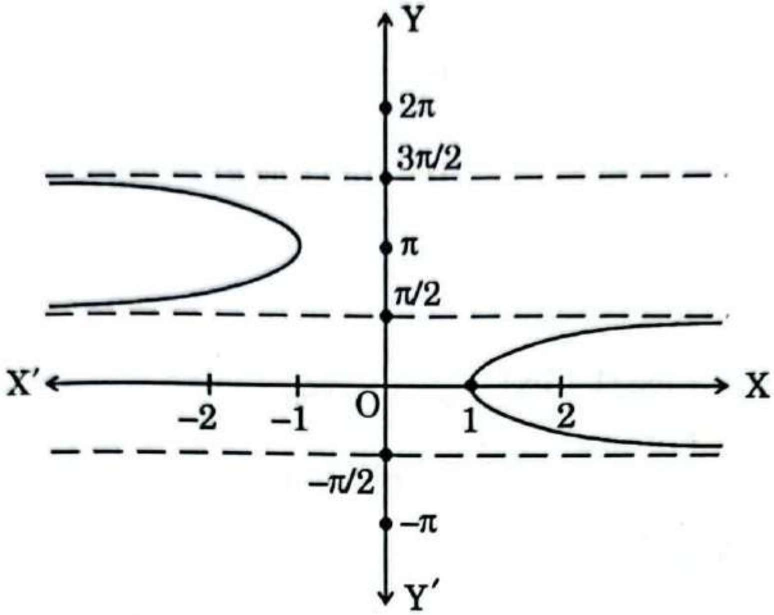
13.

Assertion (A) : Set of values of $\sec^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is a null set.

Reason (R) : $\sec^{-1} x$ is defined for $x \in R - (-1, 1)$.

14.

The graph shown below depicts :

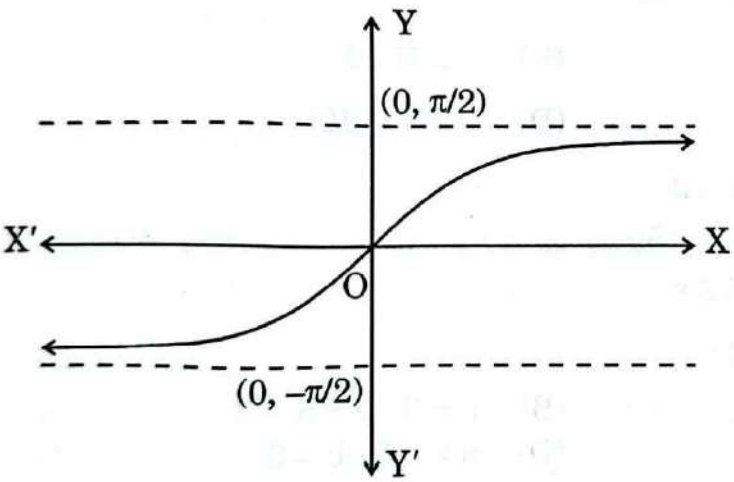


(A) $y = \sec^{-1} x$ (B) $y = \sec x$

(C) $y = \text{cosec}^{-1} x$ (D) $y = \text{cosec } x$

15.

The given graph illustrates :

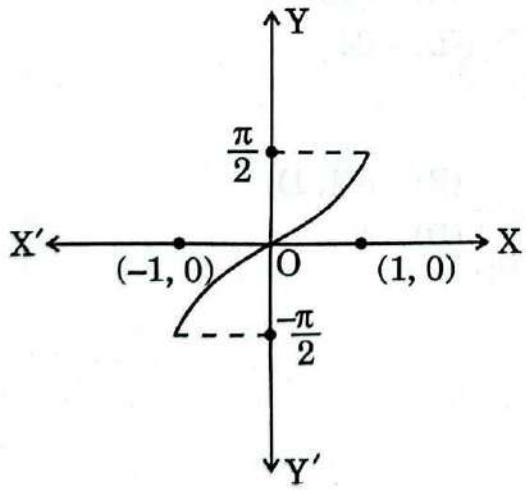


(A) $y = \tan^{-1} x$ (B) $y = \text{cosec}^{-1} x$

(C) $y = \cot^{-1} x$ (D) $y = \sec^{-1} x$

16.

Study the given graph. It illustrates :

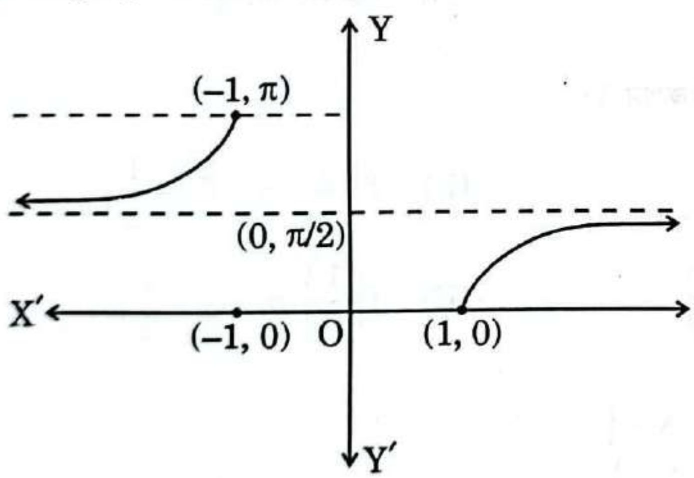


(A) $y = \tan^{-1} x$ (B) $y = \cos^{-1} x$

(C) $y = \sec^{-1} x$ (D) $y = \sin^{-1} x$

17.

