4. Determinants

(Previous Year Questions solutions from 2015-2025)

McQ's :

1.

If A is a square matrix of order 3 and $|A| = 5$, then the value of $|2A'|$ is

(A) -10
(B) 10
(C) -40
(D) 40

2.

If A is a non-singular square matrix of order 3 such that $A^2 = 3A$, then value of $|A|$ is

(A) -3
(B) 3
(C) 9
(D) 27

3.

If $A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, then the value of $|\text{adj } A|$ is

(A) 64
(B) 16
(C) 0
(D) -8

4.

If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$, then the value of x is

(a) 3 (b) 0 (c) -1 (d) 1

5.

If A is a 3×3 matrix such that $|A| = 8$, then $|3A|$ equals.

(a) 8 (b) 24 (c) 72 (d) 216

6.

$$\begin{vmatrix} 43 & 44 & 45 \\ 44 & 45 & 46 \\ 45 & 46 & 47 \end{vmatrix} \text{ equals}$$

(A) 0

(B) -1

(C) 1

(D) 2

7.

The matrix $\begin{bmatrix} 2 & -1 & 3 \\ \lambda & 0 & 7 \\ -1 & 1 & 4 \end{bmatrix}$ is not invertible for

(A) $\lambda = -1$

(B) $\lambda = 0$

(C) $\lambda = 1$

(D) $\lambda \in R - \{1\}$

8.

Let $A = \begin{bmatrix} 200 & 50 \\ 10 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 50 & 40 \\ 2 & 3 \end{bmatrix}$, then $|AB|$ is equal to

(a) 460 (b) 2000 (c) 3000 (d) -7000

9.

If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then det (adj A) equals

(a) a^{27} (b) a^9 (c) a^6 (d) a^2

10.

If A is a square matrix of order 3, such that A (adj A) = 10 I, then |adj A| is equal to

(a) 1 (b) 10 (c) 100 (d) 101

11.

If A is a skew symmetric matrix of order 3, then the value of |A| is

(a) 3 (b) 0 (c) 9 (d) 27

12.

If $\begin{vmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 0$, then the value of α is

(a) 1 (b) 2

(c) 3 (d) 4

13.

The value of the determinant $\begin{vmatrix} 6 & 0 & -1 \\ 2 & 1 & 4 \\ 1 & 1 & 3 \end{vmatrix}$ is

(a) 10 (b) 8

(c) 7 (d) –7

14.

If for a square matrix A, $A^2 - A + I = O$, then A^{-1} equals

(a) A (b) A + I

(c) I – A (d) A – I

15.

If A is a square matrix of order 3 and $|A| = 6$, then the value of $|\text{adj } A|$ is :

(a) 6 (b) 36

(c) 27 (d) 216

16.

For which value of x, are the determinants $\begin{vmatrix} 2x & -3 \\ 5 & x \end{vmatrix}$ and $\begin{vmatrix} 10 & 1 \\ -3 & 2 \end{vmatrix}$ equal ?

(a) ± 3 (b) -3 (c) ± 2 (d) 2

17.

The value of the cofactor of the element of second row and third column in the matrix $\begin{bmatrix} 4 & 3 & 2 \\ 2 & -1 & 0 \\ 1 & 2 & 3 \end{bmatrix}$ is :

(a) 5 (b) -5 (c) -11 (d) 11

18.

If $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$, then A^{-1} is given by :

(a) $\begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix}$

19.

In the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, M_{23} is :

(where M_{ij} denotes the minor of element a_{ij}

(a) 7	(b) – 13
(c) 13	(d) – 7

20.

Value of k, for which A = $\begin{bmatrix} k & 8 \\ 4 & 2k \end{bmatrix}$ is a singular matrix is:

a) 4	b) -4
c) ±4	d) 0

21.

For matrix A =$\begin{bmatrix} 2 & 5 \\ -11 & 7 \end{bmatrix}$, $(adjA)'$ is equal to:

a) $\begin{bmatrix} -2 & -5 \\ 11 & -7 \end{bmatrix}$	b) $\begin{bmatrix} 7 & 5 \\ 11 & 2 \end{bmatrix}$
c) $\begin{bmatrix} 7 & 11 \\ -5 & 2 \end{bmatrix}$	d) $\begin{bmatrix} 7 & -5 \\ 11 & 2 \end{bmatrix}$

22.

If A = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, then:

a) $A^{-1} = B$	b) $A^{-1} = 6B$
c) $B^{-1} = B$	d) $B^{-1} = \frac{1}{6}A$

23.

Given that A is a non-singular matrix of order 3 such that $A^2 = 2A$, then value of $|2A|$ is:

a) 4	b) 8
c) 64	d) 16

24.

For A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then $14A^{-1}$ is given by:

a) $14\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$	b) $\begin{bmatrix} 4 & -2 \\ 2 & 6 \end{bmatrix}$
c) $2\begin{bmatrix} 2 & -1 \\ 1 & -3 \end{bmatrix}$	d) $2\begin{bmatrix} -3 & -1 \\ 1 & -2 \end{bmatrix}$

25.

Given that A is a square matrix of order 3 and $|A| = -2$, then $|adj(2A)|$ is equal to

(a) -2^6 (b) $+4$ (c) -2^8 (d) 2^8

2023 March:

1.

If for a square matrix A, $A^2 - 3A + I = O$ and $A^{-1} = xA + yI$, then the value of x + y is :

(a) -2 (b) 2

(c) 3 (d) -3

2.

If $|A| = 2$, where A is a 2×2 matrix, then $|4A^{-1}|$ equals :

(a) 4 (b) 2

(c) 8 (d) $\frac{1}{32}$

3.

Let A be a 3×3 matrix such that $|\text{adj } A| = 64$. Then $|A|$ is equal to :

(a) 8 only (b) -8 only

(c) 64 (d) 8 or -8

4.

If $\left|\frac{A^{-1}}{2}\right| = \frac{1}{k|A|}$, where A is a 3×3 matrix, then the value of k is :

(a) $\frac{1}{8}$ (b) 8

(c) 2 (d) $\frac{1}{2}$

5.

Let A be a 3×3 matrix such that $|\text{adj } A| = 64$. Then $|A|$ is equal to :

(a) 8 only (b) – 8 only

(c) 64 (d) 8 or – 8

6.

If $\begin{bmatrix} x & 2 \\ 3 & x-1 \end{bmatrix}$ is a singular matrix, then the product of all possible values of x is :

(a) 6 (b) – 6

(c) 0 (d) – 7

7.

If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then A^{2023} is equal to

(A) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 2023 \\ 0 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 2023 & 0 \\ 0 & 2023 \end{bmatrix}$

8.

If (a, b), (c, d) and (e, f) are the vertices of ΔABC and Δ denotes the area of ΔABC, then $\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2$ is equal to

(A) $2\Delta^2$ (B) $4\Delta^2$

(C) 2Δ (D) 4Δ

9.

If $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{bmatrix}$ is non-singular matrix and $a \in A$, then the set A is

(A) $\mathbb{R}$ (B) $\{0\}$

(C) $\{4\}$ (D) $\mathbb{R} - \{4\}$

10.

If $|A| = |kA|$, where A is a square matrix of order 2, then sum of all possible values of k is

(A) 1 (B) -1

(C) 2 (D) 0

11.

Number of symmetric matrices of order 3×3 with each entry 1 or -1 is

(A) 512 (B) 64

(C) 8 (D) 4

12.

If $A . (\text{adj } A) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then the value of $|A| + |\text{adj } A|$ is equal to :

(a) 12 (b) 9

(c) 3 (d) 27

13.

Let A be a skew-symmetric matrix of order 3. If $|A| = x$, then $(2023)^x$ is equal to :

(a) 2023 (b) $\frac{1}{2023}$

(c) $(2023)^2$ (d) 1

14.

If the area of a triangle with vertices $(2, -6)$, $(5, 4)$ and $(k, 4)$ is 35 sq units, then k is

(a) 12 (b) -2

(c) $-12, -2$ (d) $12, -2$

15.

The value of $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$ is

(a) 0 (b) 1

(c) $x + y + z$ (d) $2(x + y + z)$

16.

The value of the determinant $\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$ is :

(a) 47 (b) -79

(c) 49 (d) -51

17.

Let A be the area of a triangle having vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3). Which of the following is correct ?

(a) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm A$ (b) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm 2A$

(c) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm \frac{A}{2}$ (d) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = A^2$

18.

The value of the cofactor of the element of second row and third column in the matrix $\begin{bmatrix} 4 & 3 & 2 \\ 2 & -1 & 0 \\ 1 & 2 & 3 \end{bmatrix}$ is :

(a) 5 (b) -5 (c) -11 (d) 11

19.

If A is a square matrix of order 3 and $|A| = 6$, then the value of $|\text{adj } A|$ is :

(a) 6 (b) 36

(c) 27 (d) 216

20.

For which value of x, are the determinants $\begin{vmatrix} 2x & -3 \\ 5 & x \end{vmatrix}$ and $\begin{vmatrix} 10 & 1 \\ -3 & 2 \end{vmatrix}$ equal ?

(a) ± 3 (b) -3 (c) ± 2 (d) 2

21.

A and B are square matrices each of order 3 such that $|A| = -1$ and $|B| = 3$. What is the value of $|3AB|$?

(a) -9 (b) -18

(c) -27 (d) -81

22.

If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$, then the value of x is :

(a) -1 (b) 0

(c) 1 (d) 3

23.

The value of x for which the points $(2, -1)$, $(-3, 4)$ and $(x, 5)$ are collinear, is :

(a) -4 (b) 4

(c) 2 (d) -2

24.

Assertion (A) : If A is a square matrix of order 3 such that $|\text{adj } A| = 144$, then the value of $|A|$ is ± 12.

Reason (R) : If A is an invertible matrix of order n, then $|\text{adj } A| = |A|^{n-1}$.

25.

If $A = \begin{bmatrix} 4 & -3 \\ 9 & -3 \end{bmatrix}$, then $|A|$ is equal to :

(a) 15 (b) −42

(c) 0 (d) 25

26.

If A is any square matrix of order 3×3 such that $|A| = 3$, then the value of $|\text{adj}A|$ is ?

(a) 3 (b) $\frac{1}{3}$ (c) 9 (d) 27

27.

If A be a 3×3 square matrix such that $A(adj\ A) = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ then the value of $|Adj\ A|$ is

(a) 5 (b) 25

(c) 125 (d) 625

2024 March :

Mark MCQ's :

1.

$$\begin{vmatrix} x+1 & x-1 \\ x^2+x+1 & x^2-x+1 \end{vmatrix}$$ is equal to :

(A) $2x^3$ (B) 2

(C) 0 (D) $2x^3-2$

2.

Assertion (A) : For matrix $A = \begin{bmatrix} 1 & \cos\theta & 1 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{bmatrix}$, where $\theta \in [0, 2\pi]$,

$|A| \in [2, 4]$.

Reason (R) : $\cos\theta \in [-1, 1], \forall\, \theta \in [0, 2\pi]$.

3.

If a_{ij} and A_{ij} represent the $(ij)^{th}$ element and its cofactor of $\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$ respectively, then the value of $a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23}$ is :

(A) 0 (B) -28

(C) 114 (D) -114

4.

If $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = kabc$, then the value of k is :

(A) 0 (B) 1

(C) 2 (D) 4

5.

If $\begin{vmatrix} 1 & 3 & 1 \\ k & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \pm 6$, then the value of k is :

(A) 2 (B) -2 (C) ± 2 (D) ∓ 2

6.

The value of $\begin{vmatrix} 8 & 2 & 7 \\ 12 & 3 & 5 \\ 16 & 4 & 3 \end{vmatrix}$ is :

(A) 0 (B) 2

(C) 7 (D) -2

Question Paper code: 65/4/1,2,3

7.

If $A = \begin{bmatrix} -2 & 0 & 0 \\ 1 & 2 & 3 \\ 5 & 1 & -1 \end{bmatrix}$, then the value of $|A (\text{adj. } A)|$ is

(A) 100 I (B) 10 I

(C) 10 (D) 1000

8.

Given that $A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$, matrix A is :

(A) $7\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(C) $\frac{1}{7}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ (D) $\frac{1}{49}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

9.

If $A = [a_{ij}] = \begin{bmatrix} 2 & -1 & 5 \\ 1 & 3 & 2 \\ 5 & 0 & 4 \end{bmatrix}$ and c_{ij} is the cofactor of element a_{ij}, then the value of $a_{21} \cdot c_{11} + a_{22} \cdot c_{12} + a_{23} \cdot c_{13}$ is :

(A) -57 (B) 0
(C) 9 (D) 57

10.

For the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ \lambda & 2 & 0 \\ 1 & -2 & 3 \end{bmatrix}$ to be invertible, the value of λ is :

(A) 0 (B) 10
(C) $\mathbb{R} - \{10\}$ (D) $\mathbb{R} - \{-10\}$

11.

If A is a square matrix of order 2 and $|A| = -2$, then value of $|5A'|$ is :

(A) -50 (B) -10
(C) 10 (D) 50

12.

If A is a square matrix of order 3 such that the value of $|\text{adj} \cdot A| = 8$, then the value of $|A^T|$ is :

(A) $\sqrt{2}$ (B) $-\sqrt{2}$
(C) 8 (D) $2\sqrt{2}$

13.

If inverse of matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then value of λ is :

(A) -4 (B) 1
(C) 3 (D) 4

14.

Let $f(x) = \begin{vmatrix} x^2 & \sin x \\ p & -1 \end{vmatrix}$, where p is a constant. The value of p for which $f'(0) = 1$ is :

(A) $\mathbb{R}$ (B) 1

(C) 0 (D) -1

2025 March ;

1 Mark MCQ's :

1.

If A is a square matrix of order 2 such that det (A) = 4, then det (4 adj A) is equal to :

(A) 16 (B) 64

(C) 256 (D) 512

2.

If A and B are invertible matrices, then which of the following is not correct ?

(A) $(A + B)^{-1} = B^{-1} + A^{-1}$ (B) $(AB)^{-1} = B^{-1}A^{-1}$

(C) $\text{adj}(A) = |A|\, A^{-1}$ (D) $|A|^{-1} = |A^{-1}|$

3.

If A is a square matrix of order 3 such that det(A) = 9, then det($9\,A^{-1}$) is equal to

(A) 9 (B) 9^2

(C) 9^3 (D) 9^4

4.

A system of linear equations is represented as $AX = B$, where A is coefficient matrix, X is variable matrix and B is the constant matrix. Then the system of equations is

(A) Consistent, if $|A| \neq 0$, solution is given by $X = BA^{-1}$.

(B) Inconsistent if $|A| = 0$ and $(\text{adj } A)B = 0$

(C) Inconsistent if $|A| \neq 0$

(D) May or may not be consistent if $|A| = 0$ and $(\text{adj } A)B = 0$

5.

If M and N are square matrices of order 3 such that $\det(M) = m$ and $MN = mI$, then $\det(N)$ is equal to :

(A) -1 (B) 1

(C) $-m^2$ (D) m^2

6.

Let P be a skew-symmetric matrix of order 3. If $\det(P) = \alpha$, then $(2025)^{\alpha}$ is

(A) 0 (B) 1

(C) 2025 (D) $(2025)^3$

7.

If A and B are invertible matrices of order 3×3 such that $\det(A) = 4$ and $\det[(AB)^{-1}] = \frac{1}{20}$, then $\det(B)$ is equal to :

(A) $\frac{1}{20}$ (B) $\frac{1}{5}$

(C) 20 (D) 5

8.

If $\begin{vmatrix} -1 & 2 & 4 \\ 1 & x & 1 \\ 0 & 3 & 3x \end{vmatrix} = -57$, the product of the possible values of x is :

(A) -24 (B) -16

(C) 16 (D) 24

9.

If $\begin{vmatrix} 2x & 5 \\ 12 & x \end{vmatrix} = \begin{vmatrix} 6 & -5 \\ 4 & 3 \end{vmatrix}$, then the value of x is :

(A) 3 (B) 7

(C) ± 7 (D) ± 3

10.

If $A = [a_{ij}]$ is a 3×3 diagonal matrix such that $a_{11} = 1$, $a_{22} = 5$ and $a_{33} = -2$, then $|A|$ is :

(A) 0 (B) -10

(C) 10 (D) 1

11.

If $\begin{bmatrix} 4+x & x-1 \\ -2 & 3 \end{bmatrix}$ is a singular matrix, then the value of x is :

(A) 0 (B) 1

(C) -2 (D) -4

12.

If $\begin{vmatrix} 2x & 5 \\ 12 & x \end{vmatrix} = \begin{vmatrix} 6 & -5 \\ 4 & 3 \end{vmatrix}$, then the value of x is :

(A) 3 (B) 7

(C) ± 7 (D) ± 3

13.

If $A = kB$, where A and B are two square matrices of order n and k is a scalar, then :

(A) $|A| = k|B|$ (B) $|A| = k^n|B|$

(C) $|A| = k + |B|$ (D) $|A| = |B|^k$

14.

If A and B are two square matrices each of order 3 with $|A| = 3$ and $|B| = 5$, then $|2AB|$ is :

(A) 30 (B) 120

(C) 15 (D) 225

15.

Let A be a square matrix of order 3. If $|A| = 5$, then $|\text{adj } A|$ is :

(A) 5 (B) 125

(C) 25 (D) – 5

1Mark :

1.

If A is a square matrix of order 3 such that $A(\text{adj } A) = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, then find $|A|$.

2.

If $A = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 2 & 3 \\ 3 & 3 & 5 \end{bmatrix}$, then find A (adj A).

Ans: $A \cdot \text{adj}(A) = |A|\, I$

$\therefore\ A \cdot \text{adj}\,(A) = 2I \text{ or } \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

3.

If $\begin{vmatrix} 2x & -9 \\ -2 & x \end{vmatrix} = \begin{vmatrix} -4 & 8 \\ 1 & -2 \end{vmatrix}$, then value of x is ____________ .

4.

If A is a square matrix of order 3, with $|A| = 9$, then write the value of $|2 \,.\, \text{adj } A|$.

Sol.

$$|2 \text{ adj } A| = 2^3 \, |A|^{3-1} = 8 \times 81 = 648$$

5.a

If A is a square matrix of order 2 and $|A| = 4$, then find the value of $|2 \,.\, AA'|$, where A' is the transpose of matrix A.

5.b

If A is a square matrix of order 3 with $|A| = 4$, then write the value of $|-2A|$.

Sol.

$$|-2A| = (-2)^3 \cdot |A|$$

$$= -8 \cdot 4 = -32$$

6.

Find the cofactor of the element a_{23} of the determinant $\begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$.

7.

If A is a 3×3 invertible matrix, then what will be the value of k if $\det(A^{-1}) = (\det A)^k$.

8.

If A is a square matrix of order 3 and $|A| = 2$, then find the value of $|-AA'|$.

9.

For $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ write A^{-1}.

10.

A square matrix A is said to be singular if __________ .

OR

If $A = \begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 17 \\ 0 & -10 \end{bmatrix}$, then $|AB| =$ __________ .

11.

If A and B are square matrices of the same order 3, such that $|A| = 2$ and $AB = 2I$, write the value of $|B|$.

Sol.

$$AB = 2I \Rightarrow |AB| = |2I| \Rightarrow |A| \cdot |B| = 2^3|I|$$

$$\Rightarrow 2 \times |B| = 8 \Rightarrow |B| = 4$$

12.

If A is a square matrix satisfying $A'A = I$, write the value of $|A|$.

Sol.

$$|A'|\,|A| = |I| \Rightarrow |A|^2 = 1$$

$$\therefore \quad |A| = 1 \text{ or } |A| = -1$$

13.

If A is a square matrix of order 3 such that $|\text{adj } A| = 64$, find $|A|$.

14.

If A is an invertible matrix of order 3 and |A| = 5, then find |adj. A|.

15.

If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, then write A^{-1}.

16.

If $\Delta = |a_{ij}| = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 5 & 6 & 7 \end{vmatrix}$, then write the cofactor of element a_{23}.

17.

If A is a square matrix such that $|A| = 5$, write the value of $|AA^T|$.

18.

If A is an invertible matrix of order 2 and det (A) = 4, then write the value of $\det(A^{-1})$.

19.

If for any 2×2 square matrix A, $A(\text{adj } A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of $|A|$.

20.

If $|A| = 3$ and $A^{-1} = \begin{bmatrix} 3 & -1 \\ -\frac{5}{3} & \frac{2}{3} \end{bmatrix}$, then write the adj A.

21.

If A and B are invertible matrices of order 3, $|A| = 2$ and $|(AB)^{-1}| = -\frac{1}{6}$. Find $|B|$.

22.

If A is a square matrix of order 3 such that $A^2 = 2A$, then find the value of |A|.

Ans: $|A|^2 = 8|A|$

$\Rightarrow |A| = 8$

23.

If A is a square matrix of order 3 and $|A| = 2$, then find the value of $|-AA'|$.

Ans: $|-AA'| = -|A|^2$

$= -4$

2 Marks :

1.

For the matrix $A = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$, verify the following :

$$A\,(\text{adj } A) = (\text{adj } A)\,A = |A|\,I$$

2.

Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2A^{-1} = 9I - A$.

3.

If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $A^{-1} = kA$, then find the value of k.

4.

If $\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$, then write the value of x.

5.

Evaluate: $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

6.

Write the value of the determinant $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$

7.

Find the minor of the element of second row and third column (a_{23}) in the following determinant:

$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

8.

For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular?

9.

If $A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 3 \\ -1 & 1 \end{pmatrix}$, write the value of $|AB|$.

10.

Find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

11.

In the interval $\pi/2 < x < \pi$, find the value of x for which the matrix $\begin{pmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{pmatrix}$ is singular.

12.

If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

12.a

13.

If $A = \begin{bmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{bmatrix}$, then write the cofactor of the element a_{21} of its 2^{nd} row.

14.

If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then for any natural number n, find the value of Det (A^n).

15.

If $x \in \mathrm{N}$ and $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$, then find the value of x.

Sol.

$(x + 3)2x - (-2)(-3x) = 8$

$x = 2$

16.

For what values of k, the system of linear equations

$x+y+z=2$

$2x+y-z=3$

$3x+2y+kz=4$

has a unique solution ?

17.

If A is a 3×3 matrix and $|3A| = k|A|$, then write the value of k.

18.

If $A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -4 \\ 3 & -2 \end{pmatrix}$, find $|AB|$.

19.

If A and B are square matrices of order 3 such that $|A| = -1$, $|B| = 3$, then find the value of $|2AB|$.

19.b

If A and B are square matrices, each of order 2 such that $|A| = 3$ and $|B| = -2$, then write the value of $|3AB|$.

Ans.

$|3AB| = 9|A|\,|B|$

$= -54$

20. 2025

Let A and B be two square matrices of order 3 such that det (A) = 3 and det (B) = – 4. Find the value of det (– 6AB).

Sol.

$$|-6AB| = (-6)^3 |A||B|$$
$$= -216 \times 3 \times (-4) = 2592$$

21.

Using matrices and determinants, find the value(s) of k for which the pair of equations $5x - ky = 2$; $7x - 5y = 3$ has a unique solution.

Sol.

$$\text{For unique solution,} \begin{vmatrix} 5 & -k \\ 7 & -5 \end{vmatrix} \neq 0$$

$$\Rightarrow -25 + 7k \neq 0 \Rightarrow k \neq \frac{25}{7} \text{ or } R - \left\{\frac{25}{7}\right\}$$

22.

Find a matrix A such that

$$A \begin{bmatrix} 4 & 0 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 17 & 10 \\ 0 & -16 \end{bmatrix}. \quad \text{Also, find } A^{-1}.$$

Sol.

$$A = \begin{bmatrix} 17 & 10 \\ 0 & -16 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ -1 & -2 \end{bmatrix}^{-1}$$

$$= -\frac{1}{8} \begin{bmatrix} 17 & 10 \\ 0 & -16 \end{bmatrix} \begin{bmatrix} -2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -5 \\ 2 & 8 \end{bmatrix}$$

$$\text{Also, } A^{-1} = \frac{1}{34} \begin{bmatrix} 8 & 5 \\ -2 & 3 \end{bmatrix}$$

prepared by : BALAJI KANCHI

22.