The given graph illustrates :



(A) $y = \sec^{-1} x$ (B) $y = \cot^{-1} x$

(C) $y = \tan^{-1} x$ (D) $y = \text{cosec}^{-1} x$

I. Principal value problems :

1.c

Find the value of $\sin^{-1}\left[\sin\left(-\frac{17\pi}{8}\right)\right]$.

Sol.

$$\sin^{-1}\left[\sin\left(-\frac{17\pi}{8}\right)\right] = -\sin^{-1}\left[\sin\left(2\pi+\frac{\pi}{8}\right)\right]$$

$$= -\frac{\pi}{8}$$

prepared by : BALAJI KANCHI

1.b

Using principal value, evaluate the following: $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$

1.c

Find the value of $sin^{-1}[sin\left(\frac{13\pi}{7}\right)]$

1.d 2025

Evaluate : $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$

Sol.

$$\sin^{-1}\left(\sin\frac{3\pi}{5}\right) = \sin^{-1}\left(\sin\left(\pi-\frac{2\pi}{5}\right)\right)$$

$$= \sin^{-1}\left(\sin\left(\frac{2\pi}{5}\right)\right)$$

$$= \frac{2\pi}{5}$$

prepared by : BALAJI KANCHI

2. 2023

Evaluate :

$$\cos^{-1}\left[\cos\left(-\frac{7\pi}{3}\right)\right]$$

Sol.

$$\cos^{-1}\left[cos\left(-\frac{7\pi}{3}\right)\right] = \cos^{-1}\left[cos\left(\frac{7\pi}{3}\right)\right]$$

$$= \cos^{-1}\left[cos\left(2\pi + \frac{\pi}{3}\right)\right]$$

$$= \cos^{-1}\left[cos\frac{\pi}{3}\right] = \frac{\pi}{3}.$$

3.

What is the principal value of $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$?

4.a

Find the value of $\sin^{-1}\left(\cos\left(\frac{33\pi}{5}\right)\right)$.

4.b

Find the value of $\sin^{-1}\left(\cos\frac{43\pi}{5}\right)$.

4.c

Find the value of $\sin^{-1}\left(\cos\left(\frac{33\pi}{5}\right)\right)$.

II. Inverse Trigonometric functions of standard vales:

1.a

What is the principal value of $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$?

1.b

The principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is __________ .

2.a

Find the value of $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$.

Sol.

$$\frac{\pi}{3} - \left(\pi - \frac{\pi}{6}\right) = -\frac{\pi}{2}$$

Note: $\frac{1}{2}$m. for any one of the two correct values and

$\frac{1}{2}$m. for final answer

2.b

Find the value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$.

Sol.

$$\frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$$

$\frac{1}{2}$ for any one of $\frac{\pi}{3}$ or $\frac{2\pi}{3}$

3.a

Write the principal value of $\left[\cos^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\left(-\frac{1}{2}\right)\right]$.

3.b 2023

Evaluate : $3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)+2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)+\cos^{-1}(0)$

Sol.

$$\text{Given expression} = \frac{3\pi}{4}+\frac{2\pi}{6}+\frac{\pi}{2}$$
$$= \frac{19\pi}{12}$$

3.c

Find the principal value of $\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.

Sol.

$$\tan^{-1}(1)+\left[\pi-\cos^{-1}\left(\frac{1}{2}\right)\right]-\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{4}+\left(\pi-\frac{\pi}{3}\right)-\frac{\pi}{4}$$
$$=\frac{2\pi}{3}$$

3.cc

Find the value of $\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$.

Sol.

$$\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$$
$$=\frac{\pi}{4}+\frac{2\pi}{3}-\frac{\pi}{6}$$
$$=\frac{3\pi}{4}$$

3.c

Find the value of $\cos^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \text{cosec}^{-1}(-2)$.

Sol.

$$\cos^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \text{cosec}^{-1}(-2)$$

$$= \frac{\pi}{3} + \frac{\pi}{6} - \frac{\pi}{6}$$

$$= \frac{\pi}{3}$$

3.d

Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$.

Sol.

The given expression $= \frac{-\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4}$

$$= \frac{-\pi}{12}$$

3.f

Find the value of $\sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \cot^{-1}\left(\tan\frac{4\pi}{3}\right)$.

Sol.

The given expression $= \frac{-\pi}{6} + (\pi - \frac{\pi}{6}) + \frac{\pi}{6}$

$$= \frac{5\pi}{6}$$

3.g

Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$.

Sol.

$$cos^{-1}\left(-\frac{1}{2}\right) + 2\,sin^{-1}\left(\frac{1}{2}\right)$$
$$= \left(\pi - \frac{\pi}{3}\right) + 2\left(\frac{\pi}{6}\right)$$
$$= \pi$$

3.h

If $a = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right)$ and

$b = \tan^{-1}\left(\sqrt{3}\right) - \cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

then find the value of a + b.

Sol.

$$\mathbf{a = \frac{\pi}{4} + \frac{2\pi}{3},\ b = \frac{\pi}{3} - \frac{2\pi}{3}}$$
$$\mathbf{\therefore a + b = \frac{7\pi}{12}}$$

4.a 2023

Evaluate :

$$\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \cos^{-1}\left(\cos\frac{3\pi}{4}\right) + \tan^{-1}(1)$$

Sol.

Required value = $\frac{\pi}{4} + \frac{3\pi}{4} + \frac{\pi}{4}$

4.b 2023

Evaluate $\sin^{-1}\left(\sin\frac{3\pi}{4}\right)+\cos^{-1}(\cos\pi)+\tan^{-1}(1)$.