Find the matrix A such that $A\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -1 & 6 \end{bmatrix}$.

Sol.

$$A = \begin{bmatrix} 3 & 4 \\ -1 & 6 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}^{-1}$$

$$= \frac{1}{2}\begin{bmatrix} 3 & 4 \\ -1 & 6 \end{bmatrix}\begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$$

23.

If $\begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is a symmetric matrix, then find the value of x.

Ans: $2x - 3 = x + 2$

$\Rightarrow \quad x = 5$

24.

If A is a square matrix such that $A^2 = A$, then find $(2 + A)^3 - 19A$.

Ans: $(2 + A)^3 - 19A = A^3 + 8 + 12A + 6A^2 - 19A$

$= 8$

26.

Find $|AB|$, if $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$.

Sol.

$|A| = |B| = 0$

$\Rightarrow \quad |AB| = 0$

27.

If $A = \begin{bmatrix} p & 2 \\ 2 & p \end{bmatrix}$ and $|A^3| = 125$, then find the values of p.

Sol.

$|A| = p^2 - 4$

$$|A^3| = 125 \Rightarrow |A|^3 = 125 \Rightarrow |A| = 5$$

$\therefore \quad p^2 - 4 = 5 \Rightarrow p = \pm 3$

28.

If A and B are symmetric matrices, such that AB and BA are both defined, then prove that AB – BA is a skew symmetric matrix.

Sol.

Given: $A' = A$, $B' = B$

$(AB - BA)' = (AB)' - (BA)'$

$= B'A' - A'B'$

$= BA - AB$

$= -(AB - BA)$, Hence AB – BA is shew symmetric

prepared by : **BALAJI KANCHI**

29.

A is a square matrix with $|A| = 4$. Then find the value of $|A\,.\,(\text{adj } A)|$.

Sol.

$|A \cdot \text{adj } A| = |A|^n = 4^n$ or 16 or 64

30.

If A is a square matrix of order 2 and $|A| = 4$, then find the value of $|2 \,.\, AA'|$, where A′ is the transpose of matrix A.

Sol.

$|2\ AA'| = 4|A|\ |A'|$

$= 4 \times 4 \times 4 = 64$

31.

Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2A^{-1} = 9I - A$.

Sol.

$|A| = 2, \quad \therefore\ A^{-1} = \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

$\text{LHS} = 2A^{-1} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}, \ \text{RHS} = 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

$\therefore \quad \text{LHS} = \text{RHS}$

32.

If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $A^{-1} = kA$, then find the value of k.

Sol.

Finding $A^{-1} = \frac{-1}{19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$

$\Rightarrow \frac{-1}{19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 2k & 3k \\ 5k & -2k \end{bmatrix}$

$\Rightarrow k = \frac{1}{19}$

33.

If A is a 3×3 invertible matrix, then what will be the value of k if $\det(A^{-1}) = (\det A)^k$.

Sol.

$$|A^{-1}| = \frac{1}{|A|} \Rightarrow k = -1$$

34.

Show that all the diagonal elements of a skew symmetric matrix are zero.

Sol.

Let $A = [a_{ij}]_{n \times n}$ be skew symmetric matrix

A is skew symmetric

$\therefore \quad A = -A'$

$\Rightarrow \quad a_{ij} = -a_{ji} \ \forall i, j$

For diagonal elements $i = j$,

$\Rightarrow \quad 2a_{ii} = 0$

$\Rightarrow \quad a_{ii} = 0 \Rightarrow$ diagonal elements are zero.

35.

If A is a skew-symmetric matrix of order 3, then prove that det A = 0.

Sol.

Any skew symmetric matrix of order 3 is $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

$\Rightarrow \quad |A| = -a(bc) + a(bc) = 0$

OR

Since A is a skew-symmetric matrix $\therefore\ A^T = -A$

$\therefore\quad |A^T| = |-A| = (-1)^3.|A|$

$\Rightarrow\quad |A| = -|A|$

$\Rightarrow\quad 2|A| = 0$ or $|A| = 0.$

36.

If $|A| = 3$ and $A^{-1} = \begin{bmatrix} 3 & -1 \\ -\frac{5}{3} & \frac{2}{3} \end{bmatrix}$, then write the adj A.

Sol.

$$\text{adj } A = 3\begin{bmatrix} 3 & -1 \\ \frac{-5}{3} & \frac{2}{3} \end{bmatrix} = \begin{bmatrix} 9 & -3 \\ -5 & 2 \end{bmatrix}$$

37.

If $A = \begin{pmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{pmatrix}$ and $BA = (b_{ij})$, find $b_{21} + b_{32}$.

Sol.

$b_{21} = -16,\ b_{23} = -2$ [For any one correct value]

$b_{21} + b_{23} = -16 + (-2) = -18$

5 Mark :

I.Solving Linear Equations using A inverse:

1.

Solve the following system of equations by matrix method :

$x - y + 2z = 7$

$2x - y + 3z = 12$

$3x + 2y - z = 5$

Sol.

Writing given equations in matrix form

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & -1 & 3 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 12 \\ 5 \end{bmatrix}$$

Which is of the form AX = B

Here $|A| = -2 \neq 0$

$$A^{-1} = \frac{1}{-2} \begin{bmatrix} -5 & 3 & -1 \\ 11 & -7 & 1 \\ 7 & -5 & 1 \end{bmatrix}$$

$$\therefore \quad X = A^{-1}B = \frac{1}{-2} \begin{bmatrix} -5 & 3 & -1 \\ 11 & -7 & 1 \\ 7 & -5 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 12 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$$\Rightarrow \quad x = 2,\ y = 1,\ z = 3$$

2.

Using matrices, solve the following system of linear equations :

$$x + 2y - 3z = -4$$

$$2x + 3y + 2z = 2$$

$$3x - 3y - 4z = 11$$

Sol.

$|A| = 67 \neq 0 \qquad \therefore X = A^{-1}B$

$$\text{adj } A = \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

$$\text{So } X = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$= \frac{1}{67}\begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

$x = 3, \ y = -2, \ z = 1$

3.

Using matrices, solve the following system of linear equations :

$$2x + 3y + 10z = 4$$
$$4x - 6y + 5z = 1$$
$$6x + 9y - 20z = 2$$

Sol.

The given system of equations is

$AX = B,$

$$\text{where } A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$|A| = 1200 \neq 0$

$\Rightarrow A^{-1}$ exists.

$X = A^{-1}B$

$$\text{adj } A = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj} A = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$$\therefore\ x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{5}$$

4.

solve the following system of linear equations :

$8x + 4y + 3z = 19$

$2x + y + z = 5$

$x + 2y + 2z = 7$

Sol.

$$\therefore\quad A^{-1} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix}$$

$$AX = B \Rightarrow X = A^{-1}B$$

$$\therefore\quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{pmatrix}\begin{pmatrix} 19 \\ 5 \\ 7 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

$$\therefore\quad x = 1,\ y = 2,\ z = 1$$

5.

Using matrix method, solve the following system of equations :

$$2x - 3y + 5z = 13$$

$$3x + 2y - 4z = -2$$

$$x + y - 2z = -2$$

Sol.

Given System of equation can be written as

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ -2 \\ -2 \end{bmatrix} \text{ or } AX = B$$

$$|A| = 2(0) + 3(-2) + 5(1) = -1 \neq 0$$

$$\therefore X = A^{-1} \cdot B$$

$$(\text{adj. } A) = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = -\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 13 \\ -2 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$$

$$\Rightarrow x = 2,\ y = 2,\ z = 3.$$

prepared by : BALAJI KANCHI

6.

Using matrix method, solve the following system of equations :

$$3x - 2y + 3z = 8$$

$$2x + y - z = 1$$

$$4x - 3y + 2z = 4$$

Sol.

$$\text{Let, } A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}; \ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; \ B = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$$

System of equation in Matrix form: $A \cdot X = B$

$$|A| = 3(2 - 3) + 2(4 + 4) + 3(-6 - 4) = -17 \neq 0$$

Solution matrix, $X = A^{-1} \cdot B$.

$$(\text{adj } A) = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$

$$\therefore \ A^{-1} = \frac{-1}{17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$

$$\therefore \ \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}\begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow x = 1, y = 2, x = 3$$

prepared by : **BALAJI KANCHI**

7.2023

Solve the following system of equations by matrix method :

$$x + 2y + 3z = 6$$
$$2x - y + z = 2$$
$$3x + 2y - 2z = 3$$

Sol.

Given system is

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 2 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix}$$

$A . X = B \Rightarrow X = A^{-1}B$

$|A| = 35 \neq 0$

$A_{11} = 0 \quad A_{12} = 7 \quad A_{13} = 7$

$A_{21} = 10 \quad A_{22} = -11 \quad A_{23} = 4$

$A_{31} = 5 \quad A_{32} = 5 \quad A_{33} = -5$

$$\therefore A^{-1} = \frac{1}{35}\begin{bmatrix} 0 & 10 & 5 \\ 7 & -11 & 5 \\ 7 & 4 & -5 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{35}\begin{bmatrix} 0 & 10 & 5 \\ 7 & -11 & 5 \\ 7 & 4 & -5 \end{bmatrix}\begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix} = \frac{1}{35}\begin{bmatrix} 35 \\ 35 \\ 35 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\therefore x = 1 \quad y = 1 \quad z = 1$

prepared by : BALAJI KANCHI

8.

solve the following system of equations $3x-3y+4z=21$, $2x-3y+4z=20$, $-y+z=5$.

Sol.

The matrix form of given equations

$$\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 21 \\ 20 \\ 5 \end{bmatrix}$$

$$\Rightarrow \quad AX = B, \text{ where } X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}, B=\begin{bmatrix} 21 \\ 20 \\ 5 \end{bmatrix}$$

$$\Rightarrow \quad X = A^{-1}B$$

$$\Rightarrow \quad \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}\begin{bmatrix} 21 \\ 20 \\ 5 \end{bmatrix}=\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$

$$\therefore \quad x = 1, y = -2, z = 3$$

9.

If $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$, then find A^{-1} and use it to solve the following system of the equations :

$$x + 2y - 3z = 6$$
$$3x + 2y - 2z = 3$$
$$2x - y + z = 2$$

Sol.

$$|A| = 7\,;\ \text{adj}(A) = \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix};$$

$$A^{-1} = \frac{1}{|A|}\,\text{adj}\,A = \frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$$

The system of equations in Matrix form can be written as :

$$A \cdot X = B\text{, where } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix};\ B = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$$

$$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 7 \\ -35 \\ -35 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ -5 \end{bmatrix}$$

$\therefore\ x = 1,\ y = -5\ ,\ z = -5$

10.

If $A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}$, find A^{-1} and use it to solve the following system of equations :

$$5x - y + 4z = 5$$
$$2x + 3y + 5z = 2$$
$$5x - 2y + 6z = -1$$

Sol.

$|A| = 51$

Cofactors: $\begin{matrix} A_{11} = 28 & A_{12} = 13 & A_{13} = -19 \\ A_{21} = -2 & A_{22} = 10 & A_{23} = 5 \\ A_{31} = -17 & A_{32} = -17 & A_{33} = 17 \end{matrix}$

$$A^{-1} = \frac{1}{51}\begin{bmatrix} 28 & -2 & -17 \\ 13 & 10 & -17 \\ -19 & 5 & 17 \end{bmatrix}$$

Given system is AX = B where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} B = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$

$$\Rightarrow X = A^{-1}B = \frac{1}{51}\begin{bmatrix} 28 & -2 & -17 \\ 13 & 10 & -17 \\ -19 & 5 & 17 \end{bmatrix}\begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix}$$

$x = 3,\ y = 2,\ z = -2$

prepared by : **BALAJI KANCHI**

11.

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1}.

Using A^{-1}, solve the following system of equations :

$$2x - 3y + 5z = 11$$

$$3x + 2y - 4z = -5$$

$$x + y - 2z = -3$$

Sol.

$|A| = -1 \neq 0 \Rightarrow A^{-1}$ exists.

$$\text{adj } A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{adj } A = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

Given system of equations can be written as AX = B

$$\text{where } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\Rightarrow x = 1, y = 2, z = 3$

prepared by : **BALAJI KANCHI**

12. 2023

If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the following system of linear equations :

$$x + y + z = 6$$
$$x + 2z = 7$$
$$3x + y + z = 12$$

Sol.

$|A| = 1(-2) - 1(-5) + 1(1) = 4 \neq 0 \Rightarrow A^{-1}$ exists.

$A_{11} = -2, A_{12} = 5, A_{13} = 1$

$A_{21} = 0, A_{22} = -2, A_{23} = 2$

$A_{31} = 2, A_{32} = -1, A_{33} = -1$

$$adjA = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adjA = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

Given system of equations can be written as $AX = B$, where $B = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$

$$X = A^{-1}B$$

$$= \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 12 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$\therefore x = 3, y = 1$ and $z = 2$

prepared by : BALAJI KANCHI

13.

If $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{bmatrix}$, find A^{-1}.

Hence solve the system of equations

$$x + 3y + 4z = 8$$

$$2x + y + 2z = 5$$

and $5x + y + z = 7$

Sol.

$|A| = 11$; $\text{Adj}(A) = \begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$

$$\therefore \quad A^{-1} = \frac{1}{|A|} \cdot \text{Adj}\, A = \frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$$

Taking; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $B = \begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix}$

The system of equations in matrix form is

$A \cdot X = B \qquad \therefore X = A^{-1} \cdot B$

$\therefore$ Solution is:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}\begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\therefore \quad x = 1,\ y = 1,\ z = 1$

prepared by : BALAJI KANCHI

14.

If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$, find A^{-1}.

Hence, solve the following system of equations :

$x + y + z = 6,$

$y + 3z = 11$

and $x - 2y + z = 0$

Sol.

$$|A| = 1(7) - 1(-3) + 1(-1) = 9$$

$$(\text{adj } A) = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

Given equations can be written as $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$

$$\text{or } AX = B \Rightarrow X = A^{-1}B$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\therefore x = 1, y = 2, z = 3$$

15.

If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the system of equations

$$y + 2z = 5$$
$$x + 2y + 3z = 10$$
$$3x + y + z = 9$$

Sol.

$|A| = -2 \neq 0 \Rightarrow A^{-1}$ exists

Now, $A_{11} = -1,\ A_{12} = 8,\ A_{13} = -5$

$A_{21} = 1,\ A_{22} = -6,\ A_{23} = 3$

$A_{31} = -1,\ A_{32} = 2,\ A_{33} = -1$

$$\text{adj } A = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{adj } A = \frac{-1}{2}\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$$

Given system of equations can be written as $AX = B$,

where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ 10 \\ 9 \end{bmatrix}$

Now $AX = B \Rightarrow X = A^{-1}B$

$$= \frac{-1}{2}\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}\begin{bmatrix} 5 \\ 10 \\ 9 \end{bmatrix}$$

$$= \frac{-1}{2}\begin{bmatrix} -4 \\ -2 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

$\therefore \quad x = 2,\ y = 1,\ z = 2$

15.

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Use it to solve the system of equations

$$2x - 3y + 5z = 11$$

$$3x + 2y - 4z = -5$$

$$x + y - 2z = -3.$$

Sol.

$$|A| = 2(0) + 3(-2) + 5(1) = -1$$

$$\Rightarrow A^{-1} = -\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} \text{ or } \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

Given equations can be written as $AX = B$,

where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

$$\therefore X = A^{-1}B = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\Rightarrow x = 1, \; y = 2, \; z = 3$$

prepared by : **BALAJI KANCHI**

16. 2023

If matrix $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the following system of linear equations :

$$3x + 2y + z = 2000$$
$$4x + y + 3z = 2500$$
$$x + y + z = 900$$

Sol.

$$|A| = 3(-2) - 4(1) + 1(5) = -5 \neq 0 \Rightarrow A^{-1} \text{ exists.}$$

$$A_{11} = -2, A_{12} = -1, A_{13} = 3$$
$$A_{21} = -1, A_{22} = 2, A_{23} = -1$$
$$A_{31} = 5, A_{32} = -5, A_{33} = -5$$

$$adjA = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adjA = -\frac{1}{5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

Given system of equations can be written as $AX = B$, where $B = \begin{bmatrix} 2000 \\ 2500 \\ 900 \end{bmatrix}$

$$X = A^{-1}B$$

$$= -\frac{1}{5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}\begin{bmatrix} 2000 \\ 2500 \\ 900 \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} -2000 \\ -1500 \\ -1000 \end{bmatrix} = \begin{bmatrix} 400 \\ 300 \\ 200 \end{bmatrix}$$

$$\therefore x = 400, y = 300 \text{ and } z = 200$$

prepared by : BALAJI KANCHI

17. 2023

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{bmatrix}$, find A^{-1} and hence solve the following system of linear equations :

$$\begin{aligned} x + y + z &= 5000 \\ 6x + 7y + 8z &= 35800 \\ 6x + 7y - 8z &= 7000 \end{aligned}$$

Sol.

$|A| = (-56-56) - 6(-8-7) + 6(8-7) = -16 \neq 0 \Rightarrow A^{-1}$ exists.

$A_{11} = -112, A_{12} = 96, A_{13} = 0$

$A_{21} = 15, A_{22} = -14, A_{23} = -1$

$A_{31} = 1, A_{32} = -2, A_{33} = 1$

$$adjA = \begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adjA = -\frac{1}{16}\begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

Given system of equations can be written as $AX = B$, where $B = \begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix}$

$$X = A^{-1}B$$

$$= -\frac{1}{16}\begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix}$$

$$= -\frac{1}{16}\begin{bmatrix} -16000 \\ -35200 \\ -28800 \end{bmatrix} = \begin{bmatrix} 1000 \\ 2200 \\ 1800 \end{bmatrix}$$

$\therefore x = 1000, y = 2200, z = 1800$

prepared by : BALAJI KANCHI

18.

If $A = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ -3 & 1 & -1 \end{pmatrix}$, find A^{-1} and hence solve the system of equations $2x + y - 3z = 13$, $3x + 2y + z = 4$, $x + 2y - z = 8$.

Sol.

$|A| = -16$

$$A^{-1} = \frac{-1}{16}\begin{bmatrix} -4 & 4 & 4 \\ -5 & 1 & -3 \\ 7 & -11 & 1 \end{bmatrix}$$

given equations can be written as

$$A'X = C \Rightarrow X = \left(A^{-1}\right)' C$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{16}\begin{bmatrix} -4 & -5 & 7 \\ 4 & 1 & -11 \\ 4 & -3 & 1 \end{bmatrix}\begin{bmatrix} 13 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$$

$\Rightarrow x = 1,\ y = 2,\ z = -3$

19.

If $A = \begin{pmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{pmatrix}$, find A^{-1} and hence solve the system of equations $x + 2y + 5z = 10$, $x - y - z = -2$ and $2x + 3y - z = -11$.

Sol.

$|A| = 27 \neq 0$, A^{-1} exist

$$\therefore A^{-1} = \frac{1}{27}\begin{pmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{pmatrix}$$

Given system of equations can be written as $AX = B$ where

$$A = \begin{pmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{pmatrix}, \; X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \; B = \begin{pmatrix} 10 \\ -2 \\ -11 \end{pmatrix}$$

Now, $AX = B \Rightarrow X = A^{-1}B$

$$X = \frac{1}{27}\begin{pmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{pmatrix}\begin{pmatrix} 10 \\ -2 \\ -11 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix}$$

$\therefore x = -1, y = -2, z = 3$

prepared by : **BALAJI KANCHI**

20.

If $A = \begin{pmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{pmatrix}$, find A^{-1} and hence solve the system of equations $x - 2y = 10,\ 2x + y + 3z = 8$ and $-2y + z = 7$.

Sol.

$|A| = 11 \neq 0$, A^{-1} will exist

$$A^{-1} = \frac{1}{11}\begin{pmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{pmatrix}$$

Given sysem of equations can be written as AX = B, where

$$A = \begin{pmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{pmatrix}, X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, B = \begin{pmatrix} 10 \\ 8 \\ 7 \end{pmatrix}$$

$$X = A^{-1}B = \frac{1}{11}\begin{pmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{pmatrix}\begin{pmatrix} 10 \\ 8 \\ 7 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}$$

$\therefore\ x = 4,\ y = -3,\ z = 1$

prepared by : BALAJI KANCHI

21.

Find the inverse of the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$.

Using the inverse, A^{-1}, solve the system of linear equations

$x - y + 2z = 1;\ 2y - 3z = 1;\ 3x - 2y + 4z = 3.$

Sol.

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$$

$$|A| = 1(8-6) + 1(0+9) + 2(0-6) = -1 \neq 0$$

$\therefore$ A is invertible.

$$\text{adj } A = \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{|A|} \text{adj } A = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

The given system of equation can be written as AX = B, when

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

$$\Rightarrow X = A^{-1} B$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$\Rightarrow x = 1, y = 2, z = 1$$

prepared by : BALAJI KANCHI

22.

Given $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}$, find A^{-1}. Use it to solve the following system of equations :

$$x + y + z = 1$$
$$2x + 3y + 2z = 2$$
$$x + y + 2z = 4$$

Sol.

$|A| = 1(4) - 1(2) + 1(-1) = 1$

Cofactors of the elements of A are:

$A_{11} = 4, A_{12} = -2, A_{13} = -1$

$A_{21} = -1, A_{22} = 1, A_{23} = 0$

$A_{31} = -1, A_{32} = 0, A_{33} = 1$

$$\therefore \text{adj } A = \begin{bmatrix} 4 & -1 & -1 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{adj\ A}{|A|} = \begin{bmatrix} 4 & -1 & -1 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Given system of equations can be written as AX = B, where

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

$$X = A^{-1}B = \begin{bmatrix} 4 & -1 & -1 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}$$

$x = -2, y = 0, z = 3$

prepared by : BALAJI KANCHI

23. 2024

If $A = \begin{bmatrix} 2 & 1 & -3 \\ 3 & 2 & 1 \\ 1 & 2 & -1 \end{bmatrix}$, find A^{-1} and hence solve the following system of equations :

$$\begin{aligned} 2x + y - 3z &= 13 \\ 3x + 2y + z &= 4 \\ x + 2y - z &= 8 \end{aligned}$$

Sol.

For Matrix $A = \begin{pmatrix} 2 & 1 & -3 \\ 3 & 2 & 1 \\ 1 & 2 & -1 \end{pmatrix}$, $|A| = -16 \neq 0$ so, A^{-1} exists.

$$adjA = \begin{pmatrix} -4 & -5 & 7 \\ 4 & 1 & -11 \\ 4 & -3 & 1 \end{pmatrix},$$

$$\text{Thus, } A^{-1} = \frac{-1}{16}\begin{pmatrix} -4 & -5 & 7 \\ 4 & 1 & -11 \\ 4 & -3 & 1 \end{pmatrix}$$

so, Given equation can be written into a matrix equation as

$$\begin{pmatrix} 2 & 1 & -3 \\ 3 & 2 & 1 \\ 1 & 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 13 \\ 4 \\ 8 \end{pmatrix} \Rightarrow X = A^{-1}.B$$

A X = B

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{-1}{16}\begin{pmatrix} -4 & -5 & 7 \\ 4 & 1 & -11 \\ 4 & -3 & 1 \end{pmatrix}\begin{pmatrix} 13 \\ 4 \\ 8 \end{pmatrix} = \frac{-1}{16}\begin{pmatrix} -16 \\ -32 \\ 48 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} \Rightarrow x = 1, y = 2, z = -3$$

prepared by : BALAJI KANCHI

24

If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations :

$$x + 2y - 3z = 1$$

$$2x - 3z = 2$$

$$x + 2y = 3$$

Sol.