Sol.

$$\tan^{-1}\left[2\cos\left(2.\frac{\pi}{6}\right)\right]+\frac{\pi}{4}$$

$$=\tan^{-1}\left[2\times\frac{1}{2}\right]+\frac{\pi}{4}=\frac{\pi}{4}+\frac{\pi}{4}=\frac{\pi}{2}$$

Sol.

$$\sin^{-1}\left(\sin\frac{3\pi}{4}\right)+\cos^{-1}(\cos\pi)+\tan^{-1}(1)=\frac{\pi}{4}+\pi+\frac{\pi}{4}$$

$$=\frac{3\pi}{2}$$

4.c

Evaluate : $\sin^{-1}\left(\sin\frac{13\pi}{6}\right)+\cos^{-1}\left(\cos\frac{\pi}{3}\right)+\tan^{-1}(\sqrt{3})$

Sol.

$$\sin^{-1}\left(\sin\frac{13\pi}{6}\right)+\cos^{-1}\left(\cos\frac{\pi}{3}\right)+\tan^{-1}\left(\sqrt{3}\right)$$

$$=\frac{\pi}{6}+\frac{\pi}{3}+\frac{\pi}{3}$$

$$=\frac{5\pi}{6}$$

4.d

Find the value of

$$\tan^{-1}\left(\tan\frac{3\pi}{5}\right)+\cos^{-1}\left(\cos\frac{13\pi}{6}\right)+\sin^{-1}\left(-\frac{1}{2}\right).$$

Sol.

$$\tan^{-1}\left(\tan\frac{3\pi}{5}\right)+\cos^{-1}\left(\cos\frac{13\pi}{6}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$$

$$=-\frac{2\pi}{5}+\frac{\pi}{6}-\frac{\pi}{6}$$

$$=-\frac{2\pi}{5}$$

5.a

Evaluate: $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(\frac{1}{2}\right)\right]$

Answer:

$\sin\left[\frac{\pi}{3} + \frac{\pi}{6}\right] = 1$

5.b

Write the value of $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

5.c

Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right] + \tan^{-1} 1.$

5.d 2025

Evaluate : $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$

Sol.

$$\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$$

$$=\tan^{-1}\left[2\sin\left(2\times\frac{\pi}{6}\right)\right]=\tan^{-1}\left[2\sin\frac{\pi}{3}\right]$$

$$=\tan^{-1}\left[2\times\frac{\sqrt{3}}{2}\right]=\tan^{-1}\sqrt{3}=\frac{\pi}{3}$$

prepared by : BALAJI KANCHI

6.a

Find value of k if

$$\sin^{-1}\left[k\tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]=\frac{\pi}{3}.$$

Sol.

$$\mathbf{k\tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)=\sin\frac{\pi}{3}}$$

$$\Rightarrow\ \mathbf{k\tan\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}}$$

$$\Rightarrow\ \mathbf{k\sqrt{3}=\frac{\sqrt{3}}{2}\ \therefore k=\frac{1}{2}}$$

prepared by : BALAJI KANCHI

III. substituition based :

1.a 2024

$$\text{Evaluate : } \sec^2\left(\tan^{-1}\frac{1}{2}\right)+\operatorname{cosec}^2\left(\cot^{-1}\frac{1}{3}\right)$$

Sol.

$$\sec^2\left(\tan^{-1}\frac{1}{2}\right)+\operatorname{cosec}^2\left(\cot^{-1}\frac{1}{3}\right)$$

$$=\left[1+\tan^2\left(\tan^{-1}\frac{1}{2}\right)\right]+\left[1+\cot^2\left(\cot^{-1}\frac{1}{3}\right)\right]$$

$$=\left[1+\left(\frac{1}{2}\right)^2\right]+\left[1+\left(\frac{1}{3}\right)^2\right]$$

$$=\frac{85}{36}$$

1.b 2024

$$\text{Find the value of } \left[\sin^2\left\{\cos^{-1}\left(\frac{3}{5}\right)\right\}+\tan^2\left\{\sec^{-1}(3)\right\}\right].$$

Sol.

$$\text{Required value} = \left[1-\cos^2\left(\cos^{-1}\frac{3}{5}\right)\right]+\left[\sec^2\left(\sec^{-1}3\right)-1\right]$$

$$=\left(1-\frac{9}{25}\right)+(9-1)$$

$$=\frac{216}{25}$$

1.c 2024

$$\text{Evaluate : } \cot^2\left\{\operatorname{cosec}^{-1}3\right\}+\sin^2\left\{\cos^{-1}\left(\frac{1}{3}\right)\right\}$$

Sol.

$$\left[\operatorname{cosec}^2\left(\operatorname{cosec}^{-1}3\right)-1\right]+\left[1-\cos^2\left(\cos^{-1}\frac{1}{3}\right)\right]$$

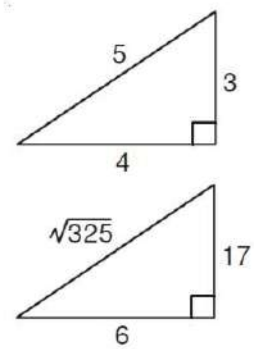
$$=(9-1)+\left(1-\frac{1}{9}\right)$$

$$=\frac{80}{9}$$

2.

Find the value of $\sin\left(\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right)$.

Sol.



$$\sin\left[\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{2}{3}\right)\right]$$

$$=\sin\left[\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{2}{3}\right)\right]$$

$$=\sin\left[\tan^{-1}\left(\frac{3/4+2/3}{1-3/4\cdot 2/3}\right)\right]=\sin\left[\tan^{-1}\left(\frac{17}{6}\right)\right]$$

$$=\sin\left[\sin^{-1}\left(\frac{17}{\sqrt{325}}\right)\right]=\frac{17}{\sqrt{325}}$$

3.

Evaluate : $\tan\left[2\tan^{-1}\frac{1}{5}-\frac{\pi}{4}\right]$

Sol.

$$\tan\left[\tan^{-1}\frac{\frac{2}{5}}{1-\frac{1}{25}}-\frac{\pi}{4}\right]=\tan\left[\tan^{-1}\frac{5}{12}-\tan^{-1}1\right]$$

$$=\tan\left[\tan^{-1}\frac{\frac{5}{12}-1}{1+\frac{5}{12}\times 1}\right]=\frac{-7}{17}$$

IV. Domain/Range Based problems :

1.a 2023

Find the domain of $y = \sin^{-1}(x^2 - 4)$.

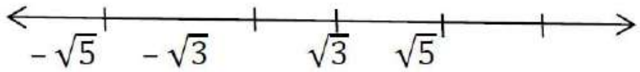
Sol.

Domain of $\sin^{-1} x$ is $-1 \le x \le 1$

$\therefore -1 \le x^2 - 4 \le 1$

or $x^2 \ge 3, x^2 \le 5$

$\Rightarrow x \ge \sqrt{3}$ or $x \le -\sqrt{3}$, $x \le \sqrt{5}$ or $x \ge -\sqrt{5}$



$\therefore$ Domain is $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

1.b 2024

Find the domain of the function $f(x) = \sin^{-1}(x^2 - 4)$. Also, find its range.

Sol.

$-1 \leqslant x^2 - 4 \leqslant 1$

$\Longrightarrow 3 \leqslant x^2 \leqslant 5$

Domain $= \left[-\sqrt{5}, -\sqrt{3}\right] \cup \left[\sqrt{3}, \sqrt{5}\right]$

1.c 2025

Find the domain of $\sin^{-1}(x^2 - 3)$.

Sol.

Domain of $\sin^{-1} x$ is $[-1, 1]$

$-1 \le x^2 - 3 \le 1 \Rightarrow 2 \le x^2 \le 4$

$\Rightarrow$ **Domain** $= [-2, -\sqrt{2}] \cup [\sqrt{2}, 2]$

1.d 2025

Find the domain of $f(x) = \sin^{-1}(-x^2)$.

Sol.

$-1 \le -x^2 \le 1 \Rightarrow -1 \le -x^2 \le 0$

$\Rightarrow 0 \le x^2 \le 1 \Rightarrow -1 \le x \le 1$

2. 2025

Find domain of $\sin^{-1}\sqrt{x-1}$.

Sol.

Here $-1 \le \sqrt{x-1} \le 1$

$\Rightarrow 0 \le x-1 \le 1 \Rightarrow 1 \le x \le 2$

Hence, domain is $x \in [1, 2]$

3.a 2024

Find the domain of $f(x) = \cos^{-1}(1 - x^2)$. Also, find its range.

Sol.

$-1 \leqslant 1 - x^2 \leqslant 1$

$\Rightarrow 0 \leqslant x^2 \leqslant 2$

Domain $= [-\sqrt{2}, \sqrt{2}]$

Range $= [0, \pi]$

3.b

Find the domain of the function $y = \cos^{-1}(x^2 - 4)$.

Sol.

Domain of $\cos^{-1}x$ is $[-1, 1]$

$\Rightarrow -1 \le x^2 - 4 \le 1 \Rightarrow 3 \le x^2 \le 5$

$\Rightarrow x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

4.

Find the domain of $\sec^{-1}(2x+1)$.

Sol.

Domain of $\sec^{-1} x$ is $(-\infty, -1] \cup [1, \infty)$

$$\Rightarrow 2x+1 \le -1 \text{ or } 2x+1 \ge 1 \Rightarrow x \le -1 \text{ or } x \ge 0$$

$$\Rightarrow \text{Domain} = (-\infty, -1] \cup [0, \infty)$$

prepared by : **BALAJI KANCHI**

5.

Write the domain and range (principle value branch) of the following functions : $f(x) = \tan^{-1} x$

Sol.

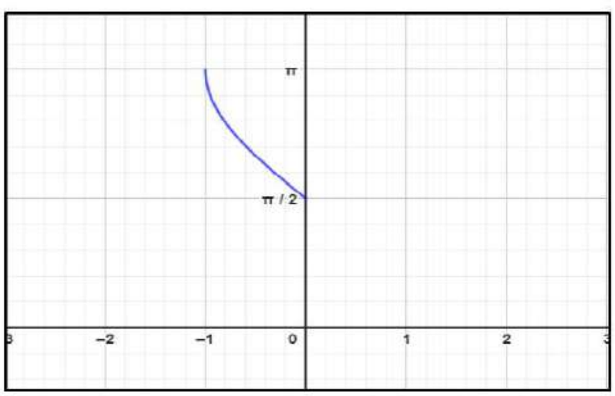
Domain = R ; Range = $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

V. Graph based problems :

1.

Draw the graph of $\cos^{-1} x$, where $x \in [-1, 0]$. Also, write its range.

Sol.