$|A| = 1(6) - 2(3) - 3(4) = -12 \neq 0$, $\therefore$ A^{-1} exist

$$\text{adj}A = \begin{bmatrix} 6 & -6 & -6 \\ -3 & 3 & -3 \\ 4 & 0 & -4 \end{bmatrix}$$

$$\therefore A^{-1} = -\frac{1}{12}\begin{bmatrix} 6 & -6 & -6 \\ -3 & 3 & -3 \\ 4 & 0 & -4 \end{bmatrix}$$

The given system of equations can be written as $AX = B$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{12}\begin{bmatrix} 6 & -6 & -6 \\ -3 & 3 & -3 \\ 4 & 0 & -4 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1/2 \\ 2/3 \end{bmatrix}$$

$\therefore$ The solution of the given system of equations is: $x = 2, y = \frac{1}{2}, z = \frac{2}{3}$

prepared by : BALAJI KANCHI

25.a 2024

Solve the following system of equations, using matrices :

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4, \quad \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1, \quad \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$$

where x, y, z $\neq 0$

Sol.

Given system of linear equations is equivalent to AX = B, where

$$A=\begin{bmatrix}2 & 3 & 10\\ 4 & -6 & 5\\ 6 & 9 & -20\end{bmatrix}, X=\begin{bmatrix}\frac{1}{x}\\ \frac{1}{y}\\ \frac{1}{z}\end{bmatrix}, B=\begin{bmatrix}4\\ 1\\ 2\end{bmatrix}$$

$|A| = 1200 \neq 0$

Cofactors of the elements of A are

$A_{11} = 75,\ A_{12} = 110,\ A_{13} = 72$

$A_{21} = 150,\ A_{22} = -100,\ A_{23} = 0$

$A_{31} = 75,\ A_{32} = 30,\ A_{33} = -24$

$$adjA=\begin{bmatrix}75 & 150 & 75\\ 110 & -100 & 30\\ 72 & 0 & -24\end{bmatrix}$$

$$A^{-1}=\frac{adjA}{|A|}=\frac{1}{1200}\begin{bmatrix}75 & 150 & 75\\ 110 & -100 & 30\\ 72 & 0 & -24\end{bmatrix}$$

$$X=A^{-1}B=\frac{1}{1200}\begin{bmatrix}75 & 150 & 75\\ 110 & -100 & 30\\ 72 & 0 & -24\end{bmatrix}\begin{bmatrix}4\\ 1\\ 2\end{bmatrix}$$

$$=\frac{1}{1200}\begin{bmatrix}600\\ 400\\ 240\end{bmatrix}=\begin{bmatrix}\frac{1}{2}\\ \frac{1}{3}\\ \frac{1}{5}\end{bmatrix}$$

prepared by : **BALAJI KANCHI**

$\therefore x = 2,\ y = 3,\ z = 5$

25.b

If $A = \begin{pmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{pmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=2;\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=5;\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=-4$$

Sol.

Here $|A| = 1200$

$$A^{-1} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

Given equation in matrix from is:

$$\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}\begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$$

$$\Rightarrow AX = B$$

$$\Rightarrow X = A^{-1}B$$

$$\Rightarrow \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{-1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$$\Rightarrow x = 2,\ y = -3,\ z = 5$$

26. 2024

Find A^{-1}, if $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 1 & 0 & 1 \end{bmatrix}$. Hence, solve the following system of equations :

$$x + 2y + z = 5$$
$$2x + 3y = 1$$
$$x - y + z = 8$$

Sol.

For Matrix $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 1 & 0 & 1 \end{pmatrix}$, Adjoint of Matrix A is

$|A| = -6 \neq 0$ so, A^{-1} exists.

$$adjA = \begin{pmatrix} 3 & -2 & -5 \\ -3 & 0 & 3 \\ -3 & 2 & -1 \end{pmatrix},$$

$$\text{Thus, } A^{-1} = \frac{-1}{6}\begin{pmatrix} 3 & -2 & -5 \\ -3 & 0 & 3 \\ -3 & 2 & -1 \end{pmatrix}$$

so, Given equation can be written into a matrix equation as

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 0 \\ 1 & -1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 8 \end{pmatrix} \Rightarrow X = (A^T)^{-1}.B = X = (A^{-1})^T.B$$

$$A^T \quad X = B$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{-1}{6}\begin{pmatrix} 3 & -3 & -3 \\ -2 & 0 & 2 \\ -5 & 3 & -1 \end{pmatrix}\begin{pmatrix} 5 \\ 1 \\ 8 \end{pmatrix} = \frac{-1}{6}\begin{pmatrix} -12 \\ 6 \\ -30 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$$

$$\therefore \boxed{x = 2, y = -1, z = 5}$$

prepared by : **BALAJI KANCHI**

27.

If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$, find A^{-1} and use it to solve the following system of equations :

$x - 2y = 10,\ 2x - y - z = 8,\ -2y + z = 7$

Sol.

$|A| = 1 \neq 0$ hence A^{-1} exists.

$$Adj\ A = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}$$

$$AX = B \Rightarrow \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

$\Rightarrow x = 0, y = -5, z = -3$

prepared by : BALAJI KANCHI

28.

If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find A^{-1}.

Hence, solve the system of linear equations :

$$x - 2y = 10$$
$$2x - y - z = 8$$
$$-2y + z = 7$$

Sol.

$|A| = 1 \neq 0 \Rightarrow A^{-1}$ exists.

$$adjA = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj}A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

The given system of equations is equivalent to the matrix equation

$A^T X = B$, where $B = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$\Rightarrow X = (A^T)^{-1} B$$

$$\Rightarrow X = (A^{-1})^T B$$

$$\Rightarrow X = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

$\therefore x = 0, y = -5, z = -3$

prepared by : BALAJI KANCHI

29. 2025

Sol.

If $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the given system of equations $3x + 4y + 7z = 14$; $2x - y + 3z = 4$; $x + 2y - 3z = 0$.

Sol.

$|A| = 3(-3) - 2(-26) + 1(19) = 62 \neq 0 \Rightarrow A^{-1}$ exists.

$$\text{cofactor Matrix} = \begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}$$

$$adjA = \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix}$$

$$A^{-1} = \frac{1}{62}\begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix}$$

Now, $A'.X = B \Rightarrow X = (A')^{-1}.B$

$$\Rightarrow X = (A^{-1})'.B = \frac{1}{62}\begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}\begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$$

$$= \frac{1}{62}\begin{bmatrix} 62 \\ 62 \\ 62 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

prepared by : BALAJI KANCHI

$\Rightarrow x = 1, y = 1, z = 1$

30. 2023

If $A = \begin{bmatrix} 3 & 2 \\ 5 & -7 \end{bmatrix}$, then find A^{-1} and use it to solve the following system of equations :

$3x + 5y = 11, \ 2x - 7y = -3.$

Sol.

$$\text{adj } A = \begin{bmatrix} -7 & -2 \\ -5 & 3 \end{bmatrix}$$

$$|A| = -31$$

$$A^{-1} = \frac{-1}{31}\begin{bmatrix} -7 & -2 \\ -5 & 3 \end{bmatrix}$$

Given system of equations is

$$\begin{bmatrix} 3 & 5 \\ 2 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11 \\ -3 \end{bmatrix}$$

which is $A'X = B$, where $X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 11 \\ -3 \end{bmatrix}$

$$\Rightarrow X = (A')^{-1}B$$

$$\Rightarrow X = (A^{-1})'B$$

$$= \frac{-1}{31}\begin{bmatrix} -7 & -5 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 11 \\ -3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$\therefore x = 2, \ \ y = 1$ prepared by : BALAJI KANCHI

II. Solving Linear Equations using product AB :

1.

Let $A=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}, B=\begin{bmatrix}4 & -6\\ -2 & 4\end{bmatrix}$. Then compute AB. Hence, solve the following system of equations: $2x+y=4, 3x+2y=1.$

2.

Use product $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ to solve the system of equations $x + 3z = 9$, $-x + 2y - 2z = 4$, $2x - 3y + 4z = -3$

Sol.

$$A=\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\cdot\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$AB = I \Rightarrow A^{-1} = B = \begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$$

Given equations in matrix form are:

$$\begin{bmatrix}1 & 0 & 3\\ -1 & 2 & -2\\ 2 & -3 & 4\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}9\\ 4\\ -3\end{bmatrix}$$

$$A'X = C$$

$$\Rightarrow \quad X = (A')^{-1} C = (A^{-1})'C$$

$$\Rightarrow \quad \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}-2 & 9 & 6\\ 0 & 2 & 1\\ 1 & -3 & -2\end{bmatrix}\begin{bmatrix}9\\ 4\\ -3\end{bmatrix}=\begin{bmatrix}0\\ 5\\ 3\end{bmatrix}$$

$$\Rightarrow \quad x = 0,\ y = 5,\ z = 3$$

3.

Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$.

Sol.

Getting $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$ ...(i)

Given equations can be written as $\begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \\ 1 \end{pmatrix}$

From (i) $A^{-1} = \frac{1}{8}\begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix}$

$\Rightarrow \quad AX = B$

$$\therefore \quad X = A^{-1}B = \frac{1}{8}\begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix}\begin{pmatrix} 4 \\ 9 \\ 1 \end{pmatrix}$$

$$= \frac{1}{8}\begin{pmatrix} 24 \\ -16 \\ -8 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$$

$\Rightarrow x = 3,\ y = -2,\ z = -1$

4.

If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ are two square matrices, find AB and hence solve the system of linear equations

$x - y = 3,\ 2x + 3y + 4z = 17$ and $y + 2z = 7$.

Sol.

$$AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$\Rightarrow AB = 6I$$

$$\Rightarrow A\left(\frac{1}{6}B\right) = I \ \Rightarrow A^{-1} = \frac{1}{6}(B)$$

The given equations can be written as

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$AX = D, \ \ where\ D = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$\Rightarrow \ X = A^{-1}D$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$$x = 2, \qquad y = -1, \qquad z = 4$$

OR

Getting $AB = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I$

Given system of equations can be written as

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

i.e., $AX = C \Rightarrow X = A^{-1}C = \frac{1}{6}\cdot BC \qquad \left(\because\ AB = 6I \Rightarrow A^{-1} = \frac{1}{6}B\right)$

$$= \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

prepared by : **BALAJI KANCHI**

$\Rightarrow x = 2,\ y = -1,\ z = 4$

5. 2023

If $A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find AB and use it to solve the following system of equations :

$$x - 2y = 3$$
$$2x - y - z = 2$$
$$-2y + z = 3$$

Sol.

$$AB = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow B^{-1} = A$

The given system of equations can be written as:

$$B^T \cdot X = C, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, C = \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}$$

$$X = \left(B^T\right)^{-1} \cdot C = \left(B^{-1}\right)^T \cdot C = A^T \cdot C$$

$$\Rightarrow X = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \therefore x = 1, y = -1, z = 1$$

prepared by : **BALAJI KANCHI**

6. 2024

Use the product of matrices $\begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{pmatrix}$ to solve the following system of equations :

$$x + 2y - 3z = 6$$
$$3x + 2y - 2z = 3$$
$$2x - y + z = 2$$

Sol.

$$AB = \begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{pmatrix} = \begin{pmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{pmatrix} = 7I$$

$$\text{Thus, } A^{-1} = \frac{1}{7}B = \frac{1}{7}\begin{pmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{pmatrix}$$

so, Given equation can be written into a matrix equation as

$$\begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix} \Rightarrow X = A^{-1}.C$$

A X = C

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{pmatrix}\begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 7 \\ -35 \\ -35 \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \\ -5 \end{pmatrix}$$

$$\therefore x = 1, y = -5, z = -5$$

prepared by : BALAJI KANCHI

7. 2024

Find the product of the matrices $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$

and hence solve the system of linear equations :

$$x + 2y - 3z = -4$$
$$2x + 3y + 2z = 2$$
$$3x - 3y - 4z = 11$$

Sol.