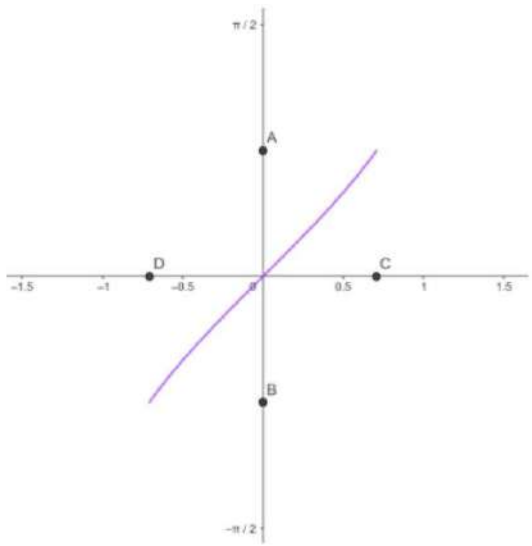


Range: $\left[\frac{\pi}{2}, \pi\right]$

2.

Draw the graph of $f(x) = \sin^{-1} x,\ x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$. Also, write range of f(x).

Sol.



Here, the points A, B, C and D are respectively $\left(0, \frac{\pi}{4}\right), \left(0, -\frac{\pi}{4}\right), \left(\frac{1}{\sqrt{2}}, 0\right), \left(-\frac{1}{\sqrt{2}}, 0\right)$.

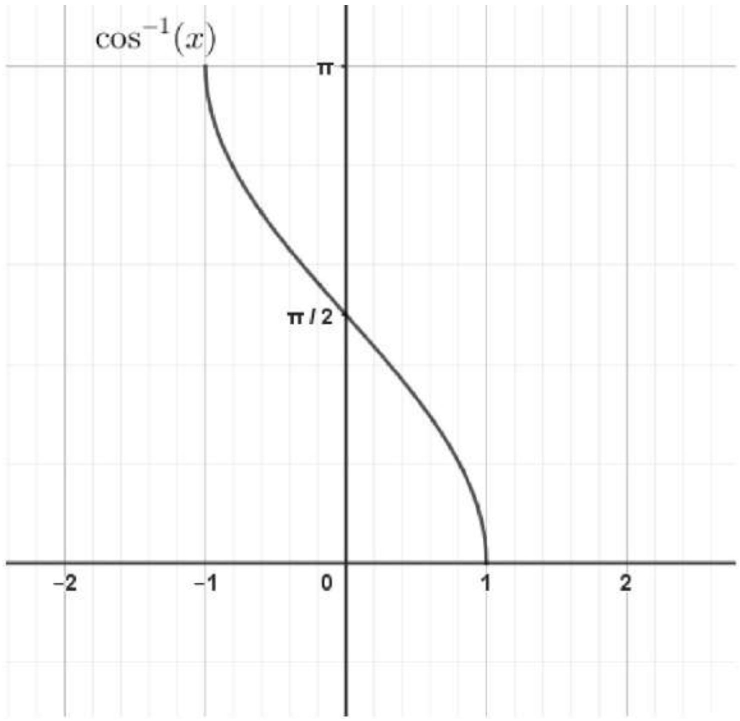
Range = $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

prepared by : **BALAJI KANCHI**

3.

Draw the graph of the principal branch of the function $f(x) = \cos^{-1} x$.

Sol.



VI. Converting to simplest form :

a. Algebraic substituition :

1.

Find the value of $\cot \frac{1}{2}\left[\cos^{-1}\frac{2x}{1+x^2} + \sin^{-1}\frac{1-y^2}{1+y^2}\right]$,

$|x| < 1,\ y > 0$ and $xy < 1$.

Sol.

$$\cot\frac{1}{2}\left[\cos^{-1}\left(\frac{2x}{1+x^2}\right)+\sin^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right]$$

$$=\cot\frac{1}{2}\left[\frac{\pi}{2}-\sin^{-1}\left(\frac{2x}{1+x^2}\right)+\frac{\pi}{2}-\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right]$$

$$=\cot\frac{1}{2}[\pi-2\tan^{-1}x-2\tan^{-1}y]$$

$$=\cot\left[\frac{\pi}{2}-(\tan^{-1}x+\tan^{-1}y)\right]$$

$$=\tan\left[\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]=\frac{x+y}{1-xy}$$

prepared by : **BALAJI KANCHI**

1.c

Prove that $3\cos^{-1}x=\cos^{-1}(4x^3-3x),\ x\in\left[\frac{1}{2},1\right]$.

1.b

Prove that : $3\sin^{-1}x = \sin^{-1}(3x - 4x^3),\ x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Sol.

In RHS, put $x = \sin\theta$

$$\text{RHS} = \sin^{-1}(3\sin\theta - 4\sin^3\theta)$$

$$= \sin^{-1}(\sin 3\theta)$$

$$= 3\theta = 3\sin^{-1}x = \text{LHS}.$$

2.

Prove that $\sin^{-1}(2x\sqrt{1-x^2}) = 2\cos^{-1}x,\ \frac{1}{\sqrt{2}} \le x \le 1.$

Sol.

Ans: Put $x = \cos\theta \Leftrightarrow \theta = \cos^{-1}x$

$$\text{L.H.S.} = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$$

$$= \sin^{-1}(2\cos\theta\sin\theta) = \sin^{-1}(\sin 2\theta) = 2\theta = 2\cos^{-1}x = \text{R.H.S.}$$

3.

Simplify $\sec^{-1}\left(\frac{1}{2x^2-1}\right),\ 0 < x < \frac{1}{\sqrt{2}}$.

4.

$$\text{Simplify} : \cos^{-1}x + \cos^{-1}\left[\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right];\ \frac{1}{2} \le x \le 1$$

Sol.

Let $\mathbf{x = \cos\theta}$,

$$\cos^{-1}x + \cos^{-1}\left[\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right]$$

$$= \theta + \cos^{-1}\left[\frac{\cos\theta}{2} + \frac{\sqrt{3}}{2}\times\sin\theta\right]$$

$$= \theta + \cos^{-1}\left[\cos\left(\frac{\pi}{3} - \theta\right)\right] = \theta + \frac{\pi}{3} - \theta = \frac{\pi}{3}$$

5.

$$\text{Prove that : } \tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x, -\frac{1}{\sqrt{2}} \le x \le 1$$

Sol.

$$\text{Put } x = \cos 2\theta \Rightarrow \theta = \frac{1}{2}\cos^{-1}x$$

$$\text{LHS} = \tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\theta\right)\right)$$

$$= \frac{\pi}{4} + \theta = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x = \text{RHS}$$

6.

Prove that :

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2;\ -1<x<1$$

Sol.

Put $x^2=\cos 2\theta \Rightarrow \theta=\frac{1}{2}\cos^{-1}x^2$

$$\text{LHS}=\tan^{-1}\left(\frac{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}\right)$$

$$=\tan^{-1}\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)=\tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$$

$$=\tan^{-1}\left(\tan\left(\frac{\pi}{4}+\theta\right)\right)=\frac{\pi}{4}+\theta$$

$$=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2,\ -1<x<1\ =\text{RHS}$$

prepared by : **BALAJI KANCHI**

7. 2025

Simplify $\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$.

Sol.

Put $x=\tan\theta \Rightarrow \theta=\tan^{-1}x$

Now $\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$

$$=\sin^{-1}\left(\frac{\tan\theta}{\sec\theta}\right)=\sin^{-1}(\sin\theta)$$

$$=\theta=\tan^{-1}x$$

prepared by : **BALAJI KANCHI**

b. Simplest form using Trigonometric formulas :

1.a 2023

Simplify :

$$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$$

Sol.

$$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)=\tan^{-1}\left(\frac{\sin\left(\frac{\pi}{2}-x\right)}{1-\cos\left(\frac{\pi}{2}-x\right)}\right)$$

$$=\tan^{-1}\left(\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin^{2}\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right)$$

$$=\tan^{-1}\left(\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)=\tan^{-1}\left(\tan\left(\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)\right)$$

$$=\frac{\pi}{4}+\frac{x}{2}$$

1.b

Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$, where $\frac{-\pi}{2}<x<\frac{\pi}{2}$ in the simplest form.

Sol.

$$y=\tan^{-1}\left[\frac{\cos x}{1-\sin x}\right]=\tan^{-1}\left[\frac{\cos^{2}\frac{x}{2}-\sin^{2}\frac{x}{2}}{(\cos\frac{x}{2}-\sin\frac{x}{2})^{2}}\right]$$

$$y=\tan^{-1}\left[\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right]=\tan^{-1}\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]=\left(\frac{\pi}{4}+\frac{x}{2}\right)$$

2.a

.Prove the following:

$$\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]=\frac{x}{2}, x\in\left(0,\frac{\pi}{4}\right)$$

2.b

Reduce $\cot^{-1}\left\{\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right\}$ where $\frac{\pi}{2}<x<\pi$ in to simplest form.

2.c

Prove that: $\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right)=\frac{\pi}{4}-\frac{x}{2}$, where $\pi<x<\frac{3\pi}{2}$

3.

Express $\sin^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right)$;where$-\frac{\pi}{4}<x<\frac{\pi}{4}$, in the simplest form.

VII. Proving Problems :

1.

Prove that :

$$\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)=\frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

Sol.

$$\text{LHS}=\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}$$

$$=\frac{9}{4}\left[\frac{\pi}{2}-\sin^{-1}\frac{1}{3}\right]=\frac{9}{4}\cos^{-1}\frac{1}{3}$$

$$=\frac{9}{4}\sin^{-1}\left(\sqrt{1-\left(\frac{1}{3}\right)^2}\right)=\frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)=\text{RHS}$$

2.

Prove the following:

$$\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x\in(0,1)$$

Sol.

$$\text{RHS}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$$

Put $x=\tan^2\theta$ or $\sqrt{x}=\tan\theta$

$$\text{RHS}=\frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)=\frac{1}{2}(2\theta)$$

$$=\theta=\tan^{-1}\sqrt{x}=\text{LHS}$$

OR

$$\text{LHS} = \tan^{-1}(\sqrt{x}) = \frac{1}{2}.2\tan^{-1}(\sqrt{x})$$

$$= \frac{1}{2}.\cos^{-1}\left(\frac{1-(\sqrt{x})^2}{1+(\sqrt{x})^2}\right) = \frac{1}{2}.\cos^{-1}\left(\frac{1-x}{1+x}\right)$$

$$= \text{RHS}$$

prepared by : **BALAJI KANCHI**

3.

Prove that : $\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$

Sol.

LHS becomes $\tan^{-1}\frac{5}{12} + \tan^{-1}\frac{3}{4}$

$$= \tan^{-1}\left(\frac{\frac{5}{12}+\frac{3}{4}}{1-\frac{5}{12}\cdot\frac{3}{4}}\right) = \tan^{-1}\frac{56}{33}$$

$$= \sin^{-1}\frac{56}{65} = \text{RHS}$$

4.

Prove that $\tan\left\{\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}+\tan\left\{\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}=\frac{2b}{a}$

Sol.