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix} = \begin{bmatrix} 67 & 0 & 0 \\ 0 & 67 & 0 \\ 0 & 0 & 67 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}^{-1} = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

Solution of the system of equations is given by:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}^{-1} \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix} = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix} \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix},$$

$$\therefore\ x = 3, y = -2, z = 1$$

prepared by : **BALAJI KANCHI**

8.

Given $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find AB. Hence, solve the system of linear equations :

$$x - y + z = 4$$
$$x - 2y - 2z = 9$$
$$2x + y + 3z = 1$$

Sol.

$$AB = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I$$

The system of equations is equivalent to the matrix equation:

$$BX = C, \text{ where } C = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\Rightarrow X = B^{-1}C$$

$$AB = 8I$$

$$\Rightarrow B^{-1} = \frac{1}{8}A$$

$$X = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

$\therefore x = 3, y = -2, z = -1$

prepared by : BALAJI KANCHI

III.Finding A^{-1} using properties from Algebraic Equation:

1.

If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$, then show that $A^3 - 4A^2 - 3A + 11\,I = O$. Hence find A^{-1}.

Sol.

$$A^2 = \begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{bmatrix}$$

$$LHS = A^3 - 4A^2 - 3A + 11I$$

$$= \begin{bmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{bmatrix} - 4\begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix} - 3\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

$$\text{Now, } A^{-1} = -\frac{1}{11}\left(A^2 - 4A - 3I\right)$$

$$= -\frac{1}{11}\begin{bmatrix} 2 & -5 & -3 \\ -7 & 1 & 5 \\ 4 & 1 & -6 \end{bmatrix}$$

prepared by : BALAJI KANCHI

2.

Show that for the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$,

$A^3 - 6A^2 + 5A + 11\,I = 0$. Hence, find A^{-1}.

Sol.

$$A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$$

$\text{LHS} = A^3 - 6A^2 + 5A + 11I$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} = O = \text{R.H.S.}$$

$A^3 - 6A^2 + 5A + 11I = O$, Pre-multiplying by A^{-1}

$$\Rightarrow \; A^2 - 6A + 5I + 11A^{-1} = O \Rightarrow A^{-1} = -\frac{1}{11}(A^2 - 6A + 5I)$$

$$\therefore \; A^{-1} = \begin{bmatrix} -3/11 & 4/11 & 5/11 \\ 9/11 & -1/11 & -4/11 \\ 5/11 & -3/11 & -1/11 \end{bmatrix}$$

3.a

If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and I is the identity matrix of order 2, then show that $A^2 = 4A - 3I$. Hence find A^{-1}.

3.b

If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then show that $A^2 - 4A - 5I = O$, and hence find A^{-1}.

4.

For the matrix $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$, show that $A^2 - 5A + 4I = O$.

Hence find A^{-1}.

Sol.

$$A^2 = \begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix}$$

$$A^2 - 5A + 4I = \begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix} - \begin{pmatrix} 10 & -5 & 5 \\ -5 & 10 & -5 \\ 5 & -5 & 10 \end{pmatrix} + \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

$$= O$$

Pre multiplying by A^{-1} and getting $A^{-1} = \frac{1}{4}(5I - A)$

and $A^{-1} = \frac{1}{4}\begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{pmatrix}$

prepared by : BALAJI KANCHI

5.

Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2A^{-1} = 9I - A$.

6.

Find the inverse of the matrix :

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

and hence show that $AA^{-1} = I$.

IV.Problems based on property $(AB)^{-1} = B^{-1}A^{-1}$:

1.a

If $A = \begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & 2 \\ 0 & -1 \end{bmatrix}$, find A^{-1} and hence find $(AB)^{-1}$.

Sol.

$$A^{-1} = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$$

$$(AB)^{-1} = B^{-1}.A^{-1}$$

$$= \begin{bmatrix} 3 & 2 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 12 & 19 \\ -3 & -5 \end{bmatrix}$$

1.b

Find $(AB)^{-1}$ if $A = \begin{bmatrix} 1 & 0 \\ -4 & 2 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$.

Sol.

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$$

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$B^{-1}A^{-1} = \frac{1}{2}\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & \frac{1}{2} \\ 9 & 1 \end{bmatrix}$$

2.

If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$.

Sol.

$$B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}, A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$(AB)^{-1} = B^{-1}A^{-1}$

$|B| = 1(3) - 2(-1) - 2(2) = 3 + 2 - 4 = 1 \neq 0$

$$\textbf{adj(B)} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$$\mathbf{B^{-1}} = \frac{1}{1}\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$$\therefore B^{-1}A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

prepared by : **BALAJI KANCHI**

3.a

If $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$, find $(AB)^{-1}$.

Sol.

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}, B^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$|A| = 1(3) - 2(-1) - 2(2) = 3 + 2 - 4 = 1 \neq 0$$

$$\textbf{adj(A)} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$$\mathbf{A^{-1}} = \frac{1}{1}\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$$\therefore B^{-1}A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 7 & 21 \\ -49 & -34 & -103 \\ 17 & 12 & 36 \end{bmatrix}$$

3.b

Given $A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}, B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, compute $(AB)^{-1}$.

$$|A| = 5(-1) + 4(1) = -1$$

$$\begin{matrix} C_{11} = -1 & C_{21} = 8 & C_{31} = -12 \\ C_{12} = 0 & C_{22} = 1 & C_{32} = -2 \\ C_{13} = 1 & C_{23} = -10 & C_{33} = 15 \end{matrix}$$

$$A^{-1} = \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$$

$$(AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix} = \begin{bmatrix} -2 & 19 & -27 \\ -2 & 18 & -25 \\ -3 & 29 & -42 \end{bmatrix}$$

4. 2024

If $A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, find $(AB)^{-1}$.

Also, find $|(AB)^{-1}|$.

sol.

We know that $(AB)^{-1} = B^{-1}A^{-1}$

$|A| = 5(-1) + 4(1) = -1 \neq 0$. Hence, A^{-1} exists.

Cofactors of the elements of A are:

$A_{11} = -1, A_{12} = 0, A_{13} = 1$

$A_{21} = 8, A_{22} = 1, A_{23} = -10$

$A_{31} = -12, A_{32} = -2, A_{33} = 15$

$$\text{adj } A = \begin{bmatrix} -1 & 8 & -12 \\ 0 & 1 & -2 \\ 1 & -10 & 15 \end{bmatrix}$$

$$A^{-1} = \frac{adj\ A}{|A|} = \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$$

$$(AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix} =$$

$$\begin{bmatrix} -2 & 19 & -27 \\ -2 & 18 & -25 \\ -3 & 29 & -42 \end{bmatrix}$$

$$|(AB)^{-1}| = |B^{-1}A^{-1}| = |B^{-1}||A^{-1}|$$

$$= 1 \times \frac{1}{-1} = -1$$

Based on Properties :

1.

Verify: A(adj A) = (adj A)A = $|A||\mathrm{I}$ for matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

2.

Let $A = \begin{bmatrix} -2 & 1 \\ 3 & 4 \end{bmatrix}$. Then verify the following: $A(\mathrm{adj}A) = (\mathrm{adj}A)A = |A|I$, where I is the identity matrix of order 2.

2.b

For the matrix $A = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$, verify the following:

$$A(\text{adj } A) = (\text{adj } A)\, A = |A|\, I$$

Sol.

$$|A| = -12 + 12 = 0, \text{ adj } A = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$$

$$A \cdot (\text{adj } A) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = (\text{adj } A) \cdot A = |A|\, I$$

3.

Find the adjoint of the matrix $A = \begin{pmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$ and hence show that $A.(\text{adj } A) = |A|\, I_3$.

4.

If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, find adj (A) and show that $A\,(\text{adj } A) = |A|\, I$.

5.

If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, find $(A')^{-1}$.

6.

If $A = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$, find adj·A and verify that $A(\text{adj}\cdot A) = (\text{adj}\cdot A)A = |A|\, I_3$.

Sol.

$|A| = 1$

$$\text{adj}\, A = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A(\text{adj}\, A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$|A|\, I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

7. 2024

If $A = \begin{bmatrix} -1 & a & 2 \\ 1 & 2 & x \\ 3 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ b & y & 3 \end{bmatrix}$,

find the value of $(a + x) - (b + y)$.

Sol.

$$AA^{-1} = I$$

$$\begin{bmatrix} -1 & a & 2 \\ 1 & 2 & x \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ b & y & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1-8a+2b & 1+7a+2y & 5-5a \\ -15+bx & 13+xy & 3x-9 \\ -5+b & 4+y & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$-5 + b = 0 \Rightarrow b = 5$, $\quad 5 - 5a = 0 \Rightarrow a = 1$

$4 + y = 0 \Rightarrow y = -4$, $\quad 3x - 9 = 0 \Rightarrow x = 3$

$\therefore (a + x) - (b + y) = (1 + 3) - (5 - 4) = 3$

prepared by : BALAJI KANCHI

8. 2024

If $A = \begin{bmatrix} 1 & \cot x \\ -\cot x & 1 \end{bmatrix}$, show that $A'A^{-1} = \begin{bmatrix} -\cos 2x & -\sin 2x \\ \sin 2x & -\cos 2x \end{bmatrix}$.

Sol.

$$|A| = 1 + cot^2x = cosec^2x$$

$$adjA = \begin{bmatrix} 1 & -cotx \\ cotx & 1 \end{bmatrix}$$

$$A^{-1} = \frac{adjA}{|A|} = \frac{1}{cosec^2x}\begin{bmatrix} 1 & -cotx \\ cotx & 1 \end{bmatrix}$$

$$A' = \begin{bmatrix} 1 & -cotx \\ cotx & 1 \end{bmatrix}$$

$$A'A^{-1} = \frac{1}{cosec^2x}\begin{bmatrix} 1 - cot^2x & -2cotx \\ 2cotx & 1 - cot^2x \end{bmatrix}$$

$$= \begin{bmatrix} sin^2x - cos^2x & -2sinxcosx \\ 2sinxcosx & sin^2x - cos^2x \end{bmatrix}$$

$$= \begin{bmatrix} -cos2x & -sin2x \\ sin2x & -cos2x \end{bmatrix}$$

prepared by : BALAJI KANCHI

9. 2025

If A is a 3 × 3 invertible matrix, show that for any scalar $k \neq 0$, $(kA)^{-1} = \frac{1}{k}A^{-1}$. Hence calculate $(3A)^{-1}$, where

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

Sol.

$$\text{Consider } (kA)\left(\frac{1}{k}A^{-1}\right) = k.\frac{1}{k}\left(A.A^{-1}\right) = I$$

$$\Rightarrow kA \text{ and } \frac{1}{k}A^{-1} \text{ are inverse of each other.}$$

$$\therefore (kA)^{-1} = \frac{1}{k}A^{-1}$$

$$\therefore (3A)^{-1} = \frac{1}{3}A^{-1}$$

$$\text{Here, } |A| = 4 \neq 0 \therefore A^{-1} \text{ exists.}$$

$$adjA = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}.adjA = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$\therefore (3A)^{-1} = \frac{1}{12}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

prepared by : BALAJI KANCHI

10.

Given a square matrix A of order 3 such that $A^2 = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$,

show that $A^3 = A^{-1}$.

Sol.

$$\mathbf{A^3 = A^{-1} \Leftrightarrow A^4 = I}$$

$$\mathbf{A^4 = A^2A^2 = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}\begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I}$$

11.

If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, find A^2 and show that $A^2 = A^{-1}$.

sol.

$$A^2 = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$$

$$A^2 \cdot A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$\text{or } A \cdot A^2 = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$\Rightarrow A^2 = A^{-1}$

a.Area of Triangle using determinant :

1. 2023

Using determinants, find the area of ΔPQR with vertices P(3, 1), Q(9, 3) and R(5, 7). Also, find the equation of line PQ using determinants.

Sol.

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 9 & 3 & 1 \\ 5 & 7 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\{3(-4) - 1(4) + 1(48)\} = 16 \text{ sq. unit}$$

$$\text{Equation of PQ is } \begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0$$

$$-2x + 6y = 0 \qquad \text{or } x - 3y = 0$$

2.