Let $\frac{1}{2}\cos^{-1}\frac{a}{b}=x$

$$\text{LHS}=\tan\left(\frac{\pi}{4}+x\right)+\tan\left(\frac{\pi}{4}-x\right)=\frac{1+\tan x}{1-\tan x}+\frac{1-\tan x}{1+\tan x}$$

$$=\frac{2(1+\tan^2 x)}{1-\tan^2 x}=\frac{2}{\cos 2x}$$

$$=\frac{2b}{a}=\text{RHS}$$

VIII. solving ITF equations :

1.a 2025

Solve for x, $2\tan^{-1}x+\sin^{-1}\left(\frac{2x}{1+x^2}\right)=4\sqrt{3}$

Sol.

$$2\tan^{-1}x+\sin^{-1}\left(\frac{2x}{1+x^2}\right)=4\sqrt{3}\Rightarrow 2\tan^{-1}x+2\tan^{-1}x=4\sqrt{3}$$

$$\Rightarrow \tan^{-1}x=\sqrt{3}\notin\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$$

$\therefore x=\tan\left(\sqrt{3}\right)$ **which has no solution.**

prepared by : BALAJI KANCHI

1.b

Solve for x : $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x, x>0$

1.c

Find the real solutions of the equation $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x,\ (x>0)$.

1.d

Prove that :

$$\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right),\ x\in[0,1]$$

Sol.

Put $x=tan^2\theta \Rightarrow \theta = tan^{-1}\sqrt{x}$

RHS $=\frac{1}{2}cos^{-1}\left(\frac{1-tan^2\theta}{1+tan^2\theta}\right)$

$=\frac{1}{2}cos^{-1}(cos\,2\theta)$

$=\frac{1}{2}(2\theta)$

$=\theta = tan^{-1}\sqrt{x}=$ LHS

2.

Solve the following for x :

$$\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$$

Sol.

$\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}\Rightarrow(1-x)=\sin\left(\frac{\pi}{2}+2\sin^{-1}x\right)$

$\Rightarrow(1-x)=\cos\left(2\sin^{-1}x\right)\Rightarrow 1-x=1-2x^2$

$\therefore 2x^2-x=0\Rightarrow x=0, x=\frac{1}{2}$

since $x=\frac{1}{2}$ does not satisty the given equation

$\therefore x=0$ is the required solution.

OR

$$\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}\Rightarrow \sin^{-1}(1-x)=\frac{\pi}{2}+2\sin^{-1}x$$

$$\Rightarrow \quad 1-x=\sin\left(\frac{\pi}{2}+2\sin^{-1}x\right)$$

$$\Rightarrow \quad 1-x=\cos(2\sin^{-1}x)=1-2\sin^{2}(\sin^{2}x)$$

$$\Rightarrow \quad 1-x=1-2x^{2} \quad \therefore x=0,\frac{1}{2}$$

$$\Rightarrow \quad \text{x}=\frac{1}{2} \text{ does not satisfy the given equation } \therefore \text{x}\neq\frac{1}{2}, x=0$$

2.b

Solve the equation for x: $\sin^{-1}\left(\frac{5}{x}\right)+\sin^{-1}\left(\frac{12}{x}\right)=\frac{\pi}{2} \quad (x\neq 0)$

Answer:

Given equation can be written as

$\sin^{-1}\left(\frac{12}{x}\right)=\frac{\pi}{2}-\sin^{-1}\left(\frac{5}{x}\right)\Rightarrow \sin^{-1}\left(\frac{12}{x}\right)=\cos^{-1}\left(\frac{5}{x}\right)$

$\therefore \sin^{-1}\left(\frac{12}{x}\right)=\sin^{-1}\left(\frac{\sqrt{x^{2}-25}}{x}\right)$

$\Rightarrow \frac{12}{x}=\frac{\sqrt{x^{2}-25}}{x}$

$\Rightarrow x^{2}-25=144 \quad \Rightarrow x=\pm 13,$

since $x=-13$ does not satisfy the given equation,

$\therefore$ required solution is $x=13$.

2.c

If $\sin^{-1}\left(\frac{3}{x}\right) + \sin^{-1}\left(\frac{4}{x}\right) = \frac{\pi}{2}$, then find the value of x.

Sol.

$$\Rightarrow \sin^{-1}\left(\frac{3}{x}\right) = \sin^{-1}\left(\sqrt{1-\frac{16}{x^2}}\right) \Rightarrow \left(\frac{3}{x}\right)^2 = \frac{x^2-16}{x^2}$$

$$\Rightarrow x^2 = 25 \Rightarrow x = \pm 5,\ x = -5 \text{ (rejected)} \ \therefore\ x = 5$$

2.d

If $\tan^{-1} x - \cot^{-1} x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$, $x > 0$, find the value of x and hence find the value of $\sec^{-1}\left(\frac{2}{x}\right)$.

Sol.

Given $\tan^{-1} x - \cot^{-1} x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right), x > 0$

$$\Rightarrow \tan^{-1} x - \left(\frac{\pi}{2} - \tan^{-1} x\right) = \frac{\pi}{6}$$

$$\Rightarrow 2\tan^{-1} x = \frac{2\pi}{3} \Rightarrow \tan^{-1} x = \frac{\pi}{3}$$

$$\Rightarrow x = \tan\left(\frac{\pi}{3}\right) = \sqrt{3} \qquad \therefore \sec^{-1}\frac{2}{x} = \sec^{-1}\frac{2}{\sqrt{3}} = \frac{\pi}{6}$$

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2.e

If $\sin^{-1}\left(\dfrac{\sin x + \cos x}{\sqrt{2}}\right) = \dfrac{\pi}{2}$, find the value of x.

Sol.

$$\sin^{-1}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right) = \frac{\pi}{2}$$

$$\sin^{-1}\left(\sin\left(x + \frac{\pi}{4}\right)\right) = \frac{\pi}{2}$$

$$x = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

2.e

Find the value of x, if $\tan\left[\sec^{-1}\left(\dfrac{1}{x}\right)\right] = \sin(\tan^{-1} 2),\ x > 0$.