Find the equation of the line joining A(1, 3) and B(0, 0), using determinants. Also, find k if D(k, 0) is a point such that the area of Δ ABD is 3 square units.

Sol.

Equation of the line through A(1, 3) and B(0, 0) is

$$\begin{vmatrix} x & y & 1 \\ 1 & 3 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 0 \Rightarrow 3x - y = 0$$

$$\frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ k & 0 & 1 \end{vmatrix} = \pm 3 \Rightarrow k = \pm 2$$

b. Determinant simplification :

1. 2023

Show that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ.

Sol.

$$\Delta = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$$

$$= x(-x^2 - 1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta)$$

$$= -x^3 - x + x\sin^2\theta + \sin\theta\cos\theta - \cos\theta\sin\theta + x\cos^2\theta$$

$$= -x^3 - x + x$$

$$= -x^3, \text{ independent of } \theta$$

prepared by : BALAJI KANCHI

Word problems :

1. 2025

A furniture workshop produces three types of furniture – chairs, tables and beds each day. On a particular day the total number of furniture pieces produced is 45. It was also found that production of beds exceeds that of chairs by 8, while the total production of beds and chairs together is twice the production of tables. Determine the units produced of each type of furniture, using matrix method.

Sol.

Let the numbers of chairs, tables and beds produced be x, y and z respectively.

$\therefore\ x + y + z = 45; \quad -x + 0.y + z = 8; \quad x - 2y + z = 0$

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix}$

$|A| = 1(0+2) - 1(-1-1) + 1(2-0) = 6 \neq 0$

$\therefore A^{-1}$ exists

$AX = B \Rightarrow X = A^{-1}B$

$\text{adj}(A) = \begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{6}\begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}$

$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}\begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix} = \begin{bmatrix} 11 \\ 15 \\ 19 \end{bmatrix}$

So, $x = 11, y = 15, z = 19$

Hence the numbers of chairs, tables and beds produced are 11, 15 and 19 respectively.

prepared by : BALAJI KANCHI

2.

An amount of ₹ 10,000 is put into three investments at the rate of 10%, 12% and 15% per annum. The combined annual income of all three investments is ₹ 1,310, however the combined annual income of the first and the second investments is ₹ 190 short of the income from the third. Use matrix method and find the investment amount in each at the beginning of the year.

Sol.

Let x, y and z (in ₹) be three investment amounts.

$$\text{Then } x + y + z = 10,000$$

$$\frac{10}{100}x + \frac{12}{100}y + \frac{15}{100}z = 1310$$

$$-\frac{10}{100}x - \frac{12}{100}y + \frac{15}{100}z = 190$$

$$\text{Let } A = \begin{bmatrix} 1 & 1 & 1 \\ 10 & 12 & 15 \\ -10 & -12 & 15 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix},\ B = \begin{bmatrix} 10,000 \\ 1,31,000 \\ 19,000 \end{bmatrix}$$

$$|A| = 60 \neq 0$$

$$\therefore A^{-1} \text{ exists}$$

$$AX = B \Rightarrow X = A^{-1}B$$

$$\text{adj}(A) = \begin{bmatrix} 360 & -27 & 3 \\ -300 & 25 & -5 \\ 0 & 2 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{60}\begin{bmatrix} 360 & -27 & 3 \\ -300 & 25 & -5 \\ 0 & 2 & 2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{60}\begin{bmatrix} 360 & -27 & 3 \\ -300 & 25 & -5 \\ 0 & 2 & 2 \end{bmatrix}\begin{bmatrix} 10,000 \\ 1,31,000 \\ 19,000 \end{bmatrix} = \begin{bmatrix} 2000 \\ 3000 \\ 5000 \end{bmatrix}$$

Hence the investments are ₹ 2000, ₹ 3000 and ₹ 5000 respectively.

3.

Three students run on a racing track such that their speeds add up to 6 km/h. However, double the speed of the third runner added to the speed of the first results in 7 km/h. If thrice the speed of the first runner is added to the original speeds of the other two, the result is 12 km/h. Using matrix method, find the original speed of each runner.

Sol.

Let original speed of three runners be x, y and z respectively.

Then $x + y + z = 6$; $x + 2z = 7$; $3x + y + z = 12$

$$\text{Let } A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$$|A| = 4 \neq 0 \implies A^{-1} \text{ exists}$$

$$AX = B \Rightarrow X = A^{-1}B$$

$$\text{adj}(A) = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

Hence the original speed of three runners are 3km/h, 1km/h and 2 km/h respectively.

prepared by : **BALAJI KANCHI**

4.

Let $2x + 5y - 1 = 0$ and $3x + 2y - 7 = 0$ represent the equations of two lines on which the ants are moving on the ground. Using matrix method, find a point common to the paths of the ants.

Sol.

The system of equations in matrices is:

$$AX = B\text{, where } A = \begin{bmatrix} 2 & 5 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

The solution is given by $X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{-1}{11}\begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

Point common to paths of the ants is $(3, -1)$.

prepared by : BALAJI KANCHI

5.

Two schools A and B decided to award prizes to their students for three values, team spirit, truthfulness and tolerance at the rate of ₹ x, ₹ y and ₹ z per student respectively. School A, decided to award a total of ₹ 1,100 for the three values to 3, 1 and 2 students respectively while school B decided to award ₹ 1,400 for the three values to 1, 2 and 3 students respectively. If one prize for all the three values together amount to ₹ 600 then

(i) Represent the above situation by a matrix equation after forming linear equations.

(ii) Is it possible to solve the system of equations so obtained using matrices ?

(iii) Which value you prefer to be rewarded most and why ?

Sol.

System of equation is

$3x + y + 2z = 1100,\ x + 2y + 3z = 1400,\ x + y + z = 600$

(i) Matrix equation is

$$\begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1100 \\ 1400 \\ 600 \end{bmatrix}$$

(ii) $|A| = -3 \neq 0$, system of equations can be solved.

(iii) Any one value with reason.

6.

A typist charges ₹ 145 for typing 10 English and 3 Hindi pages, while charges for typing 3 English and 10 Hindi pages are ₹ 180. Using matrices, find the charges of typing one English and one Hindi page separately. However typist charged only ₹ 2 per page from a poor student Shyam for 5 Hindi pages. How much less was charged from this poor boy ? Which values are reflected in this problem ?

Sol.

Getting matrix equation as $\begin{pmatrix} 10 & 3 \\ 3 & 10 \end{pmatrix}\begin{pmatrix} E \\ H \end{pmatrix} = \begin{pmatrix} 145 \\ 180 \end{pmatrix}$

$$\Rightarrow \quad \begin{pmatrix} E \\ H \end{pmatrix} = \begin{pmatrix} 10 & 3 \\ 3 & 10 \end{pmatrix}^{-1}\begin{pmatrix} 145 \\ 180 \end{pmatrix}$$

$$\Rightarrow \quad \begin{pmatrix} E \\ H \end{pmatrix} = \begin{pmatrix} 10 \\ 15 \end{pmatrix}$$

$$\Rightarrow \quad E = 10,\ H = 15$$

The poor boy was charged ₹ 65 less

Value: Helping the poor

7.

The monthly incomes of Aryan and Babban are in the ratio 3 : 4 and their monthly expenditures are in the ratio 5 : 7. If each saves ₹ 15,000 per month, find their monthly incomes using matrix method. This problem reflects which value ?

Sol.

Let the income be 3x, 4x and expenditures, 5y, 7y

$$\therefore \quad \left.\begin{aligned} 3x - 5y &= 15000 \\ 4x - 7y &= 15000 \end{aligned}\right\}$$

$$\begin{pmatrix} 3 & -5 \\ 4 & -7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 15000 \\ 15000 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = -1\begin{pmatrix} -7 & 5 \\ -4 & 3 \end{pmatrix}\begin{pmatrix} 15000 \\ 15000 \end{pmatrix}$$

$\Rightarrow \quad x = 30000,\ y = 15000$

$\therefore$ Incomes are ₹ 90000 and ₹ 120000 respectively

8.

A coaching institute of English (subject) conducts classes in two batches I and II and fees for rich and poor children are different. In batch I, it has 20 poor and 5 rich children and total monthly collection is ₹ 9,000, whereas in batch II, it has 5 poor and 25 rich children and total monthly collection is ₹ 26,000. Using matrix method, find monthly fees paid by each child of two types. What values the coaching institute is inculcating in the society ?

Sol.

Let each poor child pay ₹ x per month and each rich child pay ₹ y per month.

$$\therefore \quad 20x + 5y = 9000$$

$$5x + 25y = 26000$$

In matrix form,

$$\begin{bmatrix} 20 & 5 \\ 5 & 25 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9000 \\ 26000 \end{bmatrix}$$

$$AX = B \Rightarrow X = A^{-1}B$$

$$A^{-1} = \frac{1}{475}\begin{bmatrix} 25 & -5 \\ -5 & 20 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{475}\begin{bmatrix} 25 & -5 \\ -5 & 20 \end{bmatrix}\begin{bmatrix} 9000 \\ 26000 \end{bmatrix} = \begin{bmatrix} 200 \\ 1000 \end{bmatrix}$$

$\Rightarrow \quad x = 200, y = 1000$

Value: Compassion or any relevant value

9.

A school wants to award its students for regularity and hardwork with a total cash award of ₹ 6,000. If three times the award money for hardwork added to that given for regularity amounts to ₹ 11,000, represent the above situation algebraically and find the award money for each value, using matrix method. Suggest two more values, which the school must include for award.

Sol.

Let the award for regularily be ₹ x and for hard work be ₹ y.

$\therefore \quad x + y = 6000$ and

$x + 3y = 11000$

$$\Rightarrow \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6000 \\ 11000 \end{pmatrix} \text{ or } A.X = B$$

$\therefore \quad X = A^{-1}B$ as $|A| = 2 \neq 0$.

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 3 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 6000 \\ 11000 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3500 \\ 2500 \end{pmatrix} \quad \therefore \quad x = ₹\,3500,\ y = ₹\,2500$$

10.

A trust invested some money in two type of bonds. The first bond pays 10% interest and second bond pays 12% interest. The trust received ₹ 2,800 as interest. However, if trust had interchanged money in bonds, they would have got ₹ 100 less as interest. Using matrix method, find the amount invested by the trust. Interest received on this amount will be given to Helpage India as donation. Which value is reflected in this question ?

Sol.

$$\text{let } A = \begin{bmatrix} 5 & 6 \\ 6 & 5 \end{bmatrix}; X = \begin{bmatrix} x \\ y \end{bmatrix}; B = \begin{bmatrix} 140000 \\ 135000 \end{bmatrix}$$

$$\therefore \; A \cdot X = B$$

$$|A| = -11; A^{-1} = \frac{1}{-11}\begin{bmatrix} 5 & -6 \\ -6 & 5 \end{bmatrix}$$

$$\therefore \text{ Solution is } X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-11}\begin{bmatrix} 5 & -6 \\ -6 & 5 \end{bmatrix}\begin{bmatrix} 140000 \\ 135000 \end{bmatrix} = \begin{bmatrix} 10000 \\ 15000 \end{bmatrix}$$

$$\therefore x = 10000, \; y = 15000, \therefore \; \text{Amount invested} = ₹\, 25000$$

Value: caring elders

11.

A shopkeeper has 3 varieties of pens ‘A’, ‘B’ and ‘C’. Meenu purchased 1 pen of each variety for a total of ₹ 21. Jeevan purchased 4 pens of ‘A’ variety, 3 pens of ‘B’ variety and 2 pens of ‘C’ variety for ₹ 60. While Shikha purchased 6 pens of ‘A’ variety, 2 pens of ‘B’ variety and 3 pens of ‘C’ variety for ₹ 70. Using matrix method, find cost of each variety of pen.