Sol.

$$\tan(\sec^{-1}\frac{1}{x}) = \sin(\tan^{-1} 2) \Rightarrow$$

$$\tan\left(\tan^{-1}\frac{\sqrt{1-x^2}}{x}\right) = \sin\left(\sin^{-1}\frac{2}{\sqrt{5}}\right)$$

$$\Rightarrow \frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}} \Rightarrow \frac{1-x^2}{x^2} = \frac{4}{5}$$

$$\Rightarrow 9x^2 = 5 \Rightarrow x^2 = \frac{5}{9} \Rightarrow x = \frac{\sqrt{5}}{3}, \{x > 0\}$$

prepared by : **BALAJI KANCHI**

3.a

Solve the following equation for x :

$$\cos(\tan^{-1} x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$$

Sol.

$$\cos\,(\tan^{-1} x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$$

$$\Rightarrow \quad \cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right) = \sin\left(\sin^{-1}\frac{4}{5}\right)$$

$$\Rightarrow \quad \frac{1}{\sqrt{1+x^2}} = \frac{4}{5}$$

$$\Rightarrow \quad x = \pm\frac{3}{4}$$

3.b

Find the value of x, if

$$\sin\,[\cot^{-1}(x + 1)] = \cos\,(\tan^{-1} x).$$

4.

If $2\tan^{-1}(\cos\theta) = \tan^{-1}(2\,\mathrm{cosec}\,\theta)$, $(\theta \neq 0)$, then find the value of θ.

5.

If $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$, then find x.

6.

If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$, x, y, z, > 0, then find the value of xy + yz + zx.

7.

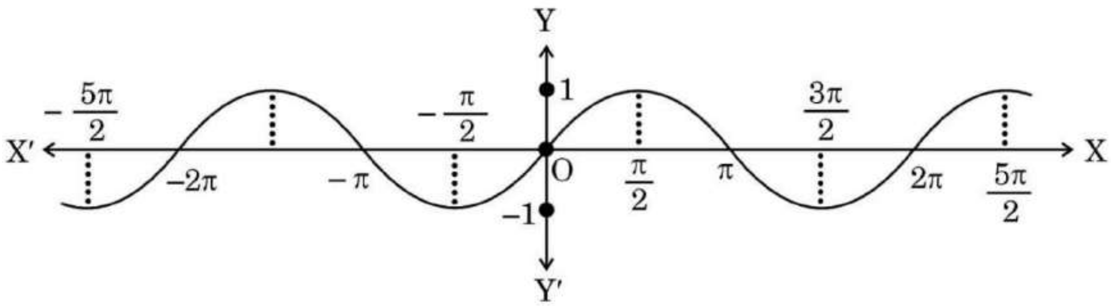
If $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$, then find the value of $\alpha(\beta+\gamma)-\beta(\gamma+\alpha)+\gamma(\alpha+\beta)$.

Case Study :

1. 2024

If a function $f : X \to Y$ defined as $f(x) = y$ is one-one and onto, then we can define a unique function $g : Y \to X$ such that $g(y) = x$, where $x \in X$ and $y = f(x)$, $y \in Y$. Function g is called the inverse of function f.

The domain of sine function is R and function sine : $R \to R$ is neither one-one nor onto. The following graph shows the sine function.



Let sine function be defined from set A to $[-1, 1]$ such that inverse of sine function exists, i.e., $\sin^{-1} x$ is defined from $[-1, 1]$ to A.

On the basis of the above information, answer the following questions :

(i) If A is the interval other than principal value branch, give an example of one such interval.

(ii) If $\sin^{-1}(x)$ is defined from $[-1, 1]$ to its principal value branch, find the value of $\sin^{-1}\left(-\frac{1}{2}\right) - \sin^{-1}(1)$.

(iii) (a) Draw the graph of $\sin^{-1} x$ from $[-1, 1]$ to its principal value branch.

Sol.

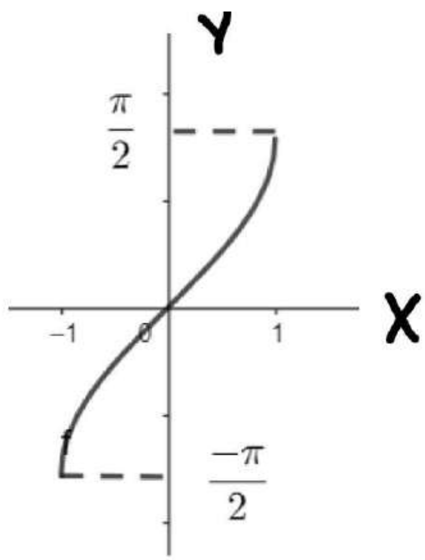
(i) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ or any other interval corresponding to the domain [-1,1]

(ii) $\sin^{-1}\left(\frac{-1}{2}\right) - \sin^{-1}(1)$

$= \frac{-\pi}{6} - \frac{\pi}{2}$

$= \frac{-4\pi}{6}$ or $\frac{-2\pi}{3}$

(iii) (a)



OR

(b) $f(x) = 2\,sin^{-1}(1 - x)$

$-1 \leq 1 - x \leq 1$

$\Rightarrow -2 \leq -x \leq 0$

$\Rightarrow 0 \leq x \leq 2$

Domain $= [0, 2]$

$\frac{-\pi}{2} \leq \sin^{-1}(1 - x) \leq \frac{\pi}{2}$

$-\pi \leq 2\,\sin^{-1}(1 - x) \leq \pi$

So range $= [-\pi, \pi]$