Sol.

let the cost of one pen of veriety ‘A’, ‘B’ and ‘C’ be ₹ x. ₹ y and ₹ z respectively then the system of equations is:

$$x + y + z = 21$$

$$4x + 3y + 2z = 60$$

$$6x + 2y + 3z = 70$$

Matrix form of the system is:

$$A\cdot X = B, \text{ where } A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$$

$$|A| = (5) - 1\ (0) + 1\ (-10) = -5$$

$$\therefore\ A^{-1} = \frac{1}{|A|} \text{Adj } A = \frac{1}{-5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$$

Solution of the matrix equation is $X = A^{-1} B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}\begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix} \therefore x = 5, y = 8, z = 8$$

prepared by : BALAJI KANCHI

12.

On her birthday Seema decided to donate some money to children of an orphanage home. If there were 8 children less, every one would have got ₹ 10 more. However, if there were 16 children more, every one would have got ₹ 10 less. Using matrix method, find the number of children and the amount distributed by Seema. What values are reflected by Seema's decision ?

Sol.

Let the number of children be x and the amount distributed by

Seema for one student be ₹ y. So, $(x - 8)(y + 10) = xy$

$$\Rightarrow \quad 5x - 4y = 40$$

and $(x + 16)(y - 10) = xy$

$$\Rightarrow \quad 5x - 8y = -80$$

$$\text{Here } A = \begin{matrix} 5 & -4 \\ 5 & -8 \end{matrix}, X = \begin{matrix} x \\ y \end{matrix}, B = \begin{matrix} 40 \\ -80 \end{matrix}$$

$$AX = B \Rightarrow X = A^{-1}B$$

$$A^{-1} = -\frac{1}{20}\begin{matrix} -8 & 4 \\ -5 & 5 \end{matrix}$$

$$\Rightarrow \quad \begin{matrix} x \\ y \end{matrix} = \begin{matrix} 32 \\ 30 \end{matrix}$$

$\Rightarrow \quad x = 32, y = 30$

No. of students = 32

Amount given to each student = ₹ 30.

Value reflected: To help needy people.

13.

A school wants to allocate students into three clubs : Sports, Music and Drama, under following conditions :

- The number of students in Sports club should be equal to the sum of the number of students in Music and Drama club.
- The number of students in Music club should be 20 more than half the number of students in Sports club.
- The total number of students to be allocated in all three clubs are 180.

Find the number of students allocated to different clubs, using matrix method.

Sol.

Let x, y and z be the no. of students allocated to Sports, Music and Drama clubs respectively.

Here, $x = y + z, y = \frac{x}{2} + 20, x + y + z = 180$

$\Rightarrow x - y - z = 0, x - 2y = -40, x + y + z = 180$

Given equations can be written as $AX = B$

where, $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -2 & 0 \\ 1 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ -40 \\ 180 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$|A| = -4 \neq 0 \Rightarrow A^{-1}$ **exists.**

$$adjA = \begin{bmatrix} -2 & 0 & -2 \\ -1 & 2 & -1 \\ 3 & -2 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times adjA = \frac{1}{4}\begin{bmatrix} 2 & 0 & 2 \\ 1 & -2 & 1 \\ -3 & 2 & 1 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \frac{1}{4}\begin{bmatrix} 2 & 0 & 2 \\ 1 & -2 & 1 \\ -3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ -40 \\ 180 \end{bmatrix} = \begin{bmatrix} 90 \\ 65 \\ 25 \end{bmatrix}$$

$\therefore x = 90, y = 65, z = 25$

Number of students allocated in sports, music and drama are 90 , 65 and 25 respectively .

prepared by : BALAJI KANCHI

14.

Ishan wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2. Using matrices, find the dimensions of the plot. Also give reason why he wants to donate the plot for a school.

Sol.

Let length be x m and breadth be y m

$\therefore \quad (x - 50)(y + 50) = xy \Rightarrow 50x - 50y = 2500$ or $x - y = 50$

and $\quad (x - 10)(y - 20) = xy - 5300 \Rightarrow 2x + y = 550$

$$\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 50 \\ 550 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 50 \\ 550 \end{pmatrix}$$

$\Rightarrow \quad x = \frac{1}{3}(600) = 200$ m, $\; y = \frac{1}{3}(450) = 150$ m

"Helping the children of his village to learn" (or any other relavent value)

Case Study :

1. 2023

Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of ₹ 160. From the same shop, Vikram buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of ₹ 190. Also Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of ₹ 250.

Based on the above information, answer the following questions :

(I) Convert the given above situation into a matrix equation of the form AX = B.

(II) Find $|A|$.

(III) Find A^{-1}.

OR

(III) Determine $P = A^2 - 5A$.

Sol.

(I) Matrix equation is AX = B, where

$$A = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 160 \\ 190 \\ 250 \end{bmatrix}$$

where x is the number of pens bought, y the number of bags and z the number of instrument boxes.

(II) $|A| = 5(4-6) - 3(8-3) + 1(4-1) = -22$

(III) $\text{adj}(A) = \begin{bmatrix} -2 & -5 & 3 \\ -10 & 19 & -7 \\ 8 & -13 & -1 \end{bmatrix}' = \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$

$$\Rightarrow A^{-1} = \frac{1}{(-22)} \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$$

OR

$$P = A^2 - 5A = \begin{bmatrix} 32 & 20 & 18 \\ 15 & 13 & 17 \\ 13 & 13 & 23 \end{bmatrix} - \begin{bmatrix} 25 & 15 & 5 \\ 10 & 5 & 15 \\ 5 & 10 & 20 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 5 & 13 \\ 5 & 8 & 2 \\ 8 & 3 & 3 \end{bmatrix}$$

2. 2025

Three students, Neha, Rani and Sam go to a market to purchase stationery items. Neha buys 4 pens, 3 notepads and 2 erasers and pays ₹ 60. Rani buys 2 pens, 4 notepads and 6 erasers for ₹ 90. Sam pays ₹ 70 for 6 pens, 2 notepads and 3 erasers.

Based upon the above information, answer the following questions :

(i) Form the equations required to solve the problem of finding the price of each item, and express it in the matrix form AX = B.

(ii) Find $|A|$ and confirm if it is possible to find A^{-1}.

(iii) (a) Find A^{-1}, if possible, and write the formula to find X.

OR

(iii) (b) Find $A^2 - 8I$, where I is an identity matrix.

sol.

Let the price of each pen, notepad, eraser be ₹x, ₹y and ₹z respectively

Given system in the form AX = B is $\begin{pmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 60 \\ 90 \\ 70 \end{pmatrix}$

(ii) $|A| = 50 \neq 0$, hence A^{-1} exists

(iii) (a) $A^{-1} = \frac{\text{adj}A}{|A|} = \frac{1}{50}\begin{pmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{pmatrix}$

$$X = A^{-1}B$$

OR

(iii)(b) $A^2 = \begin{pmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{pmatrix}\begin{pmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 34 & 28 & 32 \\ 52 & 34 & 46 \\ 46 & 32 & 33 \end{pmatrix}$

$A^2 - 8I = \begin{pmatrix} 26 & 28 & 32 \\ 52 & 26 & 46 \\ 46 & 32 & 25 \end{pmatrix}$

prepared by : BALAJI KANCHI

3. 2024

A scholarship is a sum of money provided to a student to help him or her pay for education. Some students are granted scholarships based on their academic achievements, while others are rewarded based on their financial needs.

Every year a school offers scholarships to girl children and meritorious achievers based on certain criteria. In the session 2022 – 23, the school offered monthly scholarship of ₹ 3,000 each to some girl students and ₹ 4,000 each to meritorious achievers in academics as well as sports.

In all, 50 students were given the scholarships and monthly expenditure incurred by the school on scholarships was ₹ 1,80,000.

Based on the above information, answer the following questions :

(i) Express the given information algebraically using matrices.

(ii) Check whether the system of matrix equations so obtained is consistent or not.

(iii) (a) Find the number of scholarships of each kind given by the school, using matrices.

Sol.

Ans(i)

Let No. of girl child scholarships = x

No. of meritorious achievers = y

$x + y = 50$

$3000x + 4000y = 180000$ or $3x + 4y = 180$

$$\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 180 \end{bmatrix}$$

Ans(ii)

$$\begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} = 1 \neq 0$$

Ans (iii)(a)

Let $A = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 50 \\ 180 \end{bmatrix}$

$AX = B \Rightarrow X = A^{-1}B$

$$X = \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 50 \\ 180 \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \end{bmatrix}$$

$\Rightarrow x = 20, y = 30$

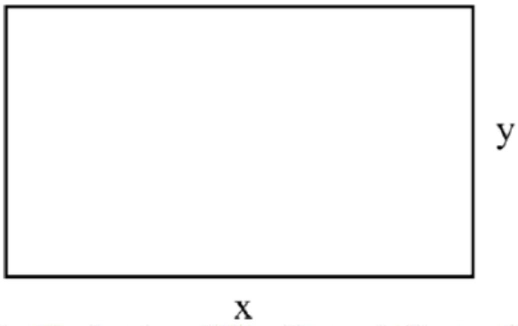
Ans (iii)(b)

Required expenditure = ₹ [30(3000) + 20(4000)]

= ₹ 1,70,000

4.

An architect is developing a plot of land for a commercial complex. When asked about the dimensions of the plot, he said that if the length is decreased by 25 m and the breadth is increased by 25 m, then its area increases by 625 m^2. If the length is decreased by 20 m and the breadth is increased by 10 m, then its area decreases by 200 m^2.



On the basis of the above information, answer the following questions :

(i) Formulate the linear equations in x and y to represent the given information.

(ii) Find the dimensions of the plot of land by matrix method.

Sol.

(i) $(x-25)(y+25) = xy + 625 \Rightarrow x - y = 50$

$(x-20)(y+10) = xy - 200 \Rightarrow x - 2y = 0$

(ii) The system of linear equations can be written in matrix form as

$$\begin{bmatrix} 1 & -1 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & -2 \end{bmatrix}^{-1}\begin{bmatrix} 50 \\ 0 \end{bmatrix} = -\begin{bmatrix} -2 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 50 \\ 0 \end{bmatrix} = \begin{bmatrix} 100 \\ 50 \end{bmatrix}$$

$x = 100m$, $y = 50m$

prepared by : BALAJI KANCHI

4.

10 students were selected from a school on the basis of values for giving awards and were divided into three groups. The first group comprises hard workers, the second group has honest and law abiding students, and the third group contains vigilant and obedient students. Double the number of students of the first group added to the number in the second group gives 13, while the combined strength of the first and the second group is four times that of the third group. Assume that x, y and z denote the number of students in first, second and third group respectively.

Based on the above information, answer the following questions :

(a) Write the system of linear equations that can be formulated from the above described situation.

(b) Write the coefficient matrix, say A.

(c) (i) Write the matrix of cofactors of every element of matrix A.

OR

(c) (ii) Determine the number of students of each group.

5.

A professional typist having his shop in a busy market charges ₹ 200 for typing 8 English and 4 Hindi pages, while he charges ₹ 275 for typing 5 English and 10 Hindi pages.

Based on the above information, answer the following questions :

(i) If he charges ₹ x for one page of English and ₹ y for one page of Hindi, express the above as a pair of linear equations.

(ii) Express the information in terms of matrix equation AX = B.

(iii) (a) Find |A|.

OR

(iii) (b) Find (adj A).

Sol.

(i) $8x+4y=200\,;\,5x+10y=275$

(ii) $\begin{bmatrix} 8 & 4 \\ 5 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 200 \\ 275 \end{bmatrix}$

(iii)(a) $|A|=60$

Or

(iii)(b) $\text{adj}A=\begin{bmatrix} 10 & -4 \\ -5 & 8 \end{bmatrix}$

Resource Materials by BALAJI KANCHI :

Class XII Maths Sample Papers (2025-26) KB Challenge series: click here

Class XII Maths Sample Papers (2025-26) KB Challenge series key : click here

Class XII Math Drive 2025-26 (Prepared by BALAJI KANCHI): Click here

Class XII MATH PYQ's model wise: Click here

CLASS XII Math PYQ's solutions: Click here

Class XII MATH important models: Click here

@kanchibalaji7
+91-8099454846
@ikbmaths